Supplement of Earth Surf. Dynam., 12, 433–448, 2024
https://doi.org/10.5194/esurf-12-433-2024-supplement

Supplement of

Quantifying the migration rate of drainage divides from high-resolution topographic data

Chao Zhou et al.

Correspondence to: Xibin Tan (tanxibin@imde.ac.cn)

Supplementary text

- The detailed derivation of Eq. (8).
- The drainage divide's migration rates at m = 0.35 and 0.55 in the Wutai Shan.

Supplementary figures

- **Figure S1**. Curves of the channel-head erosion rate (E_{ch}) against the critical area (A_{cr}) under different values of channel-head gradient (S_{ch}).
- **Figure S2.** Geological map of the natural example, Wutai Shan.
- **Figure S3.** Precipitation distribution of the natural example, Wutai Shan.
- **Figure S4**. Normalized channel steepness (k_{sn}) distribution in the Wutai Shan at m = 0.35 and 0.55.

Supplementary Tables

- **Table S1.** The comparison of drainage divide's migration rates at m = 0.35, 0.45, and 0.55 in the Wutai Shan area.

SUPPLEMENTARY TEXT

Detailed derivation of Eq. (8)

According to the detachment-limited stream power model (Howard and Kerby, 1983; Howard, 1994):

$$E = KA^m S^n \tag{1}$$

The channel gradient (*S*) can be written as:

$$S = \left(\frac{E}{K}\right)^{\frac{1}{n}} A^{-\left(\frac{m}{n}\right)} \tag{2}$$

A river's longitudinal elevation $z(x)$ can be expressed by the integration of channel gradient (*S*) in the upstream direction from a base level x_b to an observation point x:

$$z(x) = z_b + \int_{x_b}^{x} S(x)\,dx = z_b + \int_{x_b}^{x} \left(\frac{E(x)}{K(x)}\right)^{\frac{1}{n}} A(x)^{-\left(\frac{m}{n}\right)}\,dx \tag{3}$$

In the case of spatially invariant erosion rate (*E*) and erosion coefficient (*K*), Eq. (3) can be reduced to a simpler form:

$$z(x) = z_b + k_{sn}(A_0)^{-\left(\frac{m}{n}\right)}\chi \tag{4}$$

with

$$k_{sn} = \left(\frac{E}{K}\right)^{\frac{1}{n}} \tag{5}$$

and

$$\chi = \int_{x_b}^{x} \left(\frac{A_0}{A(x)}\right)^{\frac{m}{n}} dx \tag{6}$$

A_0 is an arbitrary scaling area, to make the integrand dimensionless. Assuming A_0 = 1m^2, the steepness (k_{sn}) of channel-head-segment can be written as:

$$k_{sn} = \frac{z_{ch} - z_b}{\chi} \tag{7}$$

According to the Eq. (5), the erosion rate (*E*) can be written as:

$$E = K{k_{sn}}^n = K\left(\frac{z - z_b}{(A_0)^{-\left(\frac{m}{n}\right)}\chi}\right)^n. \tag{8}$$

According to the equation of drainage-divide migration rate (D_{mr}):

$$D_{mr} = \frac{\Delta E_{ch} - \Delta U_{ch}}{tan\alpha + tan\beta} = \frac{E_{ch(\alpha)} - E_{ch(\beta)} - \Delta U_{ch}}{tan\alpha + tan\beta} \quad (9)$$

Combing Eq. (7, 8 & 9), the drainage-divide migration rate (D_{mr}) can be written as:

$$D_{mr} = \frac{E_{ch(\alpha)} - E_{ch(\beta)} - \Delta U_{ch}}{tan\alpha + tan\beta} = \frac{K{k_{sn(\alpha)}}^{n} - K{k_{sn(\beta)}}^{n} - \Delta U_{ch}}{tan\alpha + tan\beta} = \frac{K\left\{\left[\frac{(z_{ch} - z_b)_\alpha}{\chi\alpha}\right]^n - \left[\frac{(z_{ch} - z_b)_\beta}{\chi\beta}\right]^n\right\} - \Delta U_{ch}}{tan\alpha + tan\beta} \quad (10)$$

