



1 **Morphological properties of tunnel valleys of the southern sector** 2 **of the Laurentide Ice Sheet and implications for their formation**

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6 **Abstract**

7 Tunnel valleys have been widely reported on the bed of former ice sheets and are considered an
8 important expression of subglacial meltwater drainage. Although known to have been cut by erosive
9 meltwater flow, the water source and development of channels has been widely debated; ranging
10 between outburst flood events through to gradually occurring channel propagation. We have mapped
11 and analysed the spatial pattern and morphometry of tunnel valleys and associated glacial bedforms
12 along the southern sector of the former Laurentide Ice Sheet from high-resolution digital elevation
13 models. Around 2000 tunnel valleys have been mapped, revealing a well-organised pattern of sub-
14 parallel, semi-regularly spaced valleys that cluster together in distinctive networks. The tunnel valleys
15 are typically <20 km long, and 0.5-3 km wide and preferentially terminate at moraines. They tend to
16 be associated with outwash fans, eskers, giant current ripples, and hill-hole-pairs. At the ice-sheet
17 scale, we find most tunnel valleys occur on the flat portions of palaeo-ice sheet beds, where subglacial
18 water flow would have been largely unconstrained by topography, while tunnel valley morphology is
19 strongly modulated by local variations in basal conditions (e.g. thermal regime and topography) and
20 hydrology (i.e. whether conduit erosion is up into the ice or down into the sediments). Analysis of
21 cross-cutting relationships between tunnel valleys, moraines and outwash fans permits reconstruction
22 of channel development in relation to the retreating ice margin. The reconstruction demonstrates
23 incremental growth of valleys, with some used repeatedly, or for long periods, during deglaciation,
24 while others were abandoned shortly after their formation. Our data and interpretation supports
25 gradual (rather than a single-event) tunnel valley formation, with secondary contributions from flood
26 drainage of subglacial and/or supraglacially stored water.

27 **Key words:** *tunnel valleys; geomorphology; Laurentide Ice Sheet; subglacial meltwater; gradual or*
28 *catastrophic*

29 **1. Introduction**

30 Incised into bedrock or sediment, tunnel valleys and channels (hereafter referred together as tunnel
31 valleys) are elongate depressions up to several kilometres wide, with undulating long-profiles, tens of
32 kilometres long and tens to hundreds of metres deep. They are observed in many formerly glaciated



33 landscapes around the world, and tend to be orientated parallel to the direction of former ice flow (e.g.
34 Wright, 1973; Attig et al., 1989; Wingfield, 1990; Piotrowski, 1994; Patterson, 1997; Huuse &
35 Lykke-Anderson, 2000; Jørgensen & Sandersen, 2006). Features with similar dimensions have also
36 been described beneath current ice masses (e.g. Rose et al., 2014). Tunnel valley formation is
37 typically attributed to subglacial meltwater erosion at the base of ice sheets (cf. Ó Cofaigh et al.,
38 1996; Kehew et al., 2012; van der Vegt et al., 2012), and they are considered an important component
39 of the subglacial hydrological system, providing drainage routeways for large volumes of water and
40 sediment. Understanding their genesis is relevant for reconstructing former ice sheets, elucidating
41 basal processes and exploiting the geomorphological record in a way that is useful for modelling
42 subglacial hydrology. However, despite being debated for over 100 years, there is considerable
43 uncertainty about the underlying processes governing tunnel valley formation. This debate is focused
44 around two genetic models: ‘outburst’ formation and ‘gradual or steady-state’ formation (Fig. 1) (cf.
45 Ó Cofaigh et al., 1996; Kehew et al., 2012; van der Vegt et al., 2012).

46 The ‘outburst’ hypothesis (Fig. 1a) ascribes the erosion of tunnel valleys to rapid drainage of sub- or
47 supraglacially stored meltwater. Contemporary observations from the Antarctic and Greenland ice
48 sheets demonstrate the efficacy of meltwater storage and drainage in sub- and supraglacial
49 environments (Zwally et al., 2002; Wingham et al., 2006; Fricker et al., 2007; Das et al., 2008) and it
50 is reasonable to expect that the Laurentide Ice Sheet experienced similar events. In addition, the
51 impoundment of meltwater behind a frozen ice margin has been linked to tunnel valley formation, for
52 example, along the southern terrestrial margins of the former Laurentide and European ice sheets
53 where permafrost was prevalent (e.g. Piotrowski, 1994; Cutler et al., 2002; Hooke & Jennings, 2006).
54 Genesis is typically thought to occur via repeated low to moderate magnitude floods that may be at or
55 below bankfull flow (e.g. Wright, 1973; Boyd, 1988; Wingfield, 1990; Piotrowski, 1994; Cutler et al.,
56 2002; Hooke & Jennings, 2006; Jørgensen & Sandersen, 2006). Catastrophic erosion of entire tunnel
57 valley networks by massive sheet floods (bankfull flow) has also been proposed (e.g. Shaw & Gilbert,
58 1990; Brennand & Shaw, 1994; Shaw, 2002), but has been considered less likely given the very large
59 volumes of stored water required (e.g. Ó Cofaigh et al., 1996; Clarke et al., 2005).

