Reply to comments

We would like to thank the three anonymous reviewers and Marco Jorge for their comments. We are particularly pleased that three of the four reviews thought it was an important paper that should be published, and their constructive criticisms have really helped to improve it. We are disappointed that reviewer #2 thinks that we cannot extract information and meaning from the geomorphological signature of tunnel valleys. But we strongly disagree with this view and our argument is laid out below. That they ignore a large body of literature spanning many disciplines that has shown the importance of geomorphology for investigating landforms genesis we find rather perplexing. The reviewer comments are in black and our replies and revisions in blue.

- Stephen Livingstone and Chris Clark

Short Comment by Marco Jorge

INTRODUCTION

L80-84 (statement of objective): Consider rephrasing sentence (perhaps revise whole paragraph). Suggest to remove "To rectify this" (to correct this error), because it is about the lack of data rather than error. Scale -> geographic scale; Pattern -> spatial arrangement; Rectify ". . .mapping of the size, shape, pattern and [spatial] distribution to better understand spatial properties. . ." L84: "constitution" -> Composition?

We have replaced "to rectify this" with "Based on previous studies and the availability of DEMs, we are now able to undertake…" (as suggested by reviewer #3), changed "pattern" to "spatial arrangement" and "constitutes" to "is".

Reword the rationale for study area selection; e.g., that the landforms can be mapped from a DEM is unrelated to the study area.

We have deleted "they can be identified from digital elevation models (DEMs)"

Requires a study-area figure ahead of section 2. Ideally, it would include previously mapped tunnel valleys.

We have included a study area figure (new Figure 2) within which we have include sub-panels showing where the examples we present are taken from (see comments by reviewer 1).

INTRODUCTION/LIMITATIONS

Consider a more explanatory (longer) header for this section and 'challenges' as a replacement to 'limitations'. This section does not fit the introduction. It would better come under methodology.

We have moved the 'limitations' section to the methodology (now 3.4). However, we feel that 'limitations' is an accurate subheading of what this section discusses and have therefore retained it.

In what way(s) are buried tunnel valleys a limitation for the characterization of tunnel valleys? Section does not properly ponder this. What properties can be compromised?

We have extended this section to detail the limitations of infilling on each of our metrics (i.e. length, profile and width) and also stated that we cannot measure the depth of tunnel valleys.

Last sentence of paragraph L96-103: what does data from Europe show? The expectation would be for positive spatial autocorrelation – a problem for the characterization of tunnel valley spatial properties.

We compare our metrics and the distribution of tunnel valleys to those collected in Europe in the results and discussion sections and therefore do not feel that it should be included here also.

METHODS

Clarify that the USGS national elevation dataset (NED) is not a DEM per se. The NED includes several DEMs, not only the 3 m and 10 m DEMs.

We have modified the sentence to say "utilising DEMs" so that the reader is aware that the NED comprises several different resolution DEMs.

L146-147: Which criteria were used to map each of the mentioned landforms? Define each landform; use references. "Conventional criteria" is not self-explanatory

We have deleted "identified according to conventional criteria and…" as we do not want to use a lot of space defining glacial landforms that are not the major focus of what is already a long paper.

L148: Centrelines. Centrelines are not thalwegs. Revise manuscript accordingly. Were centrelines automatedly derived from valley side lines? Please describe methodology.

This should be thalwegs, which we have fixed here and throughout the paper.

L169: Justify scale chosen for analysis of longitudinal profile.

The scale chosen for analysis of the longitudinal profile was rather arbitrary, in that we wanted enough data along each line to make a decision as to whether they undulate or not and at an appropriate scale (i.e. not at metre scale where local effects become important or km-scales where we might miss smaller patterns).

L178-179: Clarify. Tributary tunnel valleys excluded from inventory?

We feel that the second sentence clarifies this point, i.e. where tributaries exist, only the longest route was used.

L181-182: Reword. It is about describing the relationship between width and local elevation gradient; not about the influence of one on the other.

We have replaced "influence of" with "relationship between".

L182: Justify 1km scale. This is a particularly important issue. Why an absolute interval, if valley length varies considerably? What about the valleys under 1 km and some kms long?

As per the comment above about scale, we had to make a decision, and felt that this scale provided a good compromise between data efficiency/processing and information. Although we may have missed some information for the very small valleys most are >5 km and so we feel the choice is justified.

L185: Reword. "Downstream distance" from what?

"from the head of the tunnel valley" added.

L191: drainage network –> tunnel valley network?

Changed as suggested.

RESULTS

L205: Perhaps would be more informative to the reader to name this section "results".

We have changed to 'results' as suggested.

Suggest reviewing usage of 'network'; 'cluster' is more appropriate; most tunnel valley clusters do not seem to be networks; rephrase, e.g., sentence L211-213.

This was also suggested by reviewer #3 (see below) and we agree that the term is not appropriate here. We have taken the reviewers suggestion and changed all references from network to cluster.

L247 vs. L399: 65 km vs. 55 km for maximum tunnel valley length. Rectify

Thanks for spotting this – it should be 65 km, which we have now fixed.

DISCUSSION

A discussion of the limitations is missing. The 'limitations' subsection under introduction is insufficient and would better be under the methodology and discussion. Include considerations on the usage of the centreline for analysing the (thalweg) longitudinal profile; and on the post-formational modification of valley bottoms, linking it to the scale chosen for longitudinal profile analysis.

As suggested above, we have moved the limitations section to the methods and expanded to include discussion of the post-formational modification of valley bottoms, and to include the initial presumption that tunnel valleys have a common genesis. See points above regarding justification of scale and use of centreline vs thalweg.

Reviewer #1

This paper is an important contribution to the understanding of the regional distribution of tunnel valleys in a Pleistocene ice sheet. The authors have conducted an intensive analysis of tunnel valleys through DEM interpretation and this paper should be published with minor revisions.

Thank you.

My main criticism of the paper is that I believe that they have underestimated the number of tunnel valleys present. It's hard to tell from their figures, but in the Saginaw lobe, where I am doing detailed mapping, I think there are more tunnel valleys than are mapped. Most of them are shallow and short. This may skew the statistics somewhat. I also believe, from just looking at DEMs and maps that they have missed tunnel valleys along the eastern side of the Lake Michigan Lobe through the Valparaiso moraine. I have not directly mapped them or studied them in the field myself.

We certainly do not pretend that we have been able to spot and map every single tunnel valley. We are covering a very large area and there are mapping problems, such as partial infilling. However, the resolution of the DEMs (3 and 9 m) now allow all but the very subtlest morphological expressions of tunnel valleys to be identified. We therefore believe that we have picked out a high percentage of all identifiable tunnel valleys. Moreover, we accept that this is a starting point, from which we hope to motivate more detailed regional studies to fill in the gaps and further progress the science.

In considering the regional distribution, it seems strange that the density of tunnel valleys in the Saginaw lobe is higher than just about any other area. And these are in the center of the lobe, where the authors suggest should have less than the margins. The original margins of the Saginaw lobe have been destroyed by overriding by the LM and Huron-Erie lobes. Do the authors have any comments on the relative abundance of TVs in the Saginaw lobe?

This is a very interesting point and although we do not know the answer, we now speculate about the importance of regional conditions in the paper. This includes the suggestion that "older or more northerly ice lobes with steeper ice-surface slopes, more extensive permafrost zones and sandier sediments … have a greater occurrence and density of tunnel valleys." [such as the Saginaw Ice Lobe] We have also toned down the discussion on the importance of ice geometry as suggested by reviewer #3.

The sections on the origins of tunnel valleys are on the right track, I think. In my area, I have found a strong association between tunnel valleys and kame-like landforms composed of sand and gravel. This supports a supraglacial meltwater source for the short tunnel valleys in this area. The outwash fans are very coarse, which suggests high discharge and perhaps a one-off type of origin. But there is no reason why they couldn't have been active for a longer period of time. Also in my area of the Saginaw lobe, I think that the tunnel valleys and outwash fans represent a change in drainage mode during ice retreat. There is a large drumlin field in which drumlins are partially buried by outwash from the tunnel valleys and I think that the drumlins were formed during advance with a distributed drainage system with high basal pore pressure and the tunnel valleys became active during retreat with a conduit-type drainage mode with overall lower basal pore pressure and higher coupling.

Thanks. Although we could not find a paper on kame deposits in the Saginaw Lobe we have commented on the evidence for supraglacial meltwater reaching the bed, referring specifically to the identification of moulin kames by Mooers (1989). The idea that tunnel valleys became active during retreat and represent a switch to channelized drainage is really neat and one that has relevance to some work I have been doing on subglacial lakes and N- and R-channels in Alberta, Canada (see Livingstone et al., 2016).

In terms of relatively minor comments, I found it annoying not to know where the detailed DEM images are located. Couldn't boxes for these figures be drawn on Fig. 3? That would help greatly in understanding the regional distribution and properties of these valleys. I was particularly interested in the catastrophic flood valley with the megaripples. I can certainly accept this as a hypothesis but I would like to know if there has been any field work done on these features. Not knowing where it is makes it more difficult to find that out.

We have included a study area figure within which we have include sub-panels showing where the examples we present are taken from.

There is a discrepancy in figure numbering between the text and figure section. Fig. 12/13 is one, and there may be others. I found a few minor typos and will upload a file with the ones that I noticed.

See below, we have made all of these corrections.

<u>Typos and additional comments from supplementary file:</u>

L276 – there rather than their

Fixed

L315 - Why is this association so rare? Out of all the tunnel valleys, it might be expected to see this more commonly.

This is intriguing and presently we are not sure why. It may be a preservation or resolution issue as they are very subtle features (<2 m high).

L334 - I disagree. Fast flow requires distributed drainage, and I don't think is consistnt with tunnel valleys.

Although this certainly runs counter to theory, we can only report what we found, i.e. that tunnel valleys occur in association with the ice lobes. However, we agree the valleys may have formed following slow-down and stagnation, and have added a caveat that they must have been fast flowing "for at least part of their history".

L451 –no "s" on "tunnels"

Fixed.

L459 – Is this based on the bedrock surface? Where glacial drift is thick and variable, the beds may not have been flat. There are bedrock features like cuestas that would not have been flat.

Not necessary – we mean the bed surface within which they formed. Certainly, the gross topography along this sector of the former Laurentide Ice Sheet is very flat, which is what has inspired this suggestion.

L462 - most are straight, trench like forms.

We agree that not all the tunnel valleys conform to this theory. We have clarified this sentence to state that it is just "tunnel valleys, which display large variations in width".

L491 - agree. Probably more likely formed during retreat rather than advance.

Thanks. We agree that they probably formed during retreat, but cannot rule out the possibility that they also form during advance and have since become buried.

L539 - this is figure 12 in the figure section.

We have corrected this.

L544 - This needs field confirmation.

We agree and have added this point at the end of the sentence.

L555 - Agree. Fining upward esker. Kehew et al. 2013

We have included this reference here.

L582 - not true for Saginaw lobe

This point is relevant at the ice sheet scale, which is what we mean here, i.e. tunnel valleys become less common towards the centre of ice masses. To help the reader we have added "at the ice sheet scale" here.

L591 - not only moraine, but non-morianal ice marginal positions

Possibly so, but this is difficult to show from our data and we have therefore left as it is.

L605 - strong association with kames in Saginaw lobe

This is a good point, and suggests contributions from supraglacial meltwater. I believe Mooers (1989) found similar deposits in the Superior Lobe and we have now commented on the evidence for supraglacial inputs to the bed in the paragraph above.

Reviewer #2

This paper is a contribution to the ongoing research on tunnel valleys, which certainly is a very important topic in paleoglaciology that needs to be deepened if the behavior of past ice sheets (stability, meltwater drainage, landforming processes) is to be better understood. Therefore, this study is a timely contribution that surely would not pass unnoticed and I fully sympathize with the authors in their effort to learn more about these fascinating landforms that have attracted a lot of attention and generated yet more controversies. The aim of this paper is to illuminate the origin of the tunnel valleys in the southern sector of the Laurentide Ice Sheet, in particular to test the catastrophic formation theory against the gradual formation theory. In order to do so around 2000 tunnel valleys have been mapped yielding valuable data on their geometrical characteristics as seen in the present relief. Unfortunately, despite my unrestricted appreciation of the great effort undertaken by the authors I find numerous methodological and conceptual flaws that resulted in interpretations and conclusions not supported by the data presented. In short, the present manuscript is an example of what one may call "extreme geomorphology", i.e. interpreting the origin of complex glacial features EXCLUSIVELY from their shapes – below I will try to demonstrate why this approach is flawed in the context of tunnel valleys.

We readily acknowledge in the manuscript that there are a range of approaches to studying tunnel valleys, including theoretical, sedimentological and morphological, and that each has its strengths and weaknesses. There have been many detailed approaches conducted by localised fieldwork and yet aspects of tunnel valleys remain enigmatic. Our approach was to assess their regional properties and of course it is not logically feasible to simultaneously conduct detailed fieldwork at these sites. Indeed, we are careful not to say at the outset that we intend to solve whether tunnel valleys form gradually or catastrophically, rather that we want to adopt an approach that can tell us something about the size, shape, pattern and distribution of tunnel valleys. We find it strange that the reviewer does not value this information on scale and shape of tunnel valleys. ***Let's be clear, tunnel valleys are erosional features and conduits for the passage of water and are wholly morphological features. To argue that it is inappropriate to address the morphology and geometry of these seems as absurd as expecting those studying rivers to ignore these aspects.*** *Furthermore, our approach isn't wholly morphological as we draw upon the rich literature of sedimentological data when discussing the implications of our results for tunnel valley formation. We are therefore disappointed that the geomorphological results of our study have been dismissed so easily. This review seems to have been badly affected by their dislike of pure geomorphological research, which has been shown to have an important place in palaeoglaciology (e.g. for palaeo-ice sheet reconstruction, identification of ice streams, interpreting subglacial bedforms).*

<u>Major issues:</u>

1. The authors mapped morphological characterististics of tunnel valleys as seen from the relief of the present land surface and use these characteristics to discuss the formation processes. This is a fundamental flaw because the present relief does not describe the geometry of the channel when it

operated but rather the geometry of the infill deposits. In particular, nothing is known about the morphology of the channel bottoms and consequently about the channel depths, longitudinal profiles (flat, adverse or undulating) and their true lengths (which may be much different from the apparent lengths seen at the land surface). In short, little is known about the actual geometry of the valleys and therefore any interpretation and conclusion based on the geometry is untenable. The authors mention this issue in passing (line 96) and yet ignore it entirely in the rest of the paper.

Although we agree that partial infilling of tunnel valleys is a limitation (and did not report on depths for this reason), we strongly contend the view that our interpretations and conclusions are untenable. Dealing with a fragmented record is part and parcel of investigating the geological and geomorphological record, and indeed extends to other disciplines such as palaeontology (e.g. Charles Darwin talked about the imperfection of the geological record in the origin of the species). However, we believe it is adequate for us to learn something about the spatial properties of tunnel valleys despite the fact that we may be missing some valleys and we cannot use their depth as a metric. Buried tunnel valleys (e.g. in the North Sea and Baltic) are nearly all pre last glacial, and we could not find reference to any Wisconsin tunnel valleys in the US literature that we could not also find. Width measured at the valley edges is not affected, and while long profiles and lengths may be contaminated by some infilling they still represent minimum bounds and likely capture a large portion of the true population given how well our results agree with more detailed field studies. To further clarify our approach regarding the issue of infilling we have expanded the limitations section to discuss some of these issues (as also suggested by the short comment). In particular, we discuss what metrics can be readily identified vs those that we cannot discern without more detailed sedimentological/geophysical data.

2. No original data are given on the nature of the deposits at the mouths of the tunnel valleys. Without such data any inferences regarding catastrophic vs. gradual water discharge through the channels are speculative at the very best. Still, the authors conclude that they favour the gradual discharge. However, the presence of coarse grained deposits including well-rounded boulders at the mouths of many tunnel valleys is well documented (e.g. in the papers referenced). This material MUST have been transported under high-flow conditions. Without describing the character of the deposits dumped at the channel mouths one cannot conclude anything about the discharge dynamics. What would help would be some data about the bottom profiles of the channels (steep adverse slopes necessitating highly pressurized, dynamic flows or shallow flat profiles suggesting more steady-state drainage), but these are not provided, either (see point 1 above).

Perhaps the referee didn't read the paper well enough! We do not disregard catastrophic discharge. In fact, we presented evidence to suggest that catastrophic drainage events did happen down tunnel valleys, drawing both on our own work and the literature specifically regarding boulders at the mouths of tunnel valleys. This includes outwash fans and giant current ripples, while we show that some valleys were only occupied for short periods of time.

3. The authors confuse incremental origin suggested for some of the tunnel valleys with steady-state, low-discharge gradual origin. These things are not the same. The fact that some of the valleys in the study area can be traced back from one moraine zone to another along the deglaciation path does not mean that these valleys must have formed "gradually" – indeed, they still could have formed in a series of cataclysmic outbursts through the same subglacial channel separated by phases of relative tranquility. But this can't be constrained because sedimentological and bottom-profile data are lacking.

This was not our intention, and we have modified our interpretation of incremental formation to include both gradual incision and/or repeated outbursts. Significantly though, this result does suggest that the whole valley was not eroded during a 'single event' and is therefore informative.

4. The authors repeatedly refer to the apparent sparsity of tunnel valleys connected to the past subglacial lakes whose location was mapped in the earlier paper (Livingstone et al. 2013) as indication of gradual rather than catastrophic drainage. This inference cannot be made because the postulated subglacial lakes were almost exclusively located in parts of the present Great Lakes basins, i.e. in areas not included in the present manuscript that only encompasses the exposed land surface. There are no data to validate the suggested lack of connection between the postulated subglacial lakes and the tunnel valleys.

We agree, as also pointed out by reviewer #3, that the subglacial lake predictions are likely to underestimate the true distribution because they do not take account of permafrost and the model makes some assumptions (e.g. water pressure = ice overburden). We have modified the discussion to make this clear (see detailed response to reviewer #3 below). As an aside, we actually started this project to specifically look at subglacial lakes in anticipation of finding associations (e.g. Livingstone et al., 2013) and therefore it was surprising to us that there was little apparent link.

5. The paper does not read well, it contains numerous loops and is not well structured. In particular, the reader lacks any information about the location of the study area (position, paleogeography, major ice lobes, etc.) and must fish this information out from bits and pieces dispersed throughout the whole paper. One first finds out where we are from Figure 3, way back into the paper. Repeatedly, there is a mix of own data with data from the literature, and descriptions are mixed with interpretations. It would be much more transparent to describe the findings separately in the context of individual major paleoglaciological systems (i.e. the ice lobes whose behavior could have been different) rather than putting all geomorphological data into one basket for most of the paper. Accordingly, Figures 4, 5 and 6 are meaningless because they contain data lumped together from different paleoglaciological systems.

