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# Macro-roughness model of bedrock-alluvial river morphodynamics

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### Abstract

The 1-D saltation-abrasion model of channel bedrock incision of Sklar and Dietrich, in which the erosion rate is buffered by the surface area fraction of bedrock covered by alluvium, was a major advance over models that treat river erosion as a function of bed slope and drainage area. Their model is, however, limited because it calculates bed cover in terms of bedload sediment supply rather than local bedload transport. It implicitly assumes that as sediment supply from upstream changes, the transport rate adjusts instantaneously everywhere downstream to match. This assumption is not valid in general, and thus can give rise unphysical consequences. Here we present a unified morphodynamic formulation of both channel incision and alluviation which specifically tracks the spatiotemporal variation of both bedload transport and alluvial thickness. It does so by relating the cover fraction not to a ratio of bedload supply rate to capacity bedload transport, but rather to the ratio of alluvium thickness to a macro-roughness characterizing the bedrock surface. The new formulation predicts waves of alluviation

- <sup>15</sup> and rarification, in addition to bedrock erosion. Embedded in it are three physical processes: alluvial diffusion, fast downstream advection of alluvial disturbances and slow upstream migration of incisional disturbances. Solutions of this formulation over a fixed bed are used to demonstrate the stripping of an initial alluvial cover, the emplacement of alluvial cover over an initially bare bed and the advection–diffusion of a sediment
- <sup>20</sup> pulse over an alluvial bed. A solution for alluvial-incisional interaction in a channel with a basement undergoing net rock uplift shows how an impulsive increase in sediment supply can quickly and completely bury the bedrock under thick alluvium, so blocking bedrock erosion. As the river responds to rock uplift or base level fall, the transition point separating an alluvial reach upstream from an alluvial-bedrock reach downstream mi-
- grates upstream in the form of a "hidden knickpoint". A solution for the case of a zone of rock subsidence (graben) bounded upstream and downstream by zones of rock uplift (horsts) yields a steady-state solution that is unattainable with the original saltationabrasion model. A solution for the case of bedrock-alluvial coevolution upstream of an



alluviated river mouth illustrates how the bedrock surface can be progressive buried not far below the alluvium. Because the model tracks the spatiotemporal variation of both bedload transport and alluvial thickness, it is applicable to the study of the incisional response of a river subject to temporally varying sediment supply. It thus has the potential to capture the response of an alluvial-bedrock river to massive impulsive sediment inputs associated with landslides or debris flows.

### 1 Introduction

The pace of river-dominated landscape evolution is set by the rate of downcutting into bedrock across the channel network. The coupled process of river incision and hillslope response is both self-promoting and self-limiting (Gilbert, 1877). Low rates of incision 10 entail some sediment supply from upstream hillslopes, which provides a modicum of abrasive material in river flows that further facilitates bedrock channel erosion. Faster downcutting leads to higher rates of hillslope sediment supply, boosting the concentration of erosion "tools" and bedrock wear rates, but also leading to greater cover of the bedrock bed with sediment (Sklar and Dietrich, 2001, 2004, 2006; Turowski et al., 15 2007; Lamb et al., 2008; Turowski, 2009). Too much sediment supply leads to choking of the channels by alluvial cover and the retardation of further channel erosion (e.g., Stark et al., 2009). This competition between incision and sedimentation leads long-term eroding channels to typically take a mixed bedrock-alluvial form in which the pattern and depth of sediment cover fluctuate over time in apposition to the pattern of 20 bedrock wear.

Theoretical approaches to treating the erosion of bedrock rivers have shifted over recent decades (see Turowski (2012) for a recent review). The pioneering work of Howard and Kerby (1983) focused on bedrock channels with little sediment cover; it led to the <sup>25</sup> detachment-limited model of Howard (1994) in which channel erosion is treated as a power function of river slope and characteristic discharge, and the "stream-powerlaw" approach in which the power-law scaling of channel slope with upstream area



underpins the way in which landscapes are thought to evolve (Whipple and Tucker, 1999; Whipple, 2004; Howard, 1971, foreshadows this approach). At the other extreme, sediment flux came into play in the transport-limited treatment of mass removal from channels of, for example, Smith and Bretherton (1972), in which no bedrock is present

- <sup>5</sup> in the channel and where the divergence of sediment flux determines the rate of lowering. Whipple and Tucker (2002) blended these approaches, and imagined a transition from detachment-limitation upstream to transport-limited behavior downstream. They also discussed, in the context of the stream-power-law approach, the idea emerging at that time (Sklar and Dietrich, 1998) of a "parabolic" form of the rate of bedrock
- wear as a function of sediment flux normalized by transport capacity. Laboratory experiments conducted by Sklar and Dietrich (2001) corroborated this idea, and they led to the first true sediment flux-dependent model of channel erosion of Sklar and Dietrich, 2004, 2006). This saltation-abrasion model was subsequently extended by Lamb et al. (2008) and Chatanantavet and Parker (2009). It was explored experimentally by a subsequent of the second second
- <sup>15</sup> Chatanantavet and Parker (2008) and Chatanantavet et al. (2013), evaluated in a field context by Johnson et al. (2009) and Chatanantavet and Parker (2009), and given a stochastic treatment by Turowski et al. (2007) and Turowski (2009).

At the heart of their saltation-abrasion model lies the idea of a cover factor  $p_c$  corresponding to the areal fraction of the bedrock bed that is covered by alluvium (Sklar

- and Dietrich, 2004). This bedrock bed is imagined as a flat surface on which sediment intermittently accumulates and degrades during bedload transport over it. The fraction of sediment cover is assumed to be a linear function of bedload transport relative to capacity. Bedrock wear takes place when bedload clasts strike the exposed bedrock. In the simplest form of the saltation-abrasion model, the subsequent rate of bedrock wear
- is treated as a linear function of the impact flux and inferred to be proportional to the bedload flux, which leads to the parabolic shape of the cover-limited abrasion curve.

The saltation-abrasion model is considerably more sophisticated and flexible (Sklar and Dietrich, 2004, 2006) than this sketch explanation can encompass. It does, however, have two major restrictions. First, it is formulated in terms of sediment supply



rather than local sediment transport. The model is thus unable to capture the interaction between processes that drive evolution of an alluvial bed and those that drive the evolution of an incising of bedrock-alluvial bed. Second, for related reasons, it cannot account for bedrock topography significant enough to affect the pattern of sediment storage and rock exposure. Such a topography is illustrated in Fig. 1 for the Shimanto River, Japan.

Here we address both these challenges in a model that allows both alluvial and incisional processes to interact and co-evolve. We do this by relating the cover factor geometrically to a measure of bedrock topography, here called macro-roughness, rather than to the ratio of sediment supply rate to capacity sediment transport rate. Our model encompasses downstream advectional alluvial behavior (e.g., waves of alluvium), diffusive alluvial behavior and upstream advecting incisional behavior (e.g., knickpoint migration). In order to distinguish between the model of Sklar and Dietrich (2004, 2006) and the present model, we refer to the former as the CSA (Capacity-based Saltation-Abrasion) model, and the latter as the MRSAA (Macro-Roughness-based Saltation-Alluviation) model.

### 2 Capacity-based Saltation-Abrasion (CSA) geomorphic incision law and its implications for channel evolution: upstream-migrating waves of incision

### 2.1 CSA geomorphic incision law

<sup>20</sup> Sklar and Dietrich (2004, 2006) present the following model, referred to here as the Capacity-based Saltation-Abrasion (CSA) model, for bedrock incision in mixed bedrock-alluvial rivers transporting gravel. Defining *E* as the vertical rate of incision into bedrock,  $q_{\rm b}$  as the volume gravel transport rate per unit width (specified in their model solely in terms of a supply, or feed rate  $q_{\rm bf}$ ) and  $q_{\rm bc}$  as the capacity volume



gravel transport per unit width such that  $q_{\rm b} < q_{\rm bc}$ ,

$$E = \beta q_{\rm b} \left( 1 - \frac{q_{\rm b}}{q_{\rm bc}} \right)$$

where  $\beta$  is an abrasion coefficient with the dimension L<sup>-1</sup>. This geomorphic law for incision can be rewritten as

 $E = \beta q_{\rm bc} p_{\rm c} (1-p_{\rm c})$ 

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where the areal fraction  $p_c$  of bedrock surface covered with alluvium (averaged over a window that is larger than a characteristic macro-scale of bedrock elevation variation) is assumed to obey the simple relation

$$p_{\rm c} = \begin{cases} \frac{q_{\rm b}}{q_{\rm bc}}, & \frac{q_{\rm b}}{q_{\rm bc}} \leq 1\\ 1, & \frac{q_{\rm b}}{q_{\rm bc}} > 1 \end{cases}$$

We refer to this formulation for cover factor  $p_c$  as "capacity based" because Eq. (2) that dictates  $p_c$  is determined in terms of the ratio of sediment supply to its capacity value in the CSA model.

Before introducing the relation of Sklar and Dietrich (2006) for  $\beta$ , it is of value to provide an interpretation for this parameter not originally given by Sklar and Dietrich (2004, 2006), but which plays a useful role in the analysis below. The abrasion coefficient has a physical interpretation in terms of Sternberg's Law (Sternberg, 1875) for downstream dimension of gravity of g

diminution of grain size (Parker, 1991, 2008; Chatanantavet et al., 2010). The analysis leading to this interpretation is given in Appendix A; salient results are summarized here. Consider a clast of material that is of identical rock type to the bedrock being abraded. Sternberg's law is

$$D = D_{\rm u} e^{-\alpha_{\rm d} x}$$

(1a)

(1b)

(2)

(3)

where *D* is gravel clast size,  $D_u$  is an upstream value of *D*, *x* is downstream distance and  $\alpha_d$  is a diminution coefficient. If all diminution results from abrasion,  $\alpha_d$  is related to  $\beta$  as

$$\alpha_{\rm d} = \frac{\beta}{3}$$

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In the case of constant  $\beta$ , and therefore  $\alpha_d$ , the distance  $L_{half}$  for such a clast to halve in size is given as

$$L_{\text{half}} = \frac{\ln(2)}{\alpha_{\text{d}}}$$

<sup>10</sup> This interpretation of abrasion coefficient  $\beta$  in terms of diminution coefficient  $\alpha_d$  allows comparison of the experimental results of Sklar and Dietrich (2001) with values of  $\alpha_d$ previously obtained from abrasion mills (Parker, 2008: see Fig. 3-41 therein; Kodama, 1994).