60 The ‘gradual’ or ‘steady-state’ hypothesis (Fig. 1b) typically invokes erosion of soft-sediment beds in
61 low pressure subglacial channels (Boulton & Hindmarsh, 1987; Mooers, 1989; Praeg, 2003; Boulton
62 et al., 2009). In this model, high water pressures transmitted through the substrate to the ice-sheet
63 terminus initiates failure and headward erosion of a conduit (by piping) (Shoemaker, 1986; Boulton &
64 Hindmarsh, 1987; Hooke & Jennings, 2006; Boulton, 2009). As the fluid pressure of the conduit is
65 lower than the surrounding substrate, meltwater flows towards the conduit, the walls are enlarged by
66 sapping (i.e. undermining and headward recession of a scarp) and the sediments are mobilized and
67 transported away by the resulting subglacial stream (Boulton & Hindmarsh, 1987). In general,
68 enlargement is suggested to occur via steady-state Darcian flow of water into the conduit (e.g.



69 Boulton & Hindmarsh, 1987; Boulton et al., 2007a,b, 2009). Hooke & Jennings (2006) adapted this
70 hypothesis, suggesting that initial headward erosion by piping was followed by more rapid
71 enlargement when the conduit tapped into a subglacial lake, thereby combining both scenarios in
72 Figure 1. Ravier et al. (2014) emphasised the potential influence of localised high porewater pressures
73 in promoting efficient erosion by hydrofracturing and brecciation, while Mooers (1989) considered
74 supraglacial drainage to the bed rather than basal meltwater as the dominant source for gradual tunnel
75 valley erosion.

76 A range of approaches can be applied to the investigation of tunnel valleys including theoretical,
77 sedimentological and morphological. Thus far, most effort has used a combination of these
78 approaches, with much data, description and detail, but for a small number of tunnel valleys (see
79 Section 2). From these it is difficult to extract representative information of the population of tunnel
80 valleys or to gain an understanding of the broader-scale distribution of landforms. To rectify this we
81 undertake a systematic and large-scale mapping campaign of the size, shape, pattern and distribution
82 of tunnel valleys to better understand the spatial properties of this phenomenon, noting that it is useful
83 to know more precisely what it is that requires explanation (e.g. Dunlop & Clark, 2006, for ribbed
84 moraine). In doing so we will answer the following questions: (1) what constitutes a tunnel valley and
85 how can they be distinguished in the geological record? (2) What are the morphological
86 characteristics of a tunnel valley? (3) Is there a characteristic distribution and network arrangement?
87 (4) Are there systematic associations between tunnel valleys and other landforms? The southern sector
88 of the Laurentide Ice Sheet was selected because it contains thousands of these landforms, they can be
89 identified from digital elevation models (DEMs) and the distinctive geometry of the ice lobes
90 provides information on the water drainage pathways. Our mapping builds on and replicates, in many
91 places, comprehensive local and regional studies, which include sedimentological details that we draw
92 on. Our data provide basic metrics on tunnel valleys and their variation in scale and pattern and
93 should promote new insights into tunnel valley formation and meltwater drainage and erosion beneath
94 ice sheets.

95 *Limitations*

96 The partial or complete burial of tunnel valleys by glacial and post-glacial infill (cf. Kehew et al.,
97 2012; van der Vegt et al., 2012) limits the mapping and measurement of tunnel valleys from DEMs.
98 Buried tunnel valleys with no surface expression cannot be identified by the mapping. However,
99 buried valleys are rare or at least fewer have been identified along the southern margin of the
100 Laurentide Ice Sheet compared to the European ice sheets (e.g. Jørgensen & Sandersen, 2006;
101 Kristensen et al., 2007, 2008; Stewart & Lonergan, 2011). Moreover, unless there is a systematic bias
102 predisposing burial in some locations over others then the mapped pattern and distribution of tunnel
103 valleys is likely to be informative.



104 2. Previous work and observations in study area

105 There is a rich history of work on tunnel valleys beneath the southern margin of the former Laurentide
 106 Ice Sheet (Wright, 1973; Attig et al., 1989; Mooers, 1989; Patterson, 1994, 1997; Clayton et al., 1999;
 107 Johnson, 1999; Kehew et al., 1999, 2013; Cutler et al., 2002; Sjogren et al., 2002; Fisher et al., 2005;
 108 Kozłowski et al., 2005; Jennings, 2006; Hooke & Jennings, 2006; Kehew & Kozłowski, 2007). In this
 109 section we briefly summarise key observations arising from this work, which need to be incorporated
 110 into any model of tunnel valley formation.