We have included a new location map (new Fig. 2), which shows where each of the example figures is in relation to the wider context. We have also modified the main map (Fig. 4) to include more details on the surficial deposits. However, as it is a morphological analysis of a large sample of tunnel valleys we are reluctant to split the findings by region. Rather, we have tried to be much more explicit in stating where regional variations occur and have added a new commentary on how variations in tunnel valley distribution relate to regional factors such as basal thermal regime, ice and bed topography, timing and climate.

<u>Minor issues:</u>

1. The abstract says that the "tunnel valley morphology is strongly modulated by local variations in basal conditions (. . .) and hydrology (. . .)". There are no data in the paper to validate this statement.

We agree that this this is conceptual and not based on data from the paper and have therefore rephrased to: "The distribution and morphology of tunnel valleys is shown to be sensitive to regional factors such as basal thermal regime, ice and bed topography, timing and climate", which relates to a more detailed discussion of the regional variations that has now been included.

2. Line 99. ". . .buried valleys are rare. . ." in the study area. This statement is unsubstantiated by any data in the paper, nor any of the earlier studies referenced.

We have re-written this statement to state that: "However, few buried tunnel valleys have been reported along the southern margin of the Laurentide Ice Sheet, and most of the completely buried tunnel valley networks in Europe relate to pre-Weichselian ice advances (e.g. Jørgensen & Sandersen, 2006; Kristensen et al., 2007, 2008; Stewart & Lonergan, 2011)."

3. Line 168. "To determine whether valley thalwegs are undulating the number of (. . .) slope segments (. . .) were calculated." This is plainly wrong and bears significantly on the outcome. Again, when applying the land surface morphology you do not calculate the relevant THALWEG morphometry but the morphometry of the top of the valley INFILL. If one claims that there is no infill to mask the bottom profile, one would have to present strong field evidence. This is entirely lacking.

It is true that there has probably been at least some infilling of all the tunnel valleys following deglaciation. However, we only calculate the valley thalweg to decipher whether it was formed subglacially or not (i.e. if it is undulating). Please note that any subsequent valley infill from subaerial lake or river deposits is likely to ***reduce*** *the undulations (meaning we can be confident in our interpretation if they are undulating). Moreover, we manually checked that undulations are not the result of post-glacial processes (e.g. slope failures, rivers cutting across the valleys). We also recognise that there could be some error by including a higher confidence level, for which valleys must also contain subglacial landforms such as eskers and/or terminate at a moraine. Finally, this technique has been widely used in the literature to identify subglacial meltwater channels (e.g. Greenwood et al., 2007 and references therein). Perhaps this reviewer is worried that we are using long profiles for some other purpose that we are not?*

4. The tunnel valleys are grouped into three classes based on subjective confidence levels, whereby class 3 consists of all kinds of channels whose subglacial origin lacks any support. Even though this class is not included in the following statistical analyses, these channels should not have been considered at all in the first place.

We prefer to include them as they likely exist and they may help future studies. The tunnel valleys we classify into class 3 may still be tunnel valleys (i.e. they are a similar form) but they require further investigation. As the reviewer states, given the uncertainty over these landforms they are shown in a different colour in Fig. 3 and not used in the statistical analysis. This seems entirely sensible to us and reflects our caution in some of the interpretations.

5. Line 195 and 279. How do you know that the "drainage-sets" you distinguish were indeed "formed during the same drainage phase"? The cross-cutting relationships interpreted exclusively from the morphology (i.e. not considering the sedimentological record below the ground surface) can be illusive – see e.g. the comment on the crosscutting channels below. Examples of moraines overlapping valleys and valleys cutting through moraines (Figs 10 and 16; all important for the interpretation) are so poorly visible that the reader can't make any own judgement.

The use of cross-cutting relationships to produce a relative history of formation is well established and routinely used in palaeoglaciology, with superimposed glacial bedforms (e.g. mega-scale glacial lineations, drumlins and moraine) used to identify dynamic shifts in ice flow. We follow the same approach, using cross-cutting tunnel valleys, moraines and outwash fans to achieve the same objective. If a series of tunnel valleys all terminate at the same moraine and contains outwash fans at the terminus we can be reasonably sure they were formed during the same phase of ice retreat.

6. Line 353. The claim that "cross-cutting relationships indicate that not all tunnel valleys were acting synchronously" can't be validated because it is exclusively based on the morphology. Even assuming

that the mapped relief indeed is the relief of valley bottoms, this is claim is still unsubstantiated. An illustrative example is given in Fig. 7D where two channels are interpreted as cross-cutting, and taken to indicate different phases of formation. This is by far not necessarily so, simply because of the possibility of an anastomosing channel network consisting of diverging and converging channels that operate simultaneously. When exposed, such a network may look like channels cross-cutting one another, yet this is not any evidence of their different ages. Such networks are well known e.g. from 3D seismic data from the North Sea but in the North Sea there is also insight into the substratum enabling proper interpretation utilizing the relationships between the infill deposits and the erosional surfaces marking channel bottoms.

See comments above, we are not making this elementary mistake (re: drainage sets). In terms of Fig. 7d, the tunnel valley trending W-E terminates only a few km further downstream at an outwash fan (see also Mooers, 1989). It therefore must relate to a period when the ice margin was at the fan. The valley that cross-cuts it continues westwards several tens of kms further and must therefore relate to an earlier period of time when the ice margin was much further west. It is this sort of detailed morphological analysis that has enabled us to identify cross-cutting relationships and therefore to show that within any cluster of tunnel valleys formation was not synchronous. In the paper we only use cross-cutting sparingly and for cases that are certain. Again the reviewer appears to be over reacting. However, as this sort of detail is maybe not immediately apparent from the figure we have extended the caption to explain why the two valleys must cross-cut and not anastomose in this example.

7. Line 369 and elsewhere. The authors repeatedly speculate about the low subglacial hydraulic gradients modulated by low ice surface gradients of the ice lobes at the southern fringe of the Laurentide Ice Sheet and use it in support of the gradual drainage. While this may have been the case in some areas, in many other areas there is clear evidence of large accumulations of rounded boulders at the mouths of the tunnel valleys (e.g. Cutler et al. 2002 referenced in the ms). In order to move boulders over 1 m in diameter the water flow velocity MUST have been high, and thus the hydraulic gradients MUST have been steep. It should be emphasized that the pressure of water in the subglacial channel is not only related to the ice thickness immediately above it (which could have been relatively small), but also to the pressure of water further up-ice as far as the systems are connected. Therefore, high pressure close to the ice margin could have been caused by high pressure of water much further under the much thicker ice.

We agree that large discharges of water must have occurred down some tunnel valleys and repeatedly reference the work of Cutler et al. (2002) to support this assertion. However, low subglacial hydraulic gradients likely existed due to the low surface slopes of many of the ice lobes (e.g. James and Des Moines – from Clark, 1992). In line with suggestions from reviewer #3 (below), we have rewritten this in the section on tunnel valley distribution. In it we now emphasise that tunnel valleys are rarer under lobes characterised by low ice-surface slopes. Indeed, despite the shallow slopes, large or high pressure discharges could still have occurred due to lake drainage events (supra and/or subglacial).

8. Line 387. The authors postulate “the paucity of tunnel valleys towards the centre of former ice sheets” and interpret it to “be indicative of a change to temperate glacier conditions” (Line 397). First, I see no logic in this statement and second, as we know from the literature the up-ice decrease in tunnel valley occurrence is accompanied by an increase in the frequency of eskers that can be considered equivalent to tunnel valleys (typically on hard beds) (see multiple papers of G. Boulton). Therefore, any straight-forward conclusions from the paucity of tunnel valleys in the up-ice areas are unsubstantiated.

We agree that this paragraph is confusing and have therefore deleted it. However, the apparent tendency for tunnel valleys to form near the southerly terrestrial margins of palaeo-ice sheets is a real feature (based on the current literature) and we therefore think is worthy of mention and some comment. Therefore, in the paragraph above where we discuss permafrost we mention that the width of the frozen toe is likely to decrease during retreat and so the decline in tunnel valleys away from the maximum limit may be indicative of a change to temperate glacier conditions.

9. Line 400 and forward. The comparison between the morphologic parameters of their tunnel valleys and those in the North Sea is misleading because there is a very high chance that the tunnel valleys mapped in this study represent only a small portion of all tunnel valleys and the small segments treated as single valleys actually are parts of one long tunnel valley. This is because that, again, the tunnel valleys in this study are not mapped from their true thalwegs, contrary to the tunnel valleys known from geophysical studies in the North Sea.

Certainly we are not able to compare the depth of tunnel valleys beneath this sector of the Laurentide Ice Sheet with those in the North Sea due to the problem of infilling, which is why we didn't. However, we are confident in our comparison of width and our minimum lengths as these are more easily observed from the geomorphological data. Although some of the valleys may have become partially infilled making them look like they are actually multiple tunnel valley segments, in the majority of cases we can use the geomorphology to reconcile this. For instance, where there are very abrupt end and start points (e.g. Fig. 8e,f). Moreover, if many of the valleys had become fragmented due to infilling we might expect a large difference in length between the North Sea and Laurentide tunnel valleys, which would be instructive in itself. Therefore, we are happy with this comparison.

10. Paragraph starting with Line 426 and elsewhere. The reasoning about channel width and volumes of water leading to the exclusion of catastrophic discharge are interesting but flawed. This is because (1) the width of the channel as seen in the landscape only refers to the width above the infill sediments, and (2) more importantly, since the depth of the channel and its cross-sectional geometry are unknown, nothing conclusive can be said about the dynamics and fluctuations of water fluxes. This is because a narrower channel can still drain more water (rather than less, as the paper postulates) than a wider channel if it is sufficiently deep.

We completely disagree with this point. Although depth clearly cannot be derived because of the issue with partial infilling following glaciation, the tops of the valley sides are still clearly visible and it is these that are mapped. We are careful to not talk about water fluxes or valley depths throughout the manuscript. But, as has been shown in fluvial geomorphology, you would still expect to be able to use the width as some function of the amount of water draining through it, and as such it is interesting that there is so much downstream variation in width. Indeed, we recognise the idea that a valley can erode down into the sediment or even up into the ice (e.g. Fig. 15) as one possible explanation, although there are also others.

11. Paragraph starting with Line 451. Here we have speculations about the influence of local basal and hydrological conditions on the tunnel valley formation. Regrettably, no DATA constraining this discussion is presented in the manuscript.

Again the referee does not seem to like a regional approach using a large sample of tunnel valleys to progress the science. Anyone is free to make detailed investigations bringing substrate data into the discussion, but it is beyond the scope of our analysis which is primarily morphological.

12. Chapter 5.3 on landform associations. This chapter is difficult to follow and understand due to the lack of thorough description of specific areas used to illustrate the examples. Rather than organizing this chapter by specific landforms, it would be much more transparent to describe specific areas and illustrate landform associations occurring there. But even then it would be weak because the only data available is the surface relief and everything on the internal composition, deposits, structures, etc. is lacking.

As per reviewer #3s comments (see below), the use of the heading "systematic landform associations" is misleading here and we have changed to "landform associations" and made clear that this section is about what associations with other landforms tell us about tunnel valley formation. Thus, it is less about the distribution of tunnel valleys and associated landforms and more about what these associations tell us about their formation. However, we have included a new figure (Figure 2), which shows where each of the examples in the study area are taken from. We have dealt with the final sentence in detail under major issues.

13. Line 583. In order to support the inference that large drainage events were not the primary mechanism of tunnel valley formation an argument is given that the lengths of tunnel valleys are "orders of magnitude less than the distance up-glacier (. . .) that supraglacial and subglacial lakes are commonly documented in Greenland and Antarctica". This can be questioned because (1) the authors only mapped tunnel valleys with a topographic expression that likely only represent a portion of the whole population of the tunnel valleys in the area (including the buried ones), and (2) as we know from the recent paradigm shift in the englacial drainage research (the cut-and-closure model; e.g. Gulley et al. 2009, J. Glaciol. vol. 55, no. 189, and some following articles) the englacial channels may be oriented nearly parallel to the ice surface instead of penetrating a glacier at a high angle (the old Shreve's theory), and therefore can drive water from supraglacial lakes for long distances englacially before it reaches the bed.

We believe the argument put forward by the reviewer is flawed. Firstly, why would the tunnel valleys preferentially become buried upstream? As far as we are aware, there is no evidence for this despite the number of geophysical studies that have been carried out on tunnel valleys around the world. Indeed, the lengths of the tunnel valleys reported here are very similar to other studies that have used both geomorphology and geophysical studies (e.g. in the North Sea, Germany and Denmark). With respect to their second point, the cut-and-closure channels proposed by Gulley et al. (2009) were found specifically on uncrevassed regions of polythermal glaciers and are associated with incision of supraglacial streams followed by roof closure. However, cut-and-fill channels have not been found on ice sheets where the ice is much thicker (resulting in greater rates of creep closure) and we are specifically referring to the drainage of supraglacial and subglacial lakes. Subglacial lakes originate at the bed, which renders the reviewer's whole argument redundant, while there is a large body of research that shows that drainage of supraglacial lakes to the bed occurs via hydrofracture and does not drain englacially for long distances (e.g. Zwally et al., 2002; van der Veen, 2007; Das et al., 2008; Das et al., 2008; Shepherd et al., 2009; Krawczinski et al., 2009; Bartholomew et al., 2010; Selmes et al., 2011; Sole et al., 2011).

14. Locations of figures with morphological examples are not shown in any reference map; Fig. 12 refers to "Giant Current Ripples" but I can't find them in this figure because they are lot labeled as such there; tunnel valleys in Fig. 14 copied in from Fig. 3 are totally out of scale of this map showing nearly the whole Northern Hemisphere and thus invisible; Fig. 15C is redundant because you know nothing about the (true) bottom profiles of the tunnel valleys; Fig. 16 lacks scale bars; Fig. 18 is trivial and brings nothing new.

We have included a location map within which we have include sub-panels showing where the examples we present are taken from (also see comments above). The term "Giant Current Ripples" has been used instead of "sinusoidal bedforms" in Fig. 12. We have added scale bars to Fig. 16. Although the tunnel valleys are out of scale in Fig. 14 the idea is to show the general distribution, hence the use of boxes to refer to other locations where tunnel valleys occur. We have therefore left these as they are. Fig. 15 presents a number of conceptual theories to explain the downstream variation in width – the aim is that these will stimulate physical modelling studies and we are therefore happy to keep Fig. 15c as it is.

15. Multiple typos and awkward/unclear expressions, e.g. Line 38, 195, 226, 411 (what is MSGLs?), 418, 592, 937, 940.

L411 - We have included "mega-scale glacial lineations (MSGLs)".

L937 - fixed.

L940 – changed from "is" to "it"

We have checked through the other lines highlighted by the reviewer but cannot determine what is meant.

Reviewer #3

This is an exciting and welcome paper; however it needs some major revision. Tunnel channels and tunnel valleys have been studied in the Upper Midwest of the United States for over 40 years. And although some regional compilations have been put together before, this paper gives us a complete map showing the distribution of topographic features the authors interpret to be tunnel valleys (and/or tunnel channels).

Thank you.

I would suggest my major criticisms are that the authors assume that (1) there is a 'one-size-fits-all' explanation to the distribution of tunnel valleys/channels and that (2) a regional geomorphic analysis is sufficient to make informed interpretations of tunnel valley/channel genesis. As a geologist who has worked on some of these tunnel valleys/channels, it is my experience that it is not often obvious what is or isn't a tunnel valley/channel. Also, it is quite clear to me that the distribution of tunnel valleys/channels is sensitive to many factors (which the authors to some degree refer to) including basal thermal regime, age, dominant grain size, timing and climate. The local geology is an essential ingredient of understanding geomorphic development. A regional geomorphic view is important, but I regard the conclusions of this paper, based solely on this map, as being tenuous. The map is a 'great map,' but it is not clear how much it tells us about tunnel valley/channel development. The article should be published, but the authors need to soft-pedal their interpretations. This pure geomorphic analysis cannot give the whole story.

Point (1) - we set out to improve knowledge on tunnel valleys by increasing their known distribution and providing information on their morphological properties, and then to see if such information can help to resolve current debates regarding gradual or catastrophic formation. In fact, our findings demonstrate that both can happen and so we do not assume one size fits all. But we did not want to put the conclusions at the beginning of paper. To try and address the question of what is or isn't a *tunnel valley/channel we have added in an extra section where we compare the morphological properties of the two. This includes a comparison of tunnel channel and tunnel valley spacing, length and width. The results show that we cannot tell them apart, and therefore that they are equifinal, and we discuss the implications of this, both for our own work and future studies.*

Point (2) - we agree that local geology and glaciological contexts are very likely to provide important influences on the formation, preservation and visibility of tunnel valleys, but we also regard that channels are morphological features primarily eroded and so it is sensible to investigate their morphology. We fully agree that the whole story cannot be written from our analysis, and hope that this contribution assists in making the next steps. To try and address this point we have put much greater emphasis on regional variations in basal thermal regime, age, dominant grain size, timing and climate throughout the paper (see series of comments below).

—The authors are inconsistent on expressing their interpretation(s) of the genesis of the TV/Cs. That is, in the abstract and elsewhere, it is clear that they interpret that a gradual genesis (TV) is more likely than an outburst one (TC). However, it is clear elsewhere, that they state both processes can work. Nonetheless, there stills seems to be a driving assumption, without stating so, that everything they identify as a TV is 'one thing.' That is, they implicitly assume that there is one explanation for all of these geomorphic features. There is actually no scientific basis to assume this, except for their impression that TVs look similar to each other. If they think that everything they identify has the same genesis, they have to argue this otherwise they are simply assuming that, just because they look alike, they must have been formed the same way.

Our intention was to improve knowledge on tunnel valleys by providing information on their distribution and morphological properties, and then to see if such information can help to resolve current debates regarding gradual (Tunnel Valleys) or catastrophic (Tunnel Channels) formation. So in that respect we do make the initial presumption that there is a common genesis and use this as a basis for exploring relationships between form, distribution and process, which could challenge this. To make this clear we have added the following sentence into the introduction:

"… we treat all linear depressions of the appropriate scale and morphological characteristics as the same thing, whilst recognising that in detail this might be a grouping of a number of types".

And another sentence in the limitations section:

"In analysing this large dataset of tunnel valleys along the southern margin of the Laurentide Ice Sheet we make the initial presumption that tunnel valleys have a common genesis and then search for circumstances and data that challenge this. This allows us to focus on possible relationships between form, distribution and process."

—Associated with the comment above is that the authors seem to discount that some 'tunnel valleys' are actually 'tunnel channels.' There are a number of papers they cite in which the papers' authors are quite convinced that the tunnel feature they are looking at is a channel, not a valley. And by this they mean that the channel was occupied bankfull when the channel formed, and the channel formed catastrophically. Though the authors mention 'tunnel channel' in the beginning, it is not at all clear whether or not they accept that they are TCs. There is no place that they say that 'we do not believe that there are any actual tunnel channels.' If TV/Cs are multigenetic, then a combined analysis of all is flawed.

