# Interactive comment on "Gravel threshold of motion: A state function of sediment transport disequilibrium?" by Joel P. L. Johnson

**D.R Parsons (Editor)**
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Dear Joel, The two reviewer reports are available for your manuscript. Both raise a few areas where they feel that the manuscript could be improved. I will be happy to recommend that the manuscript be accepted subject to the minor revisions based on these comments. If you could respond to the reviews and supply a rebuttal that details the changes made to the script that would be appreciated. Regards Dan

Thank you for the opportunity to revise the manuscript. Following the comments and recommendations of all three reviewers, I have rearranged parts of the manuscript, added more detail to make parts of it be more organized and less abstract, and have also added recommended citations. Following reviewer recommendations, I have streamlined some sections and shortened the manuscript wherever I felt possible. Because of additional clarifications and explanations requested by the reviewers the manuscript is slightly (~4.5%) longer as measured by text, but has 1 fewer figures. More importantly, I believe it is more clear and focused.

Following the recommendations of both Turowski and Reviewer2 that the physical processes causing changes to thresholds of motion be described in more detail and combined in one place, the biggest change I made to the manuscript is moving parts of two sections that discuss previous work on evolving thresholds of motion—part of previous section 3.2 that discussed the sand dependence of reference stresses in the Wilcock and Crowe (2003) model, and also most of previous section 4.1 ("Comparison to previous work"), which discussed Recking (2012) relations—into the introduction. These are now section 1.1. In this way I have one section that better describes the many various physical controls on thresholds of motion.

Because of concerns over length brought up by Reviewer2, I cut one figure (previous Figure 11) and the section of text that went along with it. I removed this part because, while interesting, I felt like it was less central to the science than the other parts of the work. The other 10 figures are essentially unchanged.

**JM Turowski (Referee)**
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In this manuscript, the author discusses the implications of the idea that the threshold of motion is an evolving function of sediment supply. This leads to a re-definition of the threshold as a state variable in analogy to thermos dynamics. The concept is interesting and provides a fascinating change of view. My major concern is that the author does not make the above-stated re-definition explicit and uses the term threshold of motion somewhat interchangeable between the new and the old version. That makes a sometimes confusing read and can be rectified by clarifying the writing and making explicit statements. Further, I think the model is insufficiently put into physical context, and the various mechanisms that can relate sediment supply to the threshold of motion are scattered amongst the different parts of the manuscript. This can be streamlined and clarified. Some further comments to this effect follow in the next few paragraphs.

As recommended, I have moved some of the "scattered" explanations to section 1.1, and expanded the physical explanations. I have also made the "redefinition" of thresholds more explicit, in two ways. First, I have slightly modified my notation: in the previous version I only used $\tau_c^*$ as the threshold variable. In the new version, I have added variable $\tau_{c(q_s)}^*$ to specifically indicate the new sediment flux-dependent model. In addition, I specifically describe the model as a redefinition of the concept of thresholds of motion (new lines 641, 646, 735).

The physical explanations that have been proposed for the observed dependence on the threshold mostly relate to properties that the author summarized as bed state controls. Recking argued that the observed variability could at least partly be connected to changes in interlocking and armoring (see e.g., his figure 6), and Bunte et al. related the variability to bed stability, which is also dependent on properties such as interlocking. There are two possible explanations that are directly dependent on transport conditions: collective entrainment, in which moving particles mobilize stationary ones by knocking them out of their position. This mechanism has been advocated recently by Ancey and co-workers in a series of paper and demonstrated in 2D-experiments (e.g., Ancey et al. 2008; there are newer articles also available), but is highly debated by researchers working on 3D systems. The second one is the effect of fine material (sand) on the mobilization of gravel (e.g., Curran and Wilcock 2005). Although the latter could be argued to be a bed state control (the sand falls into pockets between gravel grains and therefore reduces roughness). I think the physical mechanisms that lead to the equations derived in the paper need to be better worked out and discussed, and the difference between bed state controls and direct controls of sediment supply need to be clarified. I am also not sure whether the equations actually differentiate between these two mechanisms.

I have worked explicit descriptions of these processes and citations into the manuscript, both in section 1.1 and also 2.1, where previous work is reviewed, and also where the new model is presented conceptually.

The mechanism described by the author (during erosion, grains in pockets that are least stable move first, while during deposition grains stop in pockets that are most stable) could arguably be also classified as a bed state control, as it is depends on the availability of pockets of a certain degree of stability.

Good point. I am now more clear that my categorizations of threshold controls are not absolute, that the controls are interrelated, and that many controls could be categorized in different ways (section 1.1; new lines 84-87, 92-95 for example).

Further, the described mechanism in my mind only holds if either the supplied grain size distributions systematically change, or if deposition / erosion lead to systematic compaction or loosening of the bed. Consider a bed of a single grain size. By depositing a single grain, clearly it fills a pocket, but it also creates new pockets. It can be plausibly argued that the average state of the bed (roughness etc) does not change systematically in this way.

Finally, if the mechanism holds as described, there would be a feedback to roughness: deposition in stable pockets reduces the number of stable pockets, which means a smoother bed and higher flow velocity, which in turn makes each of the pockets less stable (similar to the effect of adding sand to a gravel bed, see Curran and Wilcock 2005). This would be a feedback limiting the variability of the threshold.

Good points. To address this, I have expanded the description of feedbacks in section 2.1 (the conceptual model). I now explicitly say in this section that there are physical limits of how much bed roughness and other controls can change (new lines 260-265). These limits were already built into the model equations before, but were previously not described well enough conceptually.

31 Please give some references for the statement here.

I added five references (new line 29-31)

48/50 Two consecutive sentences that are both starting with 'in practice'.

Rearranged and combined sentences to remove the repetition.

55 maybe add 'typically' here

Done (new line 60).

57 yes, but slope is a proxy for other parameters such as roughness, rather than a direct control

I agree; this is now stated directly (new lines 63-66).

53-74 Turowski et al. 2011 demonstrate both the large temporal variability of the threshold and its control by grain and bed properties for several mountain streams. Chen and Stone 2008 explained some of the variability of measured bedload transport rates with local sub-sampling of the overall grain size distribution, leading to spatially varying thresholds of motion. This is also related to recent work on patch dynamics.

I have added description and reference to these works, and also now state that patches influencing thresholds of motion and transport (new lines 66-69, 99-101).

77 I am not sure whether I totally agree. See major comment.

I have now clarified how I categorize controls on thresholds of motion, simply for the sake of describing controls in an organized manner. I have also added a separate category of sediment flux controls (new lines 84-87, 142-170).

93 comma missing after (vertical position)
Added comma

136 Individual grains each have a different threshold…
Done (cut the word "will")

142-143 inconsistent: does tau*_c follow a probability distribution (implying it is a random number) or is it constant?

I have clarified the relationship between distributions of threshold values for a population of grains on the bed surface, and the single threshold value that would best describe transport when applied in a bedload transport equation (new lines 229-239).

145 and following: overuse of future tense: Progressive erosion entrains… grains tend to preferentially deposit…

I have changed writing to be present tense, here and elsewhere.

147-148 This makes intuitive sense. Are there any data on this?

I wish there were, but I am unaware of data showing this. I have addressed this comment by adding "I assume" to make it clearer that this is an assumption of the model (new lines 244-246).

148-149 I am not entirely convinced by these arguments. It assumes that deposition systematically changes bed-averaged roughness. See major comment.

The reviewer is right, it does generally assume that bed-averaged roughness changes. I now clarify in this section that there are limits to how far thresholds of motion can evolve (new lines 260-265).

158-160 Unclear why it was necessary to make this point. Please elaborate.
I have cut this part.

207 unit missing after 4.
Added.

208 does the use of 'initial' imply here that slope was changed during the experiments?

I have rearranged text to now have the callout to Figure 2 sooner, which shows (minimal) slope evolution during the experiments. The data were included in the Figure in the previous version, but it was less clear. (new lines 319-325).

227 What does 'very low' mean here?
I have reworded the text to say that flux dropped by approximately 3 orders of magnitude, and also have a callout here to Figure 2a that shows how the transport rate changed through time. (new lines 343-344).

260 The hiding function exponent…

Added “The” as requested.

294 Which experiments? New paragraph, reference is unclear.
Edited to be clear what data is being talked about (new lines 405-407).

300-312 Curran and Wilcock 2005 should be cited somewhere here.
Added (new line 414).

304 change ‘with no’ to ‘without’.
This wording was cut during editing.

332 Undefined abbreviation RMSD.
Done (new line 433)

352 Please give the full reference.
Done (new line 451)

462 Turowski et al. 2011 should be discussed in this chapter.
I now reference this work in multiple places in the manuscript. This particular section of text has been moved to the introduction.

489-492 So, how does the model relate to the data, then?
I have expanded this paragraph a bit to more to explain how the model can be consistent with and explains previous observations of Recking (2012) and Bunte et al. (2013), but at the same time the model does not depend directly on sediment flux, but on changes in sediment flux. (new lines 561-570).

584 There needs to be at least a brief description of Phillips’ concept; it cannot be assumed that the reader is familiar with that paper.
Done, new lines 648-652.

589-605 The comparison with thermodynamics is interesting, but I wonder in how far it is novel. In the end, in river morphodynamic modelling, channels have been treated using concepts similar to state variables and state functions, they just have not been explicitly called such. Note that recently Furbish and co-workers applied concepts from statistical mechanics to bedload transport (e.g., Furbish et al. 2012, series of 4 papers in WRR and JGR).
While I thought the previous version acknowledged that similar ideas have been implicitly used, I now state this directly (new lines 647-648). I now cite the Furbish work earlier in the manuscript (new lines 217-221); it does not bear direct relevance to rate and state variables, though is excellent work applying ideas from physics.

610 This statement involves a redefinition of tau*_c, and this should be made crystal clear.
I now explicitly state that the threshold of motion is defined as a state variable (new line 641, 646, 677).

Fig. 4, caption: typo in matching, 3rd line.
Corrected.

References

Ancey, C.; Davison, A. C.; Böhm, T.; Jodeau, M. & Frey, P. Entrainment and motion of coarse particles in a shallow water stream down a steep slope, J. Fluid Mech., 2008, 595, 83-114

Chen, L. & Stone, M. C. Influence of bed material size heterogeneity on bedload transport uncertainty, Water Resources Research, 2008, 44, W01405

Curran, J. C. & Wilcock, P. R. Effect of sand supply on transport rates in a gravel-bed channel J. Hydr. Eng., 2005, 131, 961-967

Furbish, D. J.; Haff, P. K.; Roseberry, J. C. & Schmeeckle, M. W. A probabilistic description of the bed load sediment flux: 1. Theory, Journal of Geophysical Research, 2012, 117, F03031

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**Anonymous Referee #2**

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Review of 'Gravel threshold of motion: A state function of sediment transport disequilibrium' Earth Surface Dynamics (esurf-2015-52) Joel P. L. Johnson

This paper uses flume experiments and a morphodynamic model to assess the impact that sediment supply has on the evolution of thresholds of motion. The topic of the paper is of interest to readers with some interesting findings that are applicable to the wider discipline. However at present the paper is quite, long, 'dense' and difficult to read in parts meaning that the novelty of the paper is somewhat lost in places. The main comment I feel which needs addressing in this paper is the lack of emphasis on the physical underpinnings of how sediment supply affects the thresholds of motion. Whilst the author makes reference to the bed state conditions in the introduction he does not really follow those through in terms of the implications of his findings. This currently leaves the reader wanting more detail in this regard. There are many papers which talk about the effects of both bed state in terms of structure as well as sand content on entrainment thresholds. I think the latter is particularly important for this paper and the author could look at the following papers as a starting point.

I appreciate the comments, and have worked to address them by reorganizing and expanding on the physical explanations for why thresholds of motion can change over time. This topic is presented in section 1.1, and also 2.1. While the effect of sand on thresholds of motion was addressed in the previous version, it is now discussed much more prominently near the beginning of the manuscript.

Curran, J.C. and Wilcock, P.R. (2005). Effect of sand supply on transport rates in gravel bed channels. Journal of Hydraulic Engineering. 131:961-967

Ikeda, H. and Iseya, F.(1988). Experimental study of heterogeneous sediment transport. Environmental Research Centre Paper 12. University of Tsukuba; Japan.

Jackson, W. L., and Beschta,R.L. (1984). Influences of increased sand delivery on the morphology of sand and gravel channels. Journal of the American Water Resources Association. 20; 527–533.

I now reference these papers in regards to sand supply and thresholds of motion.

I also feel the paper could benefit from being shortened as it is currently quite long and loses focus in places. Detailed comments are also given below.