111 Tunnel valleys are commonly observed on the bed of the former southern margin of the Laurentide
 112 Ice Sheet and typically occur as distinct radiating sets of regularly spaced valleys associated with
 113 eskers and terminal or recessional moraines (cf. Kehew et al., 2012). At the bed of the Saginaw Lobe,
 114 for instance, valleys are typically spaced at 6-10 km intervals (Fisher et al., 2005; Kehew et al., 2013).
 115 Tunnel valleys are incised into glacial sediments up to a depth of 25 m and extend for <50 km (e.g.
 116 Jennings, 2006). However, tunnel valleys up to 150 km long have been documented in the Superior
 117 Lobe, Minnesota (Wright, 1973), and valleys are eroded up to 200 m into the bedrock floors of Lake
 118 Superior and Lake Michigan (Regis, 2003; Jennings, 2006).

119 Although tunnel valleys are typically sub-parallel, they are also observed to join, split and even cross-
 120 cut each other (e.g. Wright, 1973; Mooers, 1989; Kehew et al., 1999, 2005; Fisher et al., 2005; Kehew
 121 & Kozłowski, 2007). Cross-cutting relationships, both between tunnel valleys and with other glacial
 122 landforms (e.g. drumlins, outwash fans, moraines), record a palimpsest signature of tunnel valley
 123 erosion. In the Saginaw Lobe, Kehew et al. (1999, 2005) and Kehew & Kozłowski (2007) identified a
 124 series of palimpsest associations in which partially buried tunnel valleys pass beneath terminal
 125 moraines, diamicton and surficial outwash associated with later advances. This palimpsest style is
 126 interpreted to result from the collapse of ice and debris into the valley, which becomes (partially)
 127 buried by sediment during a re-advance and then re-emerges as the ice melts out (e.g. Kehew &
 128 Kozłowski, 2007).

129 Tunnel valley morphology ranges from sharply-defined with constant or downstream increasing
 130 dimensions (e.g. Mooers, 1989), to indistinct valleys often associated with hummocky terrain and
 131 characterised by beaded or crenulated planforms, or as a series of aligned depressions (e.g. Kehew et
 132 al., 1999; Sjogren et al., 2002). Indistinct valleys may be due to partial burial during re-advance
 133 events or by melt out of debris rich ice obscuring them (Kehew et al., 1999). Sjogren et al. (2002) also
 134 identified indistinct valleys in Michigan that are eroded into the hummocky terrain.

135 In Wisconsin, Michigan and Minnesota, bands of hills are observed to occur upstream of tunnel
 136 valleys (Johnson, 1999). These are interpreted as erosional remnants of an anastomosing subglacial
 137 meltwater system that drained along the inter-hill valleys. At their downstream end, tunnel valleys



often terminate at outwash fans (e.g. Attig et al., 1989; Mooers, 1989; Patterson, 1994; Clayton et al., 1999; Johnson, 1999; Kehew et al., 1999; Derouin, 2008), some of which contain coarse boulder-gravel material, which is interpreted as evidence for outburst events (Cutler et al., 2002).

3. Methods

3.1 Datasets and mapping

For this study, we used the National Elevation Dataset (NED) (<http://nationalmap.gov/elevation.html>), which is a seamless DEM with a resolution of 1/3 arc seconds (~10 m) across the entire study area, and 1/9 arc seconds (~3 m) in some locations. Surficial and bedrock geology maps (e.g. Fullerton et al., 2003; Soller et al., 2011) were also used to aid identification and interpretation. Glacial landforms were identified according to conventional criteria and digitised directly into a Geographical Information System (GIS). Polygons were used to map tunnel valleys sides and centrelines, eskers and moraines. Polygons were used to map hill-hole pairs, outwash fans and dissected hills.

3.2 How do we distinguish tunnel valleys in the geological record?

Apart from tunnel valleys, large elongate depressions with similar dimensions may also form by fluvial erosion (river valleys), proglacial meltwater erosion (spillways), subglacial abrasion/plucking (overdeepenings), or arise from geological structures (e.g. fault lines). These phenomena are readily observed today and the formative mechanisms are reasonably well known. In contrast, tunnel valleys have not been observed actively forming beneath, or at the margins of, modern day ice sheets, and so their genesis and properties are more enigmatic. In the geological record they have been distinguished by their large size and characteristics such as their orientation parallel to inferred ice flow, undulating thalwegs and associations with subglacial bedforms and eskers; all pointing to a subglacial origin. In particular, undulating thalwegs and their association with eskers and outwash fans, permit them to be distinguished from proglacial and fluvial rivers. However, negative evidence (e.g. no esker found in a valley) does not necessarily preclude a subglacial origin, and it is not known whether size is actually a distinguishing feature or if, for instance, much smaller meltwater channels (tens of metres in width; e.g. Greenwood et al., 2007) are less mature forms of a continuum of glacial hydrological channels.

