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Ice flow models and glacial erosion over multiple glacial-interglacial cycles

R. M. Headley¹ and T. A. Ehlers²

¹Geosciences Department, University of Wisconsin-Parkside, Kenosha, WI, USA ²Department of Geosciences, Universität Tübingen, Tübingen, Germany

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Correspondence to: R. M. Headley (headley@uwp.edu)

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Abstract

Mountain topography is constructed through a variety of interacting processes. Over glaciological time scales, even simple representations of glacial-flow physics can reproduce many of the distinctive features formed through glacial erosion. However, de-

- tailed comparisons at orogen time and length scales hold potential for quantifying the influence of glacial physics in landscape evolution models. We present a comparison using two different numerical models for glacial flow over single and multiple glaciations, within a modified version of the ICE-Cascade landscape evolution model. This model calculates not only glaciological processes but also hillslope and fluvial erosion and eadiment transport in and to processes but also hillslope and fluvial erosion
- and sediment transport, isostasy, and temporally and spatially variable orographic precipitation. We compare the predicted erosion patterns using a modified SIA as well as a nested, 3-D Stokes-flow model calculated using COMSOL Multiphysics.

Both glacial-flow models predict different patterns in time-averaged erosion rates. However, these results are sensitive to the climate and the ice temperature. For warmer

climates with more sliding, the higher-order model has a larger impact on the erosion rate, with variations of almost an order of magnitude. As the erosion influences the basal topography and the ice deformation affects the ice thickness and extent, the higher-order glacial model can lead to variations in total ice-covered that are greater than 30%, again with larger differences for temperate ice. Over multiple glaciations
 and long-time scales, these results suggest that consideration of higher-order glacial physics may be necessary, particularly in temperate, mountainous settings.

1 Introduction

Over geological time, mountainous topography is formed through a combination of erosional and tectonic processes. In many regions, mountain topography has felt the ef-

²⁵ fects of glacier erosion, in addition to other geomorphic processes. Quantifying the effects of glaciation on topography requires consideration of the physics and rheological





properties governing glacial erosion. This study builds upon previous work and evaluates how different assumptions and levels of complexity used in glacial-flow models impact the magnitude of erosion over multiple glacial-interglacial cycles. This type of study is important for evaluating what level of model complexity (and computational sophistication) is required to quantify glacial erosion processes and sediment production in both mountainous.

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Numerical models have been used to study the influence of glacial erosion on landscape development. These models have ranged from simple 2-D, glacial profile models (Anderson et al., 2006; MacGregor et al., 2009) to more complex 3-D models that incorporate a variety of processes and mechanisms (Kessler et al., 2008; Egholm et al.,

- ¹⁰ corporate a variety of processes and mechanisms (Kessler et al., 2008; Egholm et al., 2012a; Pedersen and Egholm, 2013). Other studies have incorporated glacial erosion into landscape dynamic models that also include fluvial and hillslope erosional processes (Herman and Braun, 2008; Egholm et al., 2011; Yanites and Ehlers, 2012). The evolution and continued development of glacial flow and erosion models has resulted
- in the simulation of increasingly complex processes such as the influence of subglacial hydrology (Egholm et al., 2011; Herman et al., 2011; Iverson, 2012). Despite these advances, other mechanisms are still represented by simplified assumptions and approximations, particularly the underlying physics of ice flow. Within the field of glaciology, as computing power has increased, higher-order glacial-flow models (Pattyn et al., 2008)
- ²⁰ have been made more accessible. These higher-order processes are often simulated over the time scales useful for glacial and climatic studies (10³ to 10⁴ years), yet still shorter than the timescale of orogen topographic development and Quaternary glaciations (10⁵ to 10⁷ years). The incorporation of these higher-order models into orogenic timescale models can be useful to better represent the glacial flow in Alpine settings,
- ²⁵ where the effects of longitudinal and lateral stresses on glacial flow and erosion should be important (Egholm et al., 2012a, b).

In many orogenic-scale models, glacial flow and erosion have been represented using simplifying assumptions, such as the shallow ice approximation (SIA) for glacial flow (Kessler et al., 2008; Iverson, 2012). This approximation simplifies the ice flow equation





(Glen flow law) by only considering the first-order simple shear stresses (Cuffey and Paterson, 2010). While this approximation is appropriate for some specific glacial settings where surface slopes are shallow and ice thickness is small compared to ice extent, for alpine glaciers this assumption misses effects that result from glacial flow through

- ⁵ narrow and steep topography (Egholm et al., 2011). While even the simplest approach has its merits, defining the conditions under which a higher-order model should be used warrants more detailed consideration. Recent work has shown that over the length and time scales of glacial valley formation, higher-order (HO) glacial-flow models have important feedbacks (Egholm et al., 2009, 2012b; Pedersen and Egholm, 2013). Specifi-
- ¹⁰ cally, Egholm et al. (2012a, b) showed that on the glacier valley scale and over a single glacial cycle, the incorporation of lateral and longitudinal stresses can provide an important mechanism for suppressing potential run-away problems that can come from using the SIA. In addition, larger, regional models have been investigated and stress that the physics and form of the glacier are certainly important, and that models of ¹⁵ alpine glaciers and their erosional patterns are influenced by the choice of physics,

particularly as the landforms and valley profiles evolve (Pedersen and Egholm, 2013). This study complements previous work by evaluating the effect of the glacial-flow physics model on predicted variations in glacial erosion over both single and multiple glacial cycles. Our end goal is to add to the understanding of when and under what

- ²⁰ conditions more simplified models, such as the shallow ice approximation are sufficient, as applicable to larger scale problems such as sediment production, ice-sheet stability, and tectonic-erosional interactions under alpine glaciers and continental ice sheets. We simulate 400 000 years of glaciation, including three full glaciations, with a range of different climate scenarios. Simulations using a SIA glacial-flow model and
- simulations using a nested, HO glacial-flow model are compared. We evaluate the strengths and weaknesses of SIA and Stokes flow within a 3-D landscape evolution model and address how a higher-order glacial model might affect topography and its evolution over geological time scales. Whether the higher-order model might have more or less influence than that of climate is also investigated. While there are still many





simplifications used in this comparison, from the choice of erosion and sliding laws to the use of a nested domain, this comparison yields some quantitative evidence of how HO glacial-flow physics can influence the evolution of topography.