The drainage divide's migration rate at m = 0.35 and 0.55 in the Wutai Shan

To calculate the drainage divide's migration rate at *m* = 0.35 and 0.55 in the Wutai Shan, we first calculate the normalized channel steepness (k_{sn}) values and use the Kriging interpolation method to generate the k_{sn} distribution map at m = 0.35 (Fig. S4A) and 0.55 (Fig. S4B) in this area. Then we make 20 km wide swath profiles of the k_{sn} distribution map and derive the average k_{sn} values of the upthrown side near the Northern Wutai Shan fault, which are 20 $m^{0.7}$ and 350 $m^{1.1}$ at *m* = 0.35 and 0.55, respectively (Fig. S4). Combining with the 0.5 ± 0.25 mm/yr erosion rate in the north Wutai Shan and using the equation of $K = \frac{E}{{k_{sn}}^{n}}$ follow previous studies (Kirby and Whipple, 2001; Kirkpatrick et al., 2020; Ma et al., 2020), the erosion coefficients (K) are calculated to (25.00 ± 12.50)×10^{-6} $m^{0.3}$ yr^{-1} and (1.43 ± 0.72)×10^{-6} $m^{-0.1}$ yr^{-1} at *m* = 0.35 and 0.55, respectively (Table S1). Based on the channel-head-point method (Eq. 4) and using the required parameters as well as the calculated erosion coefficients (K) at *m* = 0.35 and 0.55, we derive the drainage divide's migration rate at *m* = 0.35 and 0.55 (Table S1).