For the purposes of this study we restrict our definition of a tunnel valley to subglacially eroded channel-forms. Tunnel valleys that could clearly be differentiated as being eroded into bedrock were not mapped as their formation is more difficult to decipher from geological structures or glacial overdeepenings and valleys abraded and plucked by overlying ice. All potential tunnel valleys were mapped and then assessed to determine whether they formed subglacially. To determine whether valley thalwegs are undulating the number of negative and positive slope segments over 100 m length scale were calculated. Each valley was then assigned a confidence level from one to three, with one



171 being the most certain and three the least (Fig. 2). Channels lacking undulations and that do not
 172 contain subglacial bedforms are difficult to differentiate from proglacial or postglacial channel
 173 systems and were therefore given a confidence of 3. Valleys with an undulating long-profile, which
 174 contain eskers or terminate in outwash fans were classified as ‘certain’ tunnel valleys and given a
 175 confidence level of one (Fig. 2a-d). Only those tunnel valleys with a confidence level of one or two
 176 were used in the spatial and morphological analyses.

177 3.3 Tunnel valley measurements

178 Using the centrelines of tunnel valleys, we computed their length. Where two or more tributaries
 179 coalesce, the longest routeway was used to determine length. Tunnel valley width (distance between
 180 mapped valley sides) was measured from cross-profile transects positioned at 1 km intervals along the
 181 centreline of each tunnel valley. The influence of local elevation gradient (G_{loc}) on along valley
 182 changes in width (W_{loc}) was calculated at each 1 km interval (j) using equations [1] and [2]:

$$183 \quad G_{loc} = (E_{j+1} - E_{j-1}) / (D_{j+1} - D_{j-1}) \quad [1]$$

$$184 \quad W_{loc} = (W_{j-1} - W_{j+1}) / (D_{j+1} - D_{j-1}) \quad [2]$$

185 Where E = elevation, W = width and D = downstream distance. To calculate tunnel valley spacing it
 186 was necessary to restrict our analysis to networks comprising distinct populations of similar
 187 orientation, which were likely formed during a similar drainage phase. We calculated the spacing of
 188 966 tunnel valleys organised in 24 discrete networks. Spacing (S) was calculated from cross-profile
 189 transects orientated perpendicular to the direction of the network and positioned at 5 km intervals
 190 along each long profile. A median spacing value and the standard deviation (σ) was calculated for
 191 each drainage network. To provide an indication of tunnel valley regularity per network the
 192 coefficient of variation (σ / mean spacing), expressed as a percentage ($\sigma\%$), was also calculated
 193 (Hovius, 1996; Talling et al., 1997); tunnel valley networks with a low $\sigma\%$ exhibit low variability in
 194 spacing.

195 To investigate drainage evolution during deglaciation, a subset of meltwater features were grouped
 196 into ‘drainage-sets’, defined as a collection of features that formed during the same drainage phase.
 197 This was based on cross-cutting relationships (e.g. between channels, outwash fans and moraines) to
 198 reconstruct a relative history of drainage activity. Cross-cutting relationships between tunnel valleys
 199 and moraines were classified according to whether the tunnel valley: (1) terminates at a moraine at its
 200 downstream end and therefore formed contemporaneously with it; (2) is overlain by moraines along
 201 its length, thus suggesting that the tunnel valley was no longer active when the moraines were
 202 deposited; or (3) breaches moraines along its length, thereby indicating that the tunnel valley



203 continued to drain water, either destroying pre-existing or preventing moraines from forming during
 204 retreat.

205 **4. Properties of Tunnel Valleys**

206 *4.1 Is there a characteristic distribution and network arrangement?*

207 4.1.1 Distribution

208 Figure 3 shows the distribution of all 1931 tunnel valleys (1694 of which have a confidence of 1 or 2)
 209 mapped beneath the terrestrial southern sector of the Laurentide Sheet. We estimate that ~80% of
 210 these tunnel valleys have been previously identified and mapped during more localised investigations
 211 (e.g. Wright, 1973; Attig et al., 1989; Patterson, 1997; Fisher et al., 2005). The map reveals a
 212 tendency for tunnel valleys to cluster together in distinctive ‘networks’ with large intervening areas
 213 where no or very few valleys occur. Networks mostly avoid running down the central axes of major
 214 ice lobes. They are instead concentrated along suture zones between adjacent ice lobes or at the edge
 215 of linear to slightly lobate ice-margin positions. Tunnel valleys are rarer and more dispersed or
 216 isolated at the southernmost (LGM) margins of the James, Des Moines, Lake Michigan and Erie-
 217 Huron ice lobes (Fig. 3). Those that do occur in these ice lobes tend to be positioned up-ice, either at
 218 the lateral margins of the LGM lobes (e.g. Green Bay Ice Lobe) or at recessional moraines (e.g. Des
 219 Moines Ice Lobe).

220 Tunnel valley networks often occur down ice-flow of basins or sub-basins (Fig. 3). For example, the
 221 Saginaw Lobe tunnel valley network emanates from an arm of the present-day Lake Huron Basin, the
 222 Langlade and Chippewa tunnel valley networks are all associated with sub-basins of the present-day
 223 Lake Superior, and tunnel valleys occur downstream of the low-relief trough of the Des Moines Lobe.
 224 Based on modelled hydraulic potential surfaces, Livingstone et al. (2013) predicted that the Lake
 225 Superior Basin and NE sector of the Lake Michigan Basin were sites of several subglacial lakes
 226 during the last glacial (marked in Fig. 3). There appears to be no clear link between these lake and
 227 tunnel valleys. On the other hand, subglacial lakes may also have been present elsewhere in the Great
 228 Lake Basins and it is noteworthy that tunnel valleys are commonly downstream of these basins.