2 Methods

- ⁵ Here we build upon previous work and investigate the influence of glacial-flow physics on a developing orogen over geologic time scales using a modified version of the ICE-Cascade landscape evolution model. In order to compare the importance of the choice of ice physics, simulations are run using both the SIA model and the nested HO model. We repeat these comparisons over different climate scenarios in order to highlight temperature-dependent effects. The different climate simulations are used because glacial erosion is dependent upon the existence of liquid water at the base of the glacier, a temperature dependent property. For brevity, a summary is given of ICE-Cascade, its major components, the physics governing the ice flow, and the modeling framework behind the HO glacial-flow model. All relevant model parameters used for ICE-Cascade are presented in Table 1; readers are referred to the associated refer-
- ences for additional details.

2.1 Simulations

Four separate simulations based on climatic and ice temperature conditions are performed using both the SIA and the HO physic models described in Sect. 2.3. These simulations are summarized in Table 2, and the sea-level temperature patterns over time are shown in Fig. 3.

- Simulation 1 uses a sinusoidal temperature pattern with amplitude of 6°C and sea-level minimum temperature of 2°C, with a wavelength of 100 kyr.
- Simulation 2 has a similar pattern but with the sea-level minimum shifted to 0°C, so there are more instances of cold ice where the base is frozen.





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- Simulation 3 has a minimum sea-level temperature also of 0 °C, but the temperature pattern is based off of the most recent 500 kyr of δ^{18} O record where the 100 kyr Milankovitch cycles dominate.
- Simulation 4 uses the same temperature pattern as Simulation 1; however, in this simulation sliding occurs everywhere the ice thickness is greater than 10 m, and the ice temperature at the base of the glacier is not factored into this calculation.

Each simulation was run for over 400 kyr. Over three full glaciations are captured during this time interval. Figure 1 shows Simulation 2 (SIA) with hillshade topography and the ice coverage and thickness at T = 100 kyr in the simulation.

10 2.2 ICE-Cascade orogen development model and climate parameters

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ICE-Cascade allows us to model topographic evolution over geologic time scales, with the influences of both constructive (tectonics and sediment deposition) and destructive (erosion) processes (Herman and Braun, 2008; Yanites and Ehlers, 2012). At each time step, the topography is uplifted according to an input rate (Table 1) along with a com ¹⁵ ponent based on flexural isostasy. The isostatic response of the landscape is affected by loading and unloading due to erosion, sediment deposition, and the ice thickness. Following the uplift, the landscape is eroded according to the rates from the fluvial, hill-slope, and glacial modules. Where glacier ice is nonexistent or thinner than 10 m, fluvial and hillslope processes erode and re-distribute sediment (Yanites and Ehlers, 2012).

- Sediment transport by the glaciers occurs in regions of ice thicker than 10 m; bed material is eroded and immediately transferred to the fluvial system that emanates from the toe of the glacier. River discharge is calculated based upon the upstream precipitation and water equivalent ice melt from upstream regions. Fluvial erosion processes are calculated from this discharge and the sediment supply, local topographic slope,
- ²⁵ and the channel width, all of which also are input into a linear sediment cover model (Braun and Sambridge, 1997). Hillslope processes are simulated from diffusion and





a threshold landsliding algorithm when hillslopes steepen beyond a certain threshold (Burbank et al., 1996; Stolar et al., 2007).

Within ICE-Cascade, the climate simulation is a combination of the inputs that govern the pattern of sea-level temperature and an orographic precipitation model (Yanites

- ⁵ and Ehlers, 2012; Roe et al., 2003). The temperature and moisture content variations over all elevations is calculated using an input lapse rate and the Clausius–Clapeyron relation, and these values, along with inputs governing the wind speed and direction, are then used to calculate annual precipitation (Roe et al., 2003). When the temperature is below freezing, the precipitation takes the form of snow. A positive-degree day
- ¹⁰ algorithm is used to determine the number of days above freezing in any given year (Braithwaite, 1995), and this, in turn, is used in calculate the amount of melt of snow and ice. For any point in the landscape, the annual mass balance is then simply the difference between the amount of snowfall and the amount of melt. Climate parameters governing these processes are given in Table 1.
- ¹⁵ For these simulations, the initial topography is an equilibrium landscape generated using only fluvial and hillslope processes. This topography was built from earlier simulations (simulation m01) (Yanites and Ehlers, 2012) that started with random topography and was allowed to develop over 16 Myr with the same hillslope, fluvial, uplift and climate parameters as found in Table 1.

20 2.3 Glacial models

At the beginning of the simulations, glaciations evolve where a persistent positive mass balance exists. Glaciers flow from the orogen and its valleys onto the continental slope, where they form piedmont lobes. The shelf can be seen in Fig. 1, extending to the edges of the Y domain from 0 to 70 km and from 225 to 245 km. The shelves, with a slope of 0.001, were added to ensure that the ice velocities at the creater

with a slope of 0.001, were added to ensure that the ice velocities at the orogenparallel boundaries are numerically stable (Yanites and Ehlers, 2012). The shelf edges (Y = 0 and 245 km) have Dirichlet boundary conditions, where their elevation is fixed to





sea-level and does not change throughout the simulations. During these simulations, the ice never reached these boundaries.

The depth-averaged and sliding velocities are calculated from the glacial-flow physics (see Sects. 2.3.1 and 2.3.2 for more details). The sliding velocity is only calculated where the temperature at the base of the glacier is at the pressure-melting point associated with the local ice thickness (except in Simulation 4 where there is sliding everywhere). The basal temperature is determined from a conductive heat model for ice, where the upper boundary is the surface temperature, and the lower boundary is the geothermal heat flux (Table 1). Beyond glacial flow, other ice feedbacks are incorporated in the model. On steep slopes avalanching occurs when the snow surface

- ¹⁰ corporated in the model. On steep slopes avalanching occurs when the snow surface slope exceeds a critical stability angle (Table 1), in which case all the snow at the point is redistributed to the next downstream node. Where the glacier terminates below sealevel on the shelf, iceberg calving occurs and is a function of the depth of ice below sea-level. In addition to calving, a buoyancy feedback is also incorporated for when
- the glacier erodes an overdeepening below sea-level (Yanites and Ehlers, 2012). In an overdeepened, below sea-level reach, sea-level is assumed to be the equipotential ground water surface, such that the glacial erosion rate decreases as the ice approaches buoyancy. While this is a simplification of the complex feedbacks among the glacial flow, subglacial and englacial hydrology, and the subglacial sediment (Hooke, 1991; Alley et al., 2003), glaciated valleys are produced that fit the expectations that
- ²⁰ 1991; Alley et al., 2003), glaciated valleys are produced that fit the expectations that overdeepened glaciers tend to reach critical angles.

