***Final Response on:* Perspective – Synthetic DEMs: A vital underpinning for the quantitative future of landform analysis?**

Detailed responses to reviewers' comments.

P3L14 = Page 3 Line 14, and refer to the reviewed manuscript.
Reviewer's comments are in grey, with our responses afterwards and with a chevron.
A Word document including tracked changes is also provided.
Fig. 2 = Figure in the reviewed manuscript, unless explicitly stated otherwise.

Reviewer 1: J. Pelletier

This paper is a very thoughtful perspective piece on the efficacy of synthetic DEMs (e.g. idealized landscapes made from numerical landscape evolution models, analytic solutions to the fundamental PDEs, with and without the noise typical of nature) in geomorphic studies. The authors motivate their study by noting that landscape evolution studies often involve the inference of process from form, but making such inferences is rarely clear cut. Moreover, they note that new models and DEM analyses techniques are often demonstrated on real DEMs. Such validation exercises may not be as effective as using synthetic DEMs that have the advantage of user control on the morphology, degree of stochastic variability, etc. The authors provide a typology of synthetic DEMs that will be a useful guide to future researchers who wish to use them in their work.
> Thank you.

I enjoyed reading the paper and think it makes some excellent points. One particular strength is that it draws from an impressively wide range of geomorphology (all sub- fields are represented) and even other subfields of Earth science (e.g. examples of data processing algorithms used in deep-Earth studies).
> Thank you.

Points for the authors to consider as they revise their work:

I think the key message in this paper can be made more succinctly: the results of any proposed numerical landscape evolution model or DEM-analysis algorithm must return the exact answer for at least one case (similar to the intended application of the model) in which an analytic or exact solution is available. I agree with the basic message of the authors that this should always be done but often is not (including in my own work). One consequence of this point is that some synthetic DEMs (type 2, i.e. those created from landscape evolution models) actually require other synthetic DEMs (type 1, i.e. analytic solutions of the governing equations for simple geometries and forcings) to be established as proper synthetic DEMs. To make this validation- related point stronger, the authors could consider connecting to the large literature on the necessary conditions of model validation (or, more generally, on the kinds of confidence building that should be performed on a model to check whether it meets minimum standards of quality for its intended purpose).
> In order to make the key message more succinctly and clearly, we have re-organised the abstract and expressed it in the sentence: "A second, arguably under-utilised, role is to perform checks on accuracy and robustness that we dub 'synthetic tests'". This is not a specific as suggested by R1, largely because examination of the validation literature (as also suggested by R1) highlights additional facets of the point, which forced us to remain somewhat general.
> As R1 suggests, we have connected this work to the wider literature on model validation. Given the vast size of this literature as noted by R1, we are limited to touching upon the key points. However, we used it to completely revise Fig. 2, and move it to the start of the manuscript to provide a solid context for the manuscript.

> The framework and terminological clarifications in the revised Fig. 2 (now Fig. 1) have been carried through into the manuscript.

The paper raises some interesting issues that I would liked to have seen explored more deeply. For example, it is not clear to me how synthetic DEMs can solve the equifinaility problem (i.e. similar topography resulting from different processes or forcing histories). At several junctures (including the first bullet point in the conclusions) the paper suggest that the use of synthetic DEMS can mitigate this problem, but precisely how they can was not clear to me.
> R1 is correct that some of the 'other approaches' to predict properties of a landscape (i.e. those which do not create a DEM) might, in some instances, mitigate some aspects of the equifinality problem. Specifically, many processes could (conceptually) give drumlins of the same shape, but if growth is considered statistically only a sub-set of these processes will give the observed size-frequency distribution. The role that synthetic DEMs would have in this would be to add robustness to the observations of size-frequency distributions i.e., the 'observe' on the right hand side of the new Fig. 1. As noted by Hillier et al. [2014] in the Journal of Maps, there is a size-dependent observational bias that we should be cautious of.
> We have not expanded upon this particular aspect here as it is not central the paper, and is explored more fully in Hillier et al. [In Review] for the case of drumlins; our revisions to that paper will be submitted by the end of the year.
> We believe that the revised manuscript does not explicitly reference 'equifinality'.

The author's criticism of morphological data as typically of a quality that is weakly constrained seems outdated. We can now create bare-earth point clouds/DEMs of unvegetated landscapes on demand with ∼1 mm accuracy and comparable resolution using terrestrial laser scanning (Hodge, 2010). This is remarkable by any measure.
> Thank you for noting this. We had not intended to imply that DEMs inaccuracy is a primary concern; it is becoming less so, although in some environments it is still a key problem (e.g. deep oceans). We intended to refer to the quality of metrics derived from DEMs, or at least our knowledge of how good/reliable they are. The sentence in the abstract (P2L7) has now been modified to "Derived morphological properties, including metrics and mapping.....".

As someone who has published on the drumlin problem, I was intrigued by the statement (p. 610) that no process-based model of drumlin formation exists. In Pelletier, Quantitative Modeling of Earth Surface Processes (2008), I proposed a model of drumlin formation by deformation of subglacial till modeled as a deformable porous medium. I think there are process-based models of drumlins that are based on reasonable physics (in the case of my model on the stresses developed during till compaction and dewatering) that match observed relationships between drumlin aspect ratio and average till thickness (this relationship was shown for two US drumlin fields in Pelletier (2008)).
> We thank R1 for highlighting this work, and apologise for not being aware of this model. Pelletier (2008) is now included as a reference, and we have re-phrased the text to remove the assertion that drumlins cannot be obtained from first mathematical principles.

Reviewer 2: I. Evans

The authors make a good case that synthetic landforms are part of the armoury of geomorphological modelling. Synthetic landforms and landscapes are not widely used and this paper should encourage geomorphologists to use them. The advantages and limitations of synthetic DEMs are usefully discussed.
> Thank you.

Readers will find the combination of ideas and examples from different fields useful. The juxtaposition of statistical synthetic DEMs with landscape evolution models is interesting and could be developed in

greater depth, as could the typology: probably further distinctions can be made. For example, a different type of synthetic relief was generated by Griffin: Griffin, MW. 1987. A rapid method for simulating three-dimensional fluvial terrain. Earth Surface Processes and Landforms 12(1), 31-38. [Special issue on 'Theoretical Geomorphology']
> To explore the juxtaposition of synthetic DEMs generated by statistical and geometrical means against those synthetic DEMs created by landscape evolution models, Fig. 2 has been revised and used to provide a more detailed conceptual structure that includes their various roles in creating understanding of process from geomorphological form.
> We agree that typology could probably be further refined, and additional distinctions could be made. To this end, we have included DEMs generated by experiment. We have also included reference to Griffin (1987) within section 3.2, noting that distinctions in LEMs exist, but have chosen not to make further distinctions explicit here to achieve a balance between complexity and usability in the typology. We do state that the typology intends to cover the broad approaches (P9L6).

I would assert that models need to be tested against real topography, whether or not synthetic DEMs are used to provide complementary tests. They should not be regarded as alternatives.
> We agree with this assertion, and were not intending to indicate that they should be regarded as alternatives. We see synthetic DEMs and tests as complementary in that they can increase the quality of comparisons between model and real DEMs (i.e. used to improve accuracy, or at least to understand the level of error present when interpreting results). We hope the revised Fig 2 clarifies this.