229 4.1.2 Network arrangement

230 The overall shape of tunnel valley networks varies (Fig. 3), with both broad networks composed of
 231 many short valleys (e.g. Green Bay, James and SE edge of Superior), and narrow networks composed
 232 of long valleys (e.g. Superior, Huron-Erie and Langlade). Cross-cutting of tunnel valleys occurs both
 233 between and within networks.



Overall tunnel valley spacing (Fig. 4) displays a positively skewed, unimodal distribution with a median spacing of 4.5 km and standard deviation of 4.6 km ($\sigma\% = 81$). However, the median spacing of individual tunnel valley networks ranges from 1.9 to 9.1 km. Tunnel valleys in the Green Bay (median: 2.9 km), Superior (median: 3.7 km) and Huron-Erie (median: 1.9 km) lobes are closely spaced. Conversely, tunnel valley networks in the large Saginaw (median: 5.7 km), Michigan (median: 5.5 km) and Des Moines (median: 5.4 km) lobes and in North Dakota (median: 5.1 km) have a wider than average spacing. In all of the measured networks the standard deviation of the tunnel valley spacings is less than the mean tunnel valley spacing, and 9 of the 24 networks are $<60\%$. There is no significant correlation between the number of tunnel valleys within a network (ranging from 7 to 169) and the standard deviation, but the standard deviation increases as the mean and median network spacing increases, hence the use of the coefficient of variation ($\sigma\%$).

4.2 What are the morphological characteristics of a tunnel valley?

The lengths of mapped tunnel valleys display a unimodal, positively skewed distribution, which is approximately log-normal (Fig. 5a). Lengths range from 200 m to 65 km, with a mode of 7-9 km, median of 6.4 km and standard deviation of 8 km. Long and short tunnel valleys occur in most places, although long valleys are less common in the Green Bay and Huron Erie lobes, and dominate in the Superior, Langlade, Wadena, Michigan and Saginaw lobes.

The widths of mapped tunnel valleys display a unimodal distribution with a positive skew, which approximates normal when log-transformed (Fig. 5b). Tunnel valley widths vary considerably across the study area, ranging from 15 m to 6.7 km, with a mode of 600-800 m, median of 550 m and standard deviation of 660 m. The Chippewa, Langlade and Michigan valleys are consistently wide (typically >600 m), while the Huron-Erie, Superior, Green Bay and Des Moines valleys are narrow (<600 m). Other networks, in the Saginaw, Superior and Wadena lobes, comprise a mix of wide and narrow valleys. There is a tendency for longer tunnel valleys to be wider (power law function, $r^2 = 0.38$, $p\text{-value} = <0.001$) (Fig. 6).

Tunnel valley planform shape varies across the study area (Fig. 7). The majority consist of a single valley 'thread'; more than two orders of 'stream ordering' are rare and tributaries tend to be restricted towards valley heads (Figs. 2, 3, 7). Valley margins range from sharp to indistinct and from crenulated to straight. Straight margins are more typical of long, thin tunnel valleys (Fig. 7a,d,f). However, many margins are crenulated, with bulbous and abrupt angular morphologies that result in large down-valley changes in width (Fig. 7a-f). Figure 8 demonstrates a weak relationship between tunnel valley width and distance downstream. Valleys both widen and narrow downstream with considerable and abrupt variations in width. The variation in tunnel valley width bears no relation to the local elevation gradient (Fig. 9). Local along-valley elevation gradients are relatively low (typically $<\pm 1.5^\circ$) and valleys widen and narrow on both reverse and normal slopes.



269 Tunnel valleys and tunnel valley segments often start and end abruptly and can appear fragmented or
 270 contain bulbous depressions (Fig. 7). The gaps between segments of tunnel valleys may show no
 271 evidence of modification (Fig. 7e,f); are partially incised by narrower and more discontinuous valleys
 272 or sets of parallel valleys (Fig. 7e); or consist of a series of depressions and hummocks with indistinct
 273 valley planform (Fig. 7a,b,d). The up-glacier ends of tunnel valleys range from rounded heads with
 274 steep sides (amphitheatre) (Fig. 7a,f) to open or indistinct (Fig. 2c-d). In Figure 7e-f, tunnel valleys
 275 comprise parallel tracks of two or more tightly spaced (<1 km) valleys.