I have tried hard to improve the paper by following the comments of all of the reviewers. This includes adding material to explain many points further, hopefully making the manuscript less dense. While I have also cut text (in particular Figure 11 and the text along with it), unfortunately the manuscript is now slightly longer as measured by text. However, I also believe that it is more clear and understandable. The revised manuscript better guides the reader and explains why certain things are being presented, hopefully helping it keep its focus (for example, new lines 79-82, 203-211, 386-389, and 545-548).

Line 83- I am not sure I agree with the statement that is still only believed to be controlled by grain parameters. There is an increasing recognition that, as the author alludes, bed state controls are also important. I think at the very least this should be recognised in the current text and references made to the large body of work relating to the impact of structure on bed stability. How does this also link to the concept of mobile armours? You go on to mention this in lines 153-157 so this section could be reorganised?

Section 1.1 now more clearly lays out the relations and overlap between what I am categorizing as grain controls and bed state controls (new lines 84-87). I now specifically address and reference armoring in relation to both of these categories (new lines 92-95). Because space is limited I do not expand on the differences between mobile and static armors.

Line 93- comma missing after vertical position
Added comma.

Lines 123- 131- this section is clumsy and needs re-writing
I edited this section to use active voice and to be less awkward. (new lines 203-211).

Line 141- 143- does this not assume that the bed state does not change? You could have the same overall flux of sediment but the surface structure may change and hence the distribution of threshold stress will thus change as the bed is more stable?
I rearranged this section to have the mention of steady state at the end of the section rather than at the beginning, to more clearly explain how the proposed feedbacks work. I also expanded specifically on the case of steady state threshold of motion based on this comment: Yes, the threshold could evolve under constant flux, but because sediment transport rate and the threshold of motion are directly linked, a change in threshold would change the transport rate (new lines 268-274).

Line 155- consider revision of little additional decrease
Changed wording.

Lines 158 – 167 – I think if you are using the terms interchangeably throughout the paper then there is no need for this paragraph at all.
I got rid of some of this paragraph, and shorted and moved part of it to elsewhere (new lines 52-57). I believe it is important to explicitly state what threshold stresses and reference stresses are, and to justify using them interchangeably.

Line 185- should be dimensional not dimensionally

Changed.

Line 189 – move 'Ar is an optional dimensionless armouring parameter, described further below' to line 198 where you talk about Ar. I think the Ar should be defined as it can have different definitions.

Done; I moved text around and also expanded on this portion, to explain and justify this model parameter (new lines 307-316).

Line 202- this sentence does not make sense- do you mean large grains rather than large range?

That is exactly what I meant; I somehow put the wrong word. I'm glad the reviewer was able to figure out the intention of the sentence.

Line 205- although this paper is concentrating on step pool sequences perhaps something to consider later on in the paper is how applicable these results are to gravel bed rivers more broadly e.g. at lower slopes?

I have added a statement to the discussion that the model parameters were calibrated to these particular steep slopes, and that future work is required to validate the model over a broader range of parameter space. (new lines 635-639).

Line 207 – unit missing after flume length

Done, added m.

Line 226- can you be more specific- how much erosion?

In the interest of clarity and length, I edited the text to focus on the bed responses (coarsening and roughening and sediment transport rates) that matter more for my analysis; in doing so I cut the explicit mention of erosion. (new lines 343-344). Erosion amounts are less informative and somewhat different for the different experiments.

Line 227 – what does 'very low' mean? Can you quantify?

I have reworded the text to say that flux dropped by approximately 3 orders of magnitude, and also have a callout to Figure 2a that shows the transport rate changes through time. (new lines 341-344).

Line 228 – why was this feed rate chosen? What was this rate in comparison to the initial transport rates?

I added an explanation of why the feed rate was chosen—"this feed rate was chosen to be similar to the high initial transport rates (Fig. 2a), while not so high as to inhibit morphodynamic feedbacks by fully burying the stabilized bed surfaces." (new lines 347-349).

Lines 243- consider deleting to GSDs compared

Done

Lines 237- why was the Wilcock and Crowe model specifically used?

I now give the specific reasons that I used this model: because it can account for both surface grain size changes and also let me evaluate whether sand supply can explain the experimental transport trends. (new lines 359-365).

Lines 237- 265- can this section be shortened? Why not just reference the W&CM highlighting the changes you made to it?(lines 262-263)

I considered cutting some of the equations that are Wilcock and Crowe (2003) model, but in the end decided to leave them in. I believe that cutting Eq. 13 and 14 (previously 10 and 11), which show what nondimensional bedload transport rate means and how thresholds of motion actually go into the transport relation, would make the paper more difficult to understand for most readers, especially those not intimately familiar with W&CM. Also, since I made changes to equations 15 and 16 (previously 12 and 13) I would have to leave those equations in the manuscript; the length of writing actually cut would be pretty small.

Lines 313 -316 – what was your GSD? This is important if you are beginning to duscss sand content and the mechanisms by which sediment feed rate affects initiation of motion? Also in line 313 you mention that the % of grains smaller than 2mm was very small bu tin lines 316 you say 2.8mm was your smallest grain fraction?

I now clarify and describe more completely the full grain size distributions used in the flume experiments; relevant here is that the smallest size class used in the experiments had a D16 of 2.0 mm, D50=2.4 mm, and D84 of 2.8 mm (new lines 327-330). I clarify that this was the size class used for the calculations of sand fraction (new lines 414-422).

Lines 332 – define RMSD
Done (new line 433)

Lines 336 – 344 – this is an interesting finding but what are the implications of this in terms of bed state?
I now give a suggestion of why my model was seemingly insensitive to having combinations of D84, D50, D16 and bed roughness included as another parameter in the model—it may suggest that net erosion and deposition were more important over the range of parameter space explored in the experiments (new lines 440-442).

Line 352 – need full reference to Parker
Done, new line 451.

Line 473- change stresses to stress
Done

Line 474- I am not sure they are comparable are they? Again thinking in terms of the relative effects of bed structure and implications of grain size, structure and thresholds of motion would D50 and D84 be expected to behave the same?
I clarified the writing; I was not trying to suggest that D50 and D84 thresholds would necessarily be or should be expected to be interchangeable. I simply make the point that Recking's relation is not too far off from my thresholds of motion, although the R^2 value is still low (new lines 552-560).

Line 475- what do you mean by 'fairly comparable'?
Reworded to say I do not expect these different threshold measured to be equivalent (new lines 553-554).

Lines 503-506- I think this is one of the places where a better physical explanation behind the findings would be useful
Good point. I added a substantial amount of text. First, I now acknowledge that future work would be required to really determine specific process linkages explaining asymmetry in aggradation vs degradation effects on thresholds of motion (new lines 587-589). Second, I present a hypothesis that could be tested with future work about the differences in deposition, erosion and roughness evolution (new lines 589-606).

Lines 530 – this section is supposed to be linked to system memory but I find it hard to distinguish this and a much more explicit link needs to be made.

While the section talked about system memory, it also covered other topics. In the interest of article length I removed the section (old section 4.3 in the previous manuscript, and also figure 11) and also cut most of the content. I did however keep and expand slightly on parts related to memory, these are now at new lines 628-639.

Lines 545-546 – I would re-write to avoid asking a rhetorical question

This sentence was cut during editing.

Lines 576 – whilst I find this section an interesting concept I think it could be shortened a lot given the paper is already quite long.

I shortened this section by roughly 23% through editing (new section 4.2). However, I feel like it is an important idea to explain thoroughly, and I really do not want to remove more of it. I did cut substantial portions of old section 4.3 in the previous manuscript, and also old figure 11, in order to keep the manuscript focused and not even longer.

Line 584- expand upon the work of Phillips (2007)

Done, new lines 649-652.

Line 625 - I would re-write to avoid asking a rhetorical question

Done. I also edited out some other rhetorical questions in the manuscript.

**Anonymous Referee #3**

Received and published: 29 June 2016

I believe this is an overall excellent piece of work, written by an expert in the field. The issue of sediment transport is a long studied problem and much attention has been paid to traditional criteria, such as Shield's critical shear stresses (as the author notes himself). There are a number of problems using such criteria - as the author mentions in his work (also demonstrated in Fig. 1). However, the author still chooses to deploy this criterion focusing on the fact that data scatter (e.g. in Fig.1) is due to a range of factors, however omitting to discuss its inability to represent the rich dynamics of grain transport, as recent research has shown (Schemeeckle et al. 2003, Diplas et al. 2008).

The major novelty of the present work lays in the presentation of a state function for the description of sediment transport, which is a very much welcomed contribution as a conceptual approach. However, there is a significant concern (to this reviewer) over the suitability of the Shield's shear stress as parameter to be used in this model. Would not other more criteria that capture the full range of grain dynamics, such as instantaneous hydrodynamic forces near the bed or even better the impulse/energy content of flow structures, be more suitable as model parameters? Of course such analysis may offer enough new material for another (and perhaps more impactful) publication, but yet it may be useful to add a note about this on the discussion section.

I appreciate the review and the different perspective it provides. I now cite the work by Schmeeckle and Diplas, and also statistical mechanics descriptions of bedload transport by Furbish et al at the start of section 2.1 (new lines 219-220). In lines 214-221 I also address the reviewer's comment in another way, by more specifically defining the narrower "parameter space” of the model, and the limits of the model.  I explicitly state that the model intentionally does not describe timescales of turbulent velocity fluctuations, and I also state that the model is deterministic rather than stochastic.  I agree that there are rich bedload transport dynamics over timescales of turbulent velocity fluctuations.  I also believe that my model is new and novel in its ability to explore rich morphodynamic feedbacks that have not yet been modeled well, over timescales longer than turbulence.

Another, minor issue is with the interpretation of the data analysis. In particular, is there no better measure to assess the "amount of information embedded" between two variables than R2? R2 is rather demonstrative of the strength of association between two variables.

I also use RMSD (and define it) in the manuscript.  While I am interested in finding new and better statistical tests that could do more than determine the strength of corrrelations between variables, I do not know what other statistics would actually be better for my applications, and I respectfully note that the reviewer does not provide any specific suggestion for statistical tests to include either.

# Gravel threshold of motion: A state function of sediment transport disequilibrium?

**Joel P. L. Johnson**

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## Abstract

In most sediment transport models, a threshold variable dictates the shear stress at which non-negligible bedload transport begins. Previous work has demonstrated that nondimensional transport thresholds ($\tau_c^*$) vary with many factors related not only to grain size and shape, but also with characteristics of the local bed surface and sediment transport rate ($q_s$). I propose a new model in which $q_s$-dependent $\tau_c^*$, notated as $\tau_{c(q_s)}^*$, evolves as a power-law function of net erosion or deposition. In the model, net entrainment is assumed to progressively remove more mobile particles while leaving behind more stable grains, gradually increasing $\tau_{c(q_s)}^*$ and reducing transport rates. Net deposition tends to fill in topographic lows, progressively leading to less stable distributions of surface grains, decreasing $\tau_{c(q_s)}^*$ and increasing transport rates. Model parameters are calibrated based on laboratory flume experiments that explore transport disequilibrium. The $\tau_{c(q_s)}^*$ equation is then incorporated into a simple morphodynamic model. The evolution of $\tau_{c(q_s)}^*$ is a negative feedback on morphologic change, while also allowing reaches to equilibrate to sediment supply at different slopes. Finally, $\tau_{c(q_s)}^*$ is interpreted to be an important but nonunique state variable for morphodynamics, in a manner consistent with state variables such as temperature in thermodynamics.