Some complexities of testing different algorithms for surface metrics are illustrated in –

Minár, J., Jencˇo, M., Pacina, J., Evans, I. S., Minár, J. Jnr., Krcho, J., Kadlec, M., Burián L., & Benová, A. 2013. Third-order geomorphometric variables (derivatives) – definition, computation and utilization of changes of curvatures. International J of GIS. 27 (7), 1381-1402. http://dx.doi.org/10.1080/13658816.2013.792113 and-

Minár, Jozef, Minár Jozef Jr & Evans Ian S. 2015 Towards exactness in geomorphometry. In: Jasiewicz J., Zwolin´ski Zb., Mitasova H., Hengl T. (eds), Geomorphometry for Geosciences. Adam Mickiewicz University in Poznan´ - Institute of Geoecology and Geoinformation, International Society for Geomorphometry, Poznan´, Geomorphome- try2015. geomorphometry.org 27-30.

Real and artificial surfaces do tend to give different results!
> Thank you for these references. Minar et al [2013] use what we class a simple or geometrical synthetic DEMs for precisely one of the purposes we highlight; assessing the accuracy of a measure of the properties of a landscape. We have therefore added this reference. As they state, to test the methods "a test mathematical function [placed in a DEM] can be used if it has sufficient characteristics of the land surface". So, whilst real and artificial surfaces will perhaps give different results, the key (as we point out) is to make a synthetic that is sufficiently realistic for the task in hand.

Fig.3 shows that visually the simple model seems an excellent fit. Never the less the profiles in b) show a systematic deviation which is worth commenting. The real profiles have a sharper basal concavity than the model: presumably this betrays the operation of a different process.
> A comment as been added to the end of the first paragraph in Section 3.1.

DETAILS: page/line

Some sentences need to be reworded or simplified for smoother reading.

2/10-11 can this be rephrased to avoid the inverted commas, which may turn off some readers?
> We have removed one of these sets of inverted commas, and placed the other (a new definition) in context in a way that we hope will annoy readers less

3/16 rephrase: it is ‘bedforms in flow-sets’ – not size of flow-sets. 3/19 ‘fundamentally random’ is an exaggerated interpretation.
> This has been rephrased and moderated to "...... which arguably indicates that substantive elements of the ice-sediment-water system beneath ice sheets contain randomness"

3/20 replace ‘similar’ – be more specific.
> This has been rephrased to be more specific and read "Quantitative analysis can also provide constraints when applied to linear features ...."

3/22 ‘are identified’
> 'are' added as requested.

4/19 combine the brackets – don’t use ‘e.g.’ twice.
> Has been altered so that e.g. is not used twice.

6/20 ‘have uncovered’ implies availability of real examples: these should be cited, in- stead of the hypothetical ‘50%’.
> An example is now used. The overall detection rates of drumlins during manual mapping is 40-34%, a 60-66% effect [Hillier et al., 2014].

7/4 and 7/11 again, combine the brackets – don’t over-use ‘e.g.’.
> The brackets have been combined, removing one 'e.g.' in each case.

8/7 is “only possible to test their efficacy. . .” an exaggeration?
> We appreciate that this is a strong statement, but we believe that it is not an exaggeration once it has been modified to 'assess adequately'; clearly an inadequate assessment is always possible (e.g. by astrology). We are defining synthetic DEMs as those containing an a priori correct answer, and the 'observe' part of Fig. 1 requires a DEM for the methods to be applied to. And, this is our definition of a synthetic DEM. We have modified the text to clarify this and to indicate that by adequate we mean in an absolute sense, ruling out mapper inter-comparison as is noted in the bullet points just above this text.
> The only exceptions to this that we can think of are accuracy checks possible when measuring a DEM from an experimental simulator or real landscape, and we are referring to the 'observe' stage (see revised Fig. 2, now Fig. 1) at this point in the manuscript.

9/21 ‘investigation and’
> 'and' added as requested.

Section3.1, last para. An early demonstration of the inadequacies of fractal models was – Evans, I. S. & McClean, C. J., 1995 The land surface is not unifractal; variograms, cirque scale and allometry. Zeitschrift für Geomorphologie, N.F. Supplement-Band 101, 127-147
> Thank you. We have added this reference.

10/16 and 18/26 ‘Harbor’
> Changed.

11/0 Is anything perfect? Better ‘Further improvements are awaited’, Or (10/23) ‘Several difficulties prevent these models as yet from being ideal solutions.’
> In the first case we have changed to "highly accurate and widely accepted", and to R2's suggestion in the second case.

12/12 To what does ‘they’ refer? – apparently bedforms, but probably not?

> Originally, 'they' referred to the combination of 'hills' and 'noise'. This statement has now been simplified and clarified. It now focuses on just the noise, which is the particularly difficult part. At least, we do not know of it being attempted in the geomorphological community.

13/2 avoid "to test against synthetic DEMs" – not what you mean?
> Thank you. This is indeed not what we meant. The typographic error in the punctuation has been corrected.

13/26 perhaps 'are not used on the basis that. . .'
> Thank you. This has been changed as suggested.

14/7 not "and perhaps: better, '. . . property and its scale variation is key, it can . . .'
> Thank you. This has been changed.

14/15-19 sentence needs simplification.
> The sentence has been simplified and clarified by separating it into two sentences, with some rewording.

14/23 'later be'
> Thank you. This has been changed.

Fig.1 As the ln scale tends to be opaque to non-mathematicians, I would prefer a log10 scale, or best, actual counts on the y-axis.
> Scale has been changed to log10. An exponential plots linearly on a semi-log plot, so this is used instead of counts to illustrate that the data are visually consistent with an exponential distribution (e.g. instead of a power-law).

Fig.2 I did not find this compound figure useful.
> This figure has now been entirely revised. It is now Fig. 1 in the revised manuscript.

Fig.5 what is the extent of c)? a) is dimensionless but b) would appear to have dimensions.
> All the parts are at the same scale, or in effect so. This information has been added to the caption of the figure. The coefficients used in the modelling were chosen to make this direct comparison possible.

Reviewer 3: Anonymous

This paper provides an interesting perspective on the use of synthetic DEMs; it introduces the concept of using synthetic DEMs for geomorphology, provides a range of examples from different areas of geomorphology and highlights the role of synthetic DEMs in improving process understanding. I really enjoyed the paper and think that it raised some interesting points and feel that this would make a valuable contribution to ESurf, however some further expansion on some of the points that were introduced in the paper is required.
> Thank you, and please see below.

1. Can you quantify the error difference between observational measurements and synthetic hybrid DEM generation? Each of these have inherent errors within the measurement/computations that are the result of the method used rather than the noise and it would be useful to highlight this in the article.
> We note that these errors exist, but the magnitude of the measurement error depends on the techniques and circumstances of each study, and computational errors (e.g. floating point rounding) should ideally be fixed in the implementation of a model. We have revised Fig. 2 to clarify the procedure in geomorphology intended to link processes to form, which now explicitly includes the step from reality to an observed DEM. This distinguishes between noise and other forms of error. We have also highlighted that variability in both synthetics and observed DEMs can be both random and systematic (P13L4).
> As we note, the use of multiple synthetic DEMs can reduce random errors, and be used to understand the role this might play in observations.

2. What about the importance of initial and boundary conditions? These will influence the generation of the DEM whether it is synthetic or based on observational data, and for process understanding it is important to state the influence that these will potentially have.
> Whether of not initial or boundary conditions have a role in the generation of a synthetic DEM depends upon the type of synthetic DEM (e.g. no role in those of Hillier et al. [2012] or Wessel [1998]). In terms of LEMs, we note the initial and boundary conditions for the simulation we show (Fig. 5b), and a sentence has been added to note that these influence the outcome as well as other model parameters (P8L1).