276 4.3 *Are their systematic associations between tunnel valleys and other landforms?*

277 4.3.1 Moraines

278 The association between moraines and tunnel valleys varies with some valleys cutting through
 279 moraines (Fig. 10a); while in other locations moraines are superimposed on the valley or the valley
 280 terminates at a moraine (Fig. 10b). In Figure 10a, tunnel valleys cutting through an end moraine are
 281 observed to narrow and then trend down-glacier into esker and outwash fan deposits. Up-glacier of
 282 the end moraine are low relief (1-2 m) and regularly spaced transverse ridges ('washboard' moraine).
 283 They have a cusped geometry with the horns pointing up-glacier and converging on tunnel valley
 284 positions (see also Stewart et al., 1988; Cline et al., 2015). Fig. 10b shows examples of tunnel valleys
 285 terminating at, cutting through and overlain by recessional moraine. The tunnel valley network does
 286 not show a consistent pattern, with neighbouring channels exhibiting different moraine associations.
 287 Some valleys are continuous or semi-continuous, with a single outwash fan at, or just down-glacier
 288 from the terminus, and a series of on-lapping recessional moraines up-glacier. Elsewhere, valleys
 289 contain multiple outwash fans deposited at successive moraine positions.

290 4.3.2 Hill-hole pairs

291 We mapped 12 hill-hole-pairs (Bluemle and Clayton, 1984), 11 of which are found in North Dakota.
 292 Typically, hill-hole pairs comprise isolated features, but 4 of them are associated with tunnel valleys
 293 (e.g. Figs. 2c, 11). These seem to occur at the down-glacier end of the valleys, with smaller channels
 294 and eskers emanating from and diverging around the ice-thrust hill (Fig. 11a,b). In Fig. 11a, an esker
 295 emanating from one of the hill-hole pairs trends into another tunnel valley segment further down-
 296 glacier.

297 4.3.3 Outwash Fans

298 We mapped 187 outwash fans across the study area, predominantly at the downstream end of, but also
 299 within and between segments of tunnel valleys at moraine positions (Fig. 10b). Many of the outwash
 300 fans are connected upstream to an esker. Multiple trains of outwash fans occur along some tunnel
 301 valleys, but not all tunnel valleys are associated with outwash fans.



302 4.3.4 Giant Current Ripples

303 In Minnesota the floor of one tunnel valley is shown to contain regularly spaced sinusoidal bedforms
 304 orientated roughly perpendicular to the valley long profile (Fig. 12). The bedforms are 0.2-1.9 m high
 305 (H), 10-60 m long (L) and their crests are straight to slightly sinuous. Our data show that longer
 306 bedforms tend to be higher (linear regression, $r^2 = 0.5$), and that the H:L ratio is ~ 0.02 . The tunnel
 307 valley that the bedforms are constrained within is partially incised into underlying drumlins orientated
 308 obliquely to the valley long axis. An esker running NW-SE is overprinted on the bedforms. The
 309 southern end of the valley is bisected by a large (1 km diameter) circular incision with an intact
 310 central island.

311 The dimensions and shape of the transverse sinusoidal bedforms, the tendency for longer bedforms to
 312 be higher and their association with the tunnel valley is consistent with giant current ripples (e.g.
 313 Bretz et al., 1956; Carling, 1996; Rudoy, 2005). Given the undulating valley thalweg and
 314 superimposition of an esker on top of the ripple-forms, we suggest the simplest explanation is that the
 315 valley, circular incision and ripples were formed subglacially.

316 5. Discussion

317 5.1 *Distribution and pattern of tunnel valleys*

318 5.1.1 Southern sector of the former Laurentide Ice Sheet

319 The large-scale distribution of tunnel valleys is strongly controlled by ice geometry. Tunnel valleys
 320 are rare or absent at the terminus of major ice lobes, particularly those that are long and thin (e.g.
 321 James and Des Moines lobes), and are more common in interlobate regions, at the side of lobes or
 322 where the lobe exhibits a broader geometry (Fig. 3). This is consistent with theoretical drainage of
 323 meltwater beneath an ice lobe, which is strongly controlled by the ice-surface slope (e.g. Shoemaker,
 324 1999). Meltwater is theorised to radiate out from the centre of lobes, and converge along interlobate
 325 regions where the subglacial hydraulic gradient and ice surface are relatively steep (Fig. 13). Indeed,
 326 tunnel valley networks associated with lobate margins often have a distinctive divergent geometry
 327 (Fig. 3).

328 The locations of ice lobes along the southern sector of the Laurentide Ice Sheet are topographically
 329 controlled and are inferred to have been fast-flowing (e.g. Mickelson and Colgan, 2003; Margold et
 330 al., 2015). Fast ice-flow is likely to have been promoted by thermomechanical feedbacks, enhancing
 331 basal meltwater production and lubricating the bed (cf. Winsborrow et al., 2010 and references
 332 therein). It is therefore no surprise that tunnel valleys are typically found down-glacier of basins,
 333 where the greatest volumes of basal meltwater were focused. However, there is no clear link to
 334 predicted subglacial lake locations or with their obvious drainage corridors except for the Langlade



335 Lobe tunnel valley network (Fig. 3) (Livingstone et al., 2013). This suggests that the drainage of
 336 subglacially stored water was not the main control on tunnel valley formation, or that we have yet to
 337 discover the true extent of subglacial lakes (i.e. the prediction in Fig. 3 is an underestimate). For
 338 example, the predictions do not account for the possibility of water ponding behind frozen margins as
 339 suggested by Cutler et al., (2002) and Hooke & Jennings, (2006).