Building on the points above, we have now extended the introduction to be very clear in that we initially use the term 'Tunnel Valley' in its broadest sense (sensu lato) such that it includes depressions that could actually be tunnel channels. Thus, initially we group all linear depressions of the appropriate scale and morphological characteristics as the same thing and call them as such throughout the paper. However, our results showed that actually there are different types and we make this clear in the implications section and conclusions, where we discuss gradual vs catastrophic formation.

—There are a number of terms that the authors use without explanation that have 'loaded' meanings that are not necessarily meant in the way the author's indicate, or, more seriously, they bear with them a 'conclusion' about the nature of these features. The word 'network,' so appropriate with subaerial drainages, is used to describe regions with several tunnel features. However, if the tunnel features are tunnel channels, and perhaps formed one at a time, they are not 'networks.' They are not 'networked;' they are individual channels. However, by using the word 'network,' the authors imply the tunnel features are operating simultaneously; the reader then gets the unargued view of the authors that the tunnel features are running at the same time. I ask the authors to find another term to replace 'network.' Another example of a term is 'maturity' which props up in one place (see below). A third would be 'phase;' also a loaded term (see below).

The use of the term 'network' was also brought up by the short comment and we agree that this is not appropriate here. We have therefore changed it to refer to tunnel valley 'clusters' as suggested by Marco Jorge. We have removed the term 'mature' from the manuscript, recognising that the term is loaded and quite provocative. We have also removed 'phase' and included 'time period', with an additional caveat that not all the tunnel valleys in a cluster may have formed simultaneously.

—The mapping of subglacial lakes (in modern ice sheets and for Pleistocene ice sheets) is an important development. The Livingstone et al 2013 paper is a great step forward in our understanding, but in places in this paper, the authors refer to this paper as if it is correct. One colleague of mine has modeled subglacial water in the Upper Midwest and gets subglacial lakes in many places Livingstone and others don't. This work is not published, but it simply means that the current authors should tread lightly with the 'truth' of their results.

We agree that we should be careful with how we use the results presented in Livingstone et al. (2013). We have therefore clarified in the discussion that the modelling presented by Livingstone et al. (2013) is likely to underestimate the true distribution of subglacial lakes. The specific changes are dealt with in more detail below.

—Finally, the authors have done a hell of a lot of work and have produced a terrific map. However, as I indicated above and below, I believe that their analysis is flawed and that it is difficult to come to specific generalizations about TV/Cs that apply to all. Rather, they should emphasize regional variations and be more humble about assuming that there conclusions can be applied to all features they interpret to be TV/Cs. Here are some more specific comments.

We have responded in detail to these points above.

Line 16. It is misleading to say that TV's 'tend' to be associated with giant ripples and hill-hole pairs. Most hill-hole pairs do not have TVs and most TVs do not have hill-hole pairs. Only one example of giant ripples was found. Even outwash fans and eskers are not 'tendencies' although many TVs have both. I recommend getting rid of at least the giant ripples, if not the HHs as well.

We agree this is wrong and have deleted this sentence from the abstract.

Line 25-26. The authors' viewpoints shift regarding to TV genesis. Here, it is pretty clear that they infer a 'gradual' origin over an 'outburst' origin. However, it is clear elsewhere in the paper that both occur. The abstract should be changed to reflect this. TVs are equifinal.

Certainly, we see evidence for both 'gradual' and 'outburst' formation of tunnel valleys. But the evidence for outburst formation is less widespread and we cannot determine (based on our morphological analysis) whether the high-discharge drainage events created the tunnel valley or drained down it. That is why in the abstract we state that: "Our data and interpretations support gradual tunnel valley formation with secondary contributions from flood drainage…". We have been through the rest of the paper to make sure that this is a consistent message.

Line 34. A paper on tunnel valleys should likely refer to the oldest (?) reference: Ussing, N.V.,1903, Om Jyllands hedesletter og teorierne for deres Dannelse. Oversigt over Det Kongelige daske Videnskabernes Selskabs Forhandlingar 1903, v. 2, p. 1- 152.

We have included this reference.

Line 42-43. This is quite an overstatement. The underlying processes are very well understood - most of what they call TVs are cut by subglacial meltwater erosion. We perhaps understand them better than drumlins, eskers and even end moraines. Yes, there is a discussion on water source and basal thermal regime, but there is not 'considerable uncertainty.' (And regarding their statements on water source and basal thermal regime, this paper does not seem to solve any of this 'uncertainty.')

We agree that this statement is very strong and have changed to read "there is still uncertainty over how tunnel valleys form".

Line 57. Something wrong here - 'sheet flood' implies a sheet; 'bank full' implies a channel.

Deleted "(bankfull flow)"

Line 73. This needs to be clarified - what is being hydrofractured and brecciated? The overlying ice?

We have added "of the preglacical bed" to the sentence to clarify what is being hydrofractured and brecciated.

Lines 76-80. This is good that the authors understand that an understanding of tunnel valleys/channels cannot be achieved by geomorphology alone. Sedimentology, stratigraphy, climate, glaciology and theoretical considerations must be included. However, earlier studies have also included maps with distributions; rarely were single TVs investigated. We actually know quite a bit about state-wide and interstate distributions. This paper is a welcome compilation of much of this work (and many previously unidentified TVs), but using the word 'rectify' implies that something wrong was done in the past and the authors are coming to the rescue. They could say instead something like, 'Based on previous studies and the availability of DEMs, we are now able to examine the regional distribution of TVs. . ..'

This point was also made in the short comment and we have replaced "To rectify this" with "Based on previous studies and the availability of DEMs, we are now able to undertake…"

Line 85. Yes, landforms are part of the 'geologic record,' but most uses of the term 'geologic record' by geologists means information found in rock and sediment and implying some chronologic knowledge. What the authors are identifying is tunnel valleys in a glaciated 'landscape.' 'Geological record' sounds inaccurate.

We agree and have changed 'geological record' to 'glaciated landscape' here and elsewhere throughout the paper as suggested by the reviewer.

Lines 105-110. This is welcome that the authors site the many works on TVs that have been done. Perhaps they are aware that many more TV/TCs are mapped and described in reports and maps of the state geologic surveys. For example, Kent Syverson wrote a fine report on Chippewa County (WGNHS), which is featured in one of your figures. It would be good to check his story out. In any case, the authors have done a great job in being inclusive.

Thanks for this, and for the tips about state geological surveys. We have read some of these and they provided quite a bit of help in terms of identifying tunnel valleys. In terms of the Kent Syverson report, we have found and read this document, but although it provided a useful background to the region it did not say much about the meltwater signature.

Lines 132, 133. 'Sjogren' is misspelled in many places in the paper.

Thanks for pointing this out. Fixed!

Line 140. The analysis of Cutler et al is a little more in-depth than this sentence might imply. They made paleodischarge measurements based on boulder sizes that implied large discharge rates. A reader might think that the authors saw 'big rocks' and thought 'big water.' It was more sophisticated than that.

We have expanded this section to mention how Cutler et al. (2002) used palaeodischarge estimates on boulders as evidence for large discharge flood events.

Line 156. As I imply earlier, TVs are interesting, but we know quite a bit about them, but some aspects are still unknown. I am not sure if this makes them 'enigmatic.'

We have changed "enigmatic" to "difficult to discern".

Line 160. 'Fluvial river' sounds strange. Are there any rivers that are not 'fluvial?' And are not proglacial streams 'fluvial,' too? How is a proglacial river not fluvial?

We agree this sounds strange and so have deleted "proglacial and fluvial".

Line 163. This sentence opens up a can of worms. Hidden in this comment is an assumption that there is a landform that could be seen to be an immature tunnel valley. Can this be true? Have you seen any? How do tunnel valleys evolve? And do you have examples of TVs in youth, maturity and old age? Certainly there is evidence for smaller subglacial streams beneath glaciers, as shown by imaging of the base of the Antarctic ice sheet, or as seen in 'canal' and other glacial sediments. Are these part of a 'continuum' with tunnel valleys? I think not. By using 'less mature' and 'continuum' the authors are making the danger of linking many forms together that may not be genetically similar. There certainly are geomorphic features on this planet that show evolutionary forms (alluvial fans, hillslopes) as well as ones that form continua (eolian dunes; drumlins, drainage networks), but this does not mean all landforms are evolutionary or part of a conituum. Here too is an assumption about the nature of tunnel valleys and subglacial streams that is merely implied and assumed. There seems to be an urge on the part of some geomorphologist to see everything as part of a continuum.

This is perhaps a provocative sentence, but our aim was not to suggest that we think there is a continuum, rather that we do not know whether 'meltwater channels' and 'tunnel valleys' belong to the same population. We agree that the choice of words used is perhaps not helpful and the sentence is maybe loaded towards suggesting a continuum. We have therefore rewritten the sentence so that it now reads: "Linear incisions similar to tunnel valleys but of much smaller size (tens of metres in width) and called subglacial meltwater (or Nye) channels are also common in glaciated landscapes (e.g. Greenwood et al., 2007) but it is generally presumed that these are not part of the same population as tunnel valleys; that they are different landforms distinguished by size but perhaps also by process".

Line 167. 'Potential' tunnel valleys. The term 'tunnel valley' and 'tunnel channel' are interpretations; it is important to remember this. It would be helpful to have a term like 'tunnel-valley-or-tunnel-channel-like valley' which would be a non-genetic name for these features. However, that is rather clumsy. This sentence would be better for me if it said: "All valley forms that potentially could be interpreted as tunnel valleys or tunnel channels were mapped, and then each was tested to see if it could be shown to have been formed subglacially, and thus, be interpreted to be a tunnel valley or tunnel channel." And, prior to the next sentence, it would help to add - "One way to strengthen a subglacial interpretation would be to demonstrate that the longitudinal profile slopes upward towards an associated ice margin or that the profile undulates."

We agree with rewording suggested by the reviewer here and have included the sentences in the text. To avoid confusion, we have also expanded the first sentence in this paragraph to include: "… and use the term non-genetically in reference to both tunnel valleys and tunnel channels." Moreover, as this is important to state from the outset we have included a new discussion on the terminology, where we outline the different words used to refer to these features, and then state that we use the term tunnel valley in its broadest sense (sensu lato) to include depressions that could also be tunnel channels.

Line 167. 'Thalweg' is not quite the right word. You simply mean the valley bottom,. Avoid fanciness when it is not necessary. Subaerial rivers have, in fact, thalwegs that undulate, going up and down through riffles and pools. I suggest to get rid of 'thalweg.')

We have changed here to "valley bottom" but elsewhere it is more accurate to state that we have mapped the valley thalweg (see short comment).

Line 187. 'Phase' in the way they use it, also implies that the TV/C's are operating at the same time. This term is 'loaded' and has either unintentional or, worse, unsupported implications.

This was not our intention. We just wanted to state that tunnel valleys in the same cluster were likely formed in a similar time period when the moraine was formed, but this does not mean they were all operating at the same time. To clarify this point, we have changed "drainage phase" to "time period (although they may not all have been operating at the same time)".

Line 202. Your point (3) can be explained otherwise. It is not uncommon for TV/Cs to be filled with stagnant ice during retreat, leading to collapse after retreat. A 'breached moraine' may not be a sign of continued TV/C activity, but rather collapse of buried ice into the TV/C, implying the opposite about TV/C activity. My guess is that it would be hard to differentiate these.

This is a good point, and one which we had not considered. However, we would still expect moraines or stagnation features to fill the tunnel valley if this occurred. Thus, where moraines are clearly observed and do not cut across the valley I think our interpretation is probably the simplest.

Line 212. From reading these lines and looking at Fig. 3, it is not clear to me that you are describing what is shown. First, and again, I think you need to remove 'networks' and replace it with something else. But you need to look again at you map and simply describe what you see, and not try to force coming interpretrations (Figure 13). Perhaps I would write something like "Certain ice lobes completely lack TV/Cs, or have very few (James, DML, Michigan, Huron-Erie) while others have TV/Cs that are somewhat evenly occurring along much of the lobes' margins (Wadena, Itasca, Superior, Chippewa, Saginaw). Still others have TV/Cs along lateral lobe margins (Green Bay, parts of the DML). Some TV/Cs are more prominent at retreatal positions, others at the LGM margin. In fact, it is difficult to describe clear tendencies in TV/Cs occurrences that are valid throughout the study area." It seems that the TV/Cs actual do run down the center on the lobe for the Saginaw, retreated Langlade, Chippewa, Superior, Wadena and Itasca lobes, so your statement of 'avoidance' doesn't seem to match the figure. I also fail to see any prominent TV/Cs in what you call suture zones (interlobate areas?). If you see these, you need to give an example. You should also be aware of what may be perceived as an interlobate zone on your map might not be valid because the TV/Cs may have developed at a time when there were not two lobes adjacent. To be clear, let me repeat what I say here I simply do not recognize what you say when I look at your figure and read this text.

Further to the general comment, we agree that we have gone a bit far with the description (and interpretation) of the distribution of tunnel valleys. We have therefore softened this section along the lines suggested by the reviewer. This has included deleting text on their relation to suture zones and avoidance of long axes of major ice lobes and replacing with "Indeed, while clusters of tunnel valleys occur somewhat evenly along much of the Wadena, Itasca, Superior, Chippewa and Saginaw ice lobes, they…" Further to comments by this and another reviewer we have also removed the word network and replaced with cluster.

Line 220. You need to be clear about the use of 'basin' and 'subbasin'. You mean some kind of depression that would have been basin-like when covered by ice. Certainly the Great Lakes represent such basins. The Saginaw does originate in an arm of Huron, but the Langlade and Chippewa do not originate in sub-basins of Superior. And the Des Moines does not have any basin at all, but it does have a trough (created by ice streaming, likely). In other words, I think you have expressed these 'lows' inconsistently and inaccurately.

To clarify the use of the terms in this paragraph we have replaced "basins or sub-basins" with "depressions in the landscape". We have also removed the word sub-basin altogether, instead stating the "tunnel valleys are downstream of the present-day Lake Superior Basin".

Line 224-227. See intro of this letter to see what I feel about the subglacial lakes. Here, to point out why some TV/Cs have predicted subglacial lakes and others don't, says more potentially about the modeling of subglacial lakes than it does about TV/Cs.

See comment below, in the discussion we are now careful to clarify that the modelling presented by Livingstone et al. (2013) is likely to underestimate the true distribution of subglacial lakes.

Line 229. To repeat, I think 'network' is a loaded term and bears with it unproven implications.

We have replaced 'network' with 'cluster'.

Line 245. I suggest to replace in the heading ‘a tunnel valley’ with ‘tunnel valleys’ or ‘tunnel valleys/channels’

Changed.

Line 264. Figure 8 shows a weak relationship, it says here. As I say in the introduction to this review, there is a philosophical problem with placing all TV/Cs in the same study. If they actually do have different origins (for example, gradual vs. outburst, or that some of the ‘TVs’ are actually palimpsest meltwater channels), this means that analyzing them together is somewhat illogical, or potentially less meaningful. Certainly, these relationships can be mentioned, but the authors MUST point out the potential weakness of the assumption that there is a common genesis to their TV/Cs.

This relates to the reviewer’s major criticism and we have dealt with this in detail there. However, it is worth adding here that we have now included a caveat in the limitations section that we initially presume that all tunnel valleys did form in the same way, and then set out to challenge this. Furthermore, we have added in a section in the implications section, where we look specifically at tunnel channels vs tunnel valleys sensu stricto and discuss the implications in relation to the problem of equifinality.

Line 290. This is an interesting relationship, but it seems be a good example of how different TV/Cs are in different places. The hill-hole pairs in ND are controlled by the presence of permafrost (according to Clayton) and especially the Cretaceous SS-Sh bedrock which affects the groundwater. The thrusting involved in the hill-hole formation ‘uncorks’ the subglacial water source and a TV/C is created. However, it seems for this reason odd to have these listed in a section on ‘systematic associations.’ The ND examples are an exception to the rule that TV/Cs and HHs are most often unrelated.

We agree, the question we set up in the introduction, was poorly phrased. We are not looking for commonality, rather interesting relationships between tunnel valleys and other landforms that may tell us something about how tunnel valleys form. We have therefore re-phrased the question in the introduction to: “What can associations with other landforms tell us about tunnel valley formation?” We have removed “systematic”, and instead refer to “landform associations”. In this paragraph we also now highlight the regionality of this phenomena: “This association highlights the importance of regional variations in controlling tunnel valley formation and morphology; in this case, it is the local geology (Cenozoic and Cretaceous shale and sandstone) and presence of permafrost that likely controlled the initial formation of the hill-hole pair (e.g. Bluemle & Clayton, 1984) and which subsequently triggered tunnel valley growth.”

Line 302. As with hill-hole pairs, there is even less of a general relationship between giant ripples and TV/Cs. It is very cool that they have found some in one place, BUT again, this is an exception to the rule that giant ripples and TV/Cs are usually unrelated.

See above, we have now rephrased this section to talk about how landform associations can be used to discuss how tunnel valleys form.

Line 302. The most common landforms associated with TV/Cs are not even mentioned here, including eskers, but even hummocks. Most TV/Cs that I have seen in WI and MN are associated with ice-margin positions marked by extensive hummocky topography. Why have these been excluded?

As stated above, the intention of this section was poorly phrased – we are not looking for commonality, rather interesting relationships between tunnel valleys and other landforms that may tell us something about how tunnel valleys form. As such, the association between eskers and hummocky moraine has already been made, while we feel that the landforms we have picked out and their relationship with tunnel valleys tell us something about how they formed or the processes operating (e.g. growing out of hill-hole-pairs, giant current ripples and their association with flood events and the use of outwash fans and moraines to reconstruct the relative meltwater history). We have therefore re-phrased this section to make clear what our aims are (see comment above re. L290).

Line 319. As my comment for Line 212 says, I simply cannot see the patterns the authors say they see. Even worse, by this point at line 319, this 'unsubstantiated' observation is now presented as a general rule 'strongly correlated' to ice geometry. TV/Cs ARE NOT more common in interlobate areas - where do they see this? Furthermore, it is true that there are no TV/Cs at the center of the James and DML, BUT they DO occur in the Superior, Wadena, Itasca and Saginaw. How can you make this general statement which is demonstrably false by your own Figure 3? And why are these lobes (and their TV/Cs) different? My thoughts turn to that the James and DML are younger, advanced in a warmer climate, formed by extensive ice streams, and they deposited clay loam tills, whereas as the other lobes are older, advanced into permafrost terrains, are dominated by sandy sediments. In other words, you cannot ˇ compare these lobes! You have no basis to do so. Or rather, the better answer goes beyond comparative geomorphology and has more to day with climate, sedimentology, etc.