3. The comparison between the synthetic DEMs and LEMs was touched upon but this could be expanded further with further elucidation of the methods that were used to compare accuracy. Also, although the representation of LEMs is improving, I still do not feel that you can fully test the replicability of synthetic DEMs without drawing on observational measurements from nature; again this was mentioned but more discussion could be centred around this and what impact the simplifications made in LEMs and to some extent synthetic DEMs will affect the resultant DEM and its 'representativeness'.
> As we note, and from our experience, what classes as sufficient representativeness depends almost entirely upon the task in hand and upon the requirements of practitioners in each particular field. We also note the challenge is to "determine a generalised objective framework or workflow to assess the sufficiency of the realism of synthetic DEMs"; this is a challenge that we hope will be taken up, but is beyond the intended scope of this paper.
> We entirely agree that observational measurements from nature are needed, and are strengthened rather than replaced by using synthetic DEMs (see reply to R1 above).
> Although we acknowledge that detailed discussion of the methods and metrics used to compare DEMs is useful, this is not the focus of this paper; we hope to highlight that whatever method is used, testing it with synthetic DEMs can give some better understanding of how well it performs.

4. “Hybrid DEMs” – a figure would be useful showing the DEMs produced and comparison of these with those in nature, so that the reader can visually compare and evaluate the difference between the DEMs produced from real/simple/LEM/hybrid simulations.

> 'Hybrid DEMs' are not a single process or procedure, in fact the opposite of it. Similar goes for the other classes of synthetic DEM. So, no single figure can hope to illustrate and evaluate the differences between them. Indeed, evaluation depends entirely upon the context for which they were designed, which will differ. However, we note that we have illustrated real (Fig. 3a, 5c), simple (Fig. 5a, 4) and LEM-derived DEMs (Fig. 5b), but only a profile to illustrate the process used to create one possible sort of hybrid DEM. We have therefore added a Figure comparing a hybrid DEM to the real landscape it was created from.

Perspective – Synthetic DEMs: A vital underpinning for the quantitative future of landform analysis?

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Abstract

Physical processes, including anthropogenic feedbacks, sculpt planetary surfaces (e.g., Earth's). A fundamental tenet of Geomorphology is that the shapes created, when combined with other measurements, can be used to understand those processes. Artificial or synthetic Digital Elevation Models (DEMs) might be vital in progressing further with this endeavour in two ways. First, synthetic DEMs can be built (e.g., by directly using governing equations) to encapsulate the processes, making predictions from theory. A second, arguably under-utilised, role is to perform checks on accuracy and robustness that we dub 'synthetic tests'. Specifically, synthetic DEMs can contain *a priori* known, idealised morphologies that numerical landscape evolution models, DEM-analysis algorithms, and even manual mapping can be assessed against. Some such tests, for instance examining inaccuracies caused by noise, are moderately commonly employed whilst others are much less so. Derived morphological properties, including metrics and mapping (manual and automated) are required to establish whether or not conceptual models represent reality well, but at present their quality is typically weakly constrained (e.g., by mapper inter-comparison). Relatively rare examples illustrate how synthetic tests can make strong 'absolute' statements about landform detection and quantification; e.g., 84% of valley heads in the real landscape are identified correctly. From our perspective, it is vital to verify such statistics quantifying the properties of landscapes as ultimately this is the link between physics-driven models of processes

and morphological observations that allows quantitative hypotheses to be tested. As such the additional rigor possible with this second usage of synthetic DEMs feeds directly into a problem central to the validity of much of geomorphology. Thus, this note introduces synthetic tests and DEMs, then it outlines a typology of synthetic DEMs along with their benefits, challenges and future potential to provide constraints and insights. The aim is to discuss how we best proceed with uncertainty-aware landscape analysis to examine physical processes.

1 Introduction

Physical processes sculpt planetary surfaces such as the Earth's. A fundamental tenet of Geomorphology is that the form of the surface created, when combined with other data or modelling, can be used to understand those processes. This endeavour to reconcile observation and theory is, essentially, model validation [e.g., Martin and Church, 2004; Pretty, 2009; 2010] summarized by the question: Has the right model been constructed? [Balci, 1998]. Fig. 1 illustrates the pathways towards reconciliation between observations and models, which in geomorphology is conducted through some properties or metrics diagnostic of the landscape of interest; the pathways lead to this reckoning from both physical reality and from conceptual models, which may vary in sophistication (e.g., may even be qualitative). Whilst visual comparisons of landscape properties are obviously possible, quantitative morphometrics of DEMs ('observe' in Fig. 1) are a stronger approach and these vary according to the types of study being undertaken.

Discrete landforms [cf. Evans, 2012] (e.g., craters, cirques, drumlins, volcanoes) can be delimited with a closed boundary and then isolated in order to quantify key characteristics such as height H or slope of a flank [e.g., Hillier, 2008]. Linear features (e.g., rivers) can also be measured. Equally, spatially continuous properties of Digital Elevation Models (DEMs) can be quantified (e.g., roughness, Wetness Index) [Beven and Kirkby, 1979; Grohmann et al., 2011; Eisank et al. 2014]. Such morphology-derived observational data, including metrics from mapping that is both manual and automated, add to the more qualitative assessments that may be drawn directly from geomorphological maps.

Quantifying discrete landforms can give additional insights and provide constraints on models of physical processes. For example, discrete fluvial bedforms and their variability are quantified and used to predict extremes for engineering purposes (e.g., depth to place a pipeline) [van der Mark et al., 2008]. Impact crater size-frequency distributions are used to estimate the age of the surface of the Moon and planetary bodies (e.g., Mars and Mercury) [e.g., Hartmann and Neukum, 2001; Ivanov, 2002]. Similarly, size-frequency distributions of volcanoes have been used to examine how melt penetrates the tectonic plates [e.g., Wessel, 2001; Hillier and Watts, 2007]. Aeolian dune formation can be constrained by their sizes [e.g., Duran, 2011; Bo et al., 2011]. In sub-glacial geomorphology 'flow-sets' of proximal bedforms thought to created by the same ice motion occur exponentially less often as their size increases (Fig. 2), which arguably indicates that substantive elements of the ice-sediment-water system beneath ice sheets contain randomness [Hillier et al., 2013].

Quantitative analysis can also provide constraints when applied to linear features and spatially continuous measures. Channel geometry is measured to investigate the influences of tectonic or climatic landscape forcing [e.g., Brummer and Montgomery, 2003; Wohl, 2004; Sofia et al., 2015], and channel networks are identified to evaluate hydrological responses in floodplains [e.g., Cazorzi et al. 2013]. Continuous measures such as curvature can arguably distinguish dominant geomorphic processes (e.g., diffusive vs fluvial) [e.g., Tarolli and Dalla Fontana, 2009; Lashermes et al., 2007], and can be designed to detect the presence of anthropogenic features (e.g., agricultural terraces) [Sofia, et al., 2014]. They can also be used to estimate the probability of landsliding during rainstorms or for (semi-)automated geomorhological mapping [e.g., Tarolli and Tarboton, 2006; Milledge et al., 2009; Eisank et al., 2014]. Thus, such quantifications also have value for geomorphic understanding. Importantly, these examples illustrate how a robust, reproducible and quantitative approach can be used to develop our understanding of process.

Any enhanced use of landform observations, however, relies on us being able to trust what we have mapped or quantified. Specifically, the key question is; in terms of precision, accuracy and mapping completeness to what extent is it possible to trust the metrics derived from morphometric quantification of the landforms or surface recorded in the DEMs?