Glacial erosion is performed by two major mechanisms, abrasion and quarrying (Hallet, 1979, 1996; Iverson, 1991). These processes operate on spatial and temporal scales that are orders of magnitude shorter than those of the glacial valley, the
²⁵ climate cycle, and the orogen. The erosional processes are often simply upscaled in landscape evolution models (Tomkin, 2003; Herman and Braun, 2008). Following the methods of many existing studies of glacial erosion on orogenic time scales, we use a simple relationship between erosion and sliding. For both models, glacial erosion,

 $\frac{\partial z_{\rm b}}{\partial t}$, is proportional to the sliding velocity, which comes from modeling and empirical





studies of glacial erosion (Harbor et al., 1988; Humphrey and Raymond, 1994), such that

$$\frac{\partial z_{\rm b}}{\partial t} = K |u_{\rm sl}|^{\prime},$$

where *K* is a constant that characterizes the erodibility of the subglacial material (Laitakari et al., 1985; MacGregor et al., 2000; Duhnforth et al., 2010) and / is another constant generally equal to 1 (Table 1). While a few recent studies have used more sophisticated rules for erosion (MacGregor et al., 2009; Iverson, 2012), this is the same rule as used in previous ICE-Cascade and other glacial-influenced, landscape
evolution models (e.g., Braun et al., 1999; Herman and Braun, 2008; Kessler et al., 2008; Egholm et al., 2012b; Yanites and Ehlers, 2012)

2.3.1 Shallow ice approximation

The Shallow Ice Approximation (SIA) simplifies full-stress glacial flow by assuming that the ice is significantly wider than it is thick, and the surface slopes are not large, i.e.,

- it is shallow (Fowler and Larson, 1978; Hutter, 1983). In this assumption, all stresses but the simple shear stresses in the direction of ice flow are assumed to be negligible; longitudinal and lateral stresses, including drag, are assumed insignificant. This considerably simplifies how ice flow can be modeled, which is particularly useful when used on orogenic length and time scales. However, the assumptions based on glacial modeled, which is particularly useful when used on orogenic length and time scales. However, the assumptions based on glacial
- 20 geometry and surface slope, while originally derived for use on large ice sheets, are not necessarily true when used for alpine glaciers where narrow valleys channelize flow and large surface and basal slopes are present (Hutter, 1983). Ice thickness, *H*, is computed from mass-balance equation

 $\frac{\partial H}{\partial t} = \nabla \cdot F + M_{\rm c}$

where M is the mass balance and F is the vertically integrated ice flux.



(1)

(2)

The ice velocity, u, is simplified to just two dimensions where vertical velocities are deemed negligible, so that u = ui + vj. Each directional component is the sum of two components of glacial motion, which are designated using the SIA in this definition: deformation u_d^{sia} and sliding u_{sl}^{sia} for the velocity in the X direction (v is defined in a parallel fashion for the velocity in the Y direction).

$$u = u_{d}^{sia} + u_{sl}^{sia}$$

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The deformation component is defined

$$u_{\rm d}^{\rm sia} = -\frac{2A\beta}{n+2} (\rho g)^n H^{n+1} |\nabla (H+z_{\rm b})|^{n-1} \nabla (H+z_{\rm b}), \tag{4}$$

where *A*, which is pressure and temperature dependent but treated as a constant for most of this analysis, and *n* come from the Glen Flow equation (Cuffey and Paterson, 2010), which relates the strain rate, $\dot{\varepsilon}$, to the stress, σ , by:

$$i_{15} \quad \dot{\varepsilon} = A \sigma^n, \tag{5}$$

 β is a constriction factor that is dependent upon the gradient of the subglacial topography. In a standard SIA model, β is unity. For this version of ICE-Cascade, the SIA model is modified to incorporate this factor to partially account for the stresses in steep Alpine glaciated valleys where the SIA is too simple (Herman and Braun, 2008):

$${}_{20} \quad \beta = \left(1 + k_c \frac{\partial^2 z_b}{\partial x_f^2}\right)^{-1}$$

The constant, k_c , has been evaluated over a 1 km wide glacial valley by Egholm et al. (2011). A value of 1000 was found to have reasonable agreement with a higher-order approximation, with comparisons of sliding and deformation velocities. Using the

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(6)

(3)



constriction factor, over-predictions of erosion rate from the standard SIA were diminished to under-predictions in comparison to the erosion rates from the higher-order approximation (Egholm et al., 2011, 2012b).

The sliding velocity u_{sl}^{sia} is defined in a similar form to the deformation,

$${}_{5} \quad u_{\rm sl}^{\rm sia} = -\frac{2A_{\rm sl}\beta}{N-P}(\rho g H)^{n} |\nabla (H+z_{\rm b})|^{n-1} \nabla (H+z_{\rm b}), \tag{7}$$

incorporating the sliding flow factor A_{sl}, the constriction factor discussed above (Table 1), and simple subglacial hydrology as the effective pressure, the difference between the ice overburden pressure and the water pressure, N – P. In order to focus
on just the physics of the ice flow, we do not consider variable subglacial hydrology in this model and treat N – P as 80 % of the ice overburden pressure. However, water pressure and its change over time are influences on the sliding velocity and has been used in a variety of other glacial erosion and landscape evolution models (e.g., MacGregor et al., 2000; Tomkin, 2003; Herman and Braun, 2008; Kessler et al., 2008;

2.3.2 Higher-order physics

While a variety of various higher-order simplifications have been used to represent flow in other models (Pattyn, 1996; Egholm et al., 2011), we opt for a full stress solution nested into the larger ICE-Cascade framework. Nesting of this modeling within the

- ²⁰ SIA model is required due to computational considerations, and our analysis is mostly focused on differences between the two model setups over a limited region. Figure 1 shows an example of topography during a glacial maximum with the nested region highlighted (from 100–150 km in the *X* direction and 100–160 km in the *Y* direction). In this region, we use the full stress equations and treat ice as a non-linear viscous flow.
- The sliding velocity is calculated for this region and then passed back to ICE-Cascade. To ensure that the pattern of sliding velocity is smooth, a linear interpolation is used between the sliding velocity at the edges of the nested domain and those in the nested





domain. This small region surrounding the nested domain has a width of less than 1 km.