As discussed previously, we tend to agree with the reviewer that regional conditions are important for tunnel valley genesis and the form they take (e.g. see section 5.2), and that we have overplayed the role of ice geometry relative to other factors such as permafrost in explaining the spatial distribution. We have therefore taken on board the comments above and elsewhere in this review to re-write this paragraph, picking out some of the key regional influences that might result in the observed distribution. However, we still feel that geometry has an influence, and in particular the ice-surface morphology – i.e. that very shallow ice surfaces will hinder conduit formation.

Line 333. Here is an example of what I mentioned in the introduction. First, according to the various interpretations of TV/Cs in the literature, a subglacial water source is not necessary (Mooers for example). Second, and more important here, just because Livingstone et al 2013 don't find modelled subglacial water, does not mean there wasn't any! This means the strong conclusion of lines 335-336 is overstated (and might come back to bite the authors). To soft pedal would be to say "It might be that Livingstone et al 2013 have underestimated the distribution of subglacial water, but if their analysis is correct, then the storage of subglacial meltwater is not necessary for TV/C formation." But, if Livingstone et al 2013 do not consider a frozen margin (which likely was present along so much of the LGM margin), it makes it seem illogical to apply their lake study at all. One last extension of this topic that the authors may be unaware of is that permafrost features are rarer in Iowa and Illinois (where TV/Cs are absent!).

We agree that this modelling likely underestimates the true extent of subglacial lakes, and we do discuss this possibility at the bottom of this paragraph (i.e. that the model does not include permafrost). However, given that two reviewers mention this, we have further soft-pedalled this association by changing the section to read: "If their analysis is correct, this suggests that the drainage of subglacially stored water was not the main control on tunnel valley formation. But the modelling may well underestimate the true extent of subglacial lakes (i.e. the prediction in Fig. 3 is a *minimum distribution) as the predictions do not account for the possibility of water ponding behind frozen margins as suggested by Cutler et al., (2002) and Hooke & Jennings, (2006)."*

Lines 377 and 379. The idea of permafrost was not invented by these authors to explain TV/Cs, especially in the Upper Midwest. Rather, there is abundant evidence for well-developed permafrost that can be shown to have existed before, during and after the LGM. This means that it is a clear 'boundary' condition when the ice is at the LGM margin. This also helps explain the abundant hummocky topography, by the way.

We already acknowledge the vast amount of work that has been done linking permafrost conditions and tunnel valley formation; e.g. "The prevalence of tunnel valleys along terrestrial margins hints at an important role of permafrost in their formation (e.g. Wright, 1973; Piotrowski, 1994, 1997; Cutler et al., 2002; Jørgensen & Sandersen, 2006)." However, we have expanded the paragraph to show that permafrost conditions were extensive in the Upper Midwest and that landforms such as hummocky terrain, hill-hole pairs and tunnel valleys have all been linked to frozen ground conditions.

Line 389. It is impossible to get TV/Cs where there is thin till over crystalline bedrock. That is, it is not 'partially' controlled; it is 'completely' controlled.

Although not the target of this study, tunnel valleys/channels have been shown to form in bedrock (e.g. Regis et al., 2003 – beneath Superior Lake) and therefore it does not preclude tunnel valley formation. We have therefore left this sentence as it is.

Line 394. This is a very interesting comment and one that needs some development (but not in this paper). If a frozen margin is so important for the broad geomorphology of the Midwest lobes, how does it change during the retreat of the ice? We know that permafrost conditions existed during much of LGM retreat in the Upper Midwest, but was the toe always frozen? When the ice was at the LGM, there was sliding (and drumlins) not far up ice. Could the frozen toe get re-established as the ice retreats? Interesting question.

Thanks.

Line 407. I don't want to belabor the point, but this argument works only if the authors can demonstrate that all TV/Cs have a common genesis.

See comments above.

Line 426. It is not clear to me that this argument from subaerial streams is applicable for the TV/Cs.

We only use subaerial streams as an example of how the geometry of channels is expected to change given different drainage origins (i.e. equilibrium system vs. catastrophic event). Certainly there are additional caveats associated with the formation of channels under ice (e.g. whether it will cut up into the ice or down into the sediment), but the general premise is still expected to be valid (i.e. greater discharge = wider). We go on to discuss reasons why the width of tunnel valleys does vary and why it doesn't show a general trend of widening or narrowing.

Line 492. In some places in the Midwest, the tunnel channels are considered to be a features related to surging lobes/ice streams. In other words, these are not stable ice lobes.

This is certainly true, although it remains an open question whether the tunnel valleys formed during the surge or after it (see suggestions by reviewer #1). However, this might be an additional reason *why tunnel valleys are not so prevalent beneath the James and Des Moines Ice Lobes, which surged to their maximum extents, and we speculate on the influence of rapidly advancing and retreating ice lobes in the discussion in the context of regional variations.*

Line 511 and following. For this discussion, especially about hill-hole pairs and giant ripples, please refer to my comments from Lines 290 and 302.

See previous comments and actions taken in response to comments from lines 290 and 302.

Lines 553-555. The outburst origin proposed by Clayton, Jörgenson, Colgan, Cutler and others bears little resemblance to Shaw's ideas. That is, it is unfair to pair the unpopular hypotheses of Shaw with these researchers, all of whom reject, for example, Shaw's drumlin hypothesis.

We agree, and do not mean to infer that the work of Clayton, Jörgenson, Colgan, Cutler et al. is in any way related to the Shaw hypothesis. Indeed, we are careful in this paragraph to not include any of these references. But to highlight this point we have included references to these authors after the sentence: "Despite the lack of support for a mega-flood genesis of whole tunnel valley clusters, drainage of stored water down individual valleys almost certainly did happen".

Line 555. Mooers is a good paper, but it is only about a few TV/Cs in Minnesota. He is not necessarily trying to say all TV/Cs form this way. His few are not the 'many' you describe. You cite him inaccurately. (I think, BTW, he may be correct for the TV/Cs he describes.)

We have reworded this section so that it does not sound like all TVs form in this way. It now reads: "…as valleys are also found…"

Line 570 and following. Here (suddenly? Finally?) the authors are now talking about outburst floods to form TV/Cs (channels). It is clear that they accept this genesis! However, still, in their analysis, they are assuming all TV/Cs form in the same way. Why is it not clear from the beginning that the authors think these to be equifinal?

This relates to the reviewer's major criticism and we have dealt with this in detail there, and by reordering this point in the paper.

Line 600. The Superior lobe surged, too.

We have amalgamated this section into the paragraph on regional variations in the distribution of tunnel valleys and deleted the part about the Superior Lobe.

Lines 600-603. This argument is not clear. I have reads it several times and it does not make sense to me.

We agree and have rewritten as follows: "Growth likely proceeded up-ice from the margin rather than down-ice from a stored water body because tunnel valleys preferentially terminate at ice-margin positions irrespective of their size (e.g. see very small tunnel valleys along the southern margin of the Green Bay Lobe, Fig. 4)."

Line 606. Could your amphitheater heads be 'plunge pools' of supraglacial lakes that hydrofractured to the glacier bed? Can these exist? What would be the geomorphic evidence for them and how would you distinguish them from your headward migrating channels?

We agree that this is a bit speculative and that they are probably difficult to differentiate from 'plunge pools' (e.g. seen on the channelled scablands). We have therefore deleted this part of the sentence, and moved the other part about hill-hole pairs to further support our argument for headward growth.

Line 609. Delete 'paradigm.' There have been two primary 'explanations.' 'Paradigm' is not the right word.

We have deleted 'paradigm' and re-written as "...been two explanations for the formation of tunnel valleys:"

Line 615. Again, I think the authors are standing on thin ice by too strongly applying the results from Livingstone et al 2013 without appropriate caveats.

In this paragraph we do meant to refer to the results of Livingstone et al. (2013). Rather, that because of our history in looking for subglacial lakes we went into this work with a hypothesis that the tunnel valleys were formed by subglacial lake drainage events. But the data did not support this idea. So as not to confuse the reader we have modified the reference to: "(e.g. Livingstone et al., 2013, 2016)" and deleted "link with predicted lake locations".

Line 619. OK, I can buy the ideas that in some areas the TV/Cs seem to be 'organized.' But 'well-organized?' Sand dunes are well organized; drumlin fields tend to be also, but it is overstating it here to say that TV/Cs are.

We have deleted "well" here and elsewhere throughout the manuscript.

Line 635. Once again, how do the authors want it? They cannot admit to the idea that there is more than one way to make a TV/Cs and then treat them analytically as if they are all the same. It is illogical. Also, here, this outburst origin seems to be rather 'hidden' in the back of the article. If they really truly believe that these form in more than one way, then this needs to be a theme expressed at the beginning (and in the abstract). And every time they introduce a graph that shows all collectively, they must express a caveat.

This relates to the reviewer's major criticism and we have dealt with this in detail there. With regards to 'hiding' the outburst origin, this was not on purpose and probably reflects the fact that most of the evidence supports gradual formation. For instance, we do mention the outburst origin before this point with regards to the giant current ripples and outwash fans. In the abstract we do state that "Our data and interpretation supports gradual (rather than a single-event) tunnel valley formation with secondary contributions from flood drainage of subglacial and or supraglacially stored water down individual tunnel valleys". Rather than making the caveat about formation for every graph, we now make the point about initially assuming all tunnel valleys have a common genesis in the limitations section.

Line 933. Figure 2. This is an excellent figure! But I would like another figure in this article that shows the location of these maps; just saying 'Superior Lobe' is not satisfactory, I want to know where these exactly are from. This will help the reader convince herself if she were to check it out.

This was a comment of all the reviewers and to rectify this we have included a new figure (Figure 2) that shows the field area and the locations of all examples shown in later figures.

Line 942. I would remove F. It is not even a tunnel valley. It is not clear why this is here; perhaps as a contrast to the others?

As intimated by the reviewer it is there as a contrast and allows us to highlight the different geometry and planform that allow us to differentiate it from a subglacial tunnel valley. We would therefore rather keep it.

Line 947. Despite all my critical comments, I really like this map. However, it is weakened with some minor comments. (1) It is difficult to believe that the ‘long lines’ above the two ‘ee’s in Green Bay and SW of the ‘L’ in Huron-Erie Lobe are actually TV/Cs. (2) the thin black outline, what does that show? It is odd, where is it from? It seems to show glacial Lake Wisconsin; why? It also goes ‘inside’ the LGM moraines in Illinois - it looks like a mistake (also in the James Lobe). (3) It is alright to use Fullerton, but his map of moraines and hummocky zones is not quite correct.

Point (1) - the valleys mentioned display the criteria of a tunnel valley and we therefore think they should be included. We felt it was important to be consistent in the application of our criteria so that the mapping was not biased but our own prejudices.

Point (2) – We have removed the thin black line.

Point (3) – we have stuck with the Fullerton map, but have expanded it to include all superficial deposits of the last glaciation so that regional variations in grain size can be discussed.

Line 963. As in Figure 2, I would like to see these maps indicated in a reference map.

See comment above (re: Line 933).

Line 972. Is the Lamb reference appropriate here? Doesn’t it deal with bedrock rivers? Is it applicable?

It does deal with a bedrock canyon, but it is an example of an observed single large flood event carving a channel.

Line 956. This Langlade image seems not be represent TV/Cs. The drumlin-cutting swath in the middle may be a deglacial outwash stream (judging by the ‘valley’ sides and the geometry). Why do you think it is a TV/C?

We agree that the Langlade image does not look like a typical tunnel valley and is very wide (up to 8 km). However, it has eskers and outwash fans along its length and a gently undulating profile. We therefore have given it a confidence of 2 (probable tunnel valley).

Line 990. Where is this in Minnesota? Since most giant ripples in the world are associated with catastrophic discharges, (and there have been several in Minnesota!), is it possible that this is a spillway? I cannot judge if I don’t know where it is from.

See comment above (Line 963) – we have included a new figure which shows the locations of all our example ‘landforms’.

Line 997. Here now I understand why you emphasize interlobate areas! However, I still fail to see a pattern on your map that looks anything like how you ‘would like’ it to show.

Given previous comments we have deleted this figure and the related discussion.

Thanks for doing this impressive work! And I hope you can incorporate my criticisms to make it even better.

Many thanks for this really useful and informative review. It has really helped to clarify the paper and we think the revision is now much improved because of it.

Morphological properties of tunnel valleys of the southern sector of the Laurentide Ice Sheet and implications for their formation

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Abstract

Tunnel valleys have been widely reported on the bed of former ice sheets and are considered an important expression of subglacial meltwater drainage. Although known to have been cut by erosive meltwater flow, the water source and development of channels has been widely debated; ranging between outburst flood events through to gradually occurring channel propagation. We have mapped and analysed the spatial pattern and morphometry of tunnel valleys and associated glacial ~~bedforms~~landforms along the southern sector of the former Laurentide Ice Sheet from high-resolution digital elevation models. Around 2000 tunnel valleys have been mapped, revealing ~~a well~~ an organised pattern of sub-parallel, semi-regularly spaced valleys that ~~cluster together~~form in distinctive ~~networks~~clusters. The tunnel valleys are typically <20 km long, and 0.5-3 km wide ~~and~~ , although their width varies considerably down-valley. They preferentially terminate at moraines~~. They tend to be associated with outwash fans, eskers, giant current ripples, and hill-hole-pairs. At the ice-sheet scale, we find most~~, which suggests that formation is time dependent, while we also observe some tunnel valleys ~~occur on the flat portions of palaeo-ice sheet beds, where subglacial water flow would~~that have ~~been largely unconstrained by topography, while tunnel valley morphology is strongly modulated by local variations in basal conditions (e.g. thermal regime and topography) and hydrology (i.e. whether conduit erosion is up into the ice or down into the sediments).~~grown headwards out of hill-hole pairs. Analysis of cross-cutting relationships between tunnel valleys, moraines and outwash fans permits reconstruction of channel development in relation to the retreating ice margin. ~~The~~This palaeo-drainage reconstruction demonstrates incremental growth of most valleys, with some used repeatedly~~,~~ or for long periods, during deglaciation, while others were abandoned shortly after their formation. Our data and interpretation supports gradual (rather than a single-event) ~~tunnel valley~~ formation~~,~~ of most tunnel valleys with secondary contributions from flood drainage of subglacial and~~/~~ or supraglacially stored water~~.~~ down individual tunnel valleys. The distribution and morphology of tunnel valleys is shown to be sensitive to regional factors such as basal thermal regime, ice and bed topography, timing and climate.

Key words: *tunnel valleys; geomorphology; Laurentide Ice Sheet; subglacial meltwater; gradual or catastrophic*

1. Introduction

Incised into bedrock or sediment, tunnel valleys ~~and channels (hereafter referred together as tunnel valleys)~~ are elongate depressions up to several kilometres wide, often with undulating long-profiles, tens of kilometres long and tens to hundreds of metres deep. They are observed in many formerly glaciated landscapes around the world, and tend to be orientated parallel to the direction of former ice flow (e.g. Ussing, 1903; Wright, 1973; Attig et al., 1989; Wingfield, 1990; Piotrowski, 1994; Patterson, 1997; Huuse & Lykke-Anderson, 2000; Jørgensen & Sandersen, 2006). Features with similar dimensions have also been described beneath current ice masses (e.g. Rose et al., 2014). Since first being described (Gottsche, 1897) and then attributed to erosion by subglacial meltwater (Ussing, 1903) such phenomena have attracted a variety of names, which vary according to interpretations about how they form: Tunnel Channels (implies whole depressions occupied and cut by water), Linear Incision (purely descriptive term), Tunnel Valley *sensu stricto* (taken by some to imply a large linear depression created by activity of a smaller channel which occupied part of it, as in a river channel producing a valley); or the rather vague term Palaeo-valley. In this paper we stick to the most widely used term - Tunnel Valley - but use it in its broadest sense (*sensu lato*) to include depressions that could actually be tunnel channels. In this manner we initially treat all linear depressions of the appropriate scale and morphological characteristics as the same thing, whilst recognising that in detail this might be a grouping of a number of types.

Tunnel valley formation is typically attributed to subglacial meltwater erosion at the base of ice sheets (cf. Ó Cofaigh et al., 1996; Kehew et al., 2012; van der Vegt et al., 2012), and they are considered an important component of the subglacial hydrological system, providing drainage routeways for large volumes of water and sediment. Understanding their genesis is relevant for reconstructing former ice sheets, elucidating basal processes and exploiting the geomorphological record in a way that is useful for modelling subglacial hydrology. However, despite being debated for over 100 years, there is ~~considerable~~still uncertainty ~~about the underlying processes governing~~over how tunnel ~~valley formation~~valleys form. This debate is focused around two genetic models: 'outburst' formation and 'gradual or steady-state' formation (Fig. 1) (cf. Ó Cofaigh et al., 1996; Kehew et al., 2012; van der Vegt et al., 2012).

The 'outburst' hypothesis (Fig. 1a) ascribes the erosion of tunnel valleys to rapid drainage of sub- or supraglacially stored meltwater. Contemporary observations from the Antarctic and Greenland ice sheets demonstrate the efficacy of meltwater storage and drainage in sub- and supraglacial environments (Zwally et al., 2002; Wingham et al., 2006; Fricker et al., 2007; Das et al., 2008) and it is reasonable to expect that the Laurentide Ice Sheet experienced similar events. In addition, the impoundment of meltwater behind ~~a frozen~~an ice margin frozen to its bed has been linked to tunnel valley formation, for example, along the southern terrestrial margins of the former Laurentide and

European ice sheets where permafrost was prevalent (e.g. Piotrowski, 1994; Cutler et al., 2002; Hooke & Jennings, 2006). Genesis is typically thought to occur via repeated low to moderate magnitude floods that may be at or below bankfull flow (e.g. Wright, 1973; Boyd, 1988; Wingfield, 1990; Piotrowski, 1994; Cutler et al., 2002; Hooke & Jennings, 2006; Jørgensen & Sandersen, 2006). Catastrophic erosion of entire tunnel valley networks by massive sheet floods ~~(bankfull flow)~~ has also been proposed (e.g. Shaw & Gilbert, 1990; Brennand & Shaw, 1994, Shaw, 2002), but has been considered less likely given the very large volumes of stored water required (e.g. Ó Cofaigh et al., 1996; Clarke et al., 2005).