One way around this difficulty is to derive descriptive statistics that are as robust as possible to observational shortcomings [e.g., Hillier et al., 2013; Sofia et al, 2013; Tseng et al., 2015]. Another solution is to assess the quality of the morphological mapping and quantification, perhaps either by an estimate of data completeness or quality [e.g., Hillier & Watts, 2007] or by traditional inter-comparisons between mappers [e.g., Podwysocki et al, 1975; Siegal, 1977; Smith and Clark, 2005] or techniques [e.g., Sithole et al., 2004]. The difficulty with robust statistics is that they will still be distorted if shortcomings are substantial [e.g., Hillier and Watts, 2004], and inter-comparisons can only ever yield relative levels of success and even complete agreement is inconclusive; all techniques, mappers, or techniques calibrated to mappers [e.g., Robb et al., 2015] may be systematically missing things (e.g., smaller features [Eisank et al., 2014; Hillier et al., 2014]). Furthermore, it is simply not possible to calculate or estimate the magnitude of potential systematic biases within these approaches. An alternative is to verify each method or result against suitable features or properties known *a priori* within a suitably constructed test DEM. Thus designed landscapes, or 'synthetic' DEMs, can give strong 'absolute' answers (e.g., 84% of valley heads in the real landscape are identified correctly), and may be vital in allowing us to proceed better with uncertainty-aware landscape analysis to examine physical processes.

Synthetic DEMs built by directly using postulated governing equations that encapsulate processes, or Landscape Evolution Models (LEMs) [e.g., Chase, 1992], are another key part of examining the form-process link. By altering their constants (e.g., rainfall, hillslope diffusivity) and mathematical construction they can give insights into the drivers and impacts of physical processes [e.g., Willgoose et al., 1991; Montgomery and Dietrich, 1994; Miyamoto and Sasaki, 1997]. LEMs are, however, not yet the whole solution since to be securely compared to reality equivalent landforms within both DEM types must still be robustly quantified, sometimes making validation or calibration very difficult [e.g., Martin and Church, 2004; DeLong et al., 2007]. It is also possible to use synthetic DEMs to test for inaccuracies in DEMs created by LEMs or by measuring a landscape (i.e., 'make' in Fig. 1); one example of this might be requiring that LEMs replicate analytic solutions of the governing equations for simple geometries and forcings. Ultimately, all synthetic DEMs originate in a conceptual view of at least one aspect of a landscape (e.g., drumlin shape, stream-power based fluvial behaviour).

This note introduces synthetic tests and DEMs, then it outlines a typology of synthetic DEMs along with their benefits, challenges and future potential to provide constraints and insights. Note that 'virtual' and 'artificial' are used interchangeably with 'synthetic', as they are in the literature.

2 Synthetic Tests and the Potential Uses of Synthetic DEMs

In fields such as geophysics it is standard to verify any method against its performance on some idealised or 'synthetic' data. A well-documented example is the classic 'synthetic checkerboard' test [e.g., Dziewonski et al., 1977, Saygin and Kennett, 2010] used in tomographic imaging of the Earth's interior. Broadly, there are four requisite stages for such a test based upon synthetic data [e.g., Nolet et al., 2007].

1. *Construct a synthetic input* including any features of interest (e.g., the morphology of a landform).

2. *Create the synthetic data* that resembles the observed data, for instance adding suitable noise.
3. *Invert the synthetic data* using the same numerical approach applied to the observed data.
4. *Compare the inverted result with the synthetic input* to see how well the assumed synthetic input (e.g., landform) is recovered.

The difficulty always lies in generating a suitable, statistically representative synthetic; in the case of geomorphology the task is to create an 'appropriate' synthetic landscape or DEM that is realistic enough in the aspects under investigation.

DEMs containing a synthetic component have been employed in ‘synthetic tests’ to assess approaches used to estimate the fractal dimension of topography [Malinverno, 1989; Rodriguez-Iturbe and Rinaldo, 1997; Tate, 1998a,b], slope and aspect [Zhou, 2004], land surface parameters (LSPs) [e.g., Wechsler, 2006; Sofia et al, 2013], and the reliability of DEMs [e.g., Fischer, 1998; Oksanen, 2010]. Additionally, they have been used to evaluate how well some features (e.g., river networks, terraces) are identified [Pelletier, 2013; Sofia et al., 2014] and others (e.g., submarine volcanoes and drumlins) are isolated in 3D (i.e., their volumes explicitly delimited) [Wessel, 1998; Hillier, 2008; Kim and Wessel, 2008; Hillier & Smith, 2014]. Synthetics have also been used to assess algorithms quantifying landscape processes such as flow-routing [e.g., Pelletier, 2010] and to give a first insight into how effective the manual mapping of glacial bedforms is [Hillier et al., 2014]. Often, when including randomness (e.g., in locations or noise) in a *Monte Carlo* approach multiple realisations of a landscape (e.g., n = 10 or 1,000) are used to understand uncertainty and variability and more tightly constrain results [e.g., Heuvenlink, 1998; Raaflaub and Collins, 2006; Wechsler, 2006]. The large (e.g., 60-66% in Hillier et al. [2014]) and systematic trends and biases that studies so far have uncovered indicates that the uses of synthetic tests in geomorphology should be, arguably, similar in extent and function to the current use of inferential statistics; namely they are a demonstration that the observation claimed actually exists or method actually works. Some potential applications of synthetic tests in geomorphology can be categorised as:

- Assessing the impact of ‘noise’ [e.g., Sofia et al., 2013; Zhou and Liu, 2002, 2008] that could be instrumental, anthropogenic (e.g., houses) or natural (e.g., vegetation). This applies to making DEMs from measurement, and making quantitative observations from any DEM.

- When observing, verifying that a geomorphic signature is actually characteristic of a particular landform type of interest, rather than other morphologies in a study area [e.g., Conway et al., 2011; Sofia et al., 2014].
- Quantifying extraction of features using metrics such as completeness, reliability [e.g., Hillier et al., 2014; Eisank et al., 2014]; in this the key advantage is that synthetics give 'absolute' measures of accuracy simply not possible with traditional mapper inter-comparisons (e.g., 34-40% of drumlins can be detected).
- Assessing filtering or other techniques used to manipulate a DEM [e.g., Hillier & Smith, 2014], whose choice would otherwise be subjective.
- Evaluating the sensitivity of algorithms quantifying geomorphic processes to modelling assumptions, such as DEM resolution [e.g., Pelletier, 2010].
- Determining whether or not LEMs have been correctly constructed (i.e. 'make' in Fig. 1).

Ultimately, the geomorphological intention is to use synthetic DEMs to examine more clearly the expression of physical processes. Rigor added to geomorphological observations through testing with synthetic DEMs will, we believe, ultimately link physics-driven models of processes to morphological observations, allowing quantitative hypotheses to be formulated and tested [e.g., see McCoy, 2015]. This is illustrated in Fig. 1, the crux of which is that it is necessary to quantify landscape properties to rigorously reconcile DEMs, with some main elements of this described in more detail below.

If arguably realistic forms can be generated directly by a physics-based model [e.g., Dunlop et al., 2008; Refince et al., 2012; Brown, 2015] creating a synthetic DEM, these may in principle be linked directly to reality if suitably equivalent field sites can be found, measured, and recorded in a DEM. The effects of various constants (e.g., rainfall), conditions and processes in the physical models on observables can be viewed and compared to reality by the simple expedient of turning them off or amplifying them, of course allowing carefully for appropriate initial and boundary conditions. Comparisons have been qualitative [e.g., Kaufman, 2001], but can provide more powerful insights if they apply consistent mapping or quantification procedures [e.g., Willgoose et al., 1994]. Thus, creating a form-process link will still depend critically upon understanding any errors or biases in landform morphometrics (e.g., in size-frequency distributions, dominant wavelength) for both the measured and generated landscapes (i.e., 'observe' in Fig. 1). The appropriate metrics are better understood for some landforms than for others, and it is only possible to adequately assess their efficacy (i.e. in an absolute sense) with tests involving *a priori* information and a DEM to apply the morphometric extraction method to, or by our definition using a synthetic DEM. If laboratory experiment replaces LEM-derived synthetic DEMs in the paragraph above, the same logic applies.