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## 1 Motivation

Despite over a century of quantitative study (Gilbert, 1914), it often remains challenging to predict gravel transport rates to much better than an order of magnitude because of the complexity of grain interactions with the flow and the surrounding grains (e.g., Schneider et al., 2015; Nitsche et al., 2011; Rickenmann, 2001; Wilcock and Crowe, 2003; Chen and Stone, 2008). Predictive models for complex systems often derive utility from their simplicity, as is the case with the widely-used Meyer-Peter and Müller (1948) transport equation, as modified by Wong and Parker (2006):

$$q_s^* = 3.97\left(\tau^* - \tau_c^*\right)^{1.5} \qquad \text{for} \qquad \tau^* \geq \tau_c^* \tag{1}$$

where $q_s^*$ is a nondimensional sediment transport rate per unit width, $\tau^*$ is a nondimensional shear stress imparted by the fluid on the channel bed (a Shields stress), and $\tau_c^*$ is the nondimensional threshold stress at which grains begin to move (a critical Shields stress). Variables are nondimensionalized as follows:

$$q_s^* = \frac{q_s}{D\sqrt{\left(\frac{\rho_s}{\rho} - 1\right)gD}} \tag{2}$$

$$\tau^* = \frac{\tau}{(\rho_s - \rho)gD} \tag{3}$$

where $q_s$ is volume sediment transport rate per unit width (m$^2$/s), $D$ is grain diameter (m), $\rho_s$ is sediment density (m$^3$/kg), $\rho$ is water density (m$^3$/kg), $g$ is gravitational acceleration (m/s$^2$), and $\tau$ is shear stress (Pa). In principle, these nondimensionalizations should account for differences in grain size, fluid and sediment density and gravity, allowing meaningful comparisons of transport and stress across different conditions. For a given grain diameter (and constant $\rho_s$, $\rho$ and $g$ assumed for terrestrial landscapes), the simplicity of Eq. (1) is that it predicts transport rate using just two variables, $\tau^*$ (a function of flow strength) and $\tau_c^*$ (a function of many variables). In practice, $\tau_c^*$ is often back-calculated from shear stress and bedload transport rate, essentially making it an empirical fitting parameter for a given transport model (e.g., Wong and Parker, 2006; Buffington and Montgomery, 1997). For example, using the original dataset of Meyer-Peter and Muller (1948), $\tau^*$ and $q_s^*$ give best-fit

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$\tau_c^*$=0.0495 for Eq. (1) (Wong and Parker, 2006). Other bedload transport models have been developed that do not use an absolute threshold stress below which transport is zero, but rather a "reference" stress that corresponds to a very low but non-zero transport rate (e.g. Parker, 1990; Wilcock and Crowe, 2003). For most applications the practical difference between threshold and reference stresses are negligible (Buffington and Montgomery, 1997). In the present work, threshold and reference stresses are used interchangeably.

Thresholds of motion for gravel often span an order of magnitude or more (Fig. 1). Variability in $\tau_c^*$ greatly influences bedload flux predictions in mountain rivers because transport typically occurs close to thresholds conditions, even during large floods (Phillips et al., 2013; Parker et al., 1982; Parker and Klingeman, 1982). Previous work has demonstrated that a great many factors collectively cause $\tau_c^*$ scatter (e.g., Buffington and Montgomery, 1997; Kirchner et al., 1990). Slope can empirically explain 34% of the variability shown in Fig. 1 data. However, other variables including the strength of turbulent velocity fluctuations, and flow depth relative to bed roughness, also vary with reach slope and have been interpreted to influence $\tau_c^*$ mechanistically (Lamb et al., 2008). In addition, thresholds can change temporally: using field data, Turowski et al. (2011) demonstrated that threshold discharges for the start and end of bedload transport could change by an order of magnitude during a given flood event.

Although thresholds of motion may dynamically evolve over time, we suggest several reasons why an assumption of constant $\tau_c^*$ remains ingrained in some studies. First, the traditional Shields diagram indicates that $\tau_c^*$ is rather insensitive to particle Reynolds number once flow becomes hydraulically rough around grains (Buffington, 1999). Second, because the best estimate of a given variable is usually its average, there is a tendency to attribute variability to measurement noise and uncertainty, even when that variability may be real, understandable, and important to system dynamics (Jerolmack, 2011; Buffington and Montgomery, 1997; Chen and Stone, 2008). Third, a broadly applicable model for the temporal evolution of $\tau_c^*$ has arguably not been developed, although progress has been made (Recking, 2012; Bunte et al., 2013; Wilcock and Crowe, 2003). Next in this section, I summarize previous work on $\tau_c^*$ controls, suggest ways that evolving $\tau_c^*$ may influence

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gravel-bed river morphodynamics, and then propose specific hypotheses to be explored with a new model for $\tau_c^*$ evolution.

## 1.1 Previous work: mechanistic controls on $\tau_c^*$

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In order to review previous work in an organized manner, factors affecting $\tau_c^*$ are categorized as (a) grain controls, (b) bed state controls, (c) discharge controls, and (d) sediment flux controls, while acknowledging that many specific factors are interrelated and can be classified in more than one category. The literature on thresholds of motion is vast; I highlight select papers while acknowledging that many contributions are not explicitly reviewed.

Grain controls are physical characteristics of individual clasts that influence $\tau_c^*$. In addition to diameter and density, these include shape and angularity (e.g., Prancevic and Lamb, 2015; Gogus and Defne, 2005). By controlling surface grain size, armoring acts as a grain control (e.g., Dietrich et al., 1989; Parker and Toro-Escobar, 2002). However, the grain size distribution (GSD) of the surrounding bed has also been shown to strongly influence $\tau_c^*$; armoring can therefore also be a bed state control. In many mixed grain size transport models, hiding/exposure functions quantify the observation that grains smaller than the average bed surface tend to be relatively less mobile than expected based on diameter alone, while grains larger than average tend to be relatively more mobile than expected based on their diameter (e.g., Parker, 1990; Wilcock and Crowe, 2003). Spatial heterogeneity in surface GSDs, whether randomly distributed or sorted into patches, can also influence local $\tau_c^*$ (Chen and Stone, 2008; Nelson et al., 2009). Mechanistically, contrasts in diameter between a grain and the surrounding bed affects pocket geometry. On rougher beds, grains tend to protrude less into the flow and therefore tend to be more stable (higher $\tau_c^*$).

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Sand content is a related GSD bed state control: increasing sand content of alluvial bed surfaces has been shown to decrease gravel thresholds of motion (e.g., Curran and Wilcock, 2005; Iseya and Ikeda, 1987; Jackson and Beschta, 1984). Wilcock and Crowe (2003) explicitly incorporated this sand dependence into their transport model:

$$\tau_{rm}^* = c_1 + c_2 e^{-c_3 F_s} \quad (4)$$

where $\tau^*_{rm}$ is a reference stress (rather than an absolute threshold) for the geometric mean diameter of the bed surface GSD, $F_s$ is the spatial fraction of sand on the bed surface, and constants $c_1$, $c_2$ and $c_3$ were empirically calibrated from flume data to be 0.021, 0.015 and 20 respectively. These values result in $\tau^*_{rm}$ varying between 0.021 and 0.036, which is in the range of typical $\tau^*_c$ (Figure 1). Subsequent work has shown that the effects described by Eq. 4 are not unique to sand sizes only. Thresholds of motion for intermediate surface diameters (e.g. $D_{50}$) can similarly be reduced by grains substantially smaller than the bed surface but larger than 2mm (Venditti et al., 2010; Sklar et al., 2009; Johnson et al., 2015). Mechanistically, the addition of sand or finer gravels smooths the bed surface by preferentially filling local topographic lows, which can affect pocket geometries (making it easier for larger grains to rotate out of a stable position), and also reduce local hydraulic roughness, increasing near-bed velocity and increasing drag on protruding grains.

Many studies have explored the bed state control of stabilizing structures formed by coarse grain clusters (e.g., Church et al., 1998; Strom and Papanicolaou, 2009). Other bed state controls include the degree of overlap, interlocking and imbrication among grains, and bed compaction or dilation (e.g., Parker, 1990; Wilcock and Crowe, 2003; Sanguinito and Johnson, 2012; Buscombe and Conley, 2012; Mao, 2012; Kirchner et al., 1990; Strom and Papanicolaou, 2009; Marquis and Roy, 2012; Powell and Ashworth, 1995; Richards and Clifford, 1991; Ockelford and Haynes, 2013). By combining experimental data and a numerical model, Measures and Tait (2008) show that increasing grain-scale bed roughness tends to shelter downstream grains, reducing entrainment. Mechanistically, these factors attest to how, even if grain size does not change, grains can move from less stable to more stable configurations. Coarse grain clusters can also enhance bed stability by increasing surface roughness, tending to deepen potential grain pockets.

Flow characteristics influencing $\tau^*_c$ include particle Reynolds number, flow depth relative to grain size, the intensity of turbulence, the history of prior flow both above and below transport thresholds, and the partitioning of stress into form drag and skin friction (e.g., Shvidchenko and Pender, 2000; Ockelford and Haynes, 2013; Schneider et al., 2015; Valyrakis et al., 2010; Celik et al., 2010). Most flow-dependent controls are not independent of the bed surface controls. For example, flow depths, turbulence and form drag depend on slope and bed roughness, while the stress history influences $\tau^*_c$ by changing grain interlocking

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and surface roughness. Mao (2012) showed that thresholds of motion and bed roughness both evolved during hydrograph rising and falling limbs, leading to bedload hysteresis.

Recent work also suggests that sediment transport can affect $\tau_c^*$, with higher rates of upstream supply corresponding to more mobile sediment and lower $\tau_c^*$ (Recking, 2012; Bunte et al., 2013). The idea that transport rate influences $\tau_c^*$ is an intriguing feedback and the focus of the present analysis because, by definition, $\tau_c^*$ influences transport rate (Eq. 1). Mechanistically, mobile grains impacting stationary grains have been shown to dislodge and entrain grains into the flow (Ancey et al., 2008). Empirically, Bunte et al. (2013) interpreted that lower $\tau_c^*$ corresponded to looser beds caused by higher rates of sediment supply from upstream, and noted that the stability of bed particles can be qualitatively assessed in the field while doing pebble counts. Yager et al. (2012b) demonstrated that in-channel sediment availability varied inversely with the degree of boulder protrusion, indicating preferential filling of topographic lows by mobile sediment.

Recking (2012) compared bed load monitoring records from steep natural channels (>5% slope) to differences in sediment supply interpreted from aerial photographs of surrounding hillslopes. Channels with higher supply rates had higher transport rates for a given shear stress, consistent with a dependence of transport thresholds on supply. While stating that deriving a threshold model "taking into account the sediment input as a parameter would be difficult", Recking (2012) proposed quantitative bounds on reference stress for the end-member cases of very high sediment supply ($\tau_{mss}^*$) and very low sediment supply ($\tau_m^*$) in steep mountain channels:

$$\tau_{mss}^* = (5S + 0.06)\left(\frac{D_{84}}{D_{50}}\right)^{-1.5} \quad (5)$$

$$\tau_m^* = (5S + 0.06)\left(\frac{D_{84}}{D_{50}}\right)^{4.4\sqrt{S}-1.5} \quad (6)$$

It should be noted that these reference stress equations describe transport of the $D_{84}$ grain size (rather than say $D_{50}$), using a $D_{84}$-based bedload transport model (Recking, 2012). Importantly, the ratio $D_{84}/D_{50}$ is included in Eq. 5 and 6 to represent surface armoring, which tends to vary with sediment supply (Dietrich et al., 1989), thus relating bed state controls to

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supply-dependent bounds. Overall, this review of previous work suggests that numerous interrelated variables influence $\tau_c^*$, but also that many controls on $\tau_c^*$ may share similar sensitivites to changing bed roughness and sediment supply.

## 1.2 Morphodynamics and hypotheses

Feedback between channel morphology and bedload transport defines mountain river morphodynamics. The Exner equation of sediment mass conservation quantifies how transport changes correspond to topographic changes (Paola and Voller, 2005):

$$\frac{\partial z}{\partial t} = -\left(\frac{1}{1-\lambda_p}\right)\frac{\partial q_s}{\partial x} \qquad (7)$$

where $z$ is bed elevation (vertical position), $x$ is horizontal position, $t$ is time, and $\lambda_p$ is bed porosity. In this morphodynamic equation (presented for simplicity without an uplift or subsidence term), topographic equilibrium ($\partial z/\partial t = 0$) is attained when the sediment flux into a reach equals the sediment flux out ($\partial q_s/\partial x = 0$). Channel morphology has long been recognized to influence sediment transport. Of particular relevance to the present work, Stark and Stark (2001) proposed a landscape evolution model with a variable called *channelization* that is defined as representing "the ease with which sediment can flux through a channel reach". Conceptually, channelization characterizes how changes in reach morphology influence local transport rate. However, channelization is an abstract unitless number that does not correspond physically to any measureable aspects of morphology. A fundamental feedback is imposed in the Stark and Stark (2001) model: channelization evolves through time as a function of both sediment flux and of itself, resulting in a differential equation. The combination of local slope and channelization tend to asymptote towards values such that $\partial q_s/\partial x = 0$, i.e. transport equilibrium. For a given upstream sediment supply rate, a modeled reach can evolve to equilibrium at different slopes (for different corresponding values of channelization) because both slope and channelization affect transport rate. Interestingly, the above definition of channelization could also be applied to $\tau_c^*$. Because of its control on transport rates, changes in $\tau_c^*$ should influence channel morphodynamics, both over human timescales (e.g., in response to natural and anthropogenic perturbations such as landslides,

**Deleted:** (Ancey et al., 2008)A constant $\tau_c^*$ is commonly assumed for gravel transport calculations, perhaps for several reasons. First, the traditional Shields diagram indicates that $\tau_c^*$ is rather insensitive to particle Reynolds number once flow becomes hydraulically rough around grains (Buffington, 1999). Second, a belief that $\tau_c^*$ is fundamentally a material property of a grain rather than a bed state control also remains somewhat ingrained. Third, because the best estimate of a given variable is usually its average, there is a tendency to attribute variability to measurement noise and uncertainty, even when that variability may be real, its causes understandable, and its influence potentially important to system dynamics (Jerolmack, 2011; Buffington and Montgomery, 1997).