The 'gradual' or 'steady-state' hypothesis (Fig. 1b) typically invokes erosion of soft-sediment beds in low pressure subglacial channels (Boulton & Hindmarsh, 1987; Mooers, 1989; Praeg, 2003; Boulton et al., 2009). In this model, high water pressures transmitted through the substrate to the ice-sheet terminus initiates failure and headward erosion of a conduit (by piping) (Shoemaker, 1986; Boulton & Hindmarsh, 1987; Hooke & Jennings, 2006; Boulton, 2009). As the fluid pressure of the conduit is lower than the surrounding substrate, meltwater flows towards the conduit, the walls are enlarged by sapping (i.e. undermining and headward recession of a scarp) and the sediments are mobilized and transported away by the resulting subglacial stream (Boulton & Hindmarsh, 1987). In general, enlargement is suggested to occur via steady-state Darcian flow of water into the conduit (e.g. Boulton & Hindmarsh, 1987; Boulton et al., 2007a, b, 2009). Hooke & Jennings (2006) adapted this hypothesis, suggesting that initial headward erosion by piping was followed by more rapid enlargement when the conduit tapped into a subglacial lake, thereby combining both scenarios in Figure 1. Ravier et al. (2014) emphasised the potential influence of localised high porewater pressures in promoting efficient erosion by hydrofracturing and brecciation of the preglacial bed, while Mooers (1989) considered supraglacial drainage to the bed rather than basal meltwater as the dominant source for gradual tunnel valley erosion.

A range of approaches can be applied to the investigation of tunnel valleys including theoretical, sedimentological and morphological. Thus far, most effort has used a combination of these approaches, with much data, description and detail, but for a small number of tunnel valleys (see Section 2). From these it is difficult to extract representative information of the population of tunnel valleys or to gain an understanding of the broader-scale distribution of landforms. ~~To rectify this we~~Based on previous studies and the availability of digital elevation models (DEMs), we are now able to undertake a systematic and large-scale mapping campaign of the size, shape, ~~pattern~~spatial arrangement and distribution of tunnel valleys to better understand the spatial properties of this phenomenon, noting that it is useful to know more precisely what it is that requires explanation (e.g. Dunlop & Clark, 2006, for ribbed moraine). In doing so we will ~~answer~~address the following questions: (1) ~~what constitutes~~how do we define a tunnel valley and how can they be distinguished in ~~the geological record?~~a glaciated landscape? (2) What are ~~the~~their morphological characteristics ~~of a tunnel valley?~~? (3) Is there a characteristic distribution and ~~network~~ arrangement? (4) ~~Are there systematic~~What can associations ~~between tunnel valleys and~~with other landforms tell us about tunnel valley formation? The southern sector of the Laurentide Ice Sheet was selected because it contains thousands of these landforms~~, they can be identified from digital elevation models (DEMs)~~ and the distinctive geometry of the ice lobes provides information on the water drainage pathways~~.~~ (Fig. 2). Our mapping builds on, and replicates~~,~~ in many places, comprehensive local and regional studies, which include sedimentological details that we draw on. ~~Our data provide basic metrics on tunnel valleys and their variation in scale and pattern and should promote new insights into tunnel valley formation and meltwater drainage and erosion beneath ice sheets~~The main purpose of this paper is to present systematic mapping of a large sample of tunnel valleys and to provide basic metrics on their variation in size, morphological characteristics and distribution. We do so to advance knowledge about these landforms and provide representative data that modellers of subglacial hydrology might find useful. We then use our morphological observations and data, along with published field observations to assess existing theories on tunnel valley formation. An obvious limitation to using DEMs of the current land surface is that any post-formational deposition (infilling) of tunnel valleys will mask or modify aspects of the morphology we can see. That it is easy to identify thousands of tunnel valleys in DEMs however, shows that infilling is often only partial and allows us to assess their presence, distribution, width and minimum length, and some properties about undulating long profiles.

~~*Limitations*~~

~~The partial or complete burial of tunnel valleys by glacial and post-glacial infill (cf. Kehew et al., 2012; van der Vegt et al., 2012) limits the mapping and measurement of tunnel valleys from DEMs. Buried tunnel valleys with no surface expression cannot be identified by the mapping. However, buried valleys are rare or at least fewer have been identified along the southern margin of the Laurentide Ice Sheet compared to the European ice sheets (e.g. Jørgensen & Sandersen, 2006; Kristensen et al., 2007, 2008; Stewart & Lonergan, 2011). Moreover, unless there is a systematic bias predisposing burial in some locations over others then the mapped pattern and distribution of tunnel valleys is likely to be informative.~~

2. Previous work and observations in study area

There is a rich history of work on tunnel valleys beneath the southern margin of the former Laurentide Ice Sheet (Wright, 1973; Attig et al., 1989; Mooers, 1989; Patterson, 1994, 1997; Clayton et al., 1999; Johnson, 1999; Kehew et al., 1999, 2013; Cutler et al., 2002; Sjogren et al., 2002; Fisher et al., 2005; Kozlowski et al., 2005; Jennings, 2006; Hooke & Jennings, 2006; Kehew & Kozlowski, 2007). In this section we briefly summarise key observations arising from this work, which need to be incorporated into any model of tunnel valley formation.

Tunnel valleys are commonly observed on the bed of the~~ former~~ southern margin of the Laurentide Ice Sheet and typically occur as distinct radiating sets of regularly spaced ~~valleys~~depressions associated with eskers and terminal or recessional moraines (cf. Kehew et al., 2012). At the bed of the Saginaw Lobe, for instance, valleys are typically spaced at 6-10 km intervals (Fisher et al., 2005; Kehew et al., 2013). Tunnel valleys are incised into glacial sediments up to a depth of 25 m and extend for <50 km (e.g. Jennings, 2006). However, tunnel valleys~~-~~ up to 150 km long have been documented in the Superior Lobe, Minnesota (Wright, 1973), and valleys are eroded up to 200 m into the bedrock floors of Lake Superior and Lake Michigan (Regis, 2003; Jennings, 2006).

Although tunnel valleys are typically sub-parallel, they are also observed to join, split and even cross-cut each other (e.g. Wright, 1973; Mooers, 1989; Kehew et al., 1999, 2005; Fisher et al., 2005; Kehew & Kozlowski, 2007). Cross-cutting relationships, both between tunnel valleys and with other glacial landforms (e.g. drumlins, outwash fans, moraines), record a palimpsest signature of tunnel valley erosion. In the Saginaw Lobe, Kehew et al. (1999, 2005) and Kehew & Kozlowski (2007) identified a series of palimpsest associations in which partially buried tunnel valleys pass beneath terminal moraines, diamicton and surficial outwash associated with later advances. This palimpsest style is interpreted to result from the collapse of ice and debris into the valley, which becomes (partially) buried by sediment during a re-advance and then re-emerges as the ice melts out (e.g. Kehew & Kozlowski, 2007).

Tunnel valley cross-sectional morphology ranges from sharply-defined with constant or downstream increasing dimensions (e.g. Mooers, 1989), to indistinct valleys often associated with hummocky terrain and characterised by beaded or crenulated planforms, or as a series of aligned depressions (e.g. Kehew et al., 1999; ~~Sjorgen~~Sjogren et al., 2002). Indistinct valleys may be due to partial burial during re-advance events or by melt out of debris rich ice obscuring them (Kehew et al., 1999). ~~Sjorgen~~Sjogren et al. (2002) also identified indistinct valleys in Michigan that are eroded into the hummocky terrain.

In Wisconsin, Michigan and Minnesota, bands of hills are observed to occur upstream of tunnel valleys (Johnson, 1999). These are interpreted as erosional remnants of an anastomosing subglacial meltwater system that drained ~~along~~through the inter-hill ~~valleys~~depressions. At their downstream end, tunnel valleys often terminate at outwash fans ~~(e.g.~~some of which contain coarse boulder-gravel material (e.g. Attig et al., 1989; Mooers, 1989; Patterson, 1994; Clayton et al., 1999; Johnson, 1999; Kehew et al., 1999; Cutler et al., 2002; Derouin, 2008~~), some of which contain coarse~~). Palaeo-discharge estimates from the boulder~~-gravel material, which is interpreted~~ deposits imply large discharges, and this has been used as evidence for outburst flood events (Cutler et al., 2002).

3. Methods

3.1 Datasets and mapping

For this study, we used the National Elevation Dataset (NED) (http://nationalmap.gov/elevation.html), ~~which is a seamless DEM~~utilising DEMs with a resolution of 1/3 arc seconds (~10 m) across the entire study area, and 1/9 arc seconds (~3 m) in some locations. Surficial and bedrock geology maps (e.g. Fullerton et al., 2003; Soller et al., 2011) were also used to aid identification and interpretation. Glacial landforms were ~~identified according to conventional criteria and~~ digitised directly into a Geographical Information System (GIS). Polylines were used to map tunnel valleys sides and ~~centrelines~~thalwegs, eskers and moraines. Polygons were used to map hill-hole pairs, outwash fans and dissected hills.

3.2 How do we distinguish tunnel valleys in ~~the geological record~~a glaciated landscape?

Apart from tunnel valleys, large elongate depressions with similar dimensions may also form by fluvial erosion (river valleys), proglacial meltwater erosion (spillways), subglacial abrasion/plucking (overdeepenings), or arise from geological structures (e.g. fault lines). These phenomena are readily observed today and the formative mechanisms are reasonably well known. In contrast, tunnel valleys have ~~not been~~yet to be observed actively forming beneath, or at the margins of, modern day ice sheets, and so their genesis and properties are more ~~enigmatic. In the geological record~~difficult to discern. In glacial landscapes they have been distinguished by their large size and characteristics such as their orientation parallel to inferred ice flow, undulating ~~thalwegs~~long profiles and associations with subglacial bedforms and eskers; all pointing to a subglacial origin. In particular, undulating thalwegs and their association with eskers and outwash fans, permit them to be distinguished from ~~proglacial and fluvial~~subaerial rivers. However, negative evidence (e.g. no esker found in a valley) does not necessarily preclude a subglacial origin~~, and it is not known whether size is actually a distinguishing feature or if, for instance, ~~. Linear incisions similar to tunnel valleys but of much smaller ~~meltwater channels~~ size (tens of metres in width~~;~~) and generally called subglacial meltwater (or Nye) channels are also common in glaciated landscapes (e.g. Greenwood et al., 2007) ~~are less mature forms of a continuum~~but it is generally presumed that these are not part of ~~glacial hydrological channels.~~the same population as tunnel valleys; that they are different landforms distinguished by size but perhaps also by process.

For the purposes of this study we restrict our definition of a tunnel valley to subglacially eroded channel-forms~~.~~ and use the term non-genetically in reference to both tunnel valleys and tunnel channels. Tunnel valleys that could clearly be differentiated as being eroded into bedrock were not mapped as their formation is more difficult to decipher from geological structures or glacial overdeepenings and valleys abraded and plucked by overlying ice. All ~~potential~~valley forms that potentially could be interpreted as tunnel valleys or tunnel channels were mapped, and then ~~assessed~~each was tested to ~~determine whether they~~see if it could be shown to have been formed subglacially~~.~~, and thus, be interpreted to be a tunnel valley or tunnel channel. One way to strengthen a subglacial interpretation would be to demonstrate that the longitudinal profile slopes upward towards an associated ice margin or that the profile undulates. To determine whether the valley ~~thalwegs are~~bottom is undulating the number of negative and positive slope segments over 100 m length scale were calculated~~.~~ (see later with regard to problem of valley infills contaminating these assessments). Each valley was then assigned a confidence level from one to three, with one being the most certain and three the least (Fig. ~~2~~3). Channels lacking undulations and that do not contain subglacial bedforms are difficult to differentiate from proglacial or postglacial channel systems and were therefore given a confidence of 3. Valleys with an undulating long-profile, which contain eskers or terminate in outwash fans were classified as ‘certain’ tunnel valleys and given a confidence level of one (Fig. ~~2a~~3a-d). Only those tunnel valleys with a confidence level of one or two were used in the spatial and morphological analyses.

3.3 *Tunnel valley measurements*

~~Using the centrelines of tunnel valleys, we computed their length.~~Tunnel valley length was computed in the GIS. These measurements are best treated as minimum bounds, because if some valleys have a complete infilling in their upstream reaches we would not be able to recognise this part of the valley and therefore underestimate its length. Where two or more tributaries coalesce, the longest routeway was used to determine length. Tunnel valley width (distance between mapped valley sides) was measured from cross-profile transects positioned at 1 km intervals along the ~~centreline~~course of each tunnel valley. The ~~influence of~~relationship between local elevation gradient (G_{loc}) on along valley changes in width (W_{loc}) was calculated at each 1 km interval (j) using equations [1] and [2]:

$$G_{loc} = (E_{j+1} - E_{j-1}) / (D_{j+1} - D_{j-1}) \quad [1]$$

$$W_{loc} = (W_{j-1} - W_{j+1}) / (D_{j+1} - D_{j-1}) \quad [2]$$

Where E = elevation, W = width and D = downstream distance~~.~~ from the head of the tunnel valley. To calculate tunnel valley spacing it was necessary to restrict our analysis to ~~networks~~clusters comprising distinct populations of similar orientation, which were likely formed during a similar ~~drainage phase.~~time period (although they may not all have been operating at the same time). We calculated the spacing of 966 tunnel valleys organised in 24 discrete ~~networks~~clusters. Spacing (S) was calculated from cross-profile transects orientated perpendicular to the direction of the ~~network~~cluster and positioned at 5 km intervals along each long profile. A median spacing value and the standard deviation (σ) was calculated for each ~~drainage network.~~tunnel valley cluster. To provide an indication of ~~tunnel valley~~ regularity per ~~network~~cluster the coefficient of variation (σ / mean spacing), expressed as a percentage (σ%), was also calculated (Hovius, 1996; Talling et al., 1997); tunnel valley ~~networks~~clusters with a low σ% exhibit low variability in spacing.

To investigate drainage evolution during deglaciation, a subset of meltwater features in Wisconsin were grouped into ‘drainage-sets’, defined as a collection of features ~~that~~interpreted as having formed during the same drainage phase. This was based on cross-cutting relationships (e.g. between channels, outwash fans and moraines) to reconstruct a relative history of drainage activity~~.~~ as the ice sheet retreated across the area. Cross-cutting relationships between tunnel valleys and moraines were classified according to whether the tunnel valley: (1) terminates at a moraine at its downstream end and therefore formed contemporaneously with it; (2) is overlain by moraines along its length, thus suggesting that the tunnel valley was no longer active when the moraines were deposited; or (3) breaches moraines along its length, thereby indicating that the tunnel valley continued to drain water, either destroying pre-existing moraines or preventing ~~moraines~~them from forming during retreat.

3.4 Limitations

The partial or complete burial of tunnel valleys by glacial and post-glacial infill (cf. Kehew et al., 2012; van der Vegt et al., 2012) limits the mapping and measurement of tunnel valleys from DEMs. Significantly, we do not extract information on tunnel valley depth for this reason. Width measured at the valley edges is not affected. As noted earlier, our measurements of length are minimum bounds if upstream infilling sufficiently hides the valleys. The long profiles were used to determine whether the valley formed subglacially (i.e. whether there are undulations). As infilling is likely to selectively occur in hollows smoothing the long profile, this is likely to result in some tunnel valleys being missed because we could not ascertain if they had truly undulating long profiles and so our classification of linear incisions as tunnel valleys is therefore a minimum bound.

Completely buried tunnel valleys with no surface expression cannot be identified by the mapping and could therefore present a problem of bias in assessing spatial distributions. However, unlike elsewhere in the world few buried tunnel valleys have been reported along the southern margin of the Laurentide Ice Sheet, and most of the completely buried tunnel valley networks in Europe relate to pre-Weichselian ice advances (e.g. Jørgensen & Sandersen, 2006; Kristensen et al., 2007, 2008; Stewart & Lonergan, 2011), so we suppose this problem to be minimal. Moreover, unless there is a systematic bias predisposing burial in some locations over others then the mapped pattern and distribution of tunnel valleys is likely to be informative. ~~**4. Properties of Tunnel Valleys**~~

In analysing this large dataset of tunnel valleys along the southern margin of the Laurentide Ice Sheet we make the initial presumption that tunnel valleys have a common genesis and then search for circumstances and data that challenge this. This allows us to focus on possible relationships between form, distribution and process.

4. Results

4.1 Is there a characteristic spatial distribution and ~~network~~ arrangement?

4.1.1 Distribution

Figure ~~3~~4 shows the distribution of all 1931 tunnel valleys (1694 of which have a confidence of 1 or 2) mapped beneath the terrestrial southern sector of the Laurentide Sheet. We estimate that ~80% of these tunnel valleys have been previously identified and mapped during more localised investigations (e.g. Wright, 1973; Attig et al., 1989; Patterson, 1997; Fisher et al., 2005). The map reveals a tendency for tunnel valleys to cluster together ~~in distinctive 'networks'~~ with large intervening areas where no or very few valleys occur. ~~Networks mostly avoid running down the central axes~~Indeed, while clusters of ~~major~~tunnel valleys are prevalent along much of the Wadena, Itasca, Superior, Chippewa and Saginaw ice lobes~~. They are instead concentrated along suture zones between adjacent ice lobes or at the edge of linear to slightly lobate ice-margin positions. Tunnel valleys~~, they are rarer and more dispersed or isolated at the southernmost (LGM) margins of the James, Des Moines, Lake Michigan and ~~Erie-~~Huron-Erie ice lobes (Fig. ~~3~~4). Those that do occur in these ice lobes tend to be positioned up-ice, either at the lateral margins of the LGM lobes (e.g. Green Bay Ice Lobe) or at recessional moraines (e.g. Des Moines Ice Lobe).

Tunnel valley ~~networks~~clusters often occur down ice-flow of ~~basins or sub-basins~~depressions in the landscape (Fig. ~~3~~4). For example, the Saginaw Lobe tunnel ~~valley network emanates~~valleys emanate from an arm of the present-day Lake Huron Basin, the Langlade and Chippewa tunnel ~~valley networks~~valleys are ~~all associated with sub-basins~~downstream of the present-day Lake Superior Basin, and tunnel valleys occur downstream of the low-relief trough of the Des Moines Lobe. Based on modelled hydraulic potential surfaces, Livingstone et al. (2013) predicted that the Lake Superior Basin and NE sector of the Lake Michigan Basin were sites of several subglacial lakes during the last glacial (marked in Fig. ~~3~~4). There appears to be no clear link between these ~~lake~~potential lakes and tunnel valleys. On the other hand, subglacial lakes may also have been present elsewhere in the Great Lake Basins and it is noteworthy that tunnel valleys are commonly downstream of these basins.

4.1.2 ~~Network~~Spatial arrangement

The overall shape of tunnel valley ~~networks~~clusters varies (Fig. ~~3~~4), with both broad ~~networks~~clusters composed of many short valleys (e.g. Green Bay, James and SE edge of Superior), and narrow ~~networks~~clusters composed of long valleys (e.g. Superior, Huron-Erie and Langlade). Cross-cutting of tunnel valleys occurs both between and within ~~networks~~clusters.