SUPPLEMENTARY FIGURES

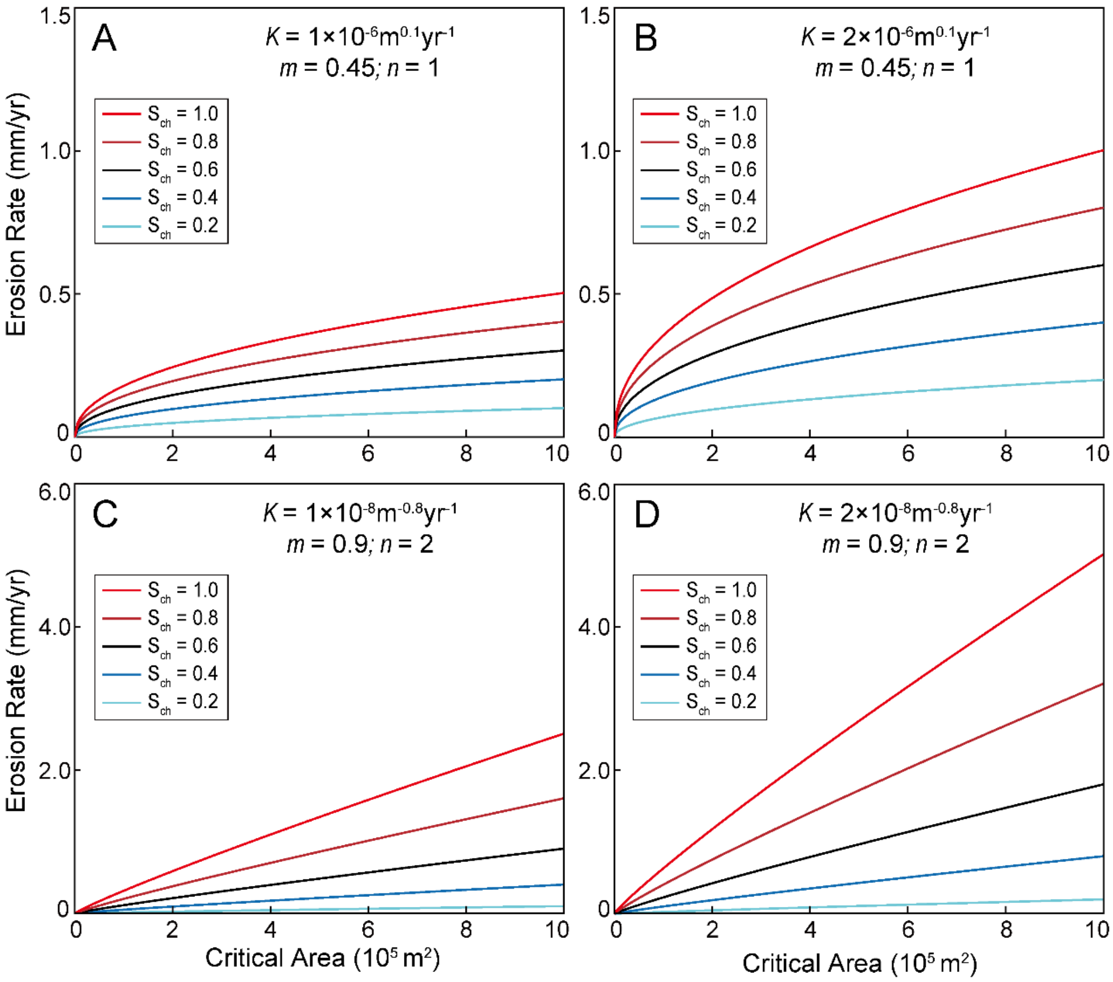

Figure S1. Curves of the channel-head erosion rate (E_{ch}) against the critical area (A_{cr}) under different values of channel-head gradient (S_{ch}). (A) Curves of m = 0.45, n = 1 and $K = 1\times10^{-6}$ m$^{0.1}$ yr^{-1}; (B) Curves of m = 0.45, n = 1 and $K = 2\times10^{-6}$ m$^{0.1}$ yr^{-1}; (C) Curves of m = 0.9, n = 2 and $K = 1\times10^{-8}$ m$^{-0.8}$ yr^{-1}; (D)Curves of m = 0.9, n = 2 and $K = 2\times10^{-8}$ m$^{-0.8}$ yr^{-1}.

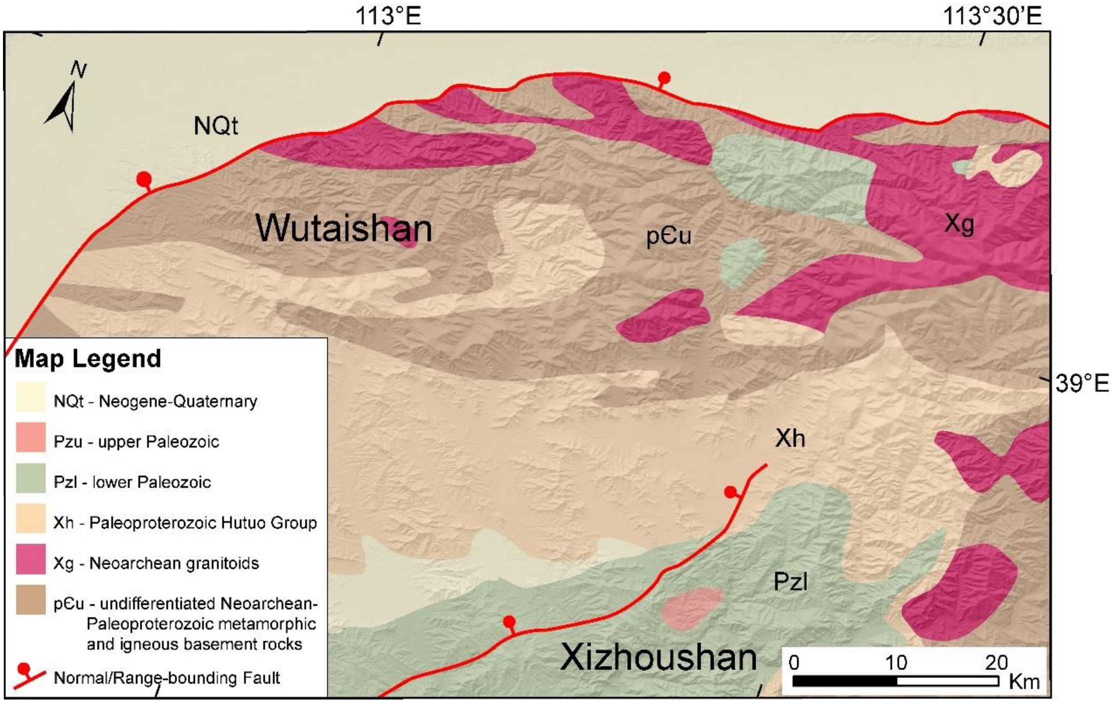

Figure S2. Geological map of the natural example, Wutai Shan. The map is revised according to Clinkscales et al. (2020). The northern catchment evolves mainly through the undifferentiated Neoarchean-Paleoproterozoic metamorphic and igneous basement rocks (pЄu) and Neoarchean granitoids (Xg), whereas the southern catchment evolves mainly through the Paleoproterozoic Hutuo Group (Xh). There is no significant difference in rock erodibility on both sides of the drainage divide.