Conservation of mass is given by

 $\nabla \cdot \boldsymbol{u} = 0,$

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where \boldsymbol{u} is the 3-D velocity vector $\boldsymbol{u} = u\hat{i} + v\hat{j} + w\hat{k}$, and is also subdivided into deformational and sliding portions: $u = u_d^{ho} + u_{sl}^{ho}$.

Ice flows as an incompressible laminar material. The Glen flow equation, when written in viscosity form for a single stress component is

10
$$\sigma_{ij}' = A\eta \dot{\varepsilon}_{ij}, \qquad (9)$$

and the viscosity η is given as

$$\eta = \frac{1}{2} A^{-1/n} \dot{\varepsilon}^{(1-n)/n},\tag{10}$$

where \dot{c} represents the effective strain rate, the second-invariant of the strain rate. *A* is a flow constant in many cases but can also be dependent upon the temperature of the ice.

The full strain rate tensor is defined as:

$$\begin{pmatrix} \dot{\varepsilon}_{xx} & \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{xz} \\ \dot{\varepsilon}_{yx} & \dot{\varepsilon}_{yy} & \dot{\varepsilon}_{xy} \\ \dot{\varepsilon}_{zx} & \dot{\varepsilon}_{zy} & \dot{\varepsilon}_{zz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{\partial v}{\partial y} & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) & \frac{\partial w}{\partial z} \end{pmatrix},$$
(11)

20

and the second-invariant that is used to define the effective strain rate in the viscosity term, Eq. (10), is

$$\dot{\varepsilon}^2 = \sum_{ij} \frac{1}{2} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}.$$
(12)

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(8)

The shear stress (τ_{ij}) can be found from Eq. (9). Combining Eqs. (9), (10), and (12), the full shear stress acting in the X direction, τ_{xz} , is then defined

$$\tau_{xz} = \frac{1}{2} A^{-1/n} \dot{\varepsilon}^{(1-n)/n} \dot{\varepsilon}_{xz},$$
(13)

s and this is the term (along with the shear stress in the Y direction τ_{vz}), when defined at the bed of the glacier $\tau_{\rm h}$, upon which the basal sliding is dependent. The sliding velocity is defined in both the X and Y directions, similar to Eq. (7). For the X direction, the sliding velocity is

$$u_{\rm sl}^{\rm ho} = \frac{A_{\rm sl}\tau_{\rm XZ}^{\prime\prime\prime}}{N-P},\tag{14}$$

where A_{sl} is the same constant as in Eq. (7), and *m* is 3 in this case (Table 1). Is in the SIA model, the effective pressure (N - P) is 80 % the ice overburden pressure.

2.4 COMSOL multiphysics

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COMSOL Multiphysics has been used for modeling ice flow in 2-D flowband and flowline profiles (Johnson and Staiger, 2007; Campbell, 2009; Headley et al., 2012). Here, 15 we modify the steady-state viscous flow module using the Glen Flow law. The flow is represented in equation form by the 3-D incompressible laminar flow, and we set the viscosity to be dependent upon the strain rates, Eq. (9). The geometry is defined from the bed topography and the ice thickness within the nested domain. The boundary con-

- ditions on the edges of the domain are open boundaries. These boundaries are only 20 used within COMSOL simulations, as the velocity distribution over depth is not output back into ICE-Cascade. The top surface is a free surface, and the bottom boundary is no slip, with the sliding velocity later calculated from the basal shear stress per Eq. (14). Within COMSOL, proper meshing is important for solution convergence. In this case,
- **Discussion** Paper we set the mesh size along the top and bottom ice surface as COMSOL setting Coarse.



Discussion



From the non-linear ice flow law, Eq. (5), velocity gradients are larger closer to the bed. For more efficient computations, we use nine boundary layers perpendicular to the bed and surface, with decreasing vertical dimension approaching the bed, using order 10⁴ tetrahedral elements. When the velocities are input back into the ICE-Cascade model,

the velocity profile of both the HO region and the surrounding SIA region is linearly interpolated over a small region (< 1 km wide) to ensure that the final results are smooth. Figure 2 shows an example of the mesh used in COMSOL for the nested model.

3 Results

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The ICE-Cascade model outputs used in this study include the glacial erosion rate,
 subglacial topography, glacier thickness, and sediment thickness; these primary outputs are analyzed for the rest of the study. Figure 1 shows example output of the topography and ice coverage at a glacial maximum. The size of the nested model region (black box in Fig. 1) was set due to computational limitations in the Stokes flow simulations. The two profiles highlighted in Fig. 1 are used for comparison of the mod els: one profile is orogen parallel (A-A'), and the other follows a valley profile (B-B').

Four separate simulations are performed and compared. The four climate and thermal settings (Simulations 1–4) are further discussed in Sect. 2.2, summarized in Table 2, and shown in Fig. 3. For these four simulations, each was performed using both the ICE-Cascade with SIA glacial-flow physics and with the HO nested subdomain.

20 3.1 Influence of glacial-flow physics on glacial area and volume

Large-scale properties of glaciations can be used to compare the different climate scenarios and the different glacial-flow models. In this case, we use the glacier-covered area and the maximum ice thickness for each time step. As expected, colder climates lead to significantly larger glacial-covered areas and persistent thick glaciers (Fig. 4), regardless of glacial-flow model. For Simulation 3, the maximum temperature is warmer



than the other simulations (Fig. 3), and the temperature leads to large glaciers that have significant short-term variation in their width. The glaciations are progressively smaller for each subsequent glaciation even though the temperature amplitude is not changing.