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floods, post-wildfire erosion, land use, changing climate) and longer timescales (landscape evolution).

The overall goal of the present work is to understand and model possible feedbacks among thresholds of motion, changes in transport rate, and the morphological evolution of channels. First, I hypothesize that variability in gravel $\tau_c^*$ is physically meaningful, and that the implicit effects of multiple processes on $\tau_c^*$ can collectively be accounted for in terms of sediment flux dependence. Second, because changes in alluvial channel morphology are strongly coupled with sediment flux (Eq. 7), I hypothesize that the evolution of $\tau_c^*$ can implicitly model effects of evolving channel morphology.

The paper is organized as follows. First, I propose a conceptual model for how $\tau_c^*$ should evolve through time as a function of sediment flux (section 2.1), and then translate this model into equations (section 2.2). Next, I describe flume experiments on disequilibrium gravel transport (section 2.3), and use these experiments to empirically calibrate $\tau_c^*$ model parameters (sections 3.1, 3.2). After that, effects of $\tau_c^*$ evolution on river channel morphodynamics are explored using a simple model for river channel longitudinal profile development (section 3.3). Finally, I argue that $\tau_c^*$ is one of many morphodynamic "state variables" that describe how river channels evolve in response to external forcing and internal feedbacks, analagous to state variables in thermodynamics (section 4.2).

## 2 Models and Methods

### 2.1 Conceptual framework for $\tau_c^*$ evolution

The $\tau_c^*$ model proposed below is designed to be applicable at the reach scale, over timescales ranging from changing discharge during floods to the morphodynamic evolution of channels and surrounding landscapes. By definition, models are useful representations of reality because many complexities are omitted. Although recent work demonstrates a richness of threshold and transport behavior caused by turbulent velocity fluctuations and the statistical mechanics of particle populations over short timescales (e.g., Schmeeckle and Nelson, 2003; Diplas et al., 2008; Furbish et al., 2012), these dynamics are not explicitly considered or parameterized in my deterministic model formulation.

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Section 1.1 shows that a great many variables and processes influence $\tau_c^*$. While separate models for every isolated control on $\tau_c^*$ would be informative, it would also be difficult to combine myriad process-specific models and still meaningfully predict the temporal evolution of $\tau_c^*$ for the morphodynamic evolution of channels. Rather than being a process “splitter“, I approach the problem as a process “lumper“: I hypothesize that many factors affecting grain mobility share common underlying dependencies on net entrainment and net deposition.

Consistent with the form of most bedload transport equations (e.g. Eq. 1), $\tau_c^*$ is defined as a particular Shields stress at which only the most mobile grains of that size become entrained. However, for a population of grains of a given size on the bed surface, there should actually be a distribution of $\tau_c^*$—notated here as a set of values $\{\tau_c^*\}$--because each individual grain has a particular pocket geometry and near-bed flow velocity at its unique location, and hence a somewhat different individual threshold. Gravel flux increases with discharge primarily because thresholds are gradually exceeded for increasing proportions of surface grains of a given size. For a given transport equation (e.g. Eq. 1), a particular $\tau_c^*$ value from the lower tail of distribution $\{\tau_c^*\}$ should best predict sediment flux. Conceptually, an underlying assumption is that net entrainment or net deposition changes the underlying $\{\tau_c^*\}$ distribution, and therefore changes the value of $\tau_c^*$ that best predicts transport rates.

In the case of a channel reach undergoing net erosion ($q_{sout} > q_{sin}$), the most mobile individual grains—i.e. the lowest $\tau_c^*$ values in the $\{\tau_c^*\}$ distribution--would preferentially be entrained first, while the grains remaining on the bed would tend to have higher thresholds. Therefore, I hypothesize that progressive erosion tends to entrain grains from increasingly more stable positions on the bed, gradually increasing $\tau_c^*$. Conversely, during net deposition ($q_{sout} \le q_{sin}$), I assume that grains tend to preferentially deposit in more stable bed positions such as local topographic lows. Continued deposition would lead to grains being deposited in progressively less stable positions, gradually decreasing $\tau_c^*$. These hypothesized $\tau_c^*$ changes represent averages for the population of grains; individual grains would exhibit great

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variability. For example, during net deposition individual grains would also both deposit and be entrained from more and less stable positions, but grains would have a greater probability of remaining deposited in the more stable positions.

Mechanistically, $\tau_c^*$ evolution would also be driven by changes in bed topography and roughness, grain clustering and stabilizing structures, compaction of the bed and interlocking of grains, etc. None of these physical variables are explicitly included in the model equations; instead their combined effects are assumed to vary with net erosion or deposition. Importantly, the amount by which $\tau_c^*$ changes should also depend on the current state of the bed surface. For example, starting from a relatively rough and interlocked bed surface, net deposition would initially cause relatively substantial decreases in bed roughness as local lows preferentially filled with loose grains, and relatively large corresponding decreases in $\tau_c^*$. However, for a given surface GSD there must be physical limits for bed roughness and grain packing. If bed surface grains are already relatively loose and mobile, additional deposition would cause less of a decrease in $\tau_c^*$, or no decrease at all if the bed is already as unstable for a given surface GSD as it can be. Thus, the change in $\tau_c^*$ should also be a function of $\tau_c^*$. The combination of processes that cause changes in $\tau_c^*$ also place physical limits on how high, and low, $\tau_c^*$ can evolve.

These $\tau_c^*$ dependencies describe negative transport feedbacks: net erosion progressively reduces rates of erosion by making grains harder to entrain, while net deposition progressively makes grains more mobile. Through these and other morphological feedbacks, it has long been recognized that channel reaches evolve towards steady-state configurations in which the sediment flux into a reach balances the flux exiting, leading to zero net erosion or deposition (Mackin, 1948). At this statistical steady state, $\tau_c^*$ should also be at equilibrium, and in fact is a key part of reaching channel reach equilibrium. If $\tau_c^*$ were still systematically evolving (e.g. from continued bed state changes), then transport rate through the reach would also change, perturbing the channel away from its statistical equilibrium.

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## 2.2 $\tau^*_{c(q_s)}$ model equations

While the above discussion makes the case that $\tau^*_c$ inevitably evolves through time due to a variety of interrelated factors, the new model proposed here is specifically in terms of sediment flux. I use the notation $\tau^*_{c(q_s)}$ to distinguish this specific model from more general representations of thresholds of motion in other models and analyses. Because longitudinal coordinate $x$ increases downstream, net erosion in a reach is indicated by $\partial q_s/\partial x > 0$ and net deposition by $\partial q_s/\partial x < 0$. The following relations are proposed:

$$\frac{\partial \tau^*_{c(q_s)}}{\partial t} = \begin{cases} kB\left(\left|\frac{\partial q_s}{\partial x}\right|\right)^{\kappa_{ent}} & if \quad \partial q_s/\partial x > 0 \\ -k(1-B)\left(\left|\frac{\partial q_s}{\partial x}\right|\right)^{\kappa_{dep}} & if \quad \partial q_s/\partial x < 0 \end{cases} \qquad (8)$$

$$B = \frac{\tau^*_{c\max} - \tau^*_{c(q_s)}}{\tau^*_{c\max} - \tau^*_{c\min}} \qquad (9)$$

where $\kappa_{ent}$ and $\kappa_{dep}$ are dimensionless exponents corresponding to entrainment and deposition, respectively, and $k$ is a scaling factor. These three parameters will be empirically fit to experiments. $\tau^*_{c\min}$ and $\tau^*_{c\max}$ represent bounds on how low or high $\tau^*_{c(q_s)}$ can plausibly evolve (assumed to be 0.02 and 0.35 respectively). Eq. (8) predicts that $\tau^*_{c(q_s)}$ incrementally decreases with net deposition, and incrementally increases during net erosion. "Feedback factor" $B$ has a value between 0 and 1 and makes Eq. (8) a differential equation. It scales the incremental change in $\tau^*_{c(q_s)}$ so that deposition on an already "loose" bed ($\tau^*_{c(q_s)}$ close to $\tau^*_{c\min}$) minimally decreases $\tau^*_{c(q_s)}$, but erosion causes a larger $\tau^*_{c(q_s)}$ increase. Conversely, if $\tau^*_{c(q_s)}$ is already high (close to $\tau^*_{c\max}$), then erosion causes a much smaller $\tau^*_{c(q_s)}$ change than deposition. Finally, I note that representing $\partial \tau^*_{c(q_s)}/\partial t$ as a function of $\partial q_s/\partial x$ (Eq. 8) is broadly analogous in form to Exner (Eq. 7).

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<#>Two additional points need to be made. First, natural sediment is a mixture of sizes. It is common to assume that a single representative grain size, such as the median ($D_{50}$), adequately describes transport of the whole mixture. Second, "thresholds" can represent different things in different models. In Eq. (1), $\tau^* = \tau^*_c$ represents a modeled transport rate of zero. In other models designed to predict measurable transport rates at very low shear stresses, a non-threshold "reference" stress is instead defined as the Shields stress at which $q^*_s$ has a very low but specific nonzero value (Parker, 1990; Wilcock and Crowe, 2003). For many applications the practical difference between threshold and reference stresses are negligible (Buffington and Montgomery, 1997), and in the present work they are largely used interchangeably. ¶

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A limitation of Eq. (8) is that, for dimensional consistency, the units of $k$ vary with $\kappa_{ent}$ and $\kappa_{dep}$. An improved equation replaces spatial changes in flux with spatial changes in the thickness of deposited or eroded sediment:

$$\frac{\partial \tau^*_{c(q_s)}}{\partial t} = \begin{cases} kA_r B\left(\left|\frac{\partial \theta_s}{\partial x}\right|\right)^{\kappa_{ent}} & if \quad \partial q_s/\partial x > 0 \\ -kA_r(1-B)\left(\left|\frac{\partial \theta_s}{\partial x}\right|\right)^{\kappa_{dep}} & if \quad \partial q_s/\partial x < 0 \end{cases} \qquad (10)$$

$\theta_s$ is the thickness of sediment deposited or eroded at a given location. $\partial\theta_s/\partial x$ is a dimensionless ratio representing spatial changes in erosion and deposition. In this case, $k$ has dimensions 1/$t$ and scales how quickly $\tau^*_{c(q_s)}$ evolves. $\theta_s$ can be calculated by integrating Eq. (7) over time interval $t_1$ to $t_2$:

$$\theta_s(t_2,x) = z(t_2,x) - z(t_1,x) = \frac{1}{1-\lambda_p}\int_{t_1}^{t_2}\frac{\partial q_s(t,x)}{\partial x}dt \qquad (11)$$

(recall that $\int_a^b(\partial f(s,t)/\partial t)dt = f(s,b) - f(s,a)$ for a generic function $f$). Using discrete flume data, $\theta_s$ is calculated over a measurement interval $\Delta t$ as $(1-\lambda_p)^{-1}(\overline{q_{sout}} - \overline{q_{\sin}})\Delta t/\Delta x$, where $\Delta x$ is the length of the flume and the sediment flux terms are averaged over $\Delta t$.

$A_r$ is a dimensionless armoring parameter, calculated in several ways in order to explore whether predictions can be improved by explicitly including bed surface grain size or bed roughness characteristics. Setting $A_r = D_{50}/D_{84}$ (the reciprical of the Recking (2012) armoring constraint in Eq. 5 and 6) means that incremental changes to $\tau^*_{c(q_s)}$ are larger where $D_{50}$ is relatively closer to $D_{84}$. Setting $A_r = 2D_{50}/(D_{84} - D_{16})$ suggests that $\tau^*_{c(q_s)}$ changes should be larger when intermediate diameters are large relative to a measure of the normalized width of the bed surface GSD. I also try $A_r = D_{50}/\sigma$, where $\sigma$ is bed surface roughness. $A_r = D_{50}/\sigma$ suggests that, relative to topographic lows and highs, large grains cause bigger $\tau^*_{c(q_s)}$ changes than small grains. Finally, $A_r$ is simply set to 1 in some calculations below.