The relations of Sklar and Dietrich (2004, 2006) to compute  $\beta$  and  $q_{\rm bc}$  can be cast 15 in the following form:

$$\beta = \frac{0.08\rho_{s}RgY}{k_{v}\sigma_{t}^{2}} \left(\frac{\tau^{*}}{\tau_{c}^{*}} - 1\right)^{-1/2} \left[1 - \frac{\tau^{*}}{R_{f}^{2}}\right]^{3/2}$$
(5a)  

$$R_{f} = \frac{v_{s}}{\sqrt{RgD}}$$
(5b)  

$$q_{bc} = \alpha_{b}\sqrt{RgD}D \left(\tau^{*} - \tau_{c}^{*}\right)^{n_{b}}$$
(5c)

<sup>20</sup> In the above relations, *D* corresponds to the characteristic size of the gravel clasts that are effective in abrading the bedrock,  $\rho_s$  is the material density of the clasts, *R* is their submerged specific gravity (~ 1.65 for quartz), *g* is gravitational acceleration,  $\tau^*$ is the dimensionless Shields number of the flow,  $\tau_c^*$  is the threshold Shields number



(4a)

(4b)

for the onset of significant bedload transport,  $\alpha_{\rm b}$  and  $n_{\rm b}$  denote, respectively, relationspecific dimensionless coefficient and exponent,  $v_{\rm s}$  is the fall velocity corresponding to size *D*, *Y* is the bedrock modulus of elasticity,  $\sigma_{\rm t}$  is the rock tensile strength, and  $k_{\rm v}$  is a dimensionless coefficient of the order of  $1 \times 10^{-6}$ . (In the above two relations and the text, several misprints in Sklar and Dietrich, (2004, 2006) have been corrected on the advice of the authors.) Equation (5c) corresponds to the bedload transport relation of Fernandez Luque and van Beek (1976) when  $\alpha_{\rm b} = 5.7$  and  $n_{\rm b} = 1.5$ ; Sklar and Dietrich (2004, 2006) used this relation with the assumed value  $\tau_{\rm c}^* = 0.03$ . It is useful to cast Eq. (5c) in the form

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$$\beta = \beta_{\rm r} \frac{\left(\frac{\tau^*}{\tau_{\rm c}^*} - 1\right)^{-1/2} \left[1 - \frac{\tau^*}{R_{\rm f}^2}\right]^{3/2}}{\left(\frac{\tau_{\rm r}^*}{\tau_{\rm c}^*} - 1\right)^{-1/2} \left[1 - \frac{\tau_{\rm r}^*}{R_{\rm f}^2}\right]^{3/2}}$$

where  $\beta_r$  is a reference value of  $\beta$ , either computed from known values of the parameters *Y*,  $k_v$ ,  $\sigma_t$ ,  $R_f$  etc., or estimated indirectly.

### 2.2 Embedding of CSA into a model of bedrock surface evolution

<sup>15</sup> A relation for the evolution of bedrock surface elevation  $\eta_b$  is obtained by substituting the CSA geomorphic law for incision of Eq. (1b) into a simplified 1-D mass conservation equation for bedrock material subjected to piston-style rock uplift or base level fall (Sklar and Dietrich, 2006):

$$\frac{\partial \eta_{\rm b}}{\partial t} = \upsilon - I_{\rm f} \beta q_{\rm bc} \rho_{\rm c} (1 - \rho_{\rm c})$$

Here t denotes time, v denotes the relative vertical velocity between the rock underlying the channel (which is assumed to undergo no deformation) and the point at which base



(5d)

(6)

level is maintained, and  $l_f$  denotes a flood intermittency factor to account for the fact that only relatively rare flow events are likely to drive incision (Chatanantavet and Parker, 2009). Also  $l_f$  is assumed to be a prescribed constant; a more generalized formulation for flow hydrograph is given in Sklar and Dietrich (2006) and DiBiase and Whipple

(2011). In interpreting Eq. (6), it should be noted that *v* denotes a rock uplift rate for the case of constant base level, or equivalently a rate of base level fall for rock undergoing neither uplift nor subsidence. Below we use the term "rock uplift" as shorthand for the relative vertical velocity between the rock and the point of base level maintenance.

### 2.3 Character of the CSA model: upstream waves of incision

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The MRSAA model (introduced below) has several new features as compared to CSA. These are best illustrated by first characterizing the mathematical nature of CSA in the context of Eq. (6). Let

$$S_{\rm b} = -\frac{\partial \eta_{\rm b}}{\partial x} \tag{7}$$

<sup>15</sup> denote the streamwise bedrock surface slope. Reducing Eq. (6) with Eq. (7) the CSA model of Eq. (6) reveals itself as a nonlinear kinematic wave equation with a source term:

$$\frac{\partial \eta_{\rm b}}{\partial t} - c_{\rm b} \frac{\partial \eta_{\rm b}}{\partial x} = \upsilon \tag{8a}$$

$$c_{\rm b} = \frac{l_{\rm f} \beta q_{\rm bc} \rho_{\rm c} (1 - \rho_{\rm c})}{S_{\rm b}} \tag{8b}$$

Here  $c_{\rm b}$  denotes the wave speed associated with bedrock incision. The form of Eq. (8a) dictates that disturbances in bedrock elevation always move upstream. We will see later that these disturbances can take the form of upstream-migrating knick-points (e.g., Chatanantavet and Parker, 2009).



Any solution of Eq. (8a, b) subject to the cover relation of Eq. (2) requires specification of a flow model. In mountain streams, backwater effects are likely to be negligible (e.g., Parker, 2004). The normal (steady, uniform) flow assumption allows simplification. Let  $Q_f$  denote water discharge during (morphodynamically active) flood flow occurring with the intermittency  $l_f$ , H denote flood depth and g denote the acceleration of gravity. Momentum and mass balance take the forms

$$\tau_{\rm b} = \rho g H S_{\rm b}, \tag{9a}$$

$$Q_{\rm f} = U B H \tag{9b}$$

where  $\tau_{\rm b}$  is boundary shear stress at flood flow, *B* is channel width and  $\rho$  is water density. The dimensionless Shields number  $\tau^*$  and dimensionless Chézy resistance coefficient *Cz* are defined as

$$\tau^* = \frac{\tau_{\rm b}}{\rho R g D},$$
$$Cz = \frac{U}{\sqrt{\tau_{\rm b}/\rho}}$$

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As shown in Parker (2004) and Chatanantavet and Parker (2009), reducing Eqs. (7), (9) and (10) yields the following relations for *H* and  $\tau^*$ :

$$H = \left(\frac{Q_{\rm f}^2}{Cz^2 g B^2 S_{\rm b}}\right)^{1/3},$$
(11a)
$$\tau^* = \left(\frac{Q_{\rm f}^2}{Cz^2 g B^2}\right)^{1/3} \frac{S_{\rm b}^{2/3}}{RD}$$
(11b)

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A comparison of Eqs. (2), (5c) and (11b) indicates that even for constant values of other parameters, the functional forms for  $q_{\rm bc}$  and thus  $p_{\rm c}$  are such that  $c_{\rm b}$  is in general a nonlinear function of  $S_{\rm b} = -\partial \eta_{\rm b}/\partial x$ .

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(10a)

(10b)

### 2.4 Limitations of CSA model

The CSA model (Sklar and Dietrich, 2004, 2006) represents a major advance in the analysis of bedrock incision due to abrasion because it (1) accounts for the effect of alluvial cover and tool availability on the incision rate through the term  $p_c(1 - p_c)$  in Eq. (1b), and Eq. (2) provides a physical basis for incision due to abrasion as gravel clasts collide with the bedrock surface. The CSA model been used, modified, adapted and extended by a number of researchers (Crosby et al., 2007; Lamb et al., 2008; Chatanantavet and Parker, 2009; Turowski et al., 2009; Lague, 2010).

The model does, however, have a significant limitation in that it does not specifically include alluvial morphodynamics. Here we study this limitation, and how to overcome it, in terms of the highly simplified configuration of a reach (HSR, highly simplified reach) with constant width, fixed, non-erodible banks, constant water discharge and sediment input only from the upstream end. For simplicity, we also neglect abrasion of the gravel itself, so that grain size *D* is a specified constant. (This condition, while

- introduced arbitrarily here, can be physically interpreted in terms of clasts that are much more resistant to abrasion than the bedrock.) The means to relax these constraints is readily available (e.g., Chatanantavet et al., 2010; DiBiase and Whipple, 2011). Such a relaxation, however obscures the first-order physics underlying the rich patterns of interaction between completely and partially alluviated conditions illustrated herein.
- In the CSA model, the bedload transport rate  $q_b$  is specified as a "supply." That is, the bedload transport rate is constrained so that it cannot change in the downstream direction, and is always equal to the bedload feed rate (supply)  $q_{bf}$  at the upstream end. When the feed rate  $q_{bf}$  increases,  $q_b$  must increase simultaneously everywhere. That is, a change in bedload supply is felt instantaneously throughout the entire reach, regardless of its length.

We illustrate this behavior in Fig. 2. The reach has length *L*. The gravel feed rate at x = 0 follows a cyclic "sedimentograph" (in analogy to a hydrograph) with period  $T = T_h + T_l$ , in which the sediment feed rate has a constant high feed  $q_{bf, h}$  rate for



time  $T_h$ , and a subsequent constant low feed rate  $q_{bf,1}$  for time  $T_l$ . According to the CSA model, at x = L corresponding to the downstream end of the reach, the temporal variation in bedload transport rate must precisely reflects the feed rate.

In a more realistic model, the effect of the same change in bedload feed rate  $q_{bf}$ would gradually diffuse and propagate downstream, so that the bedload transport rate at the downstream end of the reach would show more gradual temporal variation. This effect is also illustrated in Fig. 2. This same diffusion and propagation can be expected in the cover fraction  $p_c$ , which in general should vary in both x and t. The change in cover fraction in turn should affect the incision rate as quantified in Eq. (1a). To capture this effect, however, Eq. (1b) must be coupled with an alluvial formulation that routes sediment downstream over the bedrock.