For a landform that it is not yet possible to create numerically from first mathematical principles, other routes exist. The challenge is to securely relate the driving process (e.g., tectonic uplift rate) to a measure of morphology (i.e., 'conceptual model' to 'landscape properties' on Fig. 1), perhaps using its variability within geographical areas. For example, drumlin sizes observed for a number of flow-sets might be compared to characteristics of flow within a modelled ice sheet (e.g., flow velocity) representative of the area of the flow-set. Statistical models can be formulated that link size-frequency observations to parameters in numerical ice-flow models [Hillier et al., *in review*], but even potential empirical rules about timing (e.g., immediately before de-glaciation) and the relationships to ice flow (e.g., size directly proportional to velocity) could be tested. Robustly determined observational metrics would be needed for such an inversion; i.e., synthetic tests are needed. Realistic models are likely to contain stochastic elements [e.g., Tucker et al, 2001], thus a statistical understanding may help to identity more effectively appropriate parameterizations for size observations (e.g., Weibull) than testing a variety of established distributions [e.g., van der Mark, 2008]. Observational robustness is desirable in this case, but also for approaches that make predictions about landscape properties directly from conceptual models, for instance dominant wavelengths [e.g., Anderson, 1953; Venditti, 2013].

A final use of synthetic DEMs is examining ‘what if’ engineering scenarios as they affect behaviours such as hydrological processes [e.g., Tarolli et al., 2015]. This may be somewhat tangential, but imposing a proposed artificial geometry onto a measured DEM as a way of testing an artificial geometry to be created on the part of the Earth’s surface is clearly a legitimate pursuit.

3 Synthetic DEM Typology

Synthetic DEMs are only useful if they can be constructed, and their construction must be from or clearly identify ‘components’ (e.g., a landforms layer). In contrast to viewing a landscape as plan-view regions, height in DEMs can be described at any location (x, y)

as the sum of n 'components' (Eq. 1) [e.g., Wren, 1973; Wessel, 1998; Hillier and Smith, 2008], namely $H_{DEM} = H_1 + H_2 + ... H_n$. Conceptually, these components lie on top of each other, like geological strata, and extend across the entire DEM although they may have zero thickness for few or many parts of it.

For landform analysis the first component would typically be 'noise' (e.g., DEM error, or surface 'clutter' such as trees), the small-scale height variations not genetically related to the landform. A second component would be the landforms themselves, perhaps overlying a third component of larger-scale trends (e.g., 10 km wide smoothly undulating hills). However, in the limit, only one component is actually required, and how the components are constructed will vary depending upon the purpose of the synthetic DEM. Furthermore, the synthetic DEM might mix idealised, created components with real ones. Typically randomness is involved in the creation of statistical synthetics, and multiple realisations of landscapes may be created. The broad approaches to constructing synthetic DEMs are outlined in the typology below.

3.1 Simple and Statistical

Perhaps the simplest synthetic DEMs are those constructed by using basic geometries as building blocks such as cones, Gaussian functions, and planes or other surfaces defined by simple equations [e.g., Hodgson, 1995; Wessel, 1997; Jones, 1998; Kim and Wessel, 1998; Hillier, 2008; Pelletier, 2010; Qin et al., 2012]; admittedly, some functions may be less simple [e.g., Pelletier, 2013; Minár et al., 2013]. Typically, generalized shapes (e.g., 2D Gaussian, rotated parabola) are formulated based upon visual or statistical fitting of the functions to measured morphologies [e.g., Conway et al., 2011; Hillier and Smith, 2012; Pelletier, 2013] (Fig. 3); fits may not be perfect (Fig. 3b), highlighting that all synthetic DEMs are simplifications of reality.

These synthetics do not contain the complexity in the observed landscape, or necessarily have realistic statistical properties, but they have the advantages of being simple to construct and understand, and noise can be entirely omitted or modified with certainty in order to investigate data errors. They contain the key morphologies under investigation and are perfectly sufficient for some tests; e.g., are approximately conical submarine volcanoes of variable size effectively isolated even when upon a slope? (Fig. 4). Statistically generated 'noise' can be added to simple synthetic DEMs to assess the degradation caused [e.g., Zhou, 2004; Jordan and Watts, 2005], but for results to be meaningful its statistical distribution (e.g., Gaussian, uniform), length-scale of correlation, and any non-stationarity must be correct [e.g., Fischer, 1998, Sofia et al., 2013].

Whole landscapes can be generated statistically using fractals [e.g. Mandelbrot, 1983] or multi-fractals (Fig. 5a) [e.g., Gilbert, 1989; Schertzer and Lovejoy, 1989; Weissel, 1994; Cheng, 1996], and these can be useful if the construction matches closely the element of reality being considered (e.g., uncorrelated, fractal in Swain and Kirby [2003]). Even multi-fractal landscapes, however, may not be an adequate representation without considering properties such as anisotropy [e.g., Evans and McClean, 1995; Gagon, 2006] and characteristic scales [e.g., Perron, 2008] if they are important in a particular circumstance. A limitation of these purely statistically generated, or statistically altered, DEMs for landform analysis is that they do not explicitly contain spatially distinct, isolated features (i.e., landforms are not labelled as such during generation).

3.2 Landscape Evolution Models

DEMs resembling real landscapes can also be created by the application of mathematical characterisations of physical processes in numerical models typically known as 'Landscape Evolution Models' (LEMs) (Fig. 5b) [e.g., Chase, 1992; Braun and Sambridge, 1997]; implementation approaches can vary [see Griffin, 1987]. These now incorporate numerous processes [e.g., Tucker, 2010; Refice et al., 2012]; for example, bedrock landslides [e.g., Densmore, 1998], flexure of the lithosphere [e.g. Lane et al., 2008], and erosion by ice flow within valleys [e.g. Harbor, 1992; Brocklehurst and Wipple, 2004; Amundson and Iverson, 2006; Tomkin, 2009], including when this is thermo-mechanically coupled to ice sheets [e.g. Jamieson et al., 2008]. Models of the evolution of single classes of feature (e.g., bedforms) and simpler 2D configurations (i.e., x-z profiles) fall within this class of model [cf. Dunlop et al., 2008; Zhang et al., 2010; Brown et al., 2014]. Simple geometries or measured landscapes may be used as an input [e.g., DeLong et al., 2007; Refice et al., 2012; Baartman et al., 2015; Hancock et al., 2015].