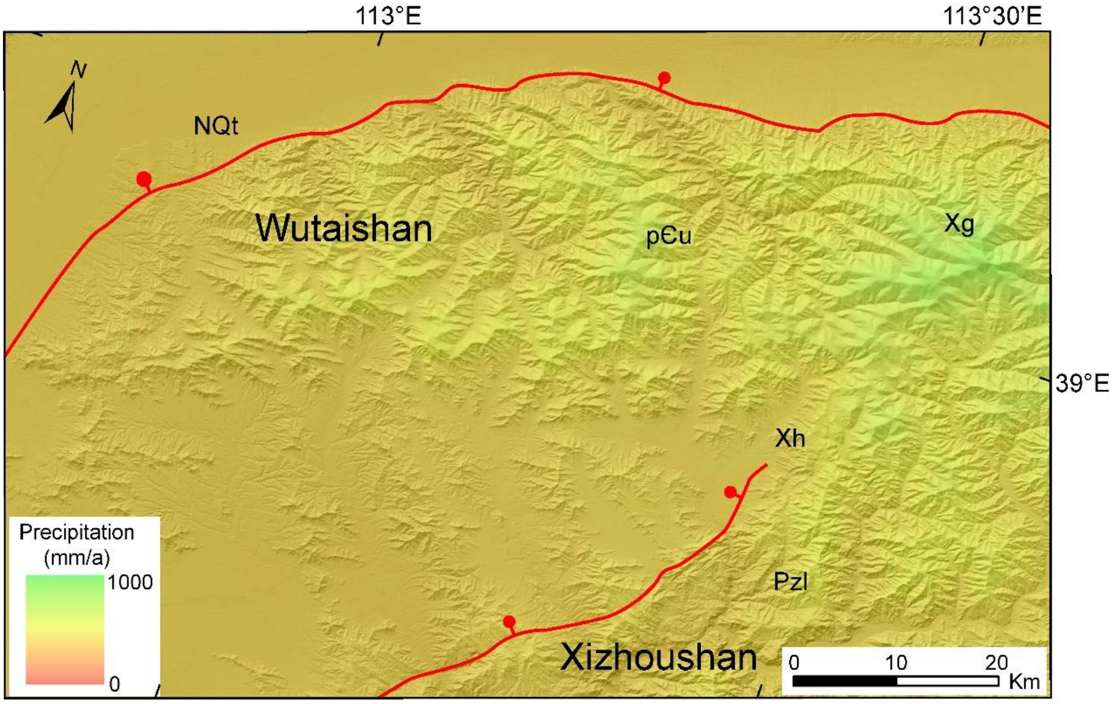

Figure S3. Precipitation distribution of the Wutai Shan. There is no significant difference in precipitation on both sides of the drainage divide, based on the precipitation data (1970-2000) from http://worldclim.org.