A second limitation concerns alluviation of the bedrock surface. Consider a wave of sediment moving over this surface, as shown in Fig. 3. The bottom of the bedrock surface is at elevation  $\eta_{\rm b}$ , the characteristic height of the roughness elements of the bedrock (Fig. 1) is  $L_{\rm mr}$  (macro-roughness length) and the alluvial thickness above the bottom of the bedrock is  $\eta_{\rm a}$ . The surface undergoes both partial ( $\eta_{\rm a} < L_{\rm mr}$ ) and then complete ( $\eta_{\rm a} \ge L_{\rm mr}$ ) alluviation, only to be excavated later as the wave passes through. Bed elevation  $\eta$  is given as

 $_{20}$   $\eta = \eta_{\rm b} + \eta_{\rm a}$ 

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Figure 3 shows that in the case of complete alluviation, the elevation of the bed  $\eta$  can be arbitrarily higher than the elevation  $\eta_{\rm b}$  of the bedrock, the difference between the two corresponding to the thickness  $\eta_{\rm a}$ . The CSA model cannot describe the variation of bed elevation  $\eta$  when the bed undergoes transitions between partial and complete alluviation; it simply infers that incision is shut down by the complete alluvial cover.

The goal of this paper is the development and implementation of a model that overcomes these limitations by: (a) capturing the spatiotemporal co-evolution of the sediment transport rate, alluvial cover thickness and bedrock incision rate, and (b) explicitly enabling spatiotemporally evolving transitions between bedrock-alluvial morphody-



(12)

namics and purely alluvial morphodynamics. The form of the model presented here is simplified in terms of the HSR outlined above, including a constant-width channel and a single sediment source upstream.

### 3 Macro-Roughness-based Saltation-Abrasion-Alluviation (MRSAA) formulation and its implications for channel evolution

### 3.1 Formulation for alluvial sediment conservation and cover factor

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The geomorphic incision law of the MRSAA model is identical to that of CSA, i.e., Eq. (1b). The essential differences are contained in (a) a formulation for the cover factor  $p_c$  that differs from Eq. (2), and (b) the inclusion of alluvial morphodynamics in a way that tracks the spatiotemporal evolution of the bedload transport rate, and allows smooth spatiotemporal transitions between the bedrock-alluvial state and the purely alluvial state.

We formulate the problem by considering a conservation equation for the alluvium, appropriately adapted to include below-capacity transport over a non-erodible surface.

- <sup>15</sup> The first model of this kind is due to Struiksma (1999), and further progress has been made by Parker et al. (2009), Izumi and Yokokawa (2011), Izumi et al. (2012), Parker et al. (2013), Tanaka and Izumi (2013) and Zhang et al. (2013). A definition diagram for the derivation of this equation is given in Fig. 4. Bedrock elevation fluctuates locally in space, as seen in Fig. 1 for the field and in Fig. 4 in schematized form. This fluctuation is
- <sup>20</sup> here characterized by a macro-roughness  $L_{mr}$ . We begin by specifying the macroscopic location of the bottom of this bedrock surface  $\eta_b$  (averaged over fluctuations) as the "base" of this rough layer, and locating the "top" of the rough layer at  $\eta_b + L_{mr}$ . The bedrock is completely exposed when  $\eta_a = 0$ , partially exposed when  $0 < \eta_a < L_{mr}$  and completely alluviated when  $\eta_a \ge L_{mr}$ . (We amend this formulation below.)
- Now let *z* be the elevation above the bedrock "base" as shown in Fig. 4, and  $\lambda_p$  be the porosity of the alluvial deposit, here assumed to be constant. The cover fraction



associated with a given elevation *z* is denoted as  $\tilde{p}_c(z)$ , a parameter that is taken to be purely geometrical, invariant in time and representative of the statistical structure of the local elevation variation of the bedrock itself (such as that shown in Fig. 1). As illustrated in Fig. 4, the volume of alluvial sediment per unit area between elevations *z* and  $z + \Delta z$  is  $(1 - \lambda_p)\tilde{p}_c(z)\Delta z$  and the bedload transport rate  $q_b$  is estimated as  $p_c q_{bc}$ , where

$$\rho_{\rm c} = \tilde{\rho}_{\rm c}|_{z=\eta_{\rm a}} \tag{13}$$

For the case of sediment of constant density, the Exner equation for mass balance of alluvial sediment can be expressed as

$$(1 - \lambda_{\rho})\frac{\partial}{\partial t} \int_{0}^{l_{a}} \tilde{\rho}_{c} dz = -l_{f} \frac{\partial \rho_{c} q_{bc}}{\partial x}$$
(14)

where the factor  $I_{\rm f}$  accounts for the fact that morphodynamics is active only during floods. Reducing Eq. (14) with Leibnitz's rule,

<sup>15</sup> 
$$(1 - \lambda_p) p_c \frac{\partial \eta_a}{\partial t} = -I_f \frac{\partial p_c q_{bc}}{\partial x}$$
 (15)

Equations (6) and (15) delineate the formulation encompassing both for bedrockalluvial rivers and alluvial rivers. In order to complete the problem, it is necessary to define a closure model for  $p_c$ . The local variation of bedrock elevation is captured by the "macro-roughness"  $L_{mr}$  of Fig. 4. Here we seek a formulation that averages over a window capturing a statistically relevant sample of this local variation. It is assumed that  $p_c$  is a specified, monotonically increasing function of  $\chi = \eta_a/L_{mr}$ , such that  $p_c = 0$ when  $\eta_a/L_{mr} = 0$  and  $p_c = 1$  when  $\eta_a/L_{mr} = 1$ , i.e.

$$p_{c}|_{\chi=0} = 0,$$
 (16a)  
 $p_{c}|_{\chi\geq 1} = 1$  (16b)

The general form of this relation  $p_c = f_c(\chi)$  is illustrated in Fig. 5. The simplest functional form for  $f_c(\chi)$  satisfying Eq. (16a, b) is a linear relation that is analogous to Eq. (2);

$$p_{c} = \begin{cases} \chi, \chi \leq 1 \\ 1, \chi > 1 \end{cases} ,$$
$$\chi = \frac{\eta_{a}}{L_{mr}}$$

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Note that this cover relation is based on the macro-roughness length  $L_{mr}$  rather than the capacity transport  $q_{\rm bc}$  of Eq. (2). This is the motivation for referring to the new model presented here as the MRSAA (Macro-Roughness-based Saltation-Abrasion-Alluviation) model.

More simplified versions of such a formulation have been previously presented by Parker et al. (2013), Zhang et al. (2013) and Tanaka and Izumi (2013). The present formulation corrects errors in Parker et al. (2013) and Zhang et al. (2013).

#### 3.2 Character of the alluvial part of the MRSAA problem: alluvial diffusion and downstream-migrating waves of alluviation

Taking  $p_c = f_c(\chi)$ , where  $f_c$  is an arbitrary function satisfying the conditions Eq. (16a, 15 b), Eq. (15) can be reduced to

$$\frac{\partial \eta_{a}}{\partial t} + c_{a} \frac{\partial \eta_{a}}{\partial x} = -\frac{1}{1 - \lambda_{p}} I_{f} \frac{\partial q_{bc}}{\partial x}$$
(18)

#### where

<sup>20</sup> 
$$C_{a} = \frac{I_{f}}{1 - \lambda_{p}} \frac{q_{bc}}{L_{mr}\rho_{c}} f_{c}',$$
  
 $f_{c}' = \frac{df_{c}}{d\chi}$ 

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(19a)

(19b)

(17a)

(17b)

Neglect of the right-hand side of Eq. (18) yields a kinematic wave equation, where  $c_a$  is the wave speed of downstream-directed alluviation.

The form of the equation can be further clarified by rewriting it as

$${}_{_{5}} \frac{\partial \eta_{a}}{\partial t} + c_{a} \frac{\partial \eta_{a}}{\partial x} - \frac{\partial}{\partial x} \left( v_{a} \frac{\partial \eta_{a}}{\partial x} \right) = \frac{\partial}{\partial x} \left( v_{a} \frac{\partial \eta_{b}}{\partial x} \right)$$
(20)

where

15

$$v_{a} = \frac{l_{f}q_{bc}}{(1 - \lambda_{p})S},$$

$$S = -\frac{\partial \eta}{\partial x} = -\frac{\partial \eta_{b}}{\partial x} - \frac{\partial \eta_{a}}{\partial x}$$
(21a)
(21b)

In the above relation,  $v_a$  has the physical meaning of a kinematic diffusivity. In general,  $q_{\rm bc}$ ,  $q_{\rm bc}/S$  and thus  $v_a$  are nonlinear functions of *S*. The alluvial problem thus takes the form of a nonlinear advective-diffusive problem with a source term arising from a bedrock term.

### 3.3 Full MRSAA formulation: alluvial diffusion, upstream-migration waves of incision, downstream-migrating waves of alluviation

The full MRSAA model consists of the kinematic wave equation with a source term Eq. (8a) for the bedrock part, Eqs. (19)–(21) for the alluvial part, and the linkage between the two embodied in the cover relation of Eq. (17). Restating these equations for emphasis, they are

<sup>20</sup> 
$$\frac{\partial \eta_{\rm b}}{\partial t} - C_{\rm b} \frac{\partial \eta_{\rm b}}{\partial x} = 0,$$
$$C_{\rm b} = \frac{I_{\rm f} \beta q_{\rm bc} \rho_{\rm c} (1 - \rho_{\rm c})}{\left( -\frac{\partial \eta_{\rm b}}{\partial x} \right)}$$

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(22a)

(22b)

$$\frac{\partial \eta_{a}}{\partial t} + c_{a} \frac{\partial \eta_{a}}{\partial x} - \frac{\partial}{\partial x} \left( v_{a} \frac{\partial \eta_{a}}{\partial x} \right) = \frac{\partial}{\partial x} \left( v_{a} \frac{\partial \eta_{b}}{\partial x} \right)$$

$$c_{a} = \frac{l_{f}}{1 - \lambda_{p}} \frac{q_{bc}}{L_{mr} \rho_{c}} f_{c}^{\prime},$$

$$f_{c}^{\prime} = \frac{d\rho_{c}}{d(\eta_{a}/L_{mr})},$$

$$(23a)$$

$$f_{c}^{\prime} = \frac{d\rho_{c}}{d(\eta_{a}/L_{mr})},$$

$$(23c)$$

$$p_{c} = \begin{cases} \frac{\eta_{a}}{L_{mr}}, & \frac{\eta_{a}}{L_{mr}} \leq 1 \\ 1, & \frac{\eta_{a}}{L_{mr}} > 1 \end{cases}$$

$$(23a)$$

$$(23a)$$

$$(23b)$$

$$(23a)$$

$$(23b)$$

$$(23c)$$

$$(23d)$$

$$(23d)$$

In this way, upstream-migrating incisional waves are combined with downstream-10 migrating alluvial waves and alluvial diffusion. The relation for cover factor  $p_c$  is amended, however, in Sect. 3.3.