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## 2.3 Experimental design

The flume experiments used to calibrate $k$, $\kappa_{ent}$ and $\kappa_{dep}$ were designed to explore feedback during disequilibrium transport in gravel-bed rivers. Fig. 2 shows how transport rates, surface $D_{50}$ and bed slope evolved in response to fine gravel pulses. Johnson et al. (2015) provide details of the experimental conditions and how they scale to natural conditions most consistent with step-pool development, and so the summary here is brief. Four experiments were conducted in a small flume 4 m long and 10 cm wide. Experiments 1 and 4 were done at 8% initial slope, and 2 and 3 at 12% initial slope; slopes subsequently evolved fairly little during morphological adjustment (Fig. 2c). Water discharge was held constant throughout to better isolate the influence of sediment supply changes on transport. Sediment transported out of the flume was caught in a downstream basket, sieved and weighed. Overall sediment diameters ranged from 0.45 to 40 mm; these sizes were sorted and painted different colors based on five size classes with $D_{50}$ = 2.4, 4.5, 8.0, 15.4, and 27.2 mm ($D_{16}$ = 2.0, 3.4, 6.7, 12.4, and 24.0 mm; $D_{84}$ = 2.8, 5.7, 10.3, 19.7, and 31.3 mm, respectively). Surface GSDs were measured using image analysis of colored bed surface grains during the experiments. Bed topography was measured using a triangulating laser, and bed roughness ($\sigma$) was calculated from longitudinal topographic swaths as the standard deviation of detrended bed elevations. Water surface elevations were measured using an ultrasonic distance sensor, and water depths were calculated by subtracting bed elevations. Total shear stress ($\tau$) was calculated assuming steady uniform flow when spatially averaged over the flume:

$$\tau = \rho g h S \qquad (12)$$

where $h$ is water depth corrected for sidewall effects following the method of Wong and Parker (2006), and $S$ is water surface slope.

The experiments started with mixed-size sediment screeded flat. Initially, all surface sizes were observed to be mobile (and therefore above thresholds of motion). At the beginning no sediment was fed into the upstream end ($q_{sfeed}$ = 0), and the bed responded by coarsening, roughening and gradually stabilizing as transport rates dropped by ≈3 orders of magnitude (Fig. 2a). After this initial stabilization, a step-function pulse of the finest gravel size ($D_{50}$=2.4 mm) was fed into the flume at $q_{sfeed}$ =1000 g/min, representing an idealization of a landslide, debris flow, post-wildfire erosion, or anthropogenic gravel augmentation that would suddenly supply sediment finer than the existing bed surface. The feed rate was chosen

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to be similar to the high initial transport rates (Fig. 2a), while not so high as to inhibit morphodynamic feedback by fully burying the existing bed surface. Initially some deposition occurred on the bed, but the channel adjusted rapidly, by both entraining coarser bed surface grains and transporting most of the finer supplied gravel, so that the outlet transport rate ($q_{sout}$) approximately matched $q_{\sin}$. After that the sediment supply pulse $q_{\sin}$ was again dropped to zero, and the bed gradually restabilized. Johnson et al. (2015) explained in detail how bed roughness evolved, and how the addition of finer gravels ultimately caused surface coarsening (Fig. 2b). Unbalanced transport rates into and out of the flume demonstrate disequilibrium conditions (Fig. 2a), although transport evolved towards equilibrium.

## 2.4 The Wilcock and Crowe (2003) transport model

To quantify thresholds of motion from these experimental data (Fig. 2) requires a transport model. The Wilcock and Crowe (2003) "Surface-based Transport Model for Mixed-Size Sediment", abbreviated as W&CM, is used for two main reasons. First, the model can, at least in principle, account for the effects of changing surface GSD on $\tau_c^*$. Second, the model should also be able to account for possible effects of sand and fine gravel abundance on thresholds of motion (Eq. 4). By using the W&CM to isolate and remove GSD effects, experimentally-constrained thresholds of motion can then be used to evaluate the proposed $\tau^*_{c(q_s)}$ functions (Eq. 8-11). A secondary goal is to evaluate how well the W&CM predicts disequilibrium transport at steeper slopes and lower water depths than Wilcock and Crowe (2003) used in their own steady-state experiments.

A key variable in the W&CM is $\tau^*_{rs50}$, the nondimensional reference stress for the median surface grain size ($D_{s50}$). $\tau^*_{rs50}$ corresponds to a very low transport rate of $W_i^*$=0.002. $W_i^*$ is a nondimensional bedload transport rate for grain size class $i$,

$$W_i^* = \left(\frac{\rho_s}{\rho} - 1\right)\frac{g q_{bi}}{F_i u_\tau^3} \quad (13)$$

where $q_{bi}$ is the volumetric transport rate per unit channel width of grains of size $i$, $F_i$ is the fraction of size $i$ on the bed surface, and $u_\tau$ is shear velocity ($u_\tau = \sqrt{\tau/\rho}$). Wilcock and Crowe (2003) presented an empirical relationship between transport and shear stress:

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$$W_i^* = \begin{cases} 0.002\left(\dfrac{\tau}{\tau_{ri}}\right)^{7.5} & for \quad \tau/\tau_{ri} < 1.35 \\ 14\left(1 - \dfrac{0.894}{\left(\dfrac{\tau}{\tau_{ri}}\right)^{0.5}}\right)^{4.5} & for \quad \tau/\tau_{ri} \geq 1.35 \end{cases} \qquad (14)$$

where $\tau_{ri}$ is a dimensional reference stress for size class *i*, with dimensionless equivalent $\tau_{ri}^*$ (Eq. 3). A "hiding function" determines how nondimensional reference stresses vary with grain size:

$$\frac{\tau_{ri}^*}{\tau_{rs50}^*} = \left(\frac{D_i}{D_{s50}}\right)^{b-1} \qquad (15)$$

The hiding function exponent *b* is calculated as

$$b = \frac{0.67}{1 + e^{(1.5 - D_i/D_{s50})}} \qquad (16)$$

Note that Eq. (16) is slightly modified from the exact Wilcock and Crowe (2003) version by replacing $D_{sm}$ (the geometric mean surface diameter) with $D_{s50}$, to more simply use just one measure of the central tendency of the surface GSD.

# 3 Results

In this section, the W&CM is used to calculate best-fit thresholds of motion. Next, the $\tau_{c(q_s)}^*$ model is shown to predict the experimentally-constrained $\tau_{rs50}^*$ trends after calibrating several parameters. Finally, the influence of $\tau_{c(q_s)}^*$ on morphodynamics is explored using a simple model for gravel-bed river profile evolution.

## 3.1 Best-fit $\tau_{rs50}^*$ and hiding functions

The experimental data are used to determine $W_i^*$ (Eq. 13) and $\tau_{ri}^*$ for each of the five grain size classes (Eq. 14). Best-fit $\tau_{rs50}^*$ is then calculated in two ways. In the first approach, *b* is calculated using Eq. (16), and $\tau_{rs50}^*$ and 95% confidence intervals are estimated using nonlinear multiple regression in Matlab (Fig. 3, "W&CM fit"). Importantly, the temporal

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While $b$ varies with relative grain size in the W&CM (Eq. 16), other proposed hiding functions have found (or assumed) that a single $b$ value applies to different grain sizes, at least for a given set of flow and surface conditions (Parker, 1990; Buscombe and Conley, 2012). My second approach for estimating $\tau^*_{rs50}$ explores whether the results are sensitive to the particular form of Eq. (16). Rather than Eq. (16), nonlinear multiple regression was used to estimate both $b$ and $\tau^*_{rs50}$ in Eq. (15), with separate regressions for each time step (Fig. 3). The temporal evolution of experimental $\tau^*_{rs50}$ is generally comparable for the two different approaches (Fig. 3).

Interestingly, Fig. 4 shows that the hiding function exponents determined using the nonlinear multiple regressions for $b$ and $\tau^*_{rs50}$ are consistent with Eq. (16) of Wilcock and Crowe (2003). In spite of substantial scatter there is a slope break which corresponds to a change in $b$ for surface grains smaller and larger than the median, suggesting that the W&CM reasonably can describe hiding and exposure relations among grains in steeper channels and for shallower flow depths than used in the Wilcock and Crowe (2003) experiments.

## 3.2 Calibration of $\tau^*_{c(q_s)}$ model parameters

Fig. 5 compares experimentally-constrained thresholds of motion to several predictions of these trends. First, I test whether surface sand fraction (Eq. 4) can explain the evolution of $\tau^*_{rs50}$ (Curran and Wilcock, 2005; Wilcock and Crowe, 2003). As described in section 1.1, the effect of finer grains on thresholds of motion of coarser grains is not limited to sand sizes alone (Venditti et al., 2010; Sklar et al., 2009; Johnson et al., 2015). In the Johnson et al. (2015) experiments, the finest grain size class has $D_{50}$=2.4 mm, $D_{16}$=2.0 mm, $D_{84}$=2.8 mm. Setting $F_s$ equal to the surface fraction of this size class, a nonlinear multiple regression of Eq. (4) to all four experiments together yielded a poor although statistically significant fit to the data ($R^2$=0.13; $p$=3x10$^{-5}$; $c_1$=0.097±0.04, $c_2$=0.103±0.11, and $c_3$=5.6±11), confirming that surface grain size changes alone cannot explain observed $\tau^*_{rs50}$ patterns (Fig. 5, “Sand

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**Deleted:** A unique aspect of the W&CM not described above is that changes in surface sand fraction could cause temporal evolution of $\tau^*_{rs50}$. In particular, Wilcock and Crowe (2003) proposed that $\tau^*_{rs50}$ varied systematically from 0.036 with no surface sand to 0.021 with abundant surface sand:¶
$$\tau^*_{rs50} = c_1 + c_2 e^{-c_3 F_s} \quad (14)$$
where $F_s$ is the fraction of sand on the bed surface, and constants $c_1$, $c_2$ and $c_3$ were empirically found by them to be 0.021, 0.015 and 20 respectively (for simplicity, the geometric mean reference stress was again replaced in their original equation with $\tau^*_{rs50}$). Subsequent work has shown that thresholds of motion can similarly be reduced by grains substantially smaller than the bed surface but larger than sand (Venditti et al., 2010; Sklar et al., 2009; Johnson et al., 2015), suggesting that the “sand fraction” effect could also be modeled for grains larger than 2 mm. ¶
In the present experiments, the surface fraction of actual sand (<2 mm) was very small. However, because grains larger than sand but smaller than the average bed surface may also enhance mobility, we evaluate whether Eq. (14) can explain the experimentally-constrained variations in $\tau^*_{rs50}$. Using the surface fraction of grains < 2.8 mm (representing the smallest grain size class in the experiments)

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fraction"). Note that I have assumed for simplicity that $\tau^*_{rs50} = \tau^*_{rm}$, i.e. substituting the surface $D_{50}$ for the geometric mean surface diameter in Eq. (4).

Various $\tau^*_{c(q_s)}$ models provide better fits to experimentally-constrained $\tau^*_{rs50}$ (Fig. 5; Eq. 10). Models with $A_r$=1 are shown using a single set of model parameters for all four experiments ("collective best fit", $k$=0.17, $\kappa_{dep}$=0.20, $\kappa_{ent}$=0.40), and also the best fit for each experiment separately. The best-fit overall model has $R^2$=0.69, suggesting statistically that effects of supply and transport disequilibrium can explain over 2/3 of the variability in $\tau^*_{rs50}$ (Table 1). Note that $\tau^*_{c(q_s)}$ and $\tau^*_{rs50}$ are assumed to be interchangeable. Because Eq. (8) and (10) are differential equations, best-fit parameters could not be calculated using nonlinear multiple regressions. Instead, I use a brute-force approach of incrementally stepping through a wide range of $k$, $\kappa_{dep}$ and $\kappa_{ent}$, and finding the combination of parameters that give the smallest root-mean-square deviation (RMSD). These calculations started at $\tau^*_{rs50}$=0.036 at $t$=0, which is consistent with the experiments, and also is the $\tau^*_{rs50}$ proposed by Wilcock and Crowe (2003) in the absence of sand dependence.

Interestingly, model fits using $A_r = D_{50}/\sigma$ are not substantially different from $A_r$=1, and $R^2$=0.69 is the same (Fig. 5). Table 1 includes additional regressions for $A_r = D_{50}/D_{84}$ and $A_r = 2D_{50}/(D_{84} - D_{16})$. These fits overlap almost perfectly with those shown on Fig. 5. As explained in section 1.1, $A_r$ should account for surface GSD and bed topography influences on thresholds. The fact that regressions are not improved by including these variables may suggest that transport disequilibrium is a more important control on threshold evolution over a broad range of surface GSD and bed roughness. Parameters estimated for dimensional $\partial q_s/\partial x$ (Eq. 8) indicate that the dimensionally balanced model performs equally well (Table 1). Because these variants do not substantially improve $\tau^*_{c(q_s)}$ model fits, we use the simplest dimensionally consistent model (Eq. 10 with $A_r$=1) in the analysis below.