- The overall pattern of ice cap growth is similar between the SIA and HO models, and the differences between the higher-order glacial-flow physics and the shallow-ice approximation models are generally difficult to see when comparing the full magnitudes of ice-covered area and maximum ice thickness (Fig. 4). However, comparing the percent difference between the two $(\frac{A_{\text{SIA}}-A_{\text{HO}}}{\overline{A_{\text{SIA}}}} \times 100 \text{ and } \frac{H_{\text{SIA}}^*-H_{\text{HO}}^*}{\overline{H_{\text{SIA}}^*}} \times 100$, where *A* is the
- ice-covered area and H^* is the maximum thickness, and the denominators are those values averaged over the glaciation center around the 100 or 150 kyr for Simulation 3) shows significant variation (Fig. 4), reaching over a 30 % difference for some of these simulations. The percent differences between the two glacial-flow models show that even a higher-order nested model that does not cover the full ice-covered domain can have an effect on the full-glaciated area.
- At the start of the simulations, the percent differences in the area are small for all the simulations (Fig. 4), and the maximum ice thickness follows a similar pattern (Fig. 5). Ice thickness tends to grow rapidly, reaches a maximum when the maximum area and minimum temperature are reached (around every 100 kyr for the sinusoidal temperature patterns), then tapers slightly before decreasing rapidly. As time progresses,
 changes are exacerbated, and the difference between SIA and HO is more readily apparent. In general, the simulations that contain warmer temperatures and/or have
- more sliding (i.e., Simulations 3 and 4) have a much larger differences in ice thickness and glaciated area due to the HO glacial-flow physics, while Simulation 2, the coldest, has minimal differences.
- ²⁵ A null simulation was also performed where the SIA and HO models were used but glacial erosion was turned off. In this case, for both the maximum ice thickness and the glaciated area, differences between the two models were on the order of 5 % or smaller during every glaciation. The differences did not increase as time progressed.





This result is presented to emphasize that the differences between the two models for the four simulations come mainly from the models' influence on the erosion rate and are not inherent from the model nesting or model choice.

3.2 Simulation 1: the influence of glacial-flow physics on glacial erosion pattern

- ⁵ Results and comparisons of the pattern of erosion rate from Simulation 1 are shown in detail in Figs. 5 and 6. Because Simulation 1 uses climate parameters (Fig. 3 and Table 2) that are generally in the middle of the range covered by all the simulations, the results from this show the effects of the different physics models without extreme behavior, such as sliding everywhere in Simulation 4 or the extensive frozen bed in Simulation 2. However, since the erosion rate is dependent upon the basal temperature
- such that in very cold excursions and at high altitudes, the glacier can be frozen to the bed and those regions are protected from erosion. We look in detail at the pattern of erosion over the area of the nested model (Fig. 1) and compare these patterns averaged over a single glaciation and over the full simulation (400 kyr).
- Figure 5 shows the erosion rate averaged over the first full glaciation centered around 100 kyr (grey-highlighted region in Figs. 3 and 4) for the nested subdomain, and the corresponding averaged erosion rates over the full simulation are shown in Fig. 6. The patterns of erosion are similar over the two time scales for each of the SIA (Figs. 5a and 6a) and HO (Figs. 5b and 6b) simulations. Within a given model, the erosion rates are
- ²⁰ more than half as low on the long term, due to the averaging including the interglaciers when no glaciers are present. On the long term (Fig. 6), the area actively eroded by the glacier is more extensive than over the single glaciation (Fig. 5), as the regions under frozen ice over a single glaciation are not necessarily always under frozen ice over the entire 400 kyr.
- Figures 5c and 6c show the differences between the SIA modeled erosion rates and those from the HO model. When comparing the SIA model with the HO model, we focus on the full 400 kyr average (Fig. 6c), as results for the single glaciation average (Fig. 5) follow a similar pattern. In Fig. 6b, the erosion rate in the HO model peaks at





over four times that of the SIA, yet the bed is frozen over more of the HO model domain. Differences are noticeably larger in regions where the basal temperature significantly varies between the two simulations, i.e., where one model or the other has no sliding (thus no erosion) occurring. While for both glacial physics models, there is generally a region of highest erosion rate in the SE corner of the model, the SIA modeled pattern has only one broad region of high erosion rates around X = 150 km and Y = 112. The HO modeled erosion rate pattern shows three specific smaller areas of higher erosion rate in the region between X = 140-150 km and Y = 110-130 km.

3.3 Influence of glacial-flow physics on glacial erosion rate in different climates

In order to compare the erosion rates over both the 400 kyr and the single glaciation time scales for all climate simulations, we average the erosion rates over the profiles within the nested domain (Fig. 1). Figure 7 shows the erosion rates for the orogen-parallel (A-A') swath, and Fig. 8 along the valley profile (B-B'). Similar to the 2-D contour plots for Simulation 1, the SIA modeled erosion rates (Figs. 7a and 8a) are generally larger and more variable than those from the HO model (Figs. 7b and 8b). The differences between the two models (Figs. 7c and 8c) can vary up to almost an order of magnitude greater than the SIA erosion rates, and the differences between the two glacial flow models are higher for the warmer and wetter simulations.

When comparing the differences, those related to purely the climate are also striking.

- ²⁰ The patterns of erosion within a given glacial flow model are generally similar in Simulations 1–3, only the magnitudes differ. The differences are largest in the valleys, where ice is thickest and moving fastest, around X = 105 km, 120-127 km, and 150 km (Figs. 7c and 10a and b). For the valley profile B-B', there is little erosion at high elevations due to the frozen bed (Figs. 8 and 10). Simulation 4, however, particularly accentuates
- ²⁵ how having a temperature dependent sliding law can play a large role in how the glacial erosion is partitioned over the landscape. Where sliding occurs even under thin, cold ice, Simulation 4 shows significant variation in the pattern of these differences, with the largest erosion rate differences occurring around X = 100 km and 132 km (Fig. 7c).





3.4 Influence of glacial-flow physics on subglacial topography and sedimentation

Subglacial topography (Figs. 9a and b and 10a and b) is composed of not only bedrock but also sediment deposited by proglacial fluvial processes. The effects of the choice

⁵ of glacial-flow physics can be seen in comparing the topography (bedrock elevation and sediment thickness combined), the bedrock topography, and the sediment layer thickness. These are all related, but there are many differences among the different physical models and the climate.