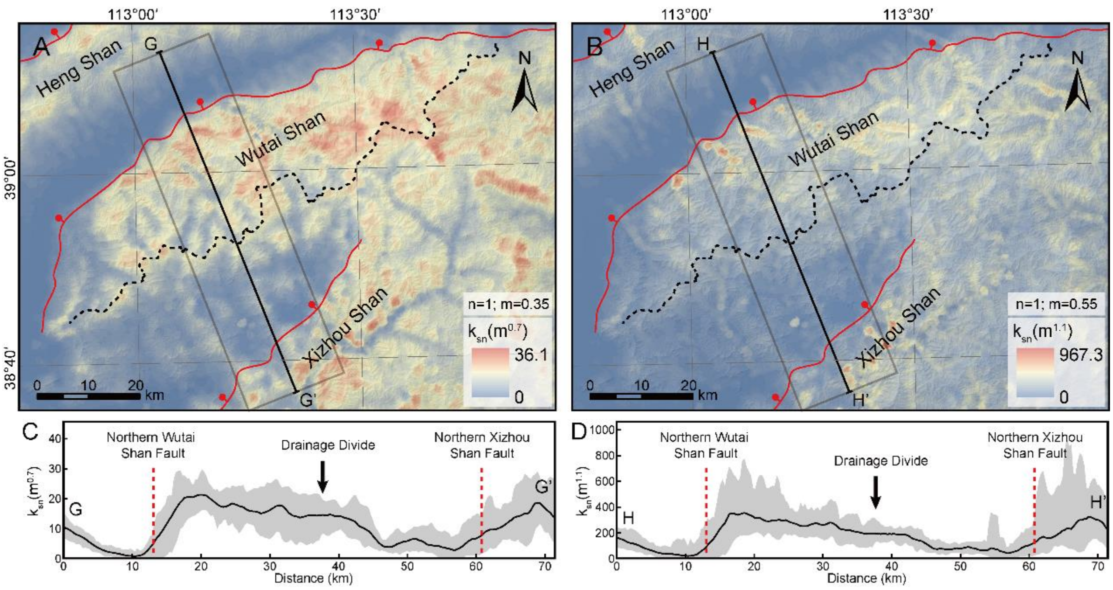

Figure S4. Normalized channel steepness (k_{sn}) distribution in the Wutai Shan. (**A**) The k_{sn} distribution at m = 0.35 and n = 1. (**B**) The k_{sn} distribution at m = 0.55 and n = 1. The black dashed line shows the location of the main drainage divide. Red lines show the main active faults. The black straight lines show the location of the profiles G-G' and H-H' and the gray rectangles show the area of the swath profiles in panel C&D. The topography data (ALOS DEM, 12.5 m resolution) is downloaded from the Alaska Satellite Facility (ASF) Data Search (https://search.asf.alaska.edu/). The k_{sn} is calculated using TopoToolbox (Schwanghart and Scherler, 2014), and the interpolation uses the Kriging method on ArcMap. (**C**) The k_{sn} swath profile along G-G' in panel A. (**D**) The k_{sn} swath profile along H-H' in panel B. The swath profiles are extracted using TopoToolbox (Schwanghart and Scherler, 2014). The red dashed lines show the location of the main active faults, and the black arrow shows the location of the main drainage divide. Both swath profiles are 20 km wide (10 km on each side). The extent of the swath profiles is represented by the grey boxes in Panel A&B.

SUPPLEMENTARY TABLES

Table S1. The comparison of drainage divide's migration rates at *m* = 0.35, 0.45, and 0.55 in the Wutai Shan area.

No.	A_{cr} ($\times10^5 m^2$)	S_{ch}	K ($\times10^{-6} m^{0.3} yr^{-1}$) (n=1; m=0.35)	K ($\times10^{-6} m^{0.1} yr^{-1}$) (n=1; m=0.45)	K ($\times10^{-6} m^{-0.1} yr^{-1}$) (n=1; m=0.55)	tanα	tanβ	D_{mr} (mm/yr) (n=1; m=0.35)	D_{mr} (mm/yr) (n=1; m=0.45)	D_{mr} (mm/yr) (n=1; m=0.55)
Fig. 4 I_α	1.75	0.16	25.00±12.50	6.25±3.13	1.43±0.72	0.14	0.66	-0.37±0.18	-0.21±0.10	-0.10±0.05
Fig. 4 I_β	0.26	0.63	25.00±12.50	6.25±3.13	1.43±0.72					
Fig. 4 II_α	0.79	0.23	25.00±12.50	6.25±3.13	1.43±0.72	0.24	0.70	-0.36±0.17	-0.23±0.11	-0.14±0.06
Fig. 4 II_β	0.30	0.67	25.00±12.50	6.25±3.13	1.43±0.72					
Fig. 4 III_α	0.67	0.29	25.00±12.50	6.25±3.13	1.43±0.72	0.28	0.65	-0.31±0.15	-0.21±0.10	-0.13±0.06
Fig. 4 III_β	0.39	0.63	25.00±12.50	6.25±3.13	1.43±0.72					

Notes: The calculation of the erosion coefficient (*K*) is using the erosion rate in the north Wutai Shan (0.5±0.25 mm/yr), combing with its k_{sn} values of 20 $m^{0.7}$, 80 $m^{0.9,}$ and 350 $m^{1.1}$ at m = 0.35, 0.45 and 0.55, respectively (k_{sn} results are in the Fig. 2 and Fig. S4). The calculation of the drainage divide's migration rates is based on the channel-head-point method (Eq. 4).