In MRSAA, then, the spatiotemporal variation of the cover fraction  $p_c(x,t)$  is specifically tied to the corresponding variation in  $\eta_a$  through Eq. (24). This variation then affects incision through Eq. (22). Consider the wave of alluvium illustrated in Fig. 3. There 15 is no incision ahead of the wave because  $p_c = 0$ . At the peak of the wave,  $\eta_a > L_{mr}$ , so  $p_{\rm c}$  = 1 and again there is no incision. Incision can only occur on the rising and falling parts of the wave, where  $0 < p_c < 1$ . It can thus be expected that the spatiotemporal variation in cover thickness  $\eta_a$  will affect the evolution of the long profile of an incising river which undergoes transitions between alluvial and mixed bedrock-alluvial states. 20

#### Amendment of flow model for MRSAA model 3.4

The flow model, and in particular Eqs. (9a) and (11), must be modified to include the alluvial formulation, so that  $S_{\rm b}$  is replaced with S, where

$$S = -\frac{\partial \eta}{\partial x} = S_{\rm b} + S_{\rm a},$$

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(25a)

$$S_{b} = -\frac{\partial \eta_{b}}{\partial x},$$
  

$$S_{a} = -\frac{\partial \eta_{a}}{\partial x}$$

Thus Eqs. (9a) and (11a, b) are amended to

5  $\tau_{\rm b} = \rho g H S$ 

10

$$H = \left(\frac{Q_{\rm f}^2}{Cz^2 g B^2 S}\right)^{1/3},$$
(27a)

$$\tau^* = \left(\frac{Q_{\rm f}^2}{Cz^2 g B^2}\right)^{1/3} \frac{S^{2/3}}{RD}$$
(27b)

The purely alluvial case, i.e.,  $p_c = 1$ ,  $f'_c = 0$  and  $\eta_b = const < \eta_a$ , results in the purely diffusional relation

$$\frac{\partial \eta_{a}}{\partial t} = \frac{\partial}{\partial x} \left( v_{a} \frac{\partial \eta_{a}}{\partial x} \right)$$
(28)

in which the diffusivity  $v_a$  is a function of  $S_a = -\partial \eta_a / \partial x$ .

In MRSAA, then, the spatiotemporal variation of the cover fraction  $p_c(x,t)$  is specifically tied to the corresponding variation in  $\eta_a$  through Eq. (24). This variation then affects incision through Eq. (22). Consider the wave of alluvium illustrated in Fig. 3. There is no incision ahead of the wave because  $p_c = 0$ . At the peak of the wave,  $\eta_a > L_{mr}$ , so

 $p_c = 1$  and again there is no incision. Incision can only occur on the rising and falling parts of the wave, where  $0 < p_c < 1$ . It can thus be expected that the spatiotemporal variation in cover thickness  $\eta_a$  will affect the evolution of the long profile of an incising river which undergoes transitions between alluvial and mixed bedrock-alluvial states.

(25b)

(25c)

(26)

### 3.5 Equivalence of MRSAA and CSA models at steady state

In the restricted case of the HSR configuration constrained by: (a) temporally constant, below-capacity sediment feed (supply) rate  $q_{\rm bf}$ ; (b) bedload transport rate  $q_{\rm b}$  everywhere equal to the feed rate  $q_{\rm bf}$ , and (c) a steady-state balance between incision and rock uplift,  $\eta_{\rm a}$ ,  $\rho_{\rm c}$  and  $S_{\rm b}$  become constant and  $S_{\rm a}$  becomes vanishing, so that Eq. (23) is identically satisfied. Equation (15) integrates to give

$$p_{\rm c} = \frac{q_{\rm bf}}{q_{\rm bc}}$$

10

25

so that  $\eta_a$  can then be back-calculated from Eq. (29). In this case, then, the MRSAA model reduces to Eqs. (22) and (29), i.e., the CSA model.

### 3.6 Amendment of the cover function of the MRSAA model

There is a problem associated with the cover formulation of the MRSAA model. According to Eqs. (23b) and (24), the cover fraction  $p_{\rm c}$  tends to 0, and thus the downstream-directed alluvial wave speed  $c_{\rm a}$  tends to infinity, as alluvial thickness  $\eta_{\rm a}$  goes to 0. That is, alluvial waves of infinitesimal amplitude travel with infinite speed. This unphysical

behavior can be resolved by considering the statistics of bedrock elevation variation. Figure 4 cannot be precisely correct: the cover fraction  $p_c$  should not be vanishing at z = 0, nor should it precisely be equal to 1 at  $z = L_{mr}$ . Instead, in so far as bedrock elevation variation has a random element, the appropriate conditions are  $p_c \rightarrow 0$  as

$$_{20}$$
  $z \rightarrow -\infty$ , and  $p_{c} \rightarrow 1$  as  $z \rightarrow \infty$ .

The amended vertical structure for  $p_c$  is schematized in Fig. 6, in which z' denotes the elevation above an arbitrary datum (as opposed to elevation above the bottom of the bedrock, which is as yet undefined in this section). The statistical formulation embodied in this figure is akin to that of Parker et al. (2000) for alluvial beds. More precisely, the parameter  $1 - p_c(z')$  denotes the probability that a point at elevation z' is in bedrock (rather than water or alluvium above).

(29)

With this in mind, the "bottom" and "top" of the bedrock, as well as the macroroughness  $L_{mr}$ , should be defined in a statistical sense. This can be done using moments or exceedance probabilities; here we use the latter. Let  $p_{c,r}$  denote some low reference cover value (e.g.,  $p_{c,r} = 0.05$ , or 5% cover),  $p_{c,1-r}$  represents a corresponding high reference cover (where e.g.,  $p_{c,1-r} = 1 - p_{c,r} = 0.95$ , or 95% cover), and  $z'_r$ and  $z'_{1-r}$  denote the corresponding bed elevations. An effective "base" of the bedrock, where  $\eta_a = 0$ , can be located at  $z'_r$ , macro-roughness height  $L_{mr}$  can be specified as

$$L_{\rm mr} = z_{1-\rm r}' - z_{\rm r}'$$

<sup>10</sup> and an effective "top" of the bedrock can be specified as  $\eta_a + L_{mr}$ . This formulation ensures that  $p_c = p_{c,r} > 0$  when  $\eta_a = 0$ .

An appropriately modified form for the cover function is

$$\rho_{\rm c} = \rho_{\rm c,r} + (\rho_{\rm c,1-r} - \rho_{\rm c,r}) f_{\rm c}(\chi) \tag{31}$$

where  $f_{\rm c}$  must satisfy the general conditions

$$f_{\rm c}(0) = 0,$$
 (32a)  
 $f_{\rm c}(1) = 1,$  (32b)  
 $f_{\rm c}(\infty) = \frac{1 - \rho_{\rm c,r}}{1 - \rho_{\rm c,r}}$  (32c)

$$f_{\rm c}(\infty) = \frac{\rho_{\rm c,1-r} - \rho_{\rm c,r}}{\rho_{\rm c,1-r} - \rho_{\rm c,r}}$$

20 and

25

 $0 < f_{\rm c}'(0) < \infty$ 

From Eqs. (23a), (31) and (32d), the wave speed at  $\eta_a = 0$  is now given as

$$c_{\rm a}|_{\eta_{\rm a}=0} = \frac{1}{1-\lambda_{\rm p}} \frac{I_{\rm f}q_{\rm bc}}{L_{\rm ma}p_{\rm c,r}} f_{\rm c}'(0) < \infty$$

(30)

(32d)

(33)

The simplest form satisfying these conditions is given below, and illustrated in Fig. 7:

$$f_{\rm c} = \begin{cases} \chi & \text{for } 0 \le \chi \le \frac{1 - \rho_{\rm c,r}}{\rho_{\rm c,1-r} - \rho_{\rm c,r}} \\ \frac{1 - \rho_{\rm c,r}}{\rho_{\rm c,1-r} - \rho_{\rm c,r}} & \text{for } \chi > \frac{1 - \rho_{\rm c,r}}{\rho_{\rm c,1-r} - \rho_{\rm c,r}} \end{cases}$$

10

The above relation is used in implementations of the MRSAA below.

### **4** The below-capacity steady-state case common to CSA and MRSAA

The steady-state form of Eq. (6) under below-capacity conditions ( $p_c < 1$ ) can be expressed with the aid of Eq. (2) in the form

$$p_{cs} = 1 - \Lambda, \qquad (35a)$$

$$\Lambda = \frac{\upsilon}{I_f \beta_s q_{bf}}, \qquad (35b)$$

$$q_{bcs} = \frac{q_{bf}}{\rho_{cs}} \qquad (35c)$$

where  $p_{cs}$ ,  $\beta_s$  and  $q_{bcs}$  denote steady-state values of  $p_c$ ,  $\beta$  and  $q_{bc}$ , respectively. Equation (35a–c) describe a balance between the incision rate and relative vertical rock velocity (e.g., constant rock uplift rate at constant base level or constant rock elevation with constant rate of base level fall). CSA and MRSAA yield the same solution for this case, which must be characterized before showing how the models differ.

Equation (35a) has an interesting character. When the value of  $\Lambda$  exceeds unity,  $p_{\rm c}$  falls below zero and no steady state solution exists. Equation (35b) reveals that  $\Lambda$  can be interpreted as a dimensionless rock uplift rate. Thus when the rock uplift rate is sufficiently large for  $\Lambda$  to exceed unity, incision cannot keep pace with rock uplift,



(34)

In solving for this steady state, and in subsequent calculations, we use the bedload transport relation of Wong and Parker (2006a) rather than the very similar formulation of Fernandez Luque and van Beek (1976); in the case of the former,  $\alpha_{\rm b} = 4$ ,  $n_{\rm b} = 1.5$  and  $\tau_{\rm c}^* = 0.0495$ . We consider two cases: one for which  $\beta_{\rm s} = \beta$  is a specified constant, and one for which only a reference value  $\beta_{\rm r}$  is specified, and  $\beta_{\rm s}$  is computed from Eq. (5d).