340 Measurements of tunnel valley spacing reveal an overall median spacing of 4.5 km with some degree
 341 of intra-network regularity (Fig. 4). Inter-network variation is greater, with median network values
 342 ranging from 1.9 to 9.1 km across the study area. The spacing metrics are within the range of
 343 previously reported values for tunnel valleys (Praeg, 2003; Jørgensen and Sandersen, 2009;
 344 Stackebrandt, 2009; Moreau et al., 2012; Kehew et al., 2013) but smaller than the average spacing of
 345 eskers (Storarr et al., 2014a, and references therein). Theory suggests that the spacing of subglacial
 346 conduits is controlled by substrate properties, basal melt rate and the hydraulic potential gradient (e.g.
 347 Boulton et al., 2007a,b, 2009; Hewitt, 2011). According to such theory the spacing between adjacent
 348 tunnel valleys should be wider if: (i) bed transmissivity is larger; (ii) melt rate/discharge is lower;
 349 and/or (iii) the subglacial hydraulic gradient is smaller. Thus the wider than average spacing towards
 350 the terminus of major ice lobes where ice surface slopes and thus hydraulic gradients are inferred to
 351 be shallower (e.g. Des Moines and Saginaw lobes – Clark, 1992), and a smaller spacing along narrow
 352 ice lobes characterised by steeper ice-surface and hydraulic gradients (e.g. Green Bay and Superior
 353 lobes – Clark, 1992) is consistent with theory. However, cross-cutting relationships indicate that not
 354 all tunnel valleys were acting synchronously, even within a drainage network (Fig. 10b), which might
 355 explain the large variations in spacing.

356 5.1.2 Geographical distribution of tunnel valleys during the last glaciation

357 Figure 14 displays the geographical distribution of tunnel valleys reported in the northern hemisphere
 358 and attributed to the last glaciation. It appears that tunnel valleys tend to be associated with the flat
 359 southern margins of terrestrial or formerly terrestrial (e.g. North Sea) palaeo-ice sheets. They also
 360 tend to occur towards the maximum limit of glaciation and are often found downstream of large
 361 basins such as the Witch Ground in the North Sea, Baltic Depression along the southern limit of the
 362 European Ice Sheet, and Great Lake basins along the southern limit of the Laurentide Ice Sheet.

363 The tendency for tunnel valleys to form on beds of low relief and gradient implies a genetic
 364 association. In particular, water flow in regions of low bed relief is largely unconstrained by
 365 topography and can therefore more easily erode laterally producing wide, shallow valley geometries.
 366 Conversely, more rugged terrain will exert a greater control on water flow, increasing network
 367 complexity and restricting valley expansion. A consequence of ice lobes along the southern margin of
 368 the Laurentide Ice Sheet having such shallow ice-surface slopes (reconstructed as 0.001 to 0.005



369 m/km Wright, 1973; Mathews, 1974; Clark, 1992), is the resulting low subglacial hydraulic gradients.
 370 Such low gradients are at odds with the development of many closely spaced large channels (Hewitt,
 371 2011). This could indicate either that: (i) large discharges of subglacial meltwater were needed to
 372 form the tunnel valleys; or (ii) tunnel valleys and their spacing were determined by initial conditions
 373 set up near the ice margin (i.e. where ice-surface slopes are steepest and the greatest volumes of
 374 meltwater are discharged). Certainly, shallow ice-surface slopes would have extended the size of the
 375 ablation zone and made it more sensitive to small changes in summer air temperature, while
 376 hydrofracture of surface meltwater to the bed is easier where ice is thin.

377 The prevalence of tunnel valleys along terrestrial margins hints at an important role of permafrost in
 378 their formation (e.g. Wright, 1973; Piotrowski, 1994, 1997; Cutler et al., 2002; Jørgensen &
 379 Sandersen, 2006). It has been proposed that the development of a frozen toe along the fringe of an ice
 380 sheet acted as a barrier to water flow facilitating tunnel valley formation by subglacial ponding and
 381 outburst cycles (e.g. Wingfield, 1990; Piotrowski, 1994, 1997; Cutler et al., 2002). Moreover, freezing
 382 of sediment deposited in channels under the thin fringe of the ice sheet during winter months may
 383 have helped to prevent creep-closure of incipient tunnel valleys, thereby stabilizing and preserving
 384 their forms from year to year.