~~Overall tunnel~~Tunnel valley spacing (Fig. ~~4~~5) displays a positively skewed, unimodal distribution with a median spacing of 4.5 km and standard deviation of 4.6 km ($\sigma\%$ = 81). However, the median spacing of individual tunnel valley ~~networks~~clusters ranges from 1.9 to 9.1 km. Tunnel valleys in the Green Bay (median: 2.9 km), Superior (median: 3.7 km) and Huron-Erie (median: 1.9 km) lobes are closely spaced. Conversely, tunnel valley ~~networks~~clusters in the large Saginaw (median: 5.7 km), Michigan (median: 5.5 km) and Des Moines (median: 5.4 km) lobes and in North Dakota (median: 5.1 km) have a wider than average spacing. In all of the measured ~~networks~~clusters the standard deviation of the ~~tunnel valley~~

~~spacings~~spacing is less than the mean ~~tunnel valley~~ spacing, and 9 of the 24 ~~networks~~clusters are <60%. There is no significant correlation between the number of tunnel valleys within a ~~network~~cluster (ranging from 7 to 169) and the standard deviation, but the standard deviation increases as the mean and median ~~network~~cluster spacing increases, hence the use of the coefficient of variation (σ%).

4.2 What are the morphological characteristics of ~~a~~ tunnel ~~valley~~valleys?

The lengths of mapped tunnel valleys display a unimodal, positively skewed distribution, which is approximately log-normal (Fig. ~~5a~~6a). Lengths range from 200 m to 65 km, with a mode of 7-9 km, median of 6.4 km and standard deviation of 8 km. Long and short tunnel valleys occur in most places, although long valleys are less common in the Green Bay and Huron Erie lobes, and dominate in the Superior, Langlade, Wadena, Michigan and Saginaw lobes.

The widths of mapped tunnel valleys display a unimodal distribution with a positive skew, which approximates normal when log-transformed (Fig. ~~5b~~6b). Tunnel valley widths vary considerably across the study area, ranging from 15 m to 6.7 km, with a mode of 600-800 m, median of ~~550~~560 m and standard deviation of 660 m. The Chippewa, Langlade and Michigan valleys are consistently wide (typically >600 m), while the Huron-Erie, Superior, Green Bay and Des Moines valleys are narrow (<600 m). Other ~~networks~~clusters, in the Saginaw, Superior and Wadena lobes, comprise a mix of wide and narrow valleys. There is a tendency for longer tunnel valleys to be wider (power law function, r^2 = 0.~~38~~34, *p-value* = <0.001) (Fig. ~~6~~7).

Tunnel valley planform shape varies across the study area (Fig. ~~7~~8). The majority consist of a single valley ‘thread’; more than two orders of ‘stream ordering’ are rare and tributaries tend to be restricted towards valley heads (Figs. ~~2,~~ 3, ~~7~~4, 8). Valley margins range from sharp to indistinct and from crenulated to straight. Straight margins are more typical of long, thin tunnel valleys (Fig. ~~7a~~8a,d,f). However, many margins are crenulated, with bulbous and abrupt angular morphologies that result in large down-valley changes in width (Fig. ~~7a~~8a-f). Figure ~~8~~9 demonstrates a weak relationship between tunnel valley width and distance downstream. Valleys both widen and narrow downstream with considerable and abrupt variations in width. The variation in tunnel valley width bears no relation to the local elevation gradient (Fig. ~~9~~10). Local along-valley elevation gradients are relatively low (typically <±1.5°) and valleys widen and narrow on both reverse and normal slopes.

Tunnel valleys and tunnel valley segments often start and end abruptly and can appear fragmented or contain bulbous depressions (Fig. ~~7~~8). The gaps between segments of tunnel valleys may show no evidence of modification (Fig. ~~7e~~8e,f); are partially incised by narrower and more discontinuous valleys or sets of parallel valleys (Fig. ~~7e~~8e); or consist of a series of depressions and hummocks with indistinct valley planform (Fig. ~~7a~~8a,b,d). The up-glacier ends of tunnel valleys range from rounded heads with steep sides (amphitheatre) (Fig. ~~7a~~8a,f) to open or indistinct (Fig. ~~2c~~3c-d). In Figure ~~7e~~8e-f, tunnel valleys comprise parallel tracks of two or more tightly spaced (<1 km) valleys.

4.3 ~~Are their systematic~~What can associations ~~between tunnel valleys and~~with other landforms tell us about tunnel valley formation?

4.3.1 Moraines

The association between moraines and tunnel valleys varies with some valleys cutting through moraines (Fig. ~~10a~~11a); while in other locations moraines are superimposed on the valley or the valley terminates at a moraine (Fig. ~~10b~~11b). In Figure ~~10a~~11a, in North Dakota, tunnel valleys cutting through an end moraine are observed to narrow and then trend down-glacier into esker and outwash fan deposits. Up-glacier of the end moraine are low relief (1-2 m) and regularly spaced transverse ridges ('washboard' moraine). They have a cuspate geometry with the horns pointing up-glacier and converging on tunnel valley positions (see also Stewart et al., 1988; Cline et al., 2015). Fig. ~~10b~~11b shows examples of tunnel valleys terminating at, cutting through and overlain by recessional ~~moraine~~moraines. The tunnel ~~valley network does~~valleys here do not show a consistent pattern, with neighbouring ~~channels~~valleys exhibiting different moraine associations. Some valleys are continuous or semi-continuous, with a single outwash fan at, or just down-glacier from the terminus, and a series of on-lapping recessional moraines up-glacier. Elsewhere, valleys contain multiple outwash fans deposited at successive moraine positions.

4.3.2 Hill-hole pairs

We mapped 12 hill-hole-pairs (Bluemle and Clayton, 1984), 11 of which are found in North Dakota. Typically, hill-hole pairs comprise isolated features, but 4 of them are associated with tunnel valleys (e.g. Figs. ~~2c, 11~~3c, 12). These seem to occur at the down-glacier end of the valleys, with smaller channels and eskers emanating from and diverging around the ice-thrust hill (Fig. ~~11a~~12a,b). In Fig. ~~11a~~12a, an esker emanating from one of the hill-hole pairs trends into another tunnel valley segment further down-glacier.

4.3.3 Outwash Fans

We mapped 187 outwash fans across the study area, predominantly at the downstream end of, but also within and between segments of tunnel valleys at moraine positions (Fig. 11b). Outwash fans are particularly common in the Chippewa, Wisconsin, Langlade, Green Bay, Superior and Wadena lobes (see also Attig et al., 1989; Clayton et al., ~~10b).~~1999; Cutler et al., 2002; Fisher and Taylor, 2002), and at the downstream end of the large bedrock tunnel valleys in Lake Superior (Regis et al., 2003; Derouin, 2008) that lie outside of our study region. Many of the outwash fans are connected upstream to an esker.

Multiple trains of outwash fans occur along some tunnel valleys, but not all tunnel valleys are associated with outwash fans.

4.3.4 Giant Current Ripples

In Minnesota the floor of one tunnel valley is shown to contain regularly spaced sinusoidal bedforms orientated roughly perpendicular to the valley ~~long profile~~axis (Fig. ~~12~~13). The bedforms are 0.2-1.9 m high (H), 10-60 m long (L) and their crests are straight to slightly sinuous. Our data show that longer bedforms tend to be higher (linear regression, $r^2 = 0.5$), and that the H:L ratio is ~0.02. The tunnel valley that the bedforms are constrained within is partially incised into underlying drumlins orientated obliquely to the valley long axis. An esker running NW-SE is overprinted on the sinusoidal bedforms. The southern end of the valley is bisected by a large (1 km diameter) circular incision with an intact central island.

The dimensions and shape of the transverse sinusoidal bedforms, the tendency for longer bedforms to be higher and their ~~association with~~context in the base of h the tunnel valley is consistent with giant current ripples (e.g. Bretz et al., 1956; Carling, 1996; Rudoy, 2005). Given the undulating valley thalweg and superimposition of an esker on top of the ripple-forms, we suggest the simplest explanation is that the valley, circular incision and ripples were formed subglacially~~.~~ by water flowing down this tunnel valley.

5. Discussion

5.1 Distribution and pattern of tunnel valleys

5.1.1 Southern sector of the former Laurentide Ice Sheet

There is spatial variation in the large-scale distribution of tunnel valleys along the southern sector of the former Laurentide Ice Sheet (Fig. 4), which likely reflects their sensitivity to regional conditions such as basal thermal regime, ice and bed topography, timing and climate. For instance, the James and Des Moines lobes, which do not contain many tunnel valleys, are younger, underlain by clay-rich tills and rapidly surged to and then retreated from their maximum positions in a warmer climate (Clayton and Moran, 1982; Clayton et al., 1985). Indeed, there is general trend of fewer tunnel valleys in the more southerly ice lobes (James, Des Moines, Lake Michigan and Huron-Erie), which may be associated with less extensive permafrost (e.g. Johnson, 1990; Mickelson & Colgan, 2003). Moreover, the very low ice-surface slopes (~0.001 m/km - Clark, 1992) reconstructed for the James and Des Moines ice lobes would have resulted in low subglacial hydraulic gradients not conducive to channel formation (e.g. Hewitt, 2011). ~~The large-scale distribution of tunnel valleys is strongly controlled by ice geometry. Tunnel valleys are rare or absent at the terminus of major ice lobes, particularly those that are long and thin (e.g. James and Des Moines lobes), and are more common in interlobate regions, at~~

~~the side of lobes or where the lobe exhibits a broader geometry (Fig. 3). This is consistent with theoretical drainage of meltwater beneath an ice lobe, which is strongly controlled by the ice-surface slope (e.g. Shoemaker, 1999). Meltwater is theorised to radiate out from the centre of lobes, and converge along interlobate regions where the subglacial hydraulic gradient and ice surface are relatively steep (Fig. 13). Indeed, tunnel valley networks associated with lobate margins often have a distinctive divergent geometry (Fig. 3).~~Conversely, older or more northerly ice lobes with steeper ice-surface slopes, more extensive permafrost zones and sandier sediments (e.g. Chippewa, Superior and Green Bay – Wright, 1973; Attig et al., 1989; Johnson, 1990; Clark, 1992; Colgan & Mickelson, 1997; Clayton et al., 2001) have a greater occurrence and density of tunnel valleys.

The locations of ice lobes along the southern sector of the Laurentide Ice Sheet are topographically controlled~~.~~, mostly by upstream basins, and ~~are~~ ice producing these lobes has been inferred to have surged or have been fast-flowing for at least part of their history (e.g. Mickelson and Colgan, 2003; Margold et al., 2015). Fast ice-flow is ~~likely~~ often thought to ~~have been~~be promoted by thermomechanical feedbacks~~, enhancing~~ which enhance basal meltwater production ~~and~~thereby lubricating the bed (cf. Winsborrow et al., 2010 and references therein). It is therefore ~~no surprise~~interesting that tunnel valleys are ~~typically~~preferentially found down-glacier of ~~basins~~depressions, where the greatest volumes of basal meltwater were ~~focused. However, there is no~~likely routed. We explored the hypothesis that some of the tunnel valleys might have been fed from subglacial lakes. No clear ~~link~~links were found to predicted subglacial lake locations or with their obvious drainage corridors except for the Langlade Lobe tunnel valley ~~network~~cluster (Fig. ~~3~~4) (Livingstone et al., 2013). ~~This~~If the predictions of lake locations of Livingstone et al., (2013) are correct, it suggests that the drainage of subglacially stored water was not the main control on tunnel valley formation~~, or~~. However, it is likely that ~~we have yet to discover~~the modelling underestimates the true extent of subglacial lakes (i.e. the prediction in Fig. ~~3 is an underestimate). For example,~~4 is a minimum distribution) as the predictions do not account for the possibility of water ponding behind frozen margins as suggested by Cutler et al., (2002) and Hooke & Jennings, (2006).

Measurements of tunnel valley spacing reveal an overall median spacing of 4.5 km with some degree of intra-~~network~~cluster regularity (Fig. ~~4~~5). Inter-~~network~~cluster variation is greater, with median ~~network~~ values ranging from 1.9 to 9.1 km for individual clusters across the study area. The spacing metrics are within the range of previously reported values for tunnel valleys (Praeg, 2003; Jørgensen and Sandersen, 2009; Stackebrandt, 2009; Moreau et al., 2012; Kehew et al., 2013) but smaller than the average spacing of eskers (Storrar et al., 2014a, and references therein). Theory suggests that the spacing of subglacial conduits is controlled by substrate properties, basal melt rate and the hydraulic potential gradient (e.g. Boulton et al., 2007a,b, 2009; Hewitt, 2011). According to such theory the spacing between adjacent tunnel valleys should be wider if: (i) bed transmissivity is larger; (ii) melt rate/discharge is lower; and/or (iii) the subglacial hydraulic gradient is smaller. Thus the wider than average spacing towards the terminus of major ice lobes where ice surface slopes and thus hydraulic gradients are inferred to be shallower (e.g. Des Moines and Saginaw lobes – Clark, 1992), and a smaller spacing along narrow ice lobes characterised by steeper ice-surface and hydraulic gradients (e.g. Green Bay and Superior lobes – Clark, 1992) is consistent with theory. However, cross-cutting relationships indicate that not all tunnel valleys were acting synchronously, even within a drainage ~~network~~cluster (Fig. ~~10b~~11b), which might explain the large variations in spacing.

5.1.2 ~~Geographical~~Wider geographical distribution of tunnel valleys during the last glaciation

Figure 14 displays the geographical distribution of tunnel valleys reported in the northern hemisphere and attributed to the last glaciation. It appears that tunnel valleys tend to be associated with the flat southern margins of terrestrial or formerly terrestrial (e.g. North Sea) palaeo-ice sheets. They also tend to occur towards the maximum limit of glaciation and are often found downstream of large basins such as the Witch Ground in the North Sea, Baltic Depression along the southern limit of the European Ice Sheet, and Great Lake basins along the southern limit of the Laurentide Ice Sheet.

The tendency for tunnel valleys to form on beds of low relief and gradient ~~implies a genetic association. In particular, water flow in regions of low bed relief is largely unconstrained by topography and can therefore more easily erode laterally producing wide, shallow valley geometries. Conversely, more rugged terrain will exert a greater control on water flow, increasing network complexity and restricting valley expansion. A consequence of ice lobes along the southern margin of the Laurentide Ice Sheet having such shallow ice-surface slopes (reconstructed as 0.001 to 0.005 m/km Wright, 1973; Mathews, 1974; Clark, 1992), is the resulting low subglacial hydraulic gradients. Such low gradients are at odds with the development of many closely spaced large channels (~~near southernmost ice limits implies a genetic association. It might be that melt volumes were sufficiently high at the warm southern extremities of the ice sheets to overcome the ability of the subglacial system to export the water by other means (e.g. groundwater), making tunnel valleys more common. Perhaps it is only in low relief settings where water flow is uninhibited by topography, and can therefore organise itself into a few large catchments, that tunnel valley forms can arise~~Hewitt, 2011). This could indicate either that: (i) large discharges of subglacial meltwater were needed to form the tunnel valleys; or (ii) tunnel valleys and their spacing were determined by initial conditions set up near the ice margin (i.e. where ice-surface slopes are steepest and the greatest volumes of meltwater are discharged). Certainly, shallow ice-surface slopes would have extended the size of the ablation zone and made it more sensitive to small changes in summer air temperature, while hydrofracture of surface meltwater to the bed is easier where ice is thin~~.

The prevalence of tunnel valleys ~~along~~close behind terrestrial margins hints at an important role of permafrost in their formation (e.g. Wright, 1973; Piotrowski, 1994, 1997; Cutler et al., 2002; Jørgensen & Sandersen, 2006). It has been proposed that the development of a toe frozen ~~toe~~to its bed along the fringe of an ice sheet acted as a barrier to water flow facilitating tunnel valley formation by subglacial ponding and outburst cycles (e.g. Wingfield, 1990; Piotrowski, 1994, 1997; Cutler et al., 2002). Moreover, freezing of sediment deposited in channels under the thin fringe of the ice sheet during winter months may have helped to prevent creep-closure of incipient tunnel valleys, thereby stabilizing and preserving their forms from year to year.

~~The occurrence of tunnel valleys near the LGM limit could indicate larger subglacial meltwater fluxes concomitant with greater catchment areas, a climatic control and or variations in basal conditions. Conversely, the paucity of tunnel valleys towards the centre of former ice sheets suggests formation is not linked to greater volumes of supraglacial meltwater production concomitant with climatic warming, although this may be partially counteracted by reduced erosion on the hard crystalline bedrock towards the centre of the Northern Hemisphere palaeo-ice sheets (Clark and Walder, 1994). Critically, the northern hemisphere Quaternary ice sheets were vastly different sizes, so it seems unlikely that tunnel valley distribution was a function of subglacial hydrological catchment size and meltwater flux, particular as the hydrological budget is likely to be dominated by supraglacial meltwater inputs during deglaciation.~~ There is abundant evidence for well-developed permafrost conditions in the southern sector of the Laurentide Ice Sheet during and after the LGM (cf. French & Millar, 2014 and references therein), and it has been associated with glacial landsystems comprising hummocky moraine, tunnel valleys and hill-hole-pairs (e.g. Wright, 1973; Clayton & Moran, 1974; Bluemle & Clayton, 1984; Attig et al., 1989; Ham & Attig, 1996; Clayton et al., 1999, 2001; Colgan et al., 2003). The width of the frozen toe is likely to decrease during retreat because adjustment of the thermal structure of the toe will lag considerably behind adjustment of the margin position to an ameliorating climate. ~~Decrease~~Thus, decrease in tunnel valley occurrence away from the maximum ice limit (Fig. 14) may~~ therefore~~ be indicative of a change to temperate glacier conditions.

5.2 Morphology of tunnel valleys

The tunnel valleys extend for up to ~~55~~65 km, although the majority (90%) are <17 km long and the median is 6.4 km (Fig. ~~5a~~6a). In comparison, reported tunnel valley lengths from the North Sea range from a few kilometres to around 100 km, with the length of individual segments not normally exceeding 20-30 km (e.g. Huuse and Lykke-Andersen, 2000). Although very wide tunnel valleys were found (maximum width ~6.7 km), the majority (90%) are 500-3000 m (Fig. ~~5b~~6b). This is similar to tunnel valley widths (500-5000 m) reported in Europe and elsewhere in North America (e.g. Brennand and Shaw, 1994; Huuse and Lykke-Andersen, 2000; Jørgensen and Sandersen, 2006; Kristensen et al., 2007).