Several difficulties prevent these models from, as yet, being ideal solutions. In terms of testing observational methods, the first difficulty is that the method of generating some landforms such as drumlins from first principles is often contested [cf., Hindmarsh, 1998; Schoof, 2007; Pelletier, 2008], and it is not computationally practical to include certain processes, such as impact crater formation in the MARSSIM model [Howard, 2007]. The simulation of rivers illustrates an area where there is progress, but also much to do [cf. Coulthard et al., 2013; Brown et al., 2014]. In general, a highly accurate and widely accepted unified model is still some way off. The second difficulty is that these models do not currently associate processes with a type of landform. For instance, a bedrock failure process is a bedrock failure process, not a bedrock failure process explicitly making a V-shaped valley. Equally, sediment is not tagged as making a floodplain. So, the number and location of defined features are not known *a priori*. This can be seen as a strength of the models, but means that creating a secure link from process to landforms as observed in reality requires a step in which consistent mapping or quantification procedures are applied to both measured and simulated DEMs. This is not easy [e.g., DeLong, 2007]. The lack of *a priori* features may also be the reason that, although LEMs have great potential to create DEMs for synthetic tests of landform mapping or extraction methodologies, we are not aware of this being done. Like simple or statistical synthetic DEMs, synthetics created by a LEM have the advantage of being free from errors associated with DEM measurement (e.g., instrumental, processing).

3.3 Laboratory derived

If LEM-derived DEMs can be considered as synthetic DEMs, then laboratory-derived ones [e.g., Hancock and Willgoose, 2001; Lauge et al. 2003; Graveleau and Dominguez, 2008; Sweeny et al. 2015] could also be considered so. Such experiments can control variables such as rainfall and uplift that are impossible to precisely control in nature [e.g. Sweeny et al 2015], but limitations in realism exist particularly in scaling [see Paola et al., 2009].

3.4 Complex geometrical

A possible class of synthetic DEM is one that uses simple or statistical building blocks, but constructed in a more complex fashion. For instance, multiple idealised shapes can be given additional observed attributes (e.g., spatial clustering, size-frequency realism) [e.g., Howard, 2007; Hillier and Smith, 2012], but such DEMs have so far contained other elements of realism as well, perhaps making them better described as hybrids.

3.5 Hybrid

A 'hybrid' class of synthetic DEM contains, for reasons of practicality, elements of the other classes. Typically, a morphology whose key properties cannot currently be readily simulated is either retained (e.g., most or all of a measured DEM), or an idealised but observationally constrained component is added (e.g., terraces [Sofia et al. 2014]), or both. The spectrum of what is possible is illustrated by the, relatively rare, studies using hybrid synthetic DEMs in geomorphology.

A first example of a hybrid synthetic DEM is impact crater formation in the MARSSIM model [Howard, 2007]. This evolution model does not dynamically model crater formation. Instead, randomly located craters are assigned shapes from a catalogue of measurements of individual fresh craters on Mars and given sizes from a power-law distribution. This introduces certain assumptions, such as the fresh craters being representative, but avoids complexity. A second example deals with the quantification of glacial bedforms, illustrated with drumlins [Hillier and Smith, 2012; Hillier and Smith, 2014; Hillier et al, 2014]. It is the association of the bedforms with underlying trends (i.e., 'hills') and complex and spatially structured 'noise' (e.g., trees, roads, houses) that makes the quantification difficult; in particular, this noise is problematic, and geomorphological analyses have yet to attempt simulating it. The approach taken was therefore to circumvent this issue entirely by leaving the hills and noise as they were, and moving the drumlins such that they were randomly positioned with respect to these problems for identification (Fig. 6). Orientations and spatial density distribution (i.e., number per km^2) were preserved, as were the geometries (i.e., height-width-length triplets) of the 173 drumlins shuffled around. In these synthetics (Fig. 7), the number and location of defined features are known *a priori* such that sizes and locations of mapped discrete landforms can be compared to synthetic ones directly. Similarly, but by assuming the highest-quality measured LiDAR DEMs were perfect, even if this is debatable, it is possible to circumvent the need to generate statistically realistic landscapes when investigating DEM errors [Raaflaub and Collins, 2006; Sofia et al., 2013]. Anthropogenic elements (e.g., open-cast mines, terraces) visually determined to be reasonable can also be added [e.g., Baartman et al., 2015], for instance to a 2D multi-fractal statistical landscape [Sofia et al., 2104; Chen et al. 2015].

4 Discussion

By providing an *a priori* known answer to test against, synthetic DEMs or DEMs containing a synthetic component have some clear and powerful advantages in geomorphological analyses. They can be used to test errors, systematic or random biases, and unpick potential sources of misinterpretation. Furthermore, they give *absolute* answers (e.g., 47% of all actual drumlins $H > 3$ m are mapped) to questions about accuracy that are simply not obtainable by other means, and are often considered 'objective'. Through this they provide a route to answering key questions about geomorphic processes (e.g., Fig. 1). There are, however, complexities surrounding these statements, which are less commonly recognised. There are issues of objectivity, realism, circularity and the cost in time and effort of constructing synthetics.

Whilst the conclusions reached through the use of synthetics may be simplistically thought of as objective, it is more accurate to say that they are quantitative, reproducible, and are likely to be significantly less subjective. Without perfect, all-purpose synthetics an element of subjectivity will remain in the choices made when designing the test DEM. Hillier and Smith [2012] illustrate some choices and a logical justification for them. Manually selecting data to test against [e.g., Sithole, 2004; Hillier & Watts, 2004] is faster in some circumstances, if more subjective. Reproducibility makes testing using synthetic DEMs superior to subjective visual verification, even if synthetic tests later indicate the visual estimate was a reasonable solution [Hillier and Smith, 2012, 2014]. Pre-existing synthetic DEMs, however, are entirely objective means for inter-comparison for future studies [e.g., Eisank et al., 2014].

A thorny question regarding synthetics is: how realistic is realistic enough? At one limit, it is notable that even extreme simplifications such as conical volcanoes can give significant and useful first-order insights [e.g., Kim and Wessel, 2008; Tarolli et al, 2015]. At the other limit, synthetic DEMs are not used on the basis that their applicability to real datasets is questioned [e.g., Robb et al., 2015]. Lacking a perfect set of properties, however, should not be taken to invalidate tests using a synthetic DEM; in statistics for instance, Student's t-test underpinned by its idealized Gaussian distribution is widely used although observations are rarely perfectly Normal. A challenge then is to determine a generalised objective framework or workflow to assess the sufficiency of the realism of synthetic DEMs, but in its absence what can be done? Deficiencies can be visually identified. For instance, if spatial resolution is raised as an issue, it can either be matched to the observed data, or varied for a sensitivity test. If a particular statistical property and its variation with scale is key, it can be measured to ensure it is realistic in the synthetic. So, if a clearly stated set of properties argued to be most relevant to any given research task are faithfully reproduced in synthetics, we believe they will provide useful insights. Ultimately, however, practitioners within a peer-group must decide what is convincing, performing additional tests if necessary. For example, Hillier and Smith [2012] did not locally align neighbouring drumlins with each other, but participants the GMapping workshop [Hillier et al., 2014] felt that this was critical. Modified DEMs with this property included were therefore provided, although in the end this proved to be a minor effect. Similarly, what must be captured well in a synthetic DEM may critically vary between studies. This is exemplified by the impact of life (e.g., buildings, earthworks, trees, eco-geomorphic work by worms), which may either be inconvenient 'noise' [e.g., Hillier & Smith, 2012] or the morphology of interest [e.g., Dietrich and Perron, 2006].

A more subtle potential issue is circularity. It is important to avoid basing aspects of a synthetic DEM on an assumption, and then using it to support the assumption. This is easily avoided in simple synthetic DEMs but a synthetic DEM based on a landscape evolution model, for instance, should not later be justified because a search algorithm trained on it finds only similar features in a real landscape; the algorithm might just be missing things in the real landscape that differ from what it has been trained to detect. A similar issue was faced by Hillier and Smith [2012], but demonstrably avoided as the filter later found to be optimal was not the one initially assumed [Hillier and Smith, 2014].