In the case of a specified constant  $\beta$ , specification of v,  $I_{\rm f}$  and  $q_{\rm bf}$  allow computation of  $\Lambda$ ,  $\rho_{\rm cs}$  and  $q_{\rm bcs}$  from Eq. (35a–c). Further specification of R (here chosen to be 1.65, the standard value for quartz) and D allows the steady-state Shields number  $\tau_{\rm s}^*$  to be computed from Eq. (5c). Steady-state bedrock slope  $S_{\rm bs}$  can then be computed from Eq. (11b) upon specification of flood discharge  $Q_{\rm f}$ , Chézy resistance coefficient Cz and channel width B. In the case of  $\beta_{\rm s}$  calculated according to Eq. (7) using a specified reference value  $\beta_{\rm r}$ , the problem can again be solved with Eqs. (35), (5c) and (11b), but the solution is implicit.

- <sup>15</sup> We performed calculations for conditions loosely based on: (a) field estimates for a reach of the bedrock Shimanto River near Tokawa, Japan (Fig. 1), for which bed slope *S* is about 0.002 and channel width is about 100 m; (b) estimates using relations in Parker et al. (2007) for alluvial gravel-bed rivers with similar slopes, and reasonable choices for otherwise poorly-constrained parameters. The input parameters, Cz = 10,
- $Q_{\rm f} = 300 \,{\rm m}^3 \,{\rm s}^{-1}$ ,  $B = 100 \,{\rm m}$ , are loosely justified in terms of bankfull characteristics of alluvial gravel-bed rivers of the same slope (Parker et al.; 2007; Wilkerson et al.; 2011) as shown in Figs. 8a and b. The value  $D = 20 \,{\rm mm}$  represents a reasonable characteristic size of the substrate (and thus the bedload) for gravel-bed rivers; a typical size for surface pavement is 2 to 3 times this (e.g., Parker et al.; 1982). Intermittency  $I_{\rm f}$  is estimated as 0.05, i.e., 18 days per year, and thus a reasonable estimate for a river subject to frequent heavy storm rainfall. Alluvial porosity is  $\lambda_{\rm p} = 0.35$ . Two sediment
- feed rates were considered. The high feed rate was  $3.5 \times 10^5$  tons year<sup>-1</sup>, corresponding to the following steady state parameters at capacity conditions: Shields number  $\tau^* = 0.12$ , depth H = 1.5 m, steady state alluvial bed slope  $S_e = 0.0026$  and Froude



number Fr = 0.51 where

$$Fr = \frac{Q_{f}}{BH\sqrt{gH}}$$

The low feed rate was  $3.5 \times 10^4$  tons year<sup>-1</sup>, corresponding to the following parameters at capacity conditions: Shields number  $\tau^* = 0.064$ , depth H = 2.1 m, steady state alluvial bed slope  $S_e = 0.0010$  and Froude number Fr = 0.32. The value  $\beta_s = 0.05$  km<sup>-1</sup> was used for the case of constant abrasion coefficient. This corresponds to a value of  $\alpha_d$  of 0.017 km<sup>-1</sup>, which falls in the middle of the range measured by Kodama (1994) for chert, quartz and andesite (see Fig. 3-41 of Parker, 2008). For the case of variable abrasion coefficient, Eq. (1a) was used with  $\beta_r$  set to 0.05 km<sup>-1</sup> and  $\tau_r^*$  set to 0.12, i.e., the value for the high feed rate. This value of  $\tau_r^*$  is about 2.5 times the threshold value of Wong and Parker (2006a).

For the high feed, predicted relations for  $\beta_s$  vs. v are shown in Fig. 9a; the corresponding predictions for  $S_{\rm bs}$  vs. v are shown in Fig. 9b; the corresponding prediction for  $p_{cs}$  is shown in Fig. 9c. Both the cases of constant and variable  $\beta_s$  are shown. There 15 are five notable aspects of these figures. (1) In Fig. 9a, the predictions for variable  $\beta_s$ are very similar to the case of constant, specified  $\beta_s$ , and indeed are nearly identical for  $\upsilon \leq 3.3 \,\mathrm{mm \, year^{-1}}$  (corresponding to  $\Lambda \leq 0.05$  in Fig. 9c). (2) In Fig. 9b and c, the predictions for  $S_{\rm bs}$ ,  $\rho_{\rm cs}$  and  $\Lambda$  for variable  $\beta_{\rm s}$  are again nearly identical to those for constant  $\beta_s$ , and again essentially independent of v for  $v \le 3.3 \text{ mm year}^{-1}$ . (3) In Fig. 9c, 20  $p_{cs}$  is only slightly below unity (i.e.,  $\geq 0.95$ ), and  $\Lambda \leq 0.05$  for  $\upsilon \leq 3.3$  mm year<sup>-1</sup>). (4) For  $v > 3.3 \,\mathrm{mm \, year^{-1}}$ , the predictions for  $S_{\rm hs}$ ,  $p_{\rm cs}$  become dependent on v, such that  $S_{\rm hs}$  increases, and  $p_{\rm cs}$  decreases, with increasing v. The values for constant  $\beta_{\rm s}$  diverge from those for variable  $\beta_s$ , but are nevertheless close to each other up to some limiting value. (5) This limiting value corresponds to  $\Lambda = 1$  and thus  $p_{cs} = 0$  from Eq. (35a); 25 larger values of  $\Lambda$  lead to hanging valley formation. Here  $\Lambda = 1$  for the very high values  $v = 65 \text{ mm year}^{-1}$  for constant  $\beta_s$  and  $v = 30 \text{ mm for variable } \beta_s$ .

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(36)

These results require interpretation. It is seen from Eq. (35a–c) that when  $v/(l_f\beta_sq_{bf}) = \Lambda \ll 1$ ,  $p_c$  becomes nearly equal to unity (very little exposed bedrock), in which case  $q_{bf}$  is constrained to be only slightly smaller than  $q_{bc}$ . From Eqs. (5c) and (11), then,  $S_{bs}$  is only slightly above the steady state alluvial bed slope  $S_e$ . Note that the steady-state bedrock slope decouples from rock uplift rate under these conditions: the predictions for  $v = 0.2 \text{ mm year}^{-1}$  are nearly identical to this for  $v = 3.3 \text{ mm year}^{-1}$ . This behavior is a specific consequence of the condition  $\Lambda \ll 1$  corresponding to a low ratio of uplift rate to a reference incision rate  $E_{ref} = l_f \beta_s q_{bf}$ . They imply a wide range of conditions for which (a) very little bedrock is exposed, and (b) bedrock slope is independent of uplift rate.

The results for the low feed rate are very similar. The values for variable  $\beta_s$  differ from the constant value  $\beta_s$  in Fig. 10a, but this is because the constant value  $\beta_s = 0.05$ was set based on the high feed rate. The results in Fig. 10b and c are qualitatively the same for Fig. 9b and c; the uplift rate below which  $\Lambda < 0.05$  is 0.33 mm year<sup>-1</sup> for the case of constant  $\beta_s$ , and 0.73 mm year<sup>-1</sup> for the case of variable  $\beta_s$ . The critical value of v beyond which a hanging valley forms is 6.8 mm year<sup>-1</sup> for constant  $\beta_s$  and 7.1 mm year<sup>-1</sup> for variable  $\beta_s$ .

15

The lack of dependence of steady-state bedrock slope  $S_{bs}$  on rock uplift rate v below a threshold value for the steady-state solutions of the CSA model (and thus the MR-SAA model as well) is in stark contrast to earlier work for which the incision rate  $E_s$  is assumed to have the following dependence on slope  $S_b$  and drainage area A (Slope– Area formulation, Howard and Kerby, 1983):

$$E_{\rm s} = K S_{\rm b}^n A^m \tag{37}$$

where A denotes drainage area, n and m are specified exponents, and K is a constant assumed to increase with increasing rock hardness.

In order to compare the steady-state predictions of the Slope–Area relation of Eq. (37) for constant v with CSA, drainage area A must be taken to be a constant value  $A_o$  so as to correspond to the HSR configuration used here. The steady-state



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slope  $S_{bs}$  corresponding to a balance between incision and rock uplift is found from Eq. (37) to be

$$S_{\rm bs} = \frac{v^{\frac{1}{n}}}{K^{\frac{1}{n}}A_o^{\frac{m}{n}}}$$

<sup>5</sup> In their Table 1, Whipple and Tucker (2000) quote a range of values of *n*, but their most quoted value is 2. We compare the results for CSA for  $S_{bs}$  with the predictions from Eq. (38) with n = 2 by normalizing against a reference value  $S_{bsr}$  corresponding to a reference rock uplift rate  $v_r$  of 0.2 mm year<sup>-1</sup>. Equation (38) yields

$$\frac{S_{\rm bs}}{S_{\rm bsr}} = \left(\frac{\upsilon}{\upsilon_{\rm r}}\right)^{1/2}$$

10

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In Fig. 11, Eq. (39) is compared against the CSA predictions of Figs. 9b and 10b (high and low feed rate, respectively), for both constant and variable  $\beta_s$ . In order to keep the plot within a realistic range, only values of v between 0.2 mm year<sup>-1</sup> and 10 mm year<sup>-1</sup> (the upper limit corresponding to Dadson et al., 2003), have been used in the CSA results. The remarkable insensitivity of the CSA predictions for steady-state slope  $S_{bs}$ 

on rock uplift rate is readily apparent from the figure.

One more difference between the CSA and Slope–Area formulations is worth noting. If the Slope–Area relation is installed into Eq. (6) in place of CSA, it is readily shown that bedrock slope gradually relaxes to zero in the absence of rock uplift. CSA does

not obey the same behavior under the constraint of constant sediment feed rate: Figs. 9b and 10b indicate that bedrock slope converges to a constant, nonzero value as rock uplift declines to zero. This is not necessarily a shortcoming of CSA; the sediment feed rate can be expected to decline as relief declines.



(38)

(39)

# 5 Boundary conditions and parameters for numerical calculations with MRSAA model

Having conducted a fairly thorough analysis of the steady state common to the CSA and MRSAA models, it is now appropriate to move on to examples of behavior that can

<sup>5</sup> be captured by the MRSAA model but not the CSA model. Before doing so, however, it is necessary to delineate the boundary conditions and other assumptions used in the MRSAA model.