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## 3.3 Influence of $\tau^*_{c(q_s)}$ on morphodynamics

Next, an idealized morphodynamic model demonstrates how the proposed $\tau^*_{c(q_s)}$ relations influence the evolution of channel profiles, focusing on reach slopes and timescales of adjustment. Because the modeling goal is to isolate and understand effects of evolving $\tau^*_{c(q_s)}$, the underlying model is arguably the simplest reasonable representation of morphodynamic feedback. Inspired by Parker (2005), the model describes a channel reach in which slope evolves through aggradation and degradation. The downstream boundary elevation is fixed (constant base level). Sediment transport and bed elevation are modeled using Eq. (1) (substituting $\tau^*_{c(q_s)}$ for $\tau^*_c$ as needed) and Eq. (7) with a single grain diameter ($D$). Unit water discharge $q_w$ is similarly held constant for simplicity. Upstream sediment supply rate ($q_{sfeed}$) is imposed, and is varied to drive channels to new steady states. Relationships among flow depth, depth-averaged velocity and discharge are imposed by assuming that hydraulic roughness remains constant, parameterized though a Darcy-Weisbach hydraulic friction coefficient:

$$f = \frac{8gq_wS}{U^3} \qquad (17)$$

For a given discharge this allows both $U$ and $h$ to be determined:

$$U = \frac{q_w}{h} \qquad (18)$$

$$h = q_w^{2/3}\left(\frac{f}{8gS}\right)^{1/3} \qquad (19)$$

Two model variations are compared: in the "Exner-only" morphodynamic model, $\tau^*_c$ is a constant. In the "Exner+$\tau^*_{c(q_s)}$" variant, $\tau^*_{c(q_s)}$ evolves through time following Eq. (10). At equilibrium, channel slope can be predicted for both model variants (and substituting $\tau^*_{c(q_s)}$ for $\tau^*_c$ where appropriate) by combining Eq. (1), (2), (12) and (19):

$$S_{eq} = \frac{2.83}{q_w}\left(\frac{g}{f}\right)^{1/2} D^{3/2}\left(\frac{\rho_s}{\rho}-1\right)^{3/2}\left[\left(\frac{q_s^*}{3.97}\right)^{2/3}+\tau_c^*\right]^{3/2} \qquad (20)$$

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For a given discharge, Eq. (20) indicates that both sediment supply and the threshold of motion influence steady-state morphology (slope).

Away from equilibrium, rates of change of bed elevation along a river profile should depend not only on the sediment flux at a given channel cross section, but also on the average velocity at which grains move downstream. This control has occasionally been ignored in previous models of profile evolution. In my model, it is crudely incorporated by assuming that average bedload velocity is a consistent fraction of water velocity, broadly consistent with previous findings that bedload velocities are proportional to shear velocity (e.g., Martin et al., 2012). The modeling timestep is set to be equal to the time it takes sediment to move from one model node (bed location) to the next, and is adjusted during simulations. While this approach makes the temporal evolution of channel changes internally consistent within the model, timescales for model response will still be much shorter than actual adjustment times in field settings because flood intermittency is not included (so the model as implemented is always at a constant flood discharge). In addition, the upstream sediment supply is imposed in the model, while in natural settings hillslope-floodplain-channel coupling could greatly affect $q_{sfeed}$ over time if significant aggradation or downcutting took place.

Table 2 provides parameters used for morphodynamic modeling. Although the highly simplified model is not intended for quantitative field comparisons, variables $D$ ($D_{50}$=50 mm), $f$ (0.1), and $q_w$ (1 m$^2$/s) were chosen to be broadly consistent with a moderate (≈2-3 year peak discharge recurrence interval) bedload-transporting flood in Reynolds Creek, Idaho (Olinde and Johnson, 2015). Reynolds creek is a snowmelt-dominated channel with reach slopes that vary widely from ~0.005 to 0.07. In an instrumented reach with a slope of 0.02, Olinde (2015) used RFID-tagged tracers and channel-spanning RFID antennas to measure $\tau^*_{rs50}$ ≈0.06. A constant $\tau^*_c$=0.06 is used for the Exner-only models, while $\tau^*_{c(q_s)}$=0.06 is used as the initial condition for Exner+$\tau^*_{c(q_s)}$ models. Field constraints on upstream sediment feed rates were not available, and so $q_{sfeed}$ values were chosen to provide reasonable model slopes. Exponents $\kappa_{dep}$ and $\kappa_{ent}$ used the experimental calibrations, while $k$ were chosen so that changes in $\tau^*_c$ occurred over the same range of timescales as topographic adjustments, to better illustrate the interplay of variables in morphodynamic evolution (Table 2).

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### 3.3.1 Morphodynamic model results

Fig. 6 and 7 compare how longitudinal profiles respond to an increase in sediment supply, for both the Exner-only (constant $\tau_c^*$) and Exner+$\tau_{c(q_s)}^*$ models. The initial condition is a channel at equilibrium ($q_{sout}$=$q_{sfeed}$). At $t$=0, sediment supply is increased by a factor 5. The Exner+$\tau_{c(q_s)}^*$ model aggrades to a new equilibrium slope that is lower than the Exner-only model. This occurs because deposition ($\partial q_s/\partial x < 0$) causes $\tau_{c(q_s)}^*$ to decrease over time, progressively increasing transport efficiency (i.e., higher transport rates at a lower slope) compared to constant $\tau_c^*$=0.06 (Fig. 7). Feedback causes the reverse effect for a decrease in $q_{sfeed}$: $\tau_{c(q_s)}^*$ progressively increases as slope decreases, leading to channel re-equilibration both sooner and at a higher slope (Fig. 7).

An equilibrium timescale ($t_{eq}$) is measured here as the amount of time it takes from a supply perturbation ($t$=0 in these models) to the slope adjusting to be within 0.0001 of its equilibrium slope (Eq. 20). In Fig. 7, $t_{eq}$ are substantially longer for the Exner-only models than for the Exner+$\tau_{c(q_s)}^*$ models. For Exner+$\tau_{c(q_s)}^*$, an increase in $q_{sfeed}$ leads to aggradation, in turn increasing local $q_s^*$ by both increasing slope and also decreasing $\tau_{c(q_s)}^*$ (Eq. 1, 10). Both factors adjusting enable equilibrium to be reached sooner.

Over a $q_{sfeed}$ range of two orders of magnitude, equilibrium slopes change less for the Exner+$\tau_{c(q_s)}^*$ model than for Exner-only (Fig. 8a). The ratio of these equilibrium slopes illustrates the magnitude of the change, where "$S_{eq}$ ratio" is $S_{eq}$ for Exner+$\tau_{c(q_s)}^*$ divided by Exner-only $S_{eq}$ (Fig. 8b). An order-of-magnitude decrease in $q_{sfeed}$ caused Exner+$\tau_{c(q_s)}^*$ $S_{eq}$ to be roughly 24% - 36% larger than Exner-only $S_{eq}$. An order-of-magnitude increase in $q_{sfeed}$ caused Exner+$\tau_{c(q_s)}^*$ to be roughly 20% smaller than the constant-$\tau_c^*$ model. Calculations are also shown for several values of scaling factor $k$. A larger $k$ means that $\tau_{c(q_s)}^*$ increases or decreases more rapidly for a given amount of aggradation or degradation (Eq. 10), which in general enables a new equilibrium to be reached with a smaller change in slope.

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Equilibrium timescales are quite sensitive to $k$ as well as to sediment supply rate (Fig 8c). Similar to the $S_{eq}$ ratio, the "$t_{eq}$ ratio" is $t_{eq}$ for Exner+$\tau^*_{c(q_s)}$ divided by $t_{eq}$ for the Exner-only model (Fig. 8d). There is an asymmetry in equilibrium times for aggradation vs. degradation; in general the difference between Exner-only and Exner+$\tau^*_{c(q_s)}$ is somewhat smaller during bed aggradation, and the difference decreases with increasing $q_{sfeed}$. Interestingly, the highest $k$ (2.8E-5) results in a threshold-like response where the $t_{eq}$ ratio rapidly increases from roughly 0.01 to 0.8 (Fig. 8d). This change occurred because $\tau^*_{c(q_s)}$ "bottomed out", i.e. reached its minimum possible value ($\tau^*_{c(q_s)} \approx \tau^*_{c\,min}$ =0.02) before the equilibrium slope had been reached (Fig. 8e). At that point, $\tau^*_{c(q_s)}$ could no longer act as a buffer to reduce slope changes, and it took much longer to reach an equilibrium slope.

Finally, Fig. 9 shows that the spatial as well as temporal evolution of $\tau^*_{c(q_s)}$ can influence river profiles. The models are the same as in Fig. 6. At $t$=0, the feed rate into the upstream-most node ($q_{sfeed}$) increases by a factor of 5. Therefore, the upstream end feels the supply perturbation both sooner and more strongly than downstream nodes. Aggradation from the supply perturbation increases upstream slopes first. In the Exner-only model, downstream slopes gradually catch up. Because $\tau^*_c$ stays constant, every location along the channel eventually asymptotes to the single slope required to transport the new $q_{sfeed}$ at the given discharge (Fig. 9a). However, for evolving thresholds, enhanced upstream aggradation caused upstream $\tau^*_{c(q_s)}$ to decrease both more rapidly and to lower values than downstream nodes. Spatial differences in $\tau^*_{c(q_s)}$ persisted at equilibrium, resulting in spatial variations in equilibrium slope (Exner+$\tau^*_{c(q_s)}$; Fig. 9b, 9c).

## 4 Discussion

In this section, the dependence of $\tau^*_{c(q_s)}$ on sediment supply is compared to previous work. $\tau^*_{c(q_s)}$ evolution is identified as a negative feedback on morphologic change that can impart a memory of previous channel "states" to the system. Finally, $\tau^*_{c(q_s)}$ is interpreted as a channel state variable, analogous to temperature in thermodynamics.

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As described in section 1.1, previous work on sediment supply-dependent thresholds of motion includes Recking (2012), who proposed high sediment supply ($\tau^*_{mss}$; Eq. 5) and low sediment supply ($\tau^*_{m}$; Eq. 6) bounds on thresholds of motion. Fig. 10 shows how these relations compare to the experimentally-constrained $\tau^*_{rs50}$. It should be noted again that these bounds were calibrated to the $D_{84}$ grain size rather than $D_{50}$ (Recking, 2012). While the actual values are therefore not expected to be equivalent, $\tau^*_{mss}$ and $\tau^*_{m}$ do tend to bound $\tau^*_{rs50}$. The low-supply bound $\tau^*_{m}$ is roughly 2-4 times larger than the experimental constraints. The high-supply bound $\tau^*_{mss}$ is similar in magnitude to $\tau^*_{rs50}$ and predicts the decrease during the feed period. The (linear) correlation between $\tau^*_{mss}$ and $\tau^*_{rs50}$ is weak ($R^2$=0.13) although statistically significant ($p$=3E-5). Nonetheless, given that threshold of motion uncertainties are typically large, Eq. (5) arguably provides a surprisingly good independent prediction of our experimental disequilibrium transport data, based on experimental slope, $D_{84}$ and $D_{50}$.

The $\tau^*_{c(q_s)}$ model is consistent with previous interpretations that high sediment supply corresponds to low thresholds of motion, and vice-versa (Recking, 2012; Bunte et al., 2013). In the $\tau^*_{c(q_s)}$ model (Eq. 10), an increase in upstream sediment supply that causes net aggradation will lower $\tau^*_{c(q_s)}$, unless $\tau^*_{c(q_s)}$ has already reached its lower physical limit ($\tau^*_{c\,min}$). Conversely, a decrease in supply that causes net erosion will increase $\tau^*_{c(q_s)}$, unless $\tau^*_{c(q_s)}$ is already high ($\approx\tau^*_{c\,max}$). However, while the $\tau^*_{c(q_s)}$ model can thus explain an inverse relation between supply and thresholds of motion, it is worth noting that Eq. (8) and (10) describe a subtly different feedback: $\tau^*_{c(q_s)}$ does not directly increase or decrease with supply, but rather with the history of sediment supply changes relative to transport capacity over time. If $q_{sin}$ equals $q_{sout}$, $\tau^*_{c(q_s)}$ could remain constant regardless of whether $q_{sin}$ is high or low.