So, subjectivity is reduced, even synthetic tests using basic DEMs can give some insight, and circularity can be avoided. On balance we argue that, if designed appropriately and used with appropriate care, tests using synthetic DEMs are worth the cost in time as they can be used to access results and insights of real significance and power. Exactly the same can be said for the application of statistical techniques, and so it seems reasonable to advocate the use of synthetic tests with similar strength.

By making observations more robust synthetic tests using synthetic DEMs containing *a priori* known landforms have the potential to strengthen the insights that can be gained through synthetic DEMs generated using physics-based numerical models, i.e. Landscape evolution models. LEMs can provide useful insights, but they are not the entire solution; firstly, they cannot model all processes yet, and secondly they are insufficient without synthetic tests to secure the observational part of the linkage between measured and generated DEMs. It is also worth noting that landscape evolution models are not the only route to creating a from-process link since the other routes described (e.g., statistical) also provide a quantitative means of establishing a form-process link even without a LEM. Thus, there are a number of valid types and specific uses of synthetic DEMs, but in combination we believe that they form a vital underpinning for the quantitative future of landform analysis [e.g., see McCoy, 2015].

5 Conclusions

From this discussion on the uses of synthetic digital landscapes (i.e., DEMs), or synthetic elements within them, the following overarching points can be drawn:

- Synthetic DEMs can help to link physics-based models of processes to morphological observations, allowing quantitative hypotheses to be formulated and tested; importantly, this is not only through the use of landscape evolution models.
- By establishing ‘absolute’ answers tests using synthetic DEMs containing *a priori* known landforms are a powerful tool with which to test and add rigor to geomorphological observations, and arguably should become as standard as statistical tests in geomorphology or synthetic test data in other arenas (e.g., Geophysics).
- A ‘perfect’ synthetic DEM faithfully representing all aspects of an environment is likely impractical or impossible to create at present, but is not necessary.
- Synthetic DEMs for tests may be easy and simple to construct, yet still provide valuable insights.
- Synthetic tests using DEM’s should be tailored to each research question, and their appropriateness to the key aspects of each inquiry (e.g., resolution, biases, and sensitivities) set out clearly and logically.

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Figures

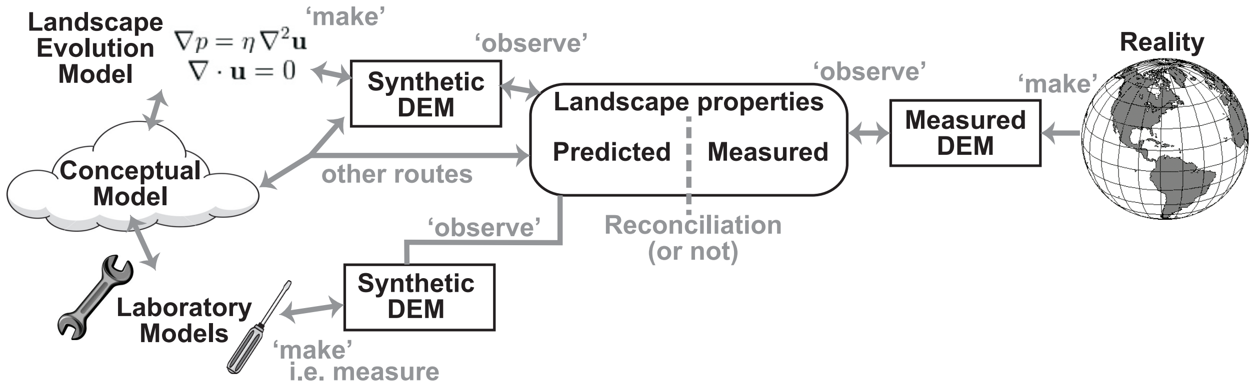

Fig 1: Illustration of the pathways and stages in reconciling geomorphological models with reality in order to understand the physical processes that sculpt planetary surfaces. Stages are in black, and tasks undertaken to move between them are in grey, with double-headed arrows indicating possible feedbacks. Synthetic DEMs may be created through various routes, and may be employed to add rigor to both the making of DEMs and the observing of them to derive landscape properties.

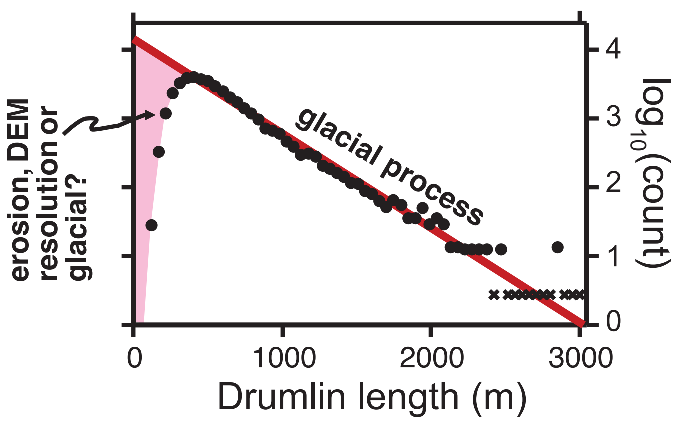

Fig. 2: Semi-logarithmic frequency plot of the lengths, *L*, of UK drumlins adapted from Hillier et al. [2013]. Black dots are data digitised from Fig. 8 of Clark et al. [2009], with a bin width of ~50 m. Red line is the exponential trend. Crosses indicate zero counts, placed at a nominal value of 1. Aspects of the curve are speculatively associated with processes, glacial or related to erosion and DEM construction.

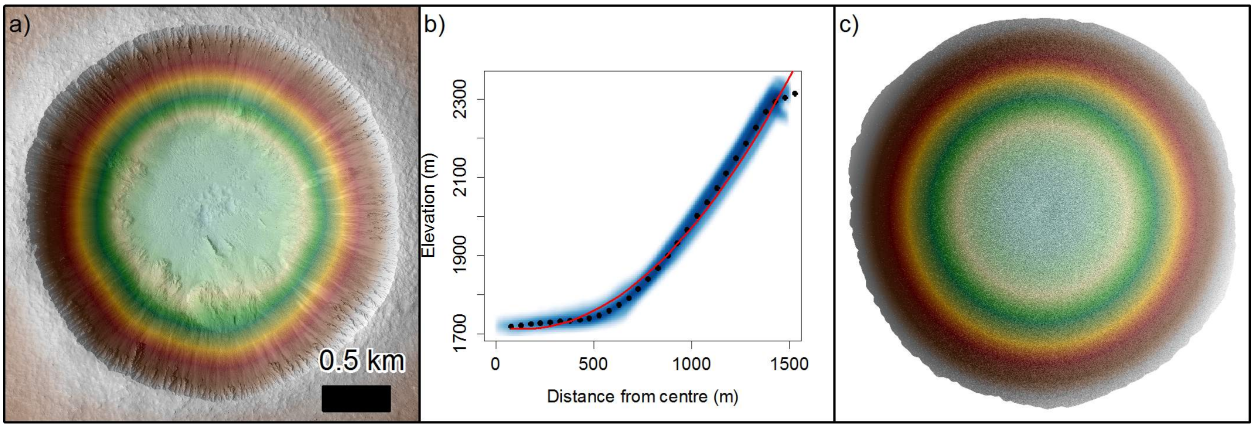

Fig. 3: a) HiRISE image of Zumba crater on Mars coloured according to elevation; HiRISE image DT2EA_002118_1510_003608_1510_A01 and DEM DTEEC_002118_1510_003608_1510_A01, credit NASA/JPL/UofA. b) Radial elevation profile; blue shading illustrates the data distribution, black dots are averages within 50 m distance bins, and the red line is a parabolic fit to those points. c) A synthetic crater created by rotating the parabolic equation, overlain by uncorrelated Gaussian noise and displayed as in a).