Let *L* denote the length of the reach. Equation (22a) indicates that the formulation for bedrock incision is first-order in x and so requires only one boundary condition. The example considered here is that of a downstream bedrock elevation, i.e., base level, that is set to 0:

$$\eta_{b}|_{x=L} = 0$$

According to Eq. (15), or alternatively Eq. (23a), the alluvial formulation is secondorder in *x* and thus requires two boundary conditions. The following boundary condition applies at the upstream end of the reach; where  $q_{bf}(t)$  denotes a feed rate which may vary in time,

$$q_{\rm b}|_{\rm x=0} = q_{\rm bf}(t) \tag{41}$$

At the downstream end, a free boundary condition is applied for  $\eta_a/L_{mr} < 1$ , and a fixed boundary condition is applied for  $\eta_a/L_{mr} \ge 1$  as follows:

$$\left[ p_{c}(1 - \lambda_{p}) \frac{\partial \eta_{a}}{\partial t} + I_{f} \frac{\partial p_{c} q_{bc}}{\partial x} \right] \Big|_{x=L} = 0 \qquad \text{if} \quad \left[ \frac{\eta_{a}}{L_{mr}} \right] \Big|_{x=L} < 1 \qquad (42a)$$

$$\eta_{a} \Big|_{x=L} = L_{mr} \qquad \text{if} \quad \left[ \frac{\eta_{a}}{L_{mr}} \right] \Big|_{x=L} \ge 1 \qquad (42b)$$

Here Eq. (42a) specifies a free boundary in the case of partial alluviation, so allowing below-capacity sediment waves to exit the reach. Equation (42b), on the other hand, fixes the maximum downstream elevation at  $\eta = \eta_a = L_{mr}$ .



(40)

In order to illustrate the essential features of the new formulation of the MRSAA model for morphodynamics of mixed bedrock-alluvial rivers, it is useful to consider the most simplified case that illustrates its expanded capabilities compared to the CSA model. Here we implement the HSR simplification. In addition, based on the results of the previous section, we approximate  $\beta_s$  as a prescribed constant. Finally, we assume that the clasts of the abrading bedload are sufficiently hard compared to the bedrock so that grain size *D* can be approximated as a constant. These constraints are easily relaxed.

### 6 Sediment waves over a fixed bed: stripping and emplacement of alluvial layer and advection-diffusion of a sediment pulse

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Here three numerical cases using the MRSAA model are studied: (1) stripping of an alluvial cover to bare bed; (2) emplacement of an alluvial cover over a bare bed; and (3) advection-diffusion of an alluvial pulse over a bare bed. Reach length *L* is 20 km. As the time for alluvial response is short compared to incisional response,  $\beta_s$  and  $\upsilon$  are set equal to zero for these calculations. In addition, flood intermittency  $I_f$  is set to unity so as to illustrate the migration from feed point to the end of the reach under the condition of continuous flow. The macro-roughness  $L_{mr}$  is set to 1 m based on visual observation of the Shimanto River near Tokawa, Japan. The values for Cz,  $Q_f$ , *B*, *D* and  $\lambda_p$  are the same as in Sect. 5, i.e., Cz = 10,  $Q_f = 300 \text{ m}^3 \text{ s}^{-1}$ , B = 100 m. D = 20 mm and  $\lambda_p = 0.35$ . Bedrock slope  $S_b$ , which is constant due to the absence of abrasion, is set to 0.004. The above numbers combined with Eqs. (5c) (using the constants of the formulation of Wong and Parker, 2006a), Eq. (27a) and (27b) yield the following values: depth H = 1.32 m, Froude number Fr = 0.63, Shields number  $\tau^* = 0.016$  and capacity

bedload transport rate  $q_{bc} = 0.0017 \text{ m}^2 \text{ s}^{-1}$ . None of these three cases can be treated using CSA. They thus illustrate capabilities unique to MRSAA.



### 6.1 Alluvial stripping

The case of stripping of an initial alluvial layer to bare bedrock is considered here. In this simulation, the bedload feed rate  $q_{bf} = 0$  and the initial thickness of alluvial cover  $\eta_a$  is set to 0.8 m, i.e., 80 % of the macro-roughness length  $L_{mr}$ . To drive stripping of

the alluvial layer, the feed rate is set equal to zero. Figure 12a shows how the alluvial cover is progressively stripped off from upstream to downstream as a wave of alluvial rarification migrates downstream. The alluvial layer is completely removed after a little more than 0.12 years.

Of interest in Fig. 12a is the fact that the wave of stripping maintains constant form <sup>10</sup> in spite of the fact that the diffusive term in Eq. (23a) should cause the wave to spread. The reason the wave does not spread is the nonlinearity of the wave speed  $c_a$  in Eq. (23b); since  $p_c$  enters into the denominator of the right-hand side of the equation, wave speed is seen to increase as  $p_c$  decreases, and thus as  $\eta_a$  decreases. As a result, the lower portion of the wave tends to migrate faster than the higher portion, so sharpening <sup>15</sup> the wave and opposing diffusion.

### 6.2 Emplacement of an alluvial layer over an initially bare bed

In this simulation, the initial thickness of alluvium  $\eta_a$  is set to zero and the sediment feed rate is set to  $0.0013 \text{ m}^2 \text{ s}^{-1}$ , i.e., 80% of the capacity value. The result of the calculation is shown in Fig. 12b. Here nonlinear advection and diffusion act in concert to cause the wave of alluviation to spread. The steady-state thickness of alluvium is 0.83 m; by 0.1 years it has been emplaced only down to about 5 km from the source.

### 6.3 Propagation of a pulse of alluvium over an initially bare bed

In this example the initial bed is bare of sediment. The sediment feed rate is set equal to  $0.0012 \text{ m}^2 \text{ s}^{-1}$ , i.e., 70% of the capacity value for 0.05 years from the start of the run, and then dropped to zero for the rest of the run. Figure 12c shows the propagation of



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a damped alluvial pulse through the reach, with complete evacuation of the pulse in a little more than 0.15 years. Nonlinear advection acts against diffusion to suppress the spreading of the upstream side of the pulse, but advection acts together with diffusion to drive spreading of the downstream side of the pulse.

# 5 7 Comparison of evolution to steady state with both rock uplift and incision using CSA and MRSAA

Here we consider three cases of channel profile evolution to steady state that include both rock uplift and incision. In the first case, the initial bedrock slope is set to a value below the steady state value, and the sediment feed rate is set to a value that is well above the steady state value for the initial bedrock slope, causing early-stage massive alluviation. The configuration for the second case is a simplified version of a graben with a horst upstream and a horst downstream. The configuration for the third case is such that there is an alluviated river mouth downstream and a bedrock-alluvial transition upstream. In all cases, MRSAA predicts evolution that cannot be predicted by CSA.

### **7.1** Evolution of bedrock profile with early-stage massive alluviation

Here we set  $Q_{\rm f}$ , *B*, *Cz*, *D* and  $\lambda_{\rm p}$  to the same values as Sect. 6. The reach length *L* is 20 km, the flood intermittency  $I_{\rm f}$  is set to 0.05, macro-roughness  $L_{\rm mr}$  is set to 1 m, initial alluvial thickness  $\eta_{\rm a}|_{t=0} = 0.5$  m, downstream bed elevation  $\eta_{\rm b}|_{x=L} = 0$  and the abrasion coefficient  $\beta_{\rm s}$  is 0.05 km<sup>-1</sup>. The initial bed slope is 0.004. The feed rate is set to 5 mm year<sup>-1</sup>. It should be noted, however, than in analogy Fig. 10b, the steady-state bedrock slope for this feed rate is independent of the uplift rate for  $v \le 5$  m year<sup>-1</sup>. This is because the steady-state value of  $\Lambda$  is 0.019, i.e.,  $\ll 1$ .

The results for the CSA model are shown in Fig. 13a. The bed slope evolves from the initial value of 0.004 to a final steady-state value of 0.0068. Evolution is achieved solely



by means of an upstream-migrating knickpoint. Only the first 4000 years of evolution are shown in the figure.

Figure 13b shows the results of the first 400 years of the calculation with MRSAA. By 100 years, the bed is completely alluviated, and by 400 years, the thickness of the
alluvial layer at the upstream end of the reach is 52 m. This massive alluviation is not predicted by CSA. Figure 13c shows the results of the first 4000 years of evolution. The upstream-migrating knickpoint takes the same form as CSA, but it is nearly completely hidden by the alluvial layer. The knickpoint gradually migrates upstream, driving the completely alluviated layer out of the domain, but this process is not complete by 4000 years. A comparison of Fig. 13a and c shows that a knickpoint that is exposed in CSA is hidden in MRSAA. CSA cannot predict the presence of a hidden knickpoint.

### 7.2 Evolution of a simplified horst-graben configuration

In this example, Cz,  $Q_f$ , B, D,  $\lambda_p$ ,  $I_f$ ,  $\beta_s$ ,  $L_{mr}$ , L,  $\eta_a|_{t=0}$  and  $\eta_b|_{x=L}$  are set to the values used in Sect. 7.1. The sediment feed rate  $q_{bf} = 0.00083 \text{ m}^2 \text{ s}^{-1}$ , and the initial bedrock slope  $S_b$  is set to the steady-state value for a rock uplift rate of 1 mm year<sup>-1</sup>, i.e., 0.0027. The model is then run for a rock uplift rate of 1 mm year<sup>-1</sup> for the domains  $0 \le x \le$ 8 km and 12 km  $\le x \le 20$  km and a rock subsidence rate of 1 mm year<sup>-1</sup> for the domain 8 km < x < 12 km. This configuration corresponds to a simplified 1-D configuration of a graben bounded by two horsts, one upstream and one downstream.

- <sup>20</sup> CSA cannot even be implemented for this case, because the model is unable to handle rock subsidence. The results for MRSAA are shown in Fig. 14. By 15 000 years, the uplifting domains evolve to a steady state in terms of both bedrock elevation and alluvial cover. The bedrock elevation of the subsiding domain never reaches steady state, because it is completely alluviated. The profile at the top of the alluvium in this do-
- main has indeed reached steady state by 15 000 years, with a bed slope that deviates only modestly for the steady state bedrock slope for the case where  $v = 1 \text{ mm year}^{-1}$ everywhere.