### 4.1 Negative feedback and asymmetric approaches to equilibrium

The evolution of $\tau^*_{c(q_s)}$ acts as a negative feedback because it reduces the morphodynamic response to perturbations. Reach slopes and $\tau^*_{c(q_s)}$ both change in the direction that brings transport back towards equilibrium, allowing smaller slope changes to

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accomodate supply changes (Fig. 6, 7, 8a,b, 9). However, as with other buffered systems, there is a limit to how large of a perturbation can be accommodated by $\tau^*_{c(q_s)}$ (as illustrated by $k$=2.8E-5 in Fig. 8c,d,e). The amount of possible $\tau^*_{c(q_s)}$ change depends on how close $\tau^*_{c(q_s)}$ is to $\tau^*_{c\,\min}$ or $\tau^*_{c\,\max}$ (Eq. 9). When changes in $\tau^*_{c(q_s)}$ are negligible but transport and morphology are not equilibrated, then the time to equilibrium ($t_{eq}$) increases because only channel morphology can adjust (Fig. 8c, d, e).

The experiments suggest that $\tau^*_{c(q_s)}$ changes faster in response to aggradation than degradation (Fig. 2, 5). This asymmetry is expressed in the best-fit exponents: $\kappa_{dep}$ is smaller than $\kappa_{ent}$ for all scenarios tested (Table 1). Note that because $\partial\theta_s/\partial x$ is much smaller than 1 (i.e, spatial changes in bed elevation are small compared to the horizontal distance the change is measured over), the smaller exponent ($\kappa_{dep}$) corresponds to a larger change in $\tau^*_{c(q_s)}$ for a given $\partial\theta_s/\partial x$ (Eq. 10). For a given increment of sediment thickness ($\theta_s$), aggradation is more efficient at decreasing $\tau^*_{c(q_s)}$ than degradation is at increasing $\tau^*_{c(q_s)}$. Future work is required to explore how specific physical processes vary during net deposition or erosion and lead to asymmetry in $\tau^*_{c(q_s)}$ change. Still, a tentative hypothesis linking bed roughness and $\tau^*_{c(q_s)}$ change asymmetry is that during deposition, clasts tend to deposit preferentially in topographic lows, because these tend to be the most sheltered locations, and simply because of the direction of gravity. Preferentially filling in lows tends to decrease bed roughness, in turn reducing topographic sheltering and hydraulic friction and increasing near-bed flow velocities. All of these factors decrease $\tau^*_{c(q_s)}$. However, erosion does not simply have an opposite but symmetric effect on bed topography as deposition. Clasts are not preferentially eroded from topographic lows, as these locations tend to remain the most sheltered. Instead, the process of increasing bed roughness during erosion is more complex and results from the more gradual development of stabilizing structures around keystones, as grains are rearranged locally to positions where they protrude into the flow but remain stable due to interlocking with surrounding grains. Thus, roughness reduction and enhancement should not equally sensitive to net erosion or deposition. Mao (2012) showed that bed roughness evolved at different rates during symmetric rising and falling limbs of hydrographs, influencing gravel

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transport hysteresis. Bed roughness due to sand ripple and dune evolution has also been shown to increase and decrease at different rates during hydrograph rising and falling limbs, leading to hysteresis in a transport system that is not threshold dominated (Martin and Jerolmack, 2013).

In Fig. 8c, the Exner+ $\tau^*_{c(q_s)}$ model indicates that equilibrium timescales are longer for aggradation ($q_{sfeed}$ / initial $q_{sfeed}$ > 1) than for degradation. At first glance this seems to contradict the argument that aggradation is more efficient at decreasing $\tau^*_{c(q_s)}$. The explanation is that the equilibrium timescale does not *only* depend on the exponents, but also on how much total aggradation or degradation occurs to attain equilibrium. Slope changed more during aggradation than degradation for these particular Exner+ $\tau^*_{c(q_s)}$ models, even though $\tau^*_{c(q_s)}$ also tended to change more during aggradation than degradation (Fig. 8a, 8e).

In the experiments, average slopes changed very little in response to changes in sediment supply and transport disequilibrium, while grain size and bed surface roughness changed much more (Fig. 2; bed roughness is presented in detail in Johnson et al., 2015). Because the W&CM accounted for surface grain size changes in determining experimental $\tau^*_{rs50}$ (Fig. 3), bed roughness and various unquantified mechanisms (such as grain interlocking) are interpreted to have physically caused the $\tau^*_{rs50}$ evolution. What does this suggest for *k*, which scales how much $\tau^*_{c(q_s)}$ changes for a given amount of aggradation or degradation? The best-fit *k* was 2.83E-3 $s^{-1}$, which reflects the rapid adjustment of experimental $\tau^*_{c(q_s)}$ compared to slope changes (Fig. 5, Table 1, Eq 10). In contrast, the morphodynamic modeling used *k* values adjusted to be 2 to 3 orders of magnitude smaller, so that the response to a perturbation in supply would involve non-negligible changes in slope (the only morphologic variable in the simple morphodynamic model) as well as in $\tau^*_{c(q_s)}$. Higher values of *k* in the morphodynamic model cause $\tau^*_{c(q_s)}$ to adjust more rapidly and slope to adjust less (Fig. 8).

An implication of $\tau^*_{c(q_s)}$ evolving with reach morphodynamics is that local channel form can retain "memories" of previous conditions, which can influence local responses to subsequent forcing. In Fig. 9b and 9c, an increase in supply led to the temporal and spatial

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evolution of $\tau^*_{c(q_s)}$, which in turn caused spatial variations in equilibrium slope. Upstream reaches acted as filters of the supply perturbation to downstream reaches. In nature, spatially and temporally-averaged morphodynamic equilibrium will reflect "channel-forming" discharges and a representative sediment supply from upstream, but floods, local supply perturbations and history add to spatial variability in both $\tau^*_{c(q_s)}$ and morphology. I also acknowledge that the model parameters were calibrated to flume experiments at steep 8% and 12% slopes with a GSD that includes scaled boulders (Table 1; Johnson et al., 2015); future work is required to determine how the surface GSD influences the strength of $\tau^*_{c(q_s)}$ evolution, and how well the model predicts $\tau^*_{c(q_s)}$ changes in lower slope gravel-bed rivers.

## 4.2 State variable framework for modeling morphodynamics

Next, I argue that $\tau^*_{c(q_s)}$ should be redefined as a state variable (or state function) for gravel-bed channels, and outline a possible state variable approach for modeling the morphodynamic evolution of channels. The term "bed state" has long been informally used to describe collective aspects of local channel morphology, such as surface GSD and armoring and clustering, that change with relative ease and influence transport rates (e.g., Church, 2006; Gomez and Church, 1989). Although explicitly defining $\tau^*_{c(q_s)}$ evolution and related feedbacks in terms of state and path variables appears to be novel (to my knowledge), channel morphodynamics have long been implicitly described using similar ideas. For example, Phillips (2007) presented a qualitative conceptual model of landscape evolution in terms of improbable system states, arguing that although deterministic process "laws" act on topography, the actual outcome (i.e., any particular landscape) depends on initial conditions and in particular is sensitive to history. Many other works have similarly generalized complex channel process and response feedbacks to understand morphodynamics (e.g., Fonstad, 2003; Phillips, 2011, 2009; Chin and Phillips, 2007; Phillips, 1991; Stark and Stark, 2001; Yanites and Tucker, 2010).

State variables are integral to many disciplines, including control systems engineering and thermodynamics. Thermodynamic state variables include temperature, pressure, enthalpy and entropy. By definition state variables are path-independent (Oxtoby et al., 2015). For example, temperature ($T$) describes the amount of thermal energy per unit of a material. A

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Natural river channels inevitably exhibit morphologic variability at reach scales. For example, although the longitudinal profile of Reynolds Creek appears smoothly concave over a spatial scale of 10 km (Fig. 11a), there is substantial slope variability when calculated for 100 m reaches (Fig. 11b). This 100 m averaging scale was chosen for the following analysis because it is sufficiently large to plausibly be used for landscape evolution modeling, while small enough to capture slope variability along a profile. ¶ ...

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change in temperature depends only on the initial and final states (i.e., $\Delta T = T_2 - T_1$), but does not depend on the path, i.e. the history of temperatures between times $t_2$ and $t_1$. In contrast, heat--the flow (transfer) of thermal energy--is a path variable (or process variable), not a state variable. Heat flow between bodies is both controlled by and changes the temperature of those bodies, but the amount of total heat transferred does depend on the path. Three other points about state variables are relevant to morphodynamics. First, state variables are rarely independent of one another. For example, Gibbs free energy is a state variable calculated from temperature, enthalpy and entropy (Hemond and Fechner, 2014). Second, although state variables are technically only defined at equilibrium, in practice they are useful for understanding gradually evolving systems (e.g., Kleidon, 2010). Third, the evolution of systems involving multiple state variables are usually described with coupled differential equations.

Channel morphodynamics can be described by a similar framework of state and path variables. Analogous to heat, the cumulative discharges of both water and sediment are path variables that drive bed state evolution. Channel morphology can be described by numerous bed state variables, including but not limited to surface GSD, slope, width, depth, bed roughness, surface grain clustering, interlocking, overlap and imbrication, and finally $\tau^*_{c(q_s)}$. Analogous to temperature, I explicitly define $\tau^*_{c(q_s)}$ as a state variable. The amount of change change in $\tau^*_{c(q_s)}$ from time $t_1$ to $t_2$ does not depend on the progression of values in between. However, the amount of sediment transported between $t_1$ and $t_2$ does depend on the history of $\tau^*_{c(q_s)}$, and also influences the history of $\tau^*_{c(q_s)}$ (Eq. 8, 10).

Entropy is the state variable perhaps used most often to characterize channel systems (e.g., Chin and Phillips, 2007; Leopold and Langbein, 1962; Rodriguez-Iturbe and Rinaldo, 1997). Entropy can provide a closure for underconstrained sets of equations, by assuming that geomorphic systems inherently maximize their entropy at equilibrium (Kleidon, 2010; Chiu, 1987). A limitation of some maximum-entropy landscape models is that physically-based surface processes are not always explicitly modeled, making them less useful for predicting landscape responses to environmental perturbations, even if they can create reasonable equilibrium morphologies (Paik and Kumar, 2010). In contrast to entropy, state variable $\tau^*_{c(q_s)}$ has a clear process-based meaning.

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I suggest that landscape evolution models could incorporate subgrid-scale channel feedbacks by treating $\tau^*_{c(q_s)}$ as a state variable. Conceptually, the $\tau^*_{c(q_s)}$ model "lumps" processes related to multiple bed state variables (sections 1.1, 2.1). Similarly, because many *channel* state variables influence transport and therefore are not independent of $\tau^*_{c(q_s)}$, I hypothesize that aspects of morphology can be implicitly subsumed into evolving $\tau^*_{c(q_s)}$ for modeling purposes, because $\tau^*_{c(q_s)}$ captures essential feedbacks over spatial and temporal scales of interest. This is similar to the channelization approach of Stark and Stark (2001).

## 4.3 Form drag vs. parsimony

Calculations of best-fit $\tau^*_{rs50}$ and transport rates used total shear stress (Eq. 12), rather than partitioning stress into form drag and a lower effective stress for calculating transport rates (skin friction). Although not a state variable, form drag is physically justifiable because larger clasts that protrude higher into the flow (e.g. stable boulders) tend to account for a disproportionate amount of the total stress through drag, turbulence generation and pressure gradients. Form drag corrections have been incorporated into many transport models to enable reasonable transport rates to be calculated using $\tau^*_c$ values typical of systems without form drag (e.g., Rickenmann and Recking, 2011; David et al., 2011; Yager et al., 2012a). Conversely, another common approach (and that taken here) is simply to use higher $\tau^*_c$ (e.g., Bunte et al., 2013; Lenzi et al., 2006), consistent with acknowledging that $\tau^*_c$ can be a physically meaningful fitting parameter to predict transport. Using field data, Schneider et al. (2015) recently compared gravel transport predictions based on (a) form drag corrections and (b) higher reference stresses. For the most part, they found that both approaches could provide similar accuracy. They also noted that "uncertainties in predicted transport rates remain huge (up to roughly 3 orders of magnitude)" (Schneider et al., 2015), and suggested that factors including supply effects may account for remaining discrepancies. Although beyond the scope of the present analysis, form drag effects could be separated from best-fit $\tau^*_{rs50}$ by using a calculated skin friction stress rather than total stress. However, doing so would add extra uncertainty to the shear stresses, while still not directly accounting for effects of sediment

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supply. Implicitly subsuming form drag into $\tau^*_{c(q_s)}$ arguably provides a simpler and more parsimonious approach for modeling transport and morphodynamics.