The topography shown in Figs. 9a and b and 10a and b is influenced by glacial, fluvial, and hillslope erosion along with sediment that is accumulated when the fluvial system lacks the carrying capacity to transport it. Hillslopes and steep areas are regions of net erosion, particularly seen in the orogen parallel swath (Fig. 9, around X = 117 km and X = 130-140 km), even when glacial erosion rates might be small or non-existent (Figs. 7 and 8) due to a frozen bed. When comparing the bedrock eleva-

¹⁵ tion in the SIA to the HO models (Figs. 9d and 10d), the bedrock differences (SIA-HO) have a similar pattern to those of the total topography, though the magnitude is slightly subdued. For the orogen parallel swath (Fig. 9, around X = 120-125 km), significant valley fill only occurs down glacier of the swath, particularly for Simulation 4, where material is eroded everywhere on the glacier, including the regions frozen to the bed in the other simulations.

The differences between the SIA and HO models for total topography, bedrock elevation, and sediment layer thickness (Figs. 9c–e and 10c–e) show similar patterns as the erosion rate (Figs. 7 and 8). In Figs. 9 and 10, Simulations 1–3 generally show very similar patterns in both the elevation and in the differences between the physical

²⁵ models. Warmer and wetter runs are associated with larger differences between the physical models. Simulation 4 shows the most extreme changes to the topography as well as the most extreme sediment accumulation (Figs. 9a and b and 10a and b) and differences between the physical models (Figs. 9c-e and 10c-e), whereas Simulation





2 shows the least change except for a large amount of deposition in the SIA model around X = 110 km (Fig. 9e).

4 Discussion

- If a simplified model can produce results similar to a more complex model, then the simpler model with fewer free parameters is preferred. Of course, this depends upon how this similarity is defined and what the area of interest is, what features or processes are being modeled, and what time and length scales are of interest. When looking at a full orogen, it seems that the modified-SIA can reproduce features similar to those found in actual orogens. However, in mountainous topography, particularly at sub-polar
 latitudes, glacier dynamics are influenced by the physical constraints of valleys and fjords and also are a strong control on the erosion and sliding rates. In this section, we discuss how the evaluation of many of these properties using the comparisons presented in the previous section and Figs. 4–10.
- The different glacial-flow models have an effect on the glacier and the topographic
 evolution of the orogen, although the magnitude of this effect is variable. These effects are dependent upon the climate. When the glacier is mostly frozen (Simulation 2), the physics chosen makes only a small influence on the glacial extent and thickness (Fig. 4) and on the topography (Figs. 9 and 10). However, if the glacier has large warm periods (Simulation 3) or is forced to be wet-based (Simulation 4), even if cold temperatures are reached over large periods, then the model choice is considerably more important. Comparisons between Simulations 1 and 4, which have the same climate
- parameters (Fig. 3) emphasize how important the choice of sliding law and reliance on basal temperature are, regardless of glacial-flow model. For example, Figs. 7 and 8 show maximum erosion rates in Simulation 4 of more than double those of Simulation
- Simulations 1 and 3 generally have more than double the erosion rates than those of Simulation 2 (Figs. 7 and 8), which stresses how important the temperature can be even without extreme erosion laws like in Simulation 4.





4.1 Glacial properties and uncertainty in differing climates and over different time scales

Regions like the Gamburtsev Subglacial Mountains underneath the East Antarctic Ice Sheet (Young et al., 2011) illustrate that no matter how large and thick ice coverage
⁵ might be, as in Simulation 2 (Fig. 4), if the basal temperature is regularly below freezing, there will be little modification of subglacial topography because there is no sliding. It follows that if the ice is generally below freezing, the glacial-flow model does not matter tremendously if interest is on the evolution of the landscape. However, if interest is in ice extent or coverage, then the glacial-flow model can be important even for
¹⁰ completely cold-based ice when time scales are long enough (Fig. 4, particularly Simulation 2). While it might be expected that on the long-term scale, differences between the two glacial-flow physics could be averaged out due to the other erosional processes reshaping the landscape during interglaciers, that does not appear to be the case.

Comparing the simulations among the different climate scenarios and not just between the two physical models allows us to consider the influence of glacial-flow physics vs. climate. The climate simulations (Fig. 3) are considerably different, and their effects on the erosional pattern and topographic evolution are substantial: the icecovered area varies by over a factor of 3 (Fig. 4a), and the erosion rates can vary by over a factor of 2 (Figs. 7a and b and 8a and b). Particularly for the orogen-wide proper-

- ties (ice-covered area and maximum thickness), the variations from the climate are substantially larger than those from the choice of glacial-flow physics model (Fig. 4). However, when the erosion rates and topographic evolution are compared over the swath profiles (Figs. 7–11), the differences from the choice of glacial-flow physics model are generally of the same magnitude or larger than those from different climates. These
- results emphasize that if interest is only on larger, orogen-wide properties, the choice of glacial-flow physics model is less important than the climate. For properties like bed topography and erosion rate on the valley scale, the choice of glacial-flow physics can make a more significant difference than even very different climate models.





4.2 Evolution of subglacial topography and sediment thickness

The subglacial topography and deposited sediment are important metrics to consider; as in real landscapes, these are the relics from previous glaciations, formed through erosion and other geomorphological processes. We evaluate not just how the glacial-

⁵ flow physics model influences the erosion of topography but also how this topography and the eroded sediment work within the other geomorphological systems. Fluvial erosion is an important mechanism, and the fluvial network is also responsible for the transport and redistribution of glacially eroded sediment once it has left the toe of the glacier (Hallet et al., 1996; Alley et al., 1997). In some cases, the existing rivers do not have the carrying capacity to support the evacuation of all the available eroded material. This material impacts the landscape not only by protecting the bedrock from further erosion but also by impacting the glacial flow, subglacial hydrology, and erosion patterns (Humphrey and Raymond, 1994; Hallet et al., 1996).

The sediment thicknesses produced vary slightly between the SIA and the HO models. Along the swath profiles, the pattern of deposition varies between the physical models. Generally, the HO model leads to no change or smaller sediment thicknesses by 5–20 m (Figs. 9c and 10c). However, Simulation 4 differs considerably, with HO model producing significantly more deposition (over 25 m) around X = 147 km in the orogen parallel swath and significantly less (almost 20 m) deposition around X = 120-125 km

²⁰ in the same swath. Along the valley profile B-B' (Fig. 10) there is also considerably less deposition for Simulation 4 in the HO model lower in the profile (Fig. 10c), despite significantly more erosion occurring upstream. These variations in amount of sedimentary fill have implications for the structure of future drainage areas, glacial flow, and isostasy.