# 7.3 Evolution of river profile with an alluviated zone corresponding to a river mouth at the downstream end

In this example *Cz*,  $Q_f$ , *B*, *D*,  $\lambda_p$ ,  $\beta_s$ ,  $L_{mr}$ , *L* and  $\eta_a|_{t=0}$  are again set to the values chosen in Sect. 7.1. The bedload feed rate is 0.00083 m<sup>2</sup>/s; the steady-state bedrock slope  $S_b$  associated with this feed rate is 0.0026 for  $\upsilon < 5 \text{ mm year}^{-1}$  (Fig. 10b). The initial bedrock slope is set, however, to the higher value 0.004. The rock uplift rate  $\upsilon$  for this case is set to 0, for which the steady-state slope is again 0.0026.

The result of CSA for this case with base level  $\eta_b|_{x=L}$  maintained at 0 is shown in Fig. 15. As in the case of Sect. 7.1, the bedrock slope evolves from the initial value of

0.004 to the steady-state value 0.0026 by means of an upstream-migrating knickpoint.
 Only 4000 years of evolution are shown in the figure, by which time the knickpoint is 4.8 km from the feed point.

MRSAA is implemented with somewhat different initial and downstream boundary conditions, in order to model the case of bed that remains alluviated at the down-15 stream end. This condition thus corresponds to an alluviated river mouth. The initial bedrock slope is again 0.004, and the downstream bedrock elevation  $\eta_{\rm b}|_{x=L}$  is again 0 m. Downstream alluvial elevation  $\eta_{\rm a}|_{x=L}$ , however, is held at 10 m, so that the downstream end is completely alluviated. The initial slope *S* for the top of the bed is 0.0021, a value chosen so that the bed elevation equals the bedrock elevation at the upstream 20 end.

Results of the MRSAA simulation are shown in Fig. 16a–f. Figure 16a–c shows the early-stage evolution, i.e., at t = 0, 10 and 100 years. Over this period, a bedrock-alluvial transition (from mixed bedrock-alluvial to purely alluvial) migrates downstream from the feed point to x = 13.6 km, i.e., 6.4 km upstream of the downstream end of the domain. Bedrock incision is negligible over this period.

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Figure 13d–f show the bedrock and top bed profiles for 1000, 2000 and 4000 years. Over this period, the bedrock-alluvial transition migrates upstream. As it does so, the bedrock slope downstream of x = 13.6 km remains alluviated and does not change.



The bedrock slope between the transition and x = 13.6 km evolves to the steady-state value of the case in Sect. 7.2, and the top bed slope downstream of the transition evolves to the same slope as the steady-state bedrock slope (because with v = 0,  $\Lambda$  is vanishing). The figures illustrate that upstream-migrating bedrock knickpoint is located at the bedrock-alluvial transition. By 4000 years, the transition has migrated out of the domain, and the bed is completely alluviated. The thickness of the alluvial cover upstream of x = 13.6 km is, however, only 1.05 m, i.e., only slightly larger than the macro-roughness height of 1 m. This means that although the reach is everywhere alluvial at 4000 years, the bedrock is only barely covered. Neither the upstream-migrating

bedrock-alluvial transition nor the thickness of the alluvial cover is captured by CSA. 10

#### Discussion 8

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The form of the MRSAA model presented here has been simplified as much as possible, i.e., to treat a HSR (highly simplified reach) with constant grain size D. It can relatively easily be extended to: (1) abrasion of the clasts that abrade the bed, so abrasional downstream fining is captured (Parker, 1991); (2) size mixtures of sediment 15 (Wilcock and Crowe, 2002); (3) multiple sediment sources; (4) channels with width variation downstream; (5) fully unsteady flow (An et al., 2014); and (6) cyclically varying hydrographs (Wong and Parker, 2006b) or "sedimentographs", the latter corresponding to events for which the sediment supply rate first increases, and then decreases cyclically (Zhang et al., 2013). Sections 6 and 7 highlight features that are captured by 20

MRSAA but not by CSA.

The MRSAA model presented here has a weakness in that the resistance coefficient Cz is a prescribed constant. The recent model of bedrock incision of Inoue et al. (2014) provides a much more detailed description of resistance. In addition to characteriz-

ing macro-roughness, it uses two micro-roughnesses, one characterizing the hydraulic 25 roughness of the alluvium and the other characterizing the hydraulic roughness of the bedrock surface. It can thus discriminate between "clast-smooth" beds for which



bedrock roughness is lower than clast roughness, and "clast-rough" beds for which bedrock roughness is higher than clast roughness. This characterization allows two innovative features: (1) both bed resistance and fraction cover become dependent on the ratio of bedrock micro-roughness to alluvial micro-roughness; (2) incision can re-

<sup>5</sup> sult from the overpassing of throughput sediment over a purely bedrock surface with no alluvial deposit. The model of Inoue et al. (2014) uses a modified form the capacity-based form for cover of Eq. (2) in order to capture these phenomena. It is thus unable to capture the co-evolution of bedrock-alluvial and purely alluvial processes of MRSAA. The amalgamation of their model and the one presented here is an attractive future future goal.

Because MRSAA tracks the spatiotemporal variation of both bedload transport and alluvial thickness, it is applicable to the study of the incisional response of a river subject to temporally varying sediment supply. It thus has the potential to capture the response of an alluvial-bedrock river to massive impulsive sediment inputs associated with land-

<sup>15</sup> slides or debris flows. A preliminary example of such an extension is given in Zhang et al. (2013). When extended to multiple sediment sources, it can encompass both the short- and long-term responses of a bedrock-alluvial river to intermittent massive sediment supply due to landslides and debris flows. As such, it has the potential to be integrated into a framework for managing sediment disasters in mountain rivers such as the Wenchuan Earthquake, Sichuan, China, 2008. Over 200 landslide dams formed during that earthquake (Xu et al., 2009; Fu et al., 2011).

### 9 Conclusions

We present a new model, the Macro-Roughness-based Saltation-Abrasion-Alluviation (MRSAA) model, which provides a unified description of both alluvial and incisional processes in rivers which may be bedrock-alluvial, purely alluvial, or may transition freely between the two morphologies. Our model specifically tracks not only bedrock

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morphodynamics, but also the morphodynamics of the alluvium over it. The key results are as follows.

1. The transport of alluvium over a bedrock surface cannot in general be described simply by a supply rate. Here we track the alluvium in terms of a spatiotemporally varying alluvial thickness.

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- 2. The area fraction of cover  $p_c$  enters into both incisional and alluvial evolution. The alluvial part allows the downstream propagation and diffusion of sediment waves, so that at any given time the alluvial bed can be above or below the top of the bedrock. The model thus allows for spatiotemporal transitions between complete cover, under which no incision occurs, partial cover, for which incision may occur, and no cover, for which no incision occurs.
- 3. The MRSAA model captures three processes: downstream alluvial advection at a fast time scale, alluvial diffusion, and upstream incisional advection at a slow time scale. Only the third of these processes is captured by the Capacity-based Saltation-Abrasion (CSA) model of Sklar and Dietrich (2004, 2006).
- 4. The MRSAA model reduces to the CSA model under the conditions of steadystate incision in balance with rock uplift and below-capacity cover. The steadystate bedrock slope predicted by both models is insensitive to the rock uplift rate over a wide range of conditions. This insensitivity is in marked contrast to the commonly used incision model in which the incision rate is a power function of bedrock slope and drainage area upstream. The two models can differ substantially under transient conditions, particularly under those that include migrating transitions between the bedrock-alluvial and purely alluvial state.
- 5. In the MRSAA model, inclusion of alluvial advection and diffusion lead to the following phenomena: (a) a wavelike stripping of antecedent alluvium over a bedrock surface in response to cessation of sediment supply; (b) advection–diffusional em-



placement of a sediment cover over initially bare bedrock; and (c) the propagation and deformation of a sediment pulse over a bedrock surface.

6. In the case of transient imbalance between rock uplift and incision with a massive increase in sediment feed, MRSAA captures an upstream-migrating transition between a purely alluvial reach upstream and a bedrock-alluvial reach downstream (here abbreviated as a alluvial-bedrock transition). The bedrock profile shows an upstream-migrating knickpoint, but this knickpoint is hidden under alluvium. CSA captures only the knickpoint, which is completely exposed, and the model thus misses the thick alluvial cover predicted by MRSAA.

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- 7. CSA fails for the case of a 1-D subsiding graben bounded by two uplifting horsts, because it cannot describe the evolution of a subsiding zone. MRSAA captures alluvial filling of the graben, and thus converges to a steady-state top-bed profile with a bedrock-alluvial transition at the upstream end of the graben and an alluvial-bedrock transition at the downstream end.
- 8. In the case studied here of an uplifting bedrock profile with an alluviated bed at the downstream end modeling a river mouth, MRSAA predicts an upstream-migrating bedrock-alluvial transition at which the bedrock undergoes a sharp transition from a higher to a lower slope. MRSAA further predicts a bedrock long-profile under the alluvium that has the same slope as the top bed. It also predicts that the cover is thin, so that the purely alluvial reach is only barely so. The steady state for this case is purely alluvial.
  - 9. The new MRSAA model provides an entry point for the study of how bedrockalluvial rivers respond to occasional large, impulsive supplies of sediment from landslides and debris flows. It thus can provide a tool for forecasting river disasters associated with such events. An example of a massive sediment disaster is the Wenchuan Earthquake in Sichuan, China in 2008, which caused over 200 landslide dams in an area of active tectonics.



### Appendix A: Interpretation of the abrasion coefficient $\beta$

Consider a clast of size *D* and volume  $V_c \sim D^3$  causing abrasion over an exposed bedrock surface. The bedload transport rate  $q_b$  is given as

 $_{5}$   $q_{b} = E_{s}L_{s}$ 

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where  $E_s$  denotes the volume rate per unit time per unit area at which particles are ejected from the bed into saltation, and  $L_s$  denotes the saltation length. In so far as a particle ejected into saltation collides into the bed a distance  $L_s$  later, the number of particles that collide with the bedrock (rather than other bed particles) per unit time per unit area is given from Eq. (A1) as

$$(1 - p_c) \frac{E_s}{V_c} = (1 - p_c) \frac{q_b}{V_c L_s}$$

The volume lost from the striking clast per strike is defined as  $\beta_c^* V_c$ , and the volume lost from the stricken bedrock per strike is similarly defined as  $\beta^* V_c$ . The parameters  $\beta_c^*$  and  $\beta^*$  could be expected to be approximately equal were the striking clast be of identical rock type as the bedrock.