Tunnel valley length and width display log-normal distributions (Fig. ~~5~~6), which is common of other glacial landforms (Fowler et al., 2013; Hillier et al., 2013; Spagnolo et al., 2014; Storrar et al., 2014). Log-normal distributions are thought to typically emerge from many independent random events in which incremental growth or fragmentation occurs (e.g. Limpert et al., 2001). For drumlins and mega-scale glacial lineations (MSGLs) a log-normal distribution has been used to suggest a growing phenomenon that occurs randomly, for random durations, or under random conditions (Hillier et al., 2013; Spagnolo et al., 2014), while for eskers it is thought to reflect ridge fragmentation (Storrar et al., 2014a). Examples of aligned tunnel valley segments characterised by abrupt start and end points implies at least some tunnel valley fragmentation, and this may occur due to partial burial during re-advance events or the melt out of debris-rich ice (Kehew et al., 1999~~), or~~). However, this fragmented appearance may also arise by differential erosion along the length of a drainage route (Fig. ~~7a~~8a,e,f; see also ~~Sjorgen~~Sjogren et al., 2002~~). However, in~~) or due to water cutting up into the ice as well as down into the sediment (e.g. Fowler, 2011; Livingstone et al., 2016). In other cases, aligned tunnel valley segments could indicate a time-transgressive origin (e.g. Mooers, 1989; Patterson, 1994; Jörgensen and Sandersen, 2006; Janszen et al., 2012). This is particularly apparent where the valley segments terminate in outwash fans, and~~/~~ or where segments cross-cut each other (Fig. ~~10~~11 and see also Mooers, 1989). The positive relationship between tunnel valley length and width (Fig. ~~6~~7) is consistent with a growing phenomenon (e.g. by headward expansion) or continuous flow (e.g. a river). In contrast, the length and width of valleys formed by floods are likely to be independent of each other; length is related to the distance that the stored water body is from the ice margin, while width is a function of the magnitude and/or frequency of drainage.

In fluvial geomorphology, channel width in an equilibrium system increases downstream (Fig. ~~7f~~8f) and has classically been related to discharge, and hence drainage area (Leopold and Maddock, 1953; Leopold et al., 1964). This may be complicated locally by the erodibility of the bed substrate and channel slope (e.g. Finnegan et al., 2005). In contrast, large single source flood events (as may occur during a subglacial or supraglacial lake drainage event), will produce a relatively constant channel width (e.g. Lamb and Fonstad, 2010~~), or even show a downstream decrease if infiltration is significant~~) (Fig. ~~7f~~8f). The downstream width of tunnel valleys in our dataset varies considerably and there is no systematic downstream trend in valley form, although general increases and decreases in width do occur (Figs. ~~6, 7a~~7, 8a-e). Thus, there is no observable signature of catastrophic (constant~~ or declining~~ width) or stable, bankfull drainage (steady widening). Moreover, the downstream variation in widths is also inconsistent with subglacial drainage channels fed by multiple supraglacial lake inputs (e.g. Palmer et al., 2011), which we would expect to produce a downstream increase in width concomitant with ~~increased~~ water ~~added~~input.

Figure ~~7a~~8a-e indicates that local variations in tunnel valley width are generally more pronounced than any downstream trend. These ~~widening's~~widenings could arise from basal conditions at the time of formation (e.g. thermal regime), catastrophic drainage (e.g. ~~Sjorgen~~Sjogren et al., 2002), or a laterally migrating stream at the base of the valley floor. Laterally migrating streams are unlikely as we do not observe terraces, bars or incised braided or meandering channels within the broader tunnel valleys, although this may partially be due to ice and post-glacial modification. The crenulated margins, circular incisions, residual hills, hummocky terrain and valley discontinuities are all analogous to features eroded during large floods by macroturbulent flow (e.g. ~~Sjorgen~~Sjogren et al., 2002), although these are typically associated with bedrock channels (Baker, 2009 and references therein). ~~Moreover~~However, we see little evidence for other characteristic features of floods, such as irregular anabranching channels (although they are observed elsewhere, e.g. Boyd, 1988; Brennand and Shaw, 1994), inner channels, furrows and large bars (e.g. Channelled Scablands: Bretz, 1923), while residual hills are not typically streamlined.

The alternative to the catastrophic hypothesis is that variations in width are strongly controlled by ~~local~~regional basal and hydrological conditions. ~~Indeed, there~~There is greater similarity between tunnel valleys from the same ~~network~~cluster (e.g. in form, size and association with other landforms) compared to ~~tunnels valleys from~~ different ~~networks~~clusters, which hints at the importance of ~~local~~regional conditions. Although there is no clear association with bed slope (Fig. ~~9~~10) or geology~~,~~ (Fig. 4), the strength and therefore stability of tunnel valleys sides ~~would~~could have been strongly modulated by variations in basal thermal regime, substrate properties ~~and~~, water flow and or ice behaviour during glaciation. ~~Using this idea, we~~We therefore propose ~~three theories~~four ideas that could produce these variations in width, and which we hope will motivate physical modelling studies or field investigations (Fig. 15). Firstly, the variations in tunnel valley width may be a consequence of the very flat beds on which they form (Fig. 14). Water flow in such a landscape will be very sensitive to small changes in bed relief and variations in discharge. Coupled with sluggish water flow due to the low hydraulic gradients, we therefore envisage ~~the~~ tunnel valleys, which display large variations in width, as a series of interconnected swampy regions (Fig. 15a). This is analogous to lakes and or swampy ground connected by overspill channels, or wide flood plains comprising dynamic river channels observed in fluvial systems flowing across similarly flat landscapes. However, not all widenings occur in bed lows (e.g. 8a,f) and so cannot account for all the variation. Secondly, tunnel valley width could relate to the rate of ice retreat, with relatively wide segments developing over longer durations when the ice is either retreating slowly or stable, and narrower segments developing during more rapid retreat (Fig. 15b). This idea is predicated on the assumption that tunnel valleys primarily form and grow close to the margin. If this were the case and width is related to time we might expect the widest segments of tunnel valleys to be associated with still-stands, and as this is not the case (e.g. Fig. ~~Secondly~~11) we therefore consider this idea unlikely. Thirdly, a basal thermal regime consisting of a mosaic of cold- and warm-based sediment patches (e.g. Kleman & Glasser, 2007) would locally influence how easily widening could happen (Fig. ~~15b~~15d). Frozen patches would inhibit channel formation and may even result in ponding of meltwater, while warm based patches would be more susceptible to erosion. Thirdly, as discharge increasesFinally, the conduit carrying water can enlarge, either by erodingcut down into the bed (typically forming tunnel valleys), melting or N-channels), up into the ice (R-channel) or both together (see Fowler, 2011) channels) or some mixture of the two (R and N channels) (Fig. 15c). What happens will vary depending uponGiven that the controls on which case occurs are likely to vary over time (e.g. water discharge) and space (e.g. varying basal conditions), we propose that the large variations in tunnel valley width might be the record of how high or low the conduit was positioned in relation to the bed. Consider a conduit with an undulatory long profile cutting deeply and widely into the bed in some places and then rising back into the ice such that the cut channel in the bed narrows and pinches out and then disappears altogether where the conduit becomes entirely englacial. Control of the conduits position in relation to the bed is likely to vary with, for example, the effective pressure, ice viscosity and and relative strength of ice versus the sediment stiffness. Consequently, the manifestation of an increase in discharge on the bed imprint is likely to vary spatially and temporally depending on the competition between sediment erosion and the melting of ice (e.g. and has been explored in Fowler (2011) and Livingstone et al., . (2016). This theory may therefore explain the fragmentation of some tunnel valleys into multiple segments (Fig. 7e,f).

5.3 Landform associations

5.3.1 Relative timing of tunnel valley formation

Cross-cutting relationships between moraines, outwash fans, and tunnel valleys in Wisconsin have enabled their relative timing of formation to be used to build a history of formation (Figs. 1011, 16). If a tunnel valley cuts through moraine positions, formation must have occurred during or after the moraine was deposited. These tunnel valleys, and those interrupted by outwash fans mid-way along their length, must therefore have been used as a drainage route either repeatedly or over a long duration during retreat (see Fig. 16b). Conversely, tunnel valleys that are cross-cut by recessional moraines were abandoned as ice retreated. In Fig. 16b these tunnel valleys correspond to the age of a single moraine position, and may have been eroded during a singular 'event' (i.e. outburst of a sub- or supra-glacial lake) or been abandoned due to a switch in drainage configuration or supply.

5.3.2 Moraines

The close link between tunnel valley networksvalleys and moraines throughout the study area (Figs. 3, 104, 11; and see also Attig et al., 1989; Mooers, 1989; Patterson, 1997; Smed, 1988; Johnson, 1999; Cutler et al., 2002; Jørgensen and Sandersen, 2006) suggests formation and growth is intimately associated with pauses or slow-downs in ice retreat or ice advancesmargin fluctuations and that meltwater drained to the ice margin. The implication is that tunnel valley formation requires a relatively stable ice-sheet configuration to allow headward growth or recharge of source storage areas. It also provides further support for the role of permafrost in tunnel valley formation given that rapid retreat will reduce the width of the frozen toe and consequently reduce the efficacy for water storage. However, whether a reconfiguration of the subglacial hydrological regime via the development of tunnel valleys behind ice margins (moraines) can influence ice retreat~~,~~ (via ice dynamics), for example causing the observed staccato jumps between still-stands (Fig. 16a), remains an open question.

Regularly spaced, low relief transverse ridges in North Dakota and the Des Moines Lobe (e.g. Fig. ~~10b~~11b), termed washboard or corrugation moraine, have been interpreted as ~~both~~ (annual) end moraine deposits ~~and~~or as subglacial crevasse ~~fill~~fills (Kemmis et al., 1981; Stewart et al., 1988; Patterson, 1997; Jennings, 2006; Cline et al., 2015; Ankersjerne et al., 2015). The deflection of transverse ridges towards the long axis of tunnel valleys (e.g. Fig. ~~10b~~11b), and buried sand and gravel deposits (see Stewart et al., 1988; Cline et al., 2015), indicates a temporal and possibly genetic relationship. One interpretation is that lower water and pore water pressures in tunnel valley and glaciofluvial deposits respectively, result in slower local ice velocities that cause the pattern of crevasses and thus ridges to be deflected (see Cline et al., 2015). However, high pressure discharges have also been inferred from coarse-grained outwash fans deposited in front of tunnel valleys (~~Section 5.3.4,~~ e.g. at the edge of the Green Bay Lobe - Cutler et al., 2002~~; Jørgensen and Sandersen, 2006).~~, see Section 5.3.4). There may therefore have been multiple modes of meltwater drainage down tunnel valleys~~;~~ (e.g. predominantly low pressure drainage interrupted by episodic high pressure outbursts~~.~~), or regional variation in tunnel valley evolution (e.g. some clusters formed predominantly by high energy drainage events and other that formed in low pressure channels).

5.3.3 Hill-hole pairs

The formation of tunnel valleys up-glacier from hill-hole-pairs ~~of similar width~~in North Dakota (Fig. 12) suggests a ~~temporal~~causative relationship. Hill-hole-pair formation is believed to require the ice to be strongly coupled to the bed so that it can exert sufficient shear stress to produce failure (Bluemle and Clayton, 1984; Aber et al., 1989). Thus, either the hill-hole pair was produced first, and the tunnel valley grew headward out of the 'hole', or once drainage through the tunnel valley had waned, ice re-coupled strongly to the bed and the downstream termination of the valley became the focus of large shear stresses that resulted in failure and formation of the hill-hole pair. We suggest the former is more likely as ~~the~~these tunnel valleys do not terminate at moraine positions as is typical elsewhere, ~~while~~and small channels and eskers emanating from and diverging around the hills appear to record the down-glacier leakage of pressurised water around the obstruction (Fig. 12b). If true, these tunnel valleys appear to be unique in having initiated up-glacier from the margin. The formation of a hill-hole pair may therefore have ~~facilitated~~seeded tunnel valley erosion by providing a low-point to attract water and a pathway for water through a frozen toe. This association highlights the importance of regional variations in controlling tunnel valley formation and morphology; in this case, it is the local geology (Cenozoic and

Cretaceous shale and sandstone) and presence of permafrost that likely controlled the initial formation of the hill-hole pair (e.g. Bluemle & Clayton, 1984; Clayton et al. 1985) and which subsequently triggered tunnel valley growth.

5.3.4 Outwash Fans

Outwash fans occur at the down-glacier end of at least 10% of the tunnel valleys in our study area (e.g. Fig. ~~10b), and are particularly common along the margins of the Green Bay, Michigan and Langlade lobes (~~11b). ~~Attig et al., 1989; Clayton et al., 1999; Cutler et al., 2002; Fisher and Taylor, 2002).~~ The fan sediments at the margin of the Green Bay Lobe include well-rounded pebbles and boulders up to 2 m diameter (Cutler et al., 2002), similar to accumulations documented in-front of European tunnel valleys (Piotrowski, 1994; Jørgensen and Sandersen, 2006; Lesemann et al., 2014). The coarse-grained sediments have been interpreted to indicate high-energy discharges and or highly pressured subglacial meltwater flow through the tunnel valleys. Cutler et al. (2002) suggested there was at least one large outburst flood just before the termination of glaciofluvial activity through each tunnel valley~~.~~ in their investigation. These high-energy floods may have been responsible for cutting the valley itself, or the valley could have acted as a preferential drainage route ~~upon tapping into a water reservoir.~~for a flood. The concentration of outwash fans in the Chippewa, Wisconsin, Langlade, Green Bay, Superior and Wadena lobes could indicate a greater or dominant influence of discrete drainage events in these regions, while more gradual processes prevailed in other lobes (e.g. North Dakota, Des Moines, Huron-Erie).

5.3.5 Giant Current Ripples

The occurrence of giant current ripples stretching across the whole width of a tunnel valley in Minnesota (Fig. 13) implies a large sub- or supra-glacial lake outburst event (e.g. Bretz et al., 1956; Carling, 1996; Rudoy, 2005). This is further supported by the circular incision at the southern end of the tunnel valley ~~(Fig. 13),~~, which is similar in form to large potholes generated by macroturbulent eddies. The flood could have cut this particular tunnel valley or the valley pre-existed and became the route of a subglacial flood ~~which completely filled it,~~ further modifying and enlarging the valley (Bretz et al., 1956; Carling, 1996; Rudoy, 2005). The unique occurrence of this landform suggests that large floods were rare or ~~the~~such landform ~~signature~~signatures are rarely preserved.

5.4 Implications for the formation of tunnel valleys

Based on our ~~large-scale~~ analysis of the morphological properties of tunnel valleys and associated ~~bedform along the southern portion of the Laurentide Ice Sheet~~landforms we ~~are able to~~ provide some ~~new~~ insights into their formation. The ~~importance of ice geometry (Fig. 3) and the semi-~~regular spacing of ~~individual tunnel valley networks (Fig. 4), implies a stable,~~most of the tunnel valleys (Fig. 5), and that particular clusters have their own characteristic spacing, suggests self-organising organisation in the basal hydrological system modulated by. Individual channels somehow 'know of' each other such that spacing can be set and this is most easily interpreted as there being an integrated system of many tunnel valleys operating at roughly the same time. It is difficult, for example, to understand how a collection of separate flood events could produce tunnel valleys that would combine to produce such regularity in spacing, unless the whole cluster of valleys were produced in single large flood events (as suggested for example by Shaw, 2002). Consistent with the argument of self-organised spacing under steady flow are a series of studies that argue that the spacing may be controlled by a combination of bed transmissivity, meltwater discharge and the hydraulic potential gradient (Piotrowski, 1997; Boulton et al., 2007a, b, 2009; Hewitt, 2011). While some tunnel valleys appear to have been short-lived, either as the preserved signature of a single event or because they were abandoned due to changes in melt delivery or ice retreat, it is inconceivable that an entire network pattern was formed during one catastrophic flood (e.g. Shaw, 2002) as many of the valleys are found to have formed incrementally (also see Mooers, 1989),We suggest that reconstructions of drainage history, as demonstrated in Figure 16, where we show tunnel valleys remaining active and relatively stable over a long period of time (Fig. 16).phases of ice retreat could significantly help advance knowledge of tunnel valley formation, especially when combined with information from field-based investigations and dating.

Recurrent outburstoutbursts of stored water responsible for incrementalto produce incision of whole networks isclusters are appealing where tunnel valleys converge towards up-glacier basins (e.g. Superior and Langlade – Figs. 2a,3a, c, 34) where one could infer that subglacial lakes periodically grew and drained (existed. In Evatt et al., (2006). However,) for example they use theory of subglacial drainage to show that lakes should undergo periodic drainage and filling episodes. Perhaps many of tunnel valleys are the networksrecord of large and repeated drainage events. Against this argument however, many of the clusters are very broad (>60 km across) and the tunnel valleys relatively parallel (e.g. Green Bay and eastern Superior – Fig. 34). To produce these networkssystems would require lakes many tens or even hundreds of kms wide. This is difficult to reconcile with mean (<1km^2) and maximum supraglacial lake areas (up to ~150 km^2 – which equates to a diameter of ~14 km if a circular lake is assumed) on the surface of the present-day Greenland Ice Sheet (e.g. Leeson et al., 2013). Moreover, while very large subglacial lakes do exist beneath the Antarctic Ice Sheet (Wright and Siegert, 2011, e.g. Lake Vostok, >250 km long by ~80 km wide) and are theorised to have existed in Hudson Bay and the Great Lake Basins (e.g. Shoemaker, 1991, 1999), they have neither been predicted by modelling or identified in the geological record (e.g. Livingstone et al., 2013).

Despite the lack of support for a mega-flood genesis of whole tunnel valley networksclusters, drainage of stored water down individual valleys almost certainly did happen. (after Piotrowski et al., 1994; Cutler et al., 2002; Jörgensen & Sandersen, 2006; Hooke & Jennings, 2006). Not all tunnel valleys formed in networksclusters or were incised time-transgressively up-glacier (Figs. 34, 16), and the simplest explanation for the formation of fans containing boulders (Figs. ~~2, 10b~~3, 11b) (e.g. Piotrowski, 1994; Cutler et al., 2002; Derouin, 2008; Lesemann et al., 2014) and for giant current-ripples (Fig. ~~12~~13) is high discharge (possibly bank-full) events. Indeed, periodic higher energy or pressurised meltwater events (e.g. during penetration of surface meltwater to the bed during summer months) were probably necessary to prevent armouring of the valley sides by coarse sediment, while bedrock tunnel valleys (e.g. in Lake Superior) are difficult to reconcile solely by gradual formation. Evidence for seasonal surface meltwater reaching the bed and draining along tunnel valleys is proffered by Mooers (1989), who identified short esker segments that frequently start at moulin kames and terminate at outwash fans at the bed of tunnel valleys in the Superior Lobe. We therefore contend that ~~large drainage events~~floods from sub- and supra-glacial lakes, and by injections of surface meltwater down moulins did occur, contributing to the formation of some tunnel valleys either by eroding new valleys or enlarging existing ones. However, ~~our data suggests they were probably not the primary mechanism by which tunnel valleys formed. Firstly, the decline in~~we note that most tunnel valley ~~incidence away from LGM margin positions~~lengths (Fig. ~~14) is inconsistent with increasing contributions of surface melt in an ameliorating climate. Secondly, their typical length distribution (Fig. 5a) is~~6a) are an order of magnitude less than the distance up-glacier (tens to hundreds of km) that supraglacial and subglacial lakes are commonly documented in Greenland and Antarctica (e.g. Selmes et al., 2011; Wright and Siegert, 2011).