25 4.3 Model caveats and limitations

The SIA and HO models used in this study have several caveats and limitations worth mention. First, this study provides only minimal estimates for the effects of the



higher-order flow model on the topographic and glacial evolution, particularly over the full domain. Since the higher-order model is nested within the standard model, not all regions of the glacier and the bed feel the effects of the flow, though the nested region covers the ice divide and spans multiple valleys. However, we suspect that incorporat-

ing all of the glacial ice would lead to larger differences in how topography develops. The full orogen comparisons (Fig. 4) might then be viewed as conservative estimates as to how much the HO model can influence the glacier extent and thickness.

Second, as mentioned briefly in the model setup section, the sliding rule used is a simple one that does not incorporate subglacial fluvial water pressures or pressure

- ¹⁰ changes. For both sliding and erosion, subglacial water has increasingly been shown to be important (Clarke, 2005; Cohen et al., 2006; Bartholomaus et al., 2008). While much current research, modeling or field-based, is focused on understanding the dynamics of the subglacial fluvial system on the short-term scale, how this system can be meaningfully scaled up to glaciological or geological time scales is still an open
- question. Related to the effects of subglacial water and the sliding velocity, the choice of erosion law, Eq. (1), is another simplification that could have an influence on the results of this study. Various other erosion laws tie the erosion rate to other powers of the sliding velocity (Hallet, 1979; Iverson, 1995; MacGregor et al., 2009), the ice flux (Kessler et al., 2008), or the basal shear stress (Pollard and Deconto, 2007), and
- ²⁰ a different choice would influence the patterns of erosion and the locations of maximum erosion rate. Over time, this could have feedbacks with the topographic and glacial evolution that are not necessarily straight-forward. Finally, another erosional-feedback that would influence the evolution of the glacier and the topography would come out of the choice of erodibility, *K* in Eq. (1). In this model, all bedrock material is treated with the
- same erodibility, with no accounting for fracture mechanics or different rock types. In addition, sediment that is eroded then deposited is reincorporated into the model with the same erodibility as the original bedrock. A variety of research related to modeling and field data of different erosional systems has shown that the rock erodibility, whether





this varies due to fracturing, rheology, or composition, can have a significant effect on the ultimate form of the landscape (Dühnforth et al., 2010; Ward et al., 2012).

On geologic time scales, glaciations can significantly change not just the extent of a future glaciation but also how future glaciers grow and retreat, which further influences the future patterns of erosion (Pederson and Egholm, 2013). Similarly, in this study, the differences between the physical models become more evident the longer the topography develops, as multiple glaciations erode the landscape and the fluvial and hillslope processes do not completely overprint these glacial signals. As models move to higher-order physical representations, the question of how well a given model represents reality need be further addressed. While this current study focuses on model comparisons, examples of regional hypsometry, patterns of glacial erosion and ice thickness, and patterns of deposition might be used to better determine if more

sophisticated models do better reproduce more realistic topography.

5 Conclusions

- ¹⁵ This research shows a comparison of a nested HO glacial-flow model with a SIA glacial-flow model within the ICE-Cascade landscape evolution model. The simulations incorporate constant tectonic uplift rate, orographic precipitation, hillslope erosion, fluvial erosion, and sedimentation in addition to the glacial erosion processes. Using multiple climate simulations, the effect of glacial-flow physics is evaluated over 400 kyr
- of topographic evolution. In general, the glacial-flow model choice makes a difference in the development of a glaciated landscape over the long-term, which corresponds with what other studies have shown in simulations without fluctuating temperatures (Egholm et al., 2012b). We have evaluated a variety of properties of landscape evolution in a glaciated orogen, from the ice-covered area and ice thickness to the bed topography and sediment thickness.

Two major conclusions can be reached. First, the climate, and whether the glacier is mostly cold-based or mostly sliding, has a large influence on glacial development;





a few degrees difference in the minimum sea-level can more than triple the glaciated area over an orogen. Changing the climate parameters can lead to large variations in average erosion rates and topography after multiple glaciations have occurred, generally by factors of 3 to 4 between warm and wet glaciers and cold-based glaciers (Figs.

- 5 7–11). Second, though these climate influences are large, comparing a HO glacial-flow model to a SIA model show that the choice of model can have as large, if not larger, an influence on the developing orogeny than climate alone for glaciers with more warm beds and less influence for colder glaciers. As topography develops, even with other processes (fluvial, hillslope) dominating over large time periods, the deviation between
- SIA and HO grows over time to make over a 30 % difference for completely wet-based glaciers (Fig. 4). The subglacial topography is similarly affected by the incorporation of the HO model, as long term erosion rates can vary by over an order of magnitude between the HO and the SIA models (Figs. 5–9), and the subglacial topography can vary by over 100 m between the physics models over 400 kyr (Figs. 9–11). Differences
- between the models for the colder climate are significantly subdued. In general, over 15 orogenic time scales and relatively warm climates, the choice of glacial-flow model can make substantial differences in the developing topography, sediment deposition, and averaged erosion rates.

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Table 1. Landscape evolution and orographic precipitation model parameters.