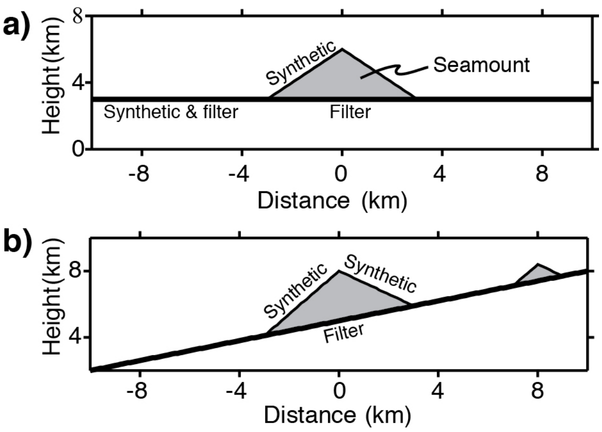

Fig. 4: a) A simple 2D (i.e., distance-height profile) synthetic seamount (grey shading) [Hillier, 2008], which following Kim & Wessel [2008] is conical with a radius of 3 km and summit height of 3 km above the surrounding seafloor. The thin black line is the synthetic topography, and the thick black line the filter's output b) A more demanding test of two, variably sized seamounts upon a sloping surface.

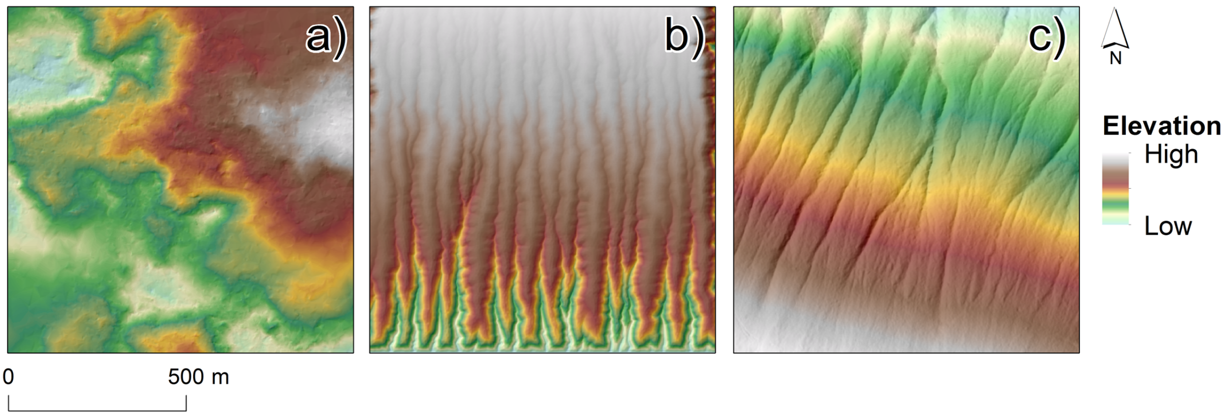

Fig 5. Comparison of simulated DEMs in a) and b) with LiDAR measurement of a real landscape in the South of Italy in c). a) Fractional Brownian motion [Mandelbrot, 1983]; initial roughness of the surface = 0.2, initial elevation of the surface = 0.0, and change of roughness over change of terrain = 0.005. Output is dimensionless, but is effectively given the same scale and resolution as c) by assigning each pixel a 2x2 m size. b) A landscape model [Refice et al. 2012] that evolves through time a southward-dipping initial topography containing small-scale randomness, with all 4 boundaries closed except lower right corner. Simulated time is ~30 kyr and the run parameters are: tectonic uplift u_f = 1 mm/yr; diffusivity constant k_d = 0.2 m^2/yr; with channelling parameters of $K_c = 10^{-4}$ $m^{(1-2m)}$/yr, m = 0.5, and n = 1. The spatial dimensions of b) are as in c). Centroid in c) is 14°37'59.46"E, 40°43'25.80"N.

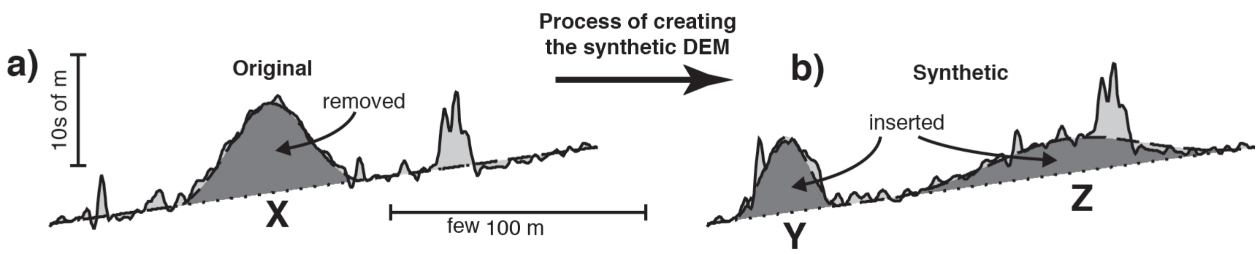

Fig. 6: Idealised distance-height profiles to illustrate the process used by Hillier and Smith [2012] to create synthetic DEMs. There are three 'components'. Drumlins, that are shaded dark grey, rise above a regional trend indicated by a dotted line. These are overprinted by 'clutter' or 'noise' shown in light grey. a) In the process the upper and lower surfaces of the drumlin (X) are estimated to define it, and its height is subtracted from the measured DEM. b) Two Gaussian shaped drumlins (Y and Z) are then inserted by adding their height to create the synthetic DEM.

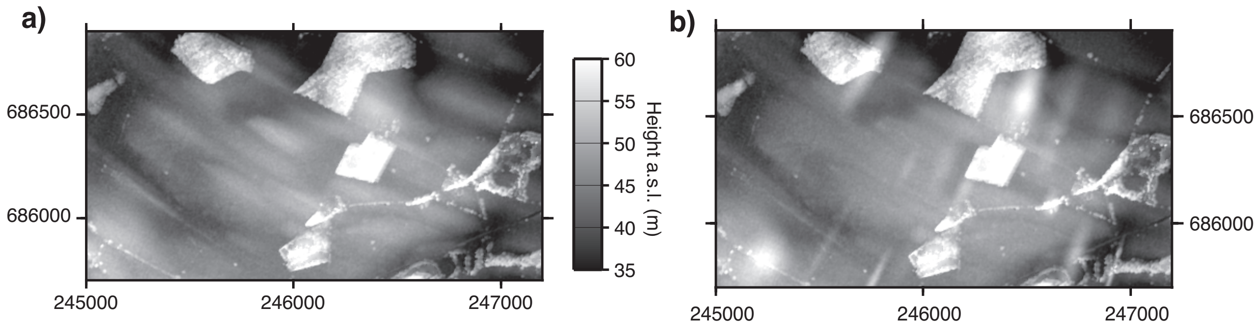

Fig. 7: Illustration of a real DEM in a) and a 'hybrid' synthetic generated from it. Method used is as in Fig. 6 [Hillier and Smith, 2012], adapted to locally align drumlins with each other [Hillier et al., 2014]. Map coordinates are of the British National Grid (5 m grid). Synthetic drumlins were orientated at 90 degrees to the original to avoid any possible confusion with any incompletely removed original ice flow fabric during the mapping exercise.