We suggest that the majority of tunnel valleys along the southern sector of the Laurentide Ice Sheet were initiated at the ice margin and then typically (although not exclusively) eroded gradually up-glacier. Tunnel valley length and width display log-normal distributions and are positively correlated, indicative of a growing phenomenon (cf. Fowler et al., 2013; Hillier et al., 2013). Their strong association with moraine positions (Fig. 3) suggests that formation is time dependent (i.e. they require time to grow), while ~~cross-cutting relationships~~our drainage history reconstruction (Fig. 16) demonstrates that many of the features remained active for extended periods~~. Thus,~~ during ice margin retreat. Growth likely proceeded up-ice from the margin rather than down-ice from a stored water body because tunnel valleys preferentially terminate at ice-margin positions irrespective of their size (e.g. very small tunnel valleys along the southern margin of the Green Bay Lobe, Fig. 4). Further support is provided by some tunnel valleys in Dakota that grew headwards out of the 'hole' of a hill-hole pair (Fig. 12). We suggest that when retreat is slow or a stable position is reached (allowing formation of a moraine), tunnel valleys have time to grow up-glacier and to widen and deepen as more water is discharged through them (Fig. 17a). A more unstable/rapid ice-retreat will limit the time for growth (headward and lateral) or may even produce a segmented tunnel valley if retreat overtakes headwards incision (Fig. 17b). ~~Indeed, the James and Des Moines ice lobes that are thought to have rapidly surged to and then retreated from their maximum positions (Clayton and Moran, 1982; Clayton et al., 1985) are relatively devoid of well-organised tunnel valley networks compared to other ice lobes, such as~~

Superior, that retreated more slowly (Dyke, 2004). We argue that growth was not a function of conditions associated with the size of a stored water body and the magnitude and frequency of its drainage because immature (smaller) tunnel valleys are also found to preferentially terminate at ice-margin positions (e.g. southern margin of Green Bay Lobe, Fig. 3). Hence, growth likely initiated and proceeded up-glacier from the ice margin rather than down-glacier from a stored water body, and there is some evidence for this, including the presence of amphitheatre-shaped tunnel valley heads (e.g. Onda, 1994; Abrams et al., 2009; Petroff, 2011) and the growth of valleys out of hill-hole pairs (Fig. 11).

Our data indicates that the formation and morphology of tunnel valleys was strongly controlled by regional variations in basal thermal regime, bed and ice topography, timing and climate. At the broad scale, tunnel valleys tend to form on beds of low relief near southern terrestrial ice sheet margins. The paucity of tunnel valleys in the James and Des Moines lobes may be a result of the very low ice-surface slopes inhibiting channel formation and because of their relatively late advance and southerly positions that would have resulted in a less extensive zone of permafrost (Clayton and Moran, 1982; Clayton et al., 1985). Indeed, there is a general trend of fewer tunnel valleys in the more southerly ice lobes (James, Des Moines, Lake Michigan and Huron-Erie), where permafrost was reconstructed as less extensive (e.g. Johnson, 1990; Mickelson & Colgan, 2003). Regionally, we observe large inter-cluster variation in tunnel valley spacing and morphology (form and size), and their association with other glacial landforms (e.g. outwash fans, hill-hole pairs), while down-valley variations in width suggests that incision was sensitive to local conditions (e.g. Fig. 15).

Despite finding evidence for both gradual formation and high discharge events (floods) down tunnel valleys *sensu lato*, those that could be identified as from floods and defined as tunnel channels are not founds to be morphologically distinct from tunnel valleys *sensu stricto* and are therefore considered as equifinal landforms. This is unfortunate as it would have been useful to find a clear distinction. For instance, we found that the spacing, width and length of potential tunnel channels, i.e. those that terminated at an outwash fan or contained giant current ripples, or from clusters thought to have experienced large drainage events (e.g. Green Bay Lobe – Cutler et al., 2002) were similar to the overall morphology of tunnel valleys *sensu lato* (e.g. Fig. 6). They were also similar to tunnel valleys *sensu stricto* (e.g. North Dakota, where tunnel valleys grew out of hill-hole-pairs). Rather, the distinction between outburst flood (tunnel channels) and gradual (tunnel valleys *sensu stricto*) origins in this and other studies (e.g. Cutler et al., 2002), is based on their association with other glacial landforms such as outwash fans, moraines, hill-hole-pairs and giant current ripples. An important next step is to use these landform associations, where they occur, to learn more precisely about the morphological characteristics that define tunnel valleys and tunnel channels and to see if unique forms can be identified. Although we have grouped landforms of supposed different origins, the large-scale distribution and arrangement of tunnel valleys *sensu lato* suggests some commonality of process (e.g.

Hooke & Jennings, 2006), and it may be, for example, that all tunnel valleys grow gradually, but that some experience occasional high-discharge, bank-full events.

6. Summary and Conclusions

To provide new information on the morphological characteristics of tunnel valleys we undertook a large-scale mapping campaign to document the distribution and morphology of about 2000 tunnel valleys and associated bedforms on the bed of the former Laurentide Ice Sheet. Our maps and analyses show that tunnel valleys are semi-regularly spaced (median of 4.5 km) and tend to cluster together. The distribution of tunnel valleys varies across the study area, with clusters of tunnel valleys common across much of the Wadena, Itasca, Superior, Chippewa and Saginaw ice lobes, but much rarer along the more southerly lobes such as James, Des Moines, Lake Michigan and Huron-Erie. The wider geographical distribution suggests that tunnel valleys tend to form on flat, terrestrial beds close to the former southern LGM extent. They are typically <20 km long and 0.5 to 3 km wide and longer valleys tend to be wider. The planform edges of tunnel valleys varies considerably across the study region, ranging from straight to crenulated and sharp to indistinct, and while there is no systematic downstream trend in valley form there are pronounced changes in width. There is a close link between tunnel valleys and moraines, while outwash fans occur at the down-glacier end of at least 10% of valleys in our study area. We also observed one tunnel valley with giant current ripples on its bed, and rare cases where tunnel valleys appear to have grown out of hill-hole-pairs.

There have traditionally been two ~~main paradigms to explain~~explanations for the formation of tunnel valleys:~~-~~ (1) outburst formation by rapid drainage of sub- and/or supraglacially stored meltwater; and (2) gradual formation by headward sapping in low pressure subglacial channels (Fig. 1) (cf. Ó Cofaigh, 1996; Kehew et al., 2012; van der Vegt et al., 2012). ~~To investigate these two models we undertook a large-scale mapping campaign to characterise the distribution and morphology of >1900 tunnel valleys and associated bedforms on the bed of the former Laurentide Ice Sheet.~~What does our mapping and analyses say about these?

Given our previous work on subglacial lakes beneath the Laurentide Ice Sheet (e.g. Livingstone et al., 2013~~)~~, 2016) we specifically explored tunnel valleys with an expectation that they might ~~link with predicted lake locations and~~mostly be the geomorphological record of outburst floods. However, to the contrary, the morphological evidence suggests that most of the tunnel valleys underwent gradual formation, but notably with some contributions from ~~large drainages of~~ floods from stored water (Fig. 18). In particular, our findings indicate that tunnel valleys comprise ~~well-~~organised ~~networks~~clusters of ~~semi-~~regularly spaced (1.9-9.1 km) valleys that formed incrementally during ice retreat. ~~This pattern is strongly controlled by ice geometry and basal properties (e.g. permafrost, flat bed and conduit erosion), and this~~This is a strong argument for ~~a~~ self-organising hydrological ~~network influenced by local conditions. Second,~~networks that mostly operated at the same time. We find that tunnel valleys preferentially terminate at moraines (irrespective of their size), which suggests that growth was initiated at and then progressed headwards from stable ice-margin positions. The concept of a growing phenomenon is further supported by log-normally distributed metrics (width, length) of valley ~~morphologies~~size, the positive correlation between length and width~~,~~ and their initiation and growth out of hill-hole-pairs~~ and the existence of amphitheatre-shaped valley heads~~. Although we favour gradual headward formation as the primary process, our results also show examples where outburst of supraglacial and or subglacial lakes have incised and/or drained down valleys. Evidence includes, giant current ripples and outwash fans with large boulders (Cutler et al., 2002), and that some valleys were only occupied for brief periods during deglaciation suggestive perhaps of a short-lived event. Indeed, ~~cross-cutting relationships~~our reconstructed drainage history (Fig. 16) demonstrate a time-transgressive origin for many tunnel valleys, with individual ~~networks~~clusters forming within the same time frame but individual valleys evolving over different spans involving multiple discrete flow events. Inter-cluster variation in tunnel valley spacing and morphology (form and size), and their association with other landforms highlights the importance of regional conditions in controlling tunnel valley formation. In particular, the presence of permafrost seems to have played a key role in determining whether tunnel valleys were produced.

Many of our observations are consistent with previous findings (e.g. Kehew et al., 2012 and references therein) and we are not the first to suggest a polygenetic origin (e.g. Hooke and Jennings, 2006). However, whilst geomorphological and sedimentological investigations in certain areas have generally advocated *either* an outburst or gradual genesis for tunnel valleys (Fig. 1), when their morphology, distribution and association with other glacial ~~bedforms~~landforms are considered at a regional-scale it suggests that both ~~processes occurred (~~tunnel channels and ~~Fig. 18).~~

~~At the ice-sheet scale, we find most~~ tunnel valleys *sensu stricto* can occur ~~on the flat portions of palaeo-ice sheet beds, where subglacial water flow would have been largely unconstrained by topography. It is on these portions of the bed, where ice geometry is the main control,~~ and that ~~subglacial water becomes organised down relatively stable and regularly-spaced drainage corridors (tunnel valleys). Once a tunnel valley has been initiated, it could provide a low pressure 'release valve' (i.e. generate a local hydraulic gradient)~~they appear to be equifinal (Fig. 18).~~evacuate basal water flowing slowly through water saturated sediments and swampy ground (after Kyrke-Smith and Fowler, 2014) in areas of the bed characterised by low hydraulic gradients, and also as a routeway for large injections of surface or stored water. These drainage corridors provide an effective means of transporting sediment and water from under the ice sheet and may thus have acted to increase basal traction across the bed and slow-down ice flow during deglaciation.~~

Author contributions

SJL and CDC designed the project. SJL generated the data on the tunnel valleys and other glacial bedforms. Both authors contributed to the analyses and interpretations of the data. SJL wrote the manuscript with input from CDC.

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Figures

Fig. 1: Cartoons depicting the two main models for tunnel valley formation: **A.** Outburst floods from supraglacial and/or subglacially stored water; and **B.** Gradual headward growth by sapping.

Fig. ~~2~~2: Location map showing the Last Glacial Maximum (LGM) extent (blue line) and major ice lobes and topography. Boxes refer to detailed examples shown in Figures 3 (red), 8 (yellow), 11 (green), 12 (black) and 13 (blue).

Fig. 3: Examples of mapped valleys and the assignment of confidence levels (1 = high confidence to 3 = low confidence) along the southern sector of the former Laurentide Ice Sheet. Valleys in panels **A** (Superior), **B** (Saginaw), **C** (North Dakota) and **D** (Green Bay) are assigned a confidence of 1. The relict valleys contain eskers, are parallel and relatively straight, and do not trend along the regional slope. In panels **A**, **C** and **D** the tunnel valley ~~networks terminates~~clusters terminate at a moraine position. The large valley in panel **E** (Superior) is assigned a confidence level of 3 as it does not contain any subglacial bedforms and exhibits a gradual and consistent change in bed slope consistent with a proglacial spillway. However, the smaller NW-SE valleys that ~~is~~it bisects is given a confidence of 2 as they have undulating thalwegs that cut across moraines. The dendritic valley ~~network~~cluster in panel **F** (North Dakota) is given a confidence of 3 as it is not associated with any subglacial bedforms and has a consistent bed slope indicating water flow towards the south. A braided channel morphology and a widening reach towards the south allows us to interpret this valley system as a proglacial spillway (fed by tunnel valleys emanating from under the ice to the north).

Fig. ~~3~~4: Distribution of mapped tunnel valleys and moraines along the southern sector of the Laurentide Ice Sheet. Likely subglacial lake locations are predictions from Livingstone et al., (2013). The Last Glacial Maximum extent is from Dyke et al., (2004) and ~~moraines~~surficial deposits are from Fullerton et al. (2003).

Fig. ~~4~~5: Frequency histogram of the spacing of 966 tunnel valleys from 24 discrete ~~networks~~clusters across the southern sector of the former Laurentide Ice Sheet.

Fig. ~~5~~6: Frequency histogram of tunnel valley length and width (for confidence levels 1 and 2). Line is the log-normal distribution for comparison. Width values were extracted at 1 km intervals along the centre-line of each tunnel valley.

Fig. ~~6~~7: Relationship between tunnel valley length and average width (for single thread valleys with a confidence level of 1 and 2, N=1135). Note, there is a tendency for longer tunnel valleys to be wider.

Fig. ~~7~~8: Examples of tunnel valley morphology of tunnel valleys with a confidence of 1 or 2. **A.** Superior Lobe (note the amphitheatre heads of some valleys); **B.** Wadena Lobe (note the large downstream changes in tunnel valley width); **C.** Langlade Lobe; **D.** Saginaw Lobe; and **E.** Wadena (note the parallel valleys) and **F** Huron-Erie (note the abrupt start and end points of the tunnel valleys and parallel organisation). In 7D, there is an example of two cross cutting tunnel valleys that formed at different times during eastward ice retreat (see also Mooers, 1989). The valley trending E-W terminates at an outwash fan, which must mark the position of the ice margin when it was formed. The valley cross-cutting it can be traced several tens of kms further to the west and therefore must have formed during an earlier phase when ice was more extensive.

Fig. ~~8~~9: Along-valley plots highlighting normalised (width of tunnel valley at a point / average width of the whole tunnel valley) tunnel valley width variations. **A.** Saginaw Lobe; **B.** North Dakota; **C.** Green Bay Lobe; **D.** Superior Lobe; and **E.** Wadena Lobe. **F.** Cartoon showing the expected relationship between width and distance downstream for a fluvial river (Leopold and Maddock, 1953; Leopold et al., 1964) and single flood event (e.g. Lamb and Fonstad, 2010). Note that the measured tunnel valley width variations conform to neither of these expectations, but instead show variations in width greatly exceeding any possible systematic trends.

Fig. ~~9~~10: Scatter plot compiled to investigate if downstream variation in channel width was controlled by variations in downstream slope gradient (see text for details). That the data are centred on zero and spread fairly evenly around this demonstrates that there is no systematic relationship between elevation gradient (i.e., whether it is a normal or reverse gradient slope) and width (i.e., whether the tunnel valley is narrowing or widening).

Fig. ~~10~~11: The varied cross-cutting associations between moraines, outwash fans and tunnel valleys in: **A.** North Dakota – note how the washboard moraines curve up-glacier towards the tunnel valleys; and **B.** Wisconsin (Chippewa Lobe).

Fig. ~~11~~12: Hill-hole-pairs in North Dakota and their association with tunnel valleys. **A.** Note the esker downstream of the hill, which trends into an aligned tunnel valley segment. **B.** Note the secondary meltwater channels and eskers that diverge around the hill.

Fig. ~~12~~13: Giant Current Ripples spanning the width of a shallow tunnel valley that is cut into an obliquely-oriented drumlin field (water flow to the south). These sinusoidal bedforms are interpreted as giant current ripples, which formed during a large subglacial flood. Note the undulating thalwegs and esker in the valley that indicates subglacial deposition, and the circular incision (with a remnant island in its centre) in the south of the valley that may have formed by a large eddy during high-energy turbulent flow.

Fig. ~~**13:** Idealised ice lobe, hydraulic potential contours (dotted lines) and drainage routes. Note how this predicts that with uniform upstream basal melting that the resultant water paths diverge down the lobe axis and away from the terminus (yielding low water delivery here) and converge in interlobate regions (high water delivery).~~

~~**Fig.**~~ **14:** Currently known Northern Hemisphere distribution of tunnel valleys that have been attributed to the last glaciation. The opaque blue shading is the Last Glacial Maximum ice sheet distribution. Black lines are the mapped tunnel valleys from Fig. 3 and black boxes are where tunnel valleys have been identified. The preference for tunnel valleys to occur in flat areas close to southern limits of the ice sheets is striking, but to what extent does this map captures the true extent of tunnel valleys, perhaps they are selectively unreported from other regions?

Fig. 15: Cartoons showing ~~three~~four theories to explain the downstream variation in tunnel valley width. A. Swampy ground (blue stipples) and overspill channels (blue lines) associated with water flow across very flat ground. In such a flat landscape tunnel valleys are able to easily expand laterally, in response to small changes in water flux, and there is little impetus for rapid vertical erosion due to shallow hydraulic gradients. B. Variable retreat rate. When the rate of retreat is slow there is more time for valleys to grow and widen, whereas more rapid retreat will produce narrower segments C.~~Tunnel valley formation is modulated by the basal thermal regime (modified from Hughes, 1995). Channels are able to develop more easily across warm sediment patches, and the mosaic of cold and warm sediment patches results in variations in width. C.~~ Undulatory conduit erosion. In this theory the width of the channel eroded into sediment depends upon the competition between erosion down into the sediment (canals) vs. melting up into the ice (R-channel) (see Fowler, 2011; Livingstone et al. ~~Sub~~2016). Note that each of the conduits (i-iii) have the roughly the same area, but that in (ii) no channel forms and in (iii) the channel width is roughly half that of (i). D. Tunnel valley formation is modulated by the basal thermal regime (modified from Hughes, 1995). Channels are able to develop more easily across warm sediment patches, and the mosaic of cold and warm sediment patches results in variations in width.

Fig. 16: Using cross-cutting relationships to reconstruct tunnel valley evolution during ice margin retreat. **A.** Mapping of tunnel valleys and associated glacial bedforms in Wisconsin (Chippewa Lobe) (from Fig. 11B). **B.** Reconstructed history of valley formation behind a back-stepping ice margin. Note that some valleys were long-lived during deglaciation and some abandoned shortly after their formation. The relative age relations help explain the variation in lengths between long continuous tunnel valleys and those comprising short fragments.

Fig. 17: Cartoon demonstrating the dependence of tunnel valley evolution (by headward growth) on ice margin retreat rate **A.** If headward growth of a tunnel valley is faster than the rate of ice retreat the valley will be able to extend continuously up-glacier and its length will only be limited by water supply and hydraulic properties of the bed. **B.** If however, headward growth of a tunnel valley is slower than the rate of ice retreat the valley is likely to be discontinuous, only being able to form and extend up-ice during slow-downs or pauses in retreat.

Fig. 18: For the southern Laurentide region we consider gradual headward erosion as the usual mechanism, but with some floods down selected valleys – note the potential for stored water to cut their own valleys (e.g. supraglacial lake drainage example) or to drain along pre-existing corridors that may have tapped into a reservoir (e.g. subglacial lake example).