Description and Parameter	Value [Units]	Reference	
Glacial Parameters			
Flow rate factor for temperate ice A Glen's Flow Law exponent n Sliding constant A_{sl} Sliding exponent m Erosion rate constant K Erosion rate exponent / Constriction constant Density of ice ρ_i Ice thermal conductivity Snow stability angle for avalanching	$\begin{array}{c} 6.8 \times 10^{24} [\text{Pa}^{-n} \text{yr}^{-1}] \\ 3 \\ 1 \times 10^{-15} [\text{Pa}^{-m} \text{s}^{-1}] \\ 3 \\ 0.001 [(\text{yr} \text{m}^{-1})^{\prime -1}] \\ 1 \\ 1000 \\ 910 [\text{kg} \text{m}^{-3}] \\ 2.4 [\text{W} \text{m}^{-1} \text{K}^{-1}] \\ 35 [^\circ] \end{array}$	Cuffey and Paterson (2010) Cuffey and Paterson (2010) Braun et al. (1999) Braun et al. (1999) Humphrey and Raymond (1994) Braun et al. (1999); Egholm et al. (2011) Herman and Braun (2008) Kessler et al. (2006)	
Tectonic Parameters			
Vertical rock uplift rate Geothermal heat flux Flexural plate length Elastic plate thickness Young's Modulus Poisson's ratio Density of crust Density of asthenosphere	0.25 [mm yr ⁻¹] 0.05 [W m ⁻²] 1000 [Km] 15 [km] 1 × 10 ¹¹ [Pa] 0.25 2750 [kg m ⁻³] 3300 [kg m ⁻³]	Herman and Braun (2008) Braun and Sambridge (1997) Braun and Sambridge (1997)	
Climate Parameters			
Sinusoidal Temperature Period Atmospheric lapse rate Positive degree day melting coefficient Annual Temperature Variation a0 a1 <i>Alf</i> Average wind speed Wind direction (angle from X-axis)		Yanites and Ehlers (2012) Braithwaite (1995) Yanites and Ehlers (2012) Roe et al. (2003) Roe et al. (2003) Roe et al. (2003) Roe et al. (2003) Roe et al. (2003)	
Fluvial and Hillslope Parameters			
Hillslope diffusivity Threshold hillslope landsliding Fluvial erosion coefficient Fluvial erosion length scale Channel width scaling coefficient Discharge threshold Alluvium length scale	$\begin{array}{l} 2\times 10^{-6} \left[{km}^2 yr^{-1} \right] \\ 35 \left[{}^{\circ} \right] \\ 3.5\times 10^{-4} \\ 1000 \left[m \right] \\ 0.1 \left[(yr m^{-1})^{0.5} \right] \\ 4 \left[m km^2 yr^{-1} \right] \\ 100 \left[m \right] \end{array}$	Braun and Sambridge (1997) Burbank et al. (1996); Stolar et al. (2007) Braun and Sambridge (1997) Braun and Sambridge (1997) Yanites and Ehlers (2012) Braun and Sambridge (1997) Braun and Sambridge (1997)	





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Table 2. Landscape evolution and orographic precipitation model parameters.

Simulation	Pattern	Amplitude	Sea-level Minimum	Other factors
1	Sinusoidal	6°C	2°C	
2	Sinusoidal	6°C	0°C	
3	δ^{18} O record-inspired	8°C (average)	0°C	
4	Sinusoidal	6°C	2°C	Temperature independent sliding.



Figure 1. Model domain with ice coverage from glacial maximum for Simulation 2 with hillshade topography at T = 100 ky. Glacial maxima for the other simulations generally follow the same form. The black box outlines region of nested, higher-order physics domain, used in Figs. 5 and 6. The shaded region (A-A') shows the location of the orogen-parallel swath used for comparisons in Figs. 7 and 9. Line B-B' gives the valley profile used for comparisons in Figs. 8 and 10.





spond to those of the black box in Fig. 1. The height corresponds to the ice thickness, and the bottom edge is at the elevation of the topography. The mesh is explained further in Sect. 2.5.

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Figure 3. Climate variations over time for the different simulations. Over three complete glaciations are simulated. Simulation 4 has the same climate as Simulation 1 and is not specifically shown. The two grey boxes show the single glaciations used in the analysis presented in Figs. 5, 7, and 8.





Figure 4. Variations in glaciated area and maximum thickness over 400 kyr. The colors correspond to the simulations in Table 2 and described in Sect. 2.2. For **(A)** and **(C)**, solid lines indicate the SIA model, and dashed lines the HO. **(A)** The area is defined at each time step by the area covered by ice greater than 10 m thick. Solid lines indicate the SIA model, and dashed lines the HO. **(B)** The percent difference between the SIA and HO. **(C)** The maximum ice thickness is defined at each time step when ice greater than 10 m thick exists. **(D)** The percent difference between the SIA and HO.





Figure 5. Pattern of erosion rates in nested region (Figs. 1 and 2) averaged over a single glaciation (grey highlighted areas in Figs. 3 and 4). The grey-scale topographic lines show the elevation of the underlying topography. **(A)** SIA erosion rates, where regions of no erosion are shown as white. **(B)** HO erosion rates, where regions of no erosion are shown as white. **(C)** Difference between the time-averaged erosion rates (SIA-HO).







Figure 6. Pattern of erosion rates in nested region (Figs. 1 and 2) averaged over the full simulation. The grey-scale topographic lines show the elevation of the underlying topography. **(A)** SIA erosion rates, where regions of no erosion are shown as white. **(B)** HO erosion rates, where regions of no erosion are shown as white. **(C)** Difference between the time-averaged erosion rates (SIA-HO).





Figure 7. Erosion rates over the A-A' orogen-parallel swath profile (Fig. 1). **(A)** SIA erosion rates averaged over the 100 kyr glaciation (dashed line; grey highlighted area in Figs. 3 and 4) and over the full simulation (solid line). **(B)** HO erosion rates averaged over the 100 kyr glaciation (dashed line) and over the full simulation (solid line). **(C)** Difference between the time-averaged erosion rates (SIA-HO).



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Figure 8. Erosion rates following the B-B' valley profile (Fig. 1). **(A)** SIA erosion rates averaged over the 100 kyr glaciation (dashed line; grey highlighted areas in Figs. 3 and 4) and over the full simulation (solid line). **(B)** HO erosion rates averaged over the 100 kyr glaciation (dashed line) and over the full simulation (solid line). **(C)** Difference between the time-averaged erosion rates (SIA-HO).











Figure 10. (A)–(C) Topographic profiles following the B-B' valley profile (Fig. 1). Solid lines indicate the ice-bed topography contact, and the dashed lines indicate the bedrock elevation (sediment is allowed to accumulate in ICE-Cascade). **(A)** SIA topographic profiles at the end of the full simulation. **(B)** HO topographic profiles at the end of the full simulation. **(C)** Differences between the topographic profiles erosion rates (SIA-HO). **(D)** Differences between the bedrock topography (SIA-HO). **(E)** Differences between the sediment thicknesses (SIA-HO).