In so far as the rate at which a clast strikes the bed per unit distance moved is  $1/L_s$ , the rate at which clast volume decreases downstream is given by the relation

$$\frac{dV_{c}}{dx} = -\beta_{c}V_{c},$$
20  $\beta_{c} = \frac{\beta_{c}^{*}}{L_{s}}$ 

Assuming that  $V_p \sim D^3$ , Eq. (A3) reduces to

 $\frac{\mathrm{d}D}{\mathrm{d}x} = -\alpha_{\mathrm{d}}D,$ 



(A1)

(A2)

(A3a)

(A3b)

(A4a)

$$\alpha_{\rm d} = \frac{1}{3}\beta_{\rm c}$$

Equation (A4a) is the differential form of Sternberg's Law;  $\alpha_d$  is a diminution coefficient with units L<sup>-1</sup>. The exponential form of Eq. (3) corresponds to the case of spatially constant  $\alpha_d$ .

The incision rate of the bedrock E is the rate of volume loss of bedrock per unit area per unit time, times the volume lost per strike, or thus

$$\Xi = \frac{q_{b}}{V_{p}L_{s}}\beta^{*}V_{p}(1-p_{c}) = \beta^{*}q_{b}(1-p_{c}) = \beta q_{bc}p_{c}(1-p_{c}),$$

$$\beta = \frac{\beta^{*}}{L_{s}}$$
(A5)

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The above relation is identical to Eq. (1b).

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### Table A1. Nomenclature.

A	upstream drainage area [L <sup>2</sup> ]
В	channel width [L]
CSA	acronym for Capacity-based Saltation-Abrasion model
Cz	dimensionless Chezy resistance coefficient [-]
Ca	speed of propagation of an alluvial disturbance (positive downstream) $[LT^{-1}]_{1}$
Cb	speed of propagation of an incisional disturbance (positive downstream) $[LT^{-1}]$
D, D <sub>u</sub>	characteristic grain size of clasts effective in abrading the bed; upstream value of D [L]
E	bedrock incision rate $[LT^{-1}]$
Fr	Froude number = $Q_{i}/[BH(qH)^{1/2}]$ [–]
f	function of y describing cover fraction [-]
a	gravitational acceleration [L $T^{-2}$ ]
э Н	flow denth [I $T^{-1}$ ]
HSR	acronym for Highly Simplified Reach
<i>I</i> <sub>f</sub>	flood intermittency = fraction of time the river is in flood $[-]$
k,	coefficient in Eq. (5a) [-]
Ĺ	reach length [L]
Lhalf	distance a clast travels to lose half its size (diameter) by abrasion [L]
Lmr	height of macro-roughness height [L]
MRSAA	acronym for Macro-Roughness-based Saltation-Abrasion-Alluviation model
n <sub>b</sub>	exponent in bedload transport relation [-]
$p_{c}$	areal fraction of bed that is covered by alluvium [-]
$p_{cs}$	steady-state value of $p_{\rm c}$ [-]
$p_{\rm c,r}$	lower reference cover fraction (0.05 herein) [–]
$p_{c,1-r}$	upper reference cover fraction (0.95 herein) [–]
$Q_{\rm f}$	flood discharge [L° T <sup>-</sup> ']
$q_{\rm b}, q_{\rm bc}, q_{\rm bcs}$	volume bedload transport rate per unit width; capacity value of $q_{\rm b}$ ; steady-state value of $q_{\rm bc}$
	[L <sup>2</sup> T <sup>-1</sup> ]
$q_{ m bf}$	feed, or supply value of $q_{\rm b} [L^2 T^{-1}]$
$q_{\sf bk}$	value of $q_{\rm b}$ at knickpoint [L <sup>2</sup> T <sup>-1</sup> ]
R	submerged specific gravity of sediment clasts [-]
R <sub>f</sub>	$= v_{\rm s} / (RgD)^{1/2}$ [-]
$S, S_{\rm h}, S_{\rm a}$	bed slope; slope of bedrock; $-\partial \eta_a / \partial x$ [-]
$S_{\rm bi,} \tilde{S}_{\rm bs}$	initial bedrock slope; steady-state bedrock slope [-]
S <sub>b, u</sub> , S <sub>b, I</sub>	bedrock slope upstream of a knickpoint; bedrock slope downstream of a knickpoint [-]
Se	steady state alluvial bed slope at capacity [-]

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**Figure 1.** Views of the Shimanto River, a mixed alluvial-bedrock river in Shikoku, Japan. (a) View of channel. (b) View of macroscopic roughness of the bed and alluvial patches. River width is about 100 m.

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**Figure 2.** Schematic diagram illustrating downstream modification of a sedimentograph. At the upstream feed point (x = 0, left), the bedload transport rate  $q_b$  takes the high feed value  $q_{bf, h}$  for time  $T_h$  and the low feed value  $q_{bf, l}$  time  $T_l$ , for a total cyclic time  $T = T_h + T_l$ . At the downstream end (x = L, right) (i) the solid line predict the unaltered sedimentograph at the downstream end of the reach, assumed to have propagated instantaneously from the supply point; and (ii) and the dashed line represents the sedimentograph as modified by advective-diffusive effects.





Figure 3. Schematic diagram illustrating the propagation of a wave of sediment over bedrock. Here  $\eta_{\rm b}$  denotes the elevation of the bottom of the bedrock,  $L_{\rm mr}$  denotes the bedrock macroroughness thickness,  $\eta_a$  denotes the thickness of the alluvial cover (which may be  $\langle \eta_b + L_{mr} \rangle$ ) and  $\eta = \eta_{\rm b} + \eta_{\rm a}$  denotes the elevation of the top of the alluvium.

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**Figure 5.** Illustration of the relation between areal fraction of alluvial cover of bedrock  $p_c$  and  $\chi = \eta_a/L_{mr}$  for MRSAA (Macro-Roughness-based Saltation-Abrasion Alluviation model). (a) Low cover. (b) Intermediate cover. (c) Complete alluviation above the top of the bedrock.





**Figure 6.** Modification of formulation to account for statistical structure of bedrock roughness. Here z' = elevation above an arbitrary datum, and  $p_c(z')$  is now interpreted as the probability that point z' does not correspond to bedrock (i.e., falls within water or alluvium rather than bedrock). The cover factor for alluvial thickness  $\eta_a$  equals  $p_c(\eta_a)$ . The points  $\eta_b$  and  $\eta_b + L_{ma}$  are determined in terms of specified exceedance fractions r and 1 - r and 1 = r of  $p_c$ .







**Figure 8. (a)** Chézy resistance coefficient estimated for the bedrock-alluvial Shimanto River, Japan, as well as alluvial rivers. **(b)** Bank-to-bank width estimated for the Shimanto River, Japan, as well as bankfull width of alluvial rivers. The ranges for characteristic bed material size of the alluvial rivers are denoted in the legends.





**Figure 9.** Variation of (a) abrasion coefficient  $\beta_s$ , (b) bedrock slope  $S_{bs}$  and (c) cover fraction  $\rho_{cs}$  with rock uplift or base lowering rate v at steady state, with a high bedload feed rate (3.5 ×  $10^5$  tons year<sup>-1</sup>). The cases of constant, specified  $\beta_s$  and  $\beta_s$  varying according to Eq. (29) are shown. The vertical lines denote the incipient conditions for the formation of a hanging valley. The predictions are the same for the CSA and MRSAA models.





**Figure 10.** Variation of **(a)** abrasion coefficient  $\beta_s$ , **(b)** bedrock slope  $S_{bs}$  and **(c)** cover fraction  $p_{cs}$  with rock uplift or base lowering rate v at steady state, with the high bedload feed rate  $(3.5 \times 10^4 \text{ tons year}^{-1})$ . The cases of constant, specified  $\beta_s$  and  $\beta_s$  varying according to Eq. (29) are shown. The vertical lines denote the incipient conditions for the formation of a hanging valley. The predictions are the same for the CSA and MRSAA models.





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**Figure 11.** Normalized steady-state bedrock slope vs. normalized rock uplift rate as predicted by the CSA model for a low and a high feed rate, and constant and variable abrasion coefficient. The results are the same for the MRSAA model. Also shown is the prediction of a model for which the incision rate is specified in terms of bedrock slope and upstream drainage area. Note that the predictions for steady-state bedrock slope of the CSA model are insensitive to the rock uplift rate over a wide range.



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Figure 12. (a) Illustration the stripping of an alluvial layer to bare bedrock. (b) Illustration of the emplacement of an alluvial cover over initially bare bedrock. (c) Illustration of the evolution of a pulse of sediment over bare bedrock. All calculations are done with the MRSAA model.





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Figure 13. (a) Progression to steady state after an impulsive increase in sediment supply: CSA model. (b) Progression to steady state after an impulsive increase in sediment supply: MRSAA model, early stage. (c) Progression to steady state after an impulsive increase in sediment supply: MRSAA model, late stage. Note the bedrock knickpoints in (a) and (b), and the migrating alluvial-bedrock transition in (c).



**Figure 14.** Evolution predicted by the MRSAA model for simplified, 1-D the case of a graben bounded by two horsts. Note the bedrock-alluvial and alluvial-bedrock transitions. By 15 000 years, the bed top has reached steady state, even though the bedrock surface in the graben continues to subside. Horst rock uplift rate and graben rock subsidence rate are assumed constant for simplicity.





Figure 15. Evolution of an initial bedrock profile to a higher steady-state profile, as predicted by CSA. This figure is the basis for comparison with the results of MRSAA shown in Fig. 16.

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**Figure 16.** Evolution of bed top and bedrock profiles with an imposed alluvial river mouth at the downstream end and an upstream-migrating bedrock-alluvial transition. The results are for: (a) t = 0 years; (b) t = 10 years; (c) t = 100 years; (d) t = 1000 years; (e) t = 2000 years; and (f) t = 4000 years (steady state). Here (a), (b) and (c) show the early response, and (d), (e) and (f) show the late response. All calculations are with MRSAA. A corresponding case using CSA is shown in Fig. 15.