## 5 Conclusions

I propose a new model in which feedback causes $\tau^*_{c(q_s)}$, the nondimensional critical shear stress for gravel transport, to evolve through time as a function of sediment transport disequilibrium (Eq. 8, 10). Net erosion tends to increase local $\tau^*_{c(q_s)}$ (reducing transport rates), while net deposition tends to decrease $\tau^*_{c(q_s)}$ (increasing transport rates). Laboratory flume experiments described by Johnson et al. (2015) are used to evaluate the proposed $\tau^*_{c(q_s)}$ model. The experiments intentionally explored disequilibrium bedload transport and morphodynamic adjustment. Thresholds of motion were back-calculated from the experimental data using the Wilcock and Crowe (2003) model for mixed grain size transport. I also show that the Wilcock and Crowe (2003) hiding function is consistent with our experimental data, supporting its applicability to steep channels.

After empirically calibrating three model parameters, the $\tau^*_{c(q_s)}$ model—a differential equation--can explain nearly 70% of the variability in experimental thresholds of motion. I then incorporate $\tau^*_{c(q_s)}$ into a simple morphodynamic model for channel profile evolution. Changes in $\tau^*_{c(q_s)}$ are negative feedbacks on morphodynamic response, because not only slope but also $\tau^*_{c(q_s)}$ evolve when perturbed.

Finally, $\tau^*_{c(q_s)}$ is redefined to be a state variable for fluvial channels. State functions and path functions are fundamental to many disciplines such as thermodynamics, because they allow the evolution of systems to be calculated. The same should be true for morphodynamics. Conceptualizing landscape evolution models in terms of feedbacks among evolving state variables and path functions may improve our ability to predict landscape responses to land use, climate change and tectonic forcing.

## Acknowledgements

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I thank Alex Aronovitz for conducting the flume experiments, Wonsuck Kim for aiding in the experimental design, Lindsay Olinde for helpful discussions, and Mike Lamb and Jeff Prancevic for sharing their $\tau_c^*$ compilation. I also thank Jens Turowski, two anonymous reviewers and AE Daniel Parsons for constructive feedback. Support came from NSF grant EAR-1053508.

## Appendix 1 List of variables

| | |
|---|---|
| $A_r$ | Dimensionless parameter for incorporating grain size or roughness ratios in Eq. (10) [1] |
| $b$ | Dimensionless hiding function exponent; either described by Eq. (16) or fit as a single value [1] |
| $B$ | Dimensionless "feedback factor"; Eq. (9) [1] |
| $c_1, c_2, c_3$ | Dimensionless empirical constants in Eq. (4) [1] |
| $D$ | Grain diameter, for model cases with a single size only [L] |
| $D_{50}$ | Median grain diameter [L] |
| $D_{s50}$ | Median grain diameter of bed surface [L] |
| $D_i$ | Grain diameter of size class *i* [L] |
| $f$ | Darcy-Weisbach hydraulic friction coefficient; Eq. (17) [1] |
| $Fr$ | Froude number [1] |
| $F_i$ | Areal fraction of grain size class *i* on the bed surface; Eq. 13 [1] |
| $F_s$ | Areal fraction of sand on the bed surface; Eq. 4 [1] |
| $g$ | Gravitational acceleration [$LT^{-2}$] |
| $h$ | Water depth [L] |
| $\kappa_{dep}$ | Exponent for net deposition in $\tau_c^*$-evolution models; Eq. (8), (10). [1] |
| $\kappa_{ent}$ | Exponent for net erosion in $\tau_c^*$-evolution models; Eq. (8), (10). [1] |

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| $k$ | Scaling factor for $\tau_c^*$ evolution. Dimensions are [1/T] for Eq. (10) |
| $\lambda_p$ | Bed porosity [1] |
| $q_{bi}$ | Volume sediment flux per unit width of size class $i$ in Wilcock and Crowe (2003); Eq. 13 [$L^2$/T] |
| $q_s$ | Volume sediment flux per unit width [$L^2$/T] |
| $q_s^*$ | Nondimensional volume sediment flux; Eq. (1) [1] |
| $q_{sin}$ | Sediment flux entering a channel bed area (reach) of interest [$L^2$/T] |
| $q_{sout}$ | Sediment flux exiting a channel bed area (reach) of interest [$L^2$/T] |
| $q_{sfeed}$ | Sediment flux entering upstream end of overall model domain [$L^2$/T] |
| $q_w$ | Volume water discharge per unit width [$L^2$/T] |
| $\rho$ | Water density [M/$L^3$] |
| $\rho_s$ | Sediment density [M/$L^3$] |
| $S$ | Water surface slope [1] |
| $S_{eq}$ | Water surface slope when reach is at equilibrium [1] |
| $\sigma$ | Bed roughness, measured here as the standard deviation of detrended bed elevations [L] |
| $\theta_s$ | Thickness of sediment deposited or eroded in a time interval; Eq. (10) [L] |
| $t$ | Time [T] |
| $t_{eq}$ | Equilibrium timescale for morphological adjustment [T] |
| $\tau$ | Shear stress; Eq. (3), (12) [$MT^{-2}L^{-1}$] |
| $\tau^*$ | Shields stress (nondimensional shear stress) [1] |
| $\tau_c^*$ | Critical Shields stress (nondimensional critical shear stress); Eq. (1) [1] |
| $\tau_{c(q_s)}^*$ | Critical Shields stress in new threshold evolution model; Eq. (8), (10) [1] |

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| $\tau^*_{c\max}$ | Imposed maximum bound for $\tau^*_{c(q_s)}$ in Eq. (9) [1] |
| $\tau^*_{c\min}$ | Imposed minimum bound for $\tau^*_{c(q_s)}$ in Eq. (9) [1] |
| $\tau^*_{mss}$ | High sediment supply nondimensional reference stress end-member bound in Recking (2012) transport model; Eq. (5) [1] |
| $\tau^*_{m}$ | Low sediment supply nondimensional reference stress end-member bound in (Recking, 2012) transport model; Eq. (6) [1] |
| $\tau^*_{ri}$ | Reference Shields stress for size class *i*, from Wilcock and Crowe (2003) (Eq. 15) [1] |
| $\tau^*_{rm}$ | Reference Shields stress for geometric mean surface diameter, Eq. (4) [1] |
| $\tau^*_{rs50}$ | Nondimensional reference Shields stress for surface grains of size $D_{s50}$ , Eq. (15) [1] |
| $U$ | Depth-averaged water velocity, Eq. (17), (18) [L] |
| $u_\tau$ | Shear velocity; Eq. (13) [L/T] |
| $x$ | Position measured horizontally (distance along channel) [L] |
| $z$ | Position measured vertically (bed elevation)[L] |
| $W_i^*$ | Nondimensional bedload transport rate for grain size class *i*, in Wilcock and Crowe (2003) , Eq. (13), (14) [1] |
| W&CM | Abbreviation for Wilcock and Crowe (2003) transport model. |

## Captions

Figure 1. Threshold of motion data from both field and experimental studies. A power law regression to these data gives $R^2$=0.34, indicating that a majority of the variability is not explained by slope alone. Dotted lines indicate common range of $\tau^*_c$=0.03 to 0.06 often assumed for modeling transport, although measured data fall well out of this range. Data have been additionally filtered to only include $D_{50}$ > 2 mm (i.e. gravel) and slopes between 0.002 and 0.2. Data were compiled and provided by Prancevic and Lamb (2015), based in part on



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Buffington and Montgomery (1997), with additional data from Olinde (2015) and Lenzi et al. (2006).

Figure 2. Flume experiment data (Johnson et al., 2015). a. Sediment transport rate in ($Q_{sfeed}$) and out of the flume. The upstream sediment supply rate was zero other than during the $Q_{sfeed}$ period. Experiment 1 was run for a longer duration than the others but shows similar trends. Note that the outlet $Q_s$ adjusts much faster to match the increase in supply than it does to decrease during periods of no input. b. Median bed surface grain diameters decreased during the feed of finer gravel, and then increase beyond their previous stable bed. c. Flume-averaged bed slopes changed relatively little even as transport rates and $D_{50}$ changed greatly in response to initial bed stabilizing and supply perturbations.

Figure 3. $\tau^*_{rs50}$ fits to the experimental data with the W&CM. ,"W&CM fit" uses Eq. (16) to calculate hiding function exponent b, while "Power-law fit" calculates a best-fit b along with $\tau^*_{rs50}$. Error bars give 95% confidence intervals on $\tau^*_{rs50}$ based on the regressions; although uncertainty can be broad the trends are clear and consistent. Shaded area indicates times of fine gravel addition (sediment feed) in each experiment.

Figure 4. Data points are based on power-law fits to exponent *b*. The W&CM hiding function (Eq. 16) does a good job matching the data, although it was not fit to these points. The first 6 measurements of each experiment (roughly the first 10 minutes) were excluded because of large scatter associated with the greatest bed instability. The axes reflect the left and right hand sides of Eq. (15), but uses dimensional stresses to be consistent with plots shown in Wilcock and Crowe (2003).

Figure 5. Best-fit models (Eq. 4, 8 and 10) compared to experimental constraints. The periods of upstream sediment supply ($Q_{sfeed}$) are indicated by the grey boxes for each experiment.

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Figure 6. Profile evolution, comparing the morphodynamic responses of models with and without threshold evolution. The initial condition is an equilibrium channel with $\tau^*_{c(q_s)}$=0.06, upstream sediment supply $q_s$=1e-3 m$^2$/s, and an initial equilibrium slope of 0.0147. Sediment supply is increased 5x at $t$=0. Lines are each 5 model days apart, and indicate the evolution to a new transport equilibrium.

Figure 7. Slope and critical shear stress evolution, for sediment supply increases (which correspond to Fig. 6 models) and decreases by factors of 5. As in figure 6, $t$=0 corresponds to an equilibrium condition where the initial slope and initial threshold are consistent with the initial upstream sediment supply. Slope and $\tau^*_{c(q_s)}$ were averaged over nodes 3-10, leaving out the first and last two nodes because of minor model boundary effects.

Figure 8. Morphodynamic model sensitivity to sediment supply perturbations and $k$. All models started at the same equilibrium condition as shown in Fig. 6 and 7. a. Slope adjustment, normalized by the initial equilibrium slope. The correspondence of Eq. 20 and the morphodynamic model calculations demonstrate that the models did asymptotically attain equilibrium slopes. b. $S_{eq}$ ratio is the ratio of equilibrium slopes of the Exner+$\tau^*_{c(q_s)}$ model divided by $S_{eq}$ for the Exner-only model, to show the relative affect that that $\tau^*_c$ evolution has on equilibrium slopes. c. Equilibrium timescales for model adjustment. d. $t_{eq}$ ratio is the ratio of $t_{eq}$ for the Exner+$\tau^*_{c(q_s)}$ model divided by $t_{eq}$ for the Exner-only model. Values are lower than 1, indicating that the $\tau^*_c$ evolution has a large influence on equilibrium timescales. e. Evolution of $\tau^*_{c(q_s)}$.

Figure 9. Spatial and temporal evolution of morphodynamic slopes, for the same models shown in Fig. 6. Slope is initially at equilibrium and responds to the 5x increase in upstream sediment supply at $t$=0. a. The Exner-only model initially has spatial slope variability, but evolves to a uniform new equilibrium slope. b, c. In the Exner+$\tau^*_{c(q_s)}$ model, spatial variability in both slope and $\tau^*_{c(q_s)}$ persist even at equilibrium.

Figure 10. Comparison of experimental and best-fit model constraints on $\tau^*_{rs50}$, compared to proposed constraints for $D_{84}$ reference stress bounds for low and high sediment supply from Recking (2012).

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**Deleted:** Figure 11. Comparison of Reynolds creek and $\tau^*_c$ data compilation predictions using Eq. (18) and (21). a. Longitudinal profile based on airborne Lidar (Olinde, 2015). Upstream end (distance=0) is an arbitrary location along the channel (a bridge). Data gap at 730 m is a gauging station weir; the slope steepens downstream of the weir where the valley becomes constricted by bedrock, although the bed remains almost entirely alluvial. b. Slopes calculated (averaged) over 100 m reaches, illustrating reach-scale slope variability. c. Histogram of $\tau^*_c$ compilation (data shown in Fig. 1), compared to $\tau^*_c$ calculated using Eq. (21), based on Reynolds Creek 100 m slopes (panel b), and assuming $q_w$=1 m2/s, $q^*_s$ =2e-3, and $f$=0.1. d. Similar calculation as panel c, but using Eq. 18 to solve for slope as a function of $\tau^*_c$ from the Fig. 1 data compilation, and compared to panel b slopes.¶
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