385 The occurrence of tunnel valleys near the LGM limit could indicate larger subglacial meltwater fluxes
 386 concomitant with greater catchment areas, a climatic control and or variations in basal conditions.
 387 Conversely, the paucity of tunnel valleys towards the centre of former ice sheets suggests formation is
 388 not linked to greater volumes of supraglacial meltwater production concomitant with climatic
 389 warming, although this may be partially counteracted by reduced erosion on the hard crystalline
 390 bedrock towards the centre of the Northern Hemisphere palaeo-ice sheets (Clark and Walder, 1994).
 391 Critically, the northern hemisphere Quaternary ice sheets were vastly different sizes, so it seems
 392 unlikely that tunnel valley distribution was a function of subglacial hydrological catchment size and
 393 meltwater flux, particular as the hydrological budget is likely to be dominated by supraglacial
 394 meltwater inputs during deglaciation. The width of the frozen toe is likely to decrease during retreat
 395 because adjustment of the thermal structure of the toe will lag considerably behind adjustment of the
 396 margin position to an ameliorating climate. Decrease in tunnel valley occurrence away from the
 397 maximum ice limit may therefore be indicative of a change to temperate glacier conditions.

398 5.2 Morphology of tunnel valleys

399 The tunnel valleys extend for up to 55 km, although the majority (90%) are <17 km long and the
 400 median is 6.4 km (Fig. 5a). In comparison, reported tunnel valley lengths from the North Sea range
 401 from a few kilometres to around 100 km, with the length of individual segments not normally
 402 exceeding 20-30 km (e.g. Huuse and Lykke-Andersen, 2000). Although very wide tunnel valleys were



403 found (maximum width ~6.7 km), the majority (90%) are 500-3000 m (Fig. 5b). This is similar to
 404 tunnel valley widths (500-5000 m) reported in Europe and elsewhere in North America (e.g.
 405 Brennand and Shaw, 1994; Huuse and Lykke-Andersen, 2000; Jørgensen and Sandersen, 2006;
 406 Kristensen et al., 2007).

407 Tunnel valley length and width display log-normal distributions (Fig. 5), which is common of other
 408 glacial landforms (Fowler et al., 2013; Hillier et al., 2013; Spagnolo et al., 2014; Storrar et al., 2014).
 409 Log-normal distributions are thought to typically emerge from many independent random events in
 410 which incremental growth or fragmentation occurs (e.g. Limpert et al., 2001). For drumlins and
 411 MSGs a log-normal distribution has been used to suggest a growing phenomenon that occurs
 412 randomly, for random durations, or under random conditions (Hillier et al., 2013; Spagnolo et al.,
 413 2014), while for eskers it is thought to reflect ridge fragmentation (Storrar et al., 2014a). Examples of
 414 aligned tunnel valley segments characterised by abrupt start and end points implies at least some
 415 tunnel valley fragmentation, and this may occur due to partial burial during re-advance events or the
 416 melt out of debris-rich ice (Kehew et al., 1999), or differential erosion along the length of a drainage
 417 route (Fig. 7a,e,f; see also Sjørgen et al., 2002). However, in other cases aligned tunnel valley
 418 segments could indicate a time-transgressive origin (e.g. Mooers, 1989; Patterson, 1994; Jørgensen
 419 and Sandersen, 2006; Janszen et al., 2012). This is particularly apparent where the valley segments
 420 terminate in outwash fans, and/or where segments cross-cut each other (Fig. 10 and see also Mooers,
 421 1989). The positive relationship between tunnel valley length and width (Fig. 6) is consistent with a
 422 growing phenomenon (e.g. by headward expansion) or continuous flow (e.g. a river). In contrast, the
 423 length and width of valleys formed by floods are likely to be independent of each other; length is
 424 related to the distance that the stored water body is from the ice margin, while width is a function of
 425 the magnitude and/or frequency of drainage.

426 In fluvial geomorphology, channel width in an equilibrium system increases downstream (Fig. 7f) and
 427 has classically been related to discharge, and hence drainage area (Leopold and Maddock, 1953;
 428 Leopold et al., 1964). This may be complicated locally by the erodibility of the bed substrate and
 429 channel slope (e.g. Finnegan et al., 2005). In contrast, large single source flood events (as may occur
 430 during a subglacial or supraglacial lake drainage event), will produce a relatively constant channel
 431 width (e.g. Lamb and Fonstad, 2010), or even show a downstream decrease if infiltration is significant
 432 (Fig. 7f). The downstream width of tunnel valleys in our dataset varies considerably and there is no
 433 systematic downstream trend in valley form, although general increases and decreases in width do
 434 occur (Figs. 6, 7a-e). Thus, there is no observable signature of catastrophic (constant or declining
 435 width) or stable, bankfull drainage (steady widening). Moreover, the downstream variation in widths
 436 is also inconsistent with subglacial drainage channels fed by multiple supraglacial lake inputs (e.g.
 437 Palmer et al., 2011), which we would expect to produce a downstream increase in width concomitant
 438 with increased water added.



Figure 7a-e indicates that local variations in tunnel valley width are generally more pronounced than any downstream trend. These widening's could arise from basal conditions at the time of formation (e.g. thermal regime), catastrophic drainage (e.g. Sjorgen et al., 2002), or a laterally migrating stream at the base of the valley floor. Laterally migrating streams are unlikely as we do not observe terraces, bars or incised braided or meandering channels within the broader tunnel valleys, although this may partially be due to ice and post-glacial modification. The crenulated margins, circular incisions, residual hills, hummocky terrain and valley discontinuities are all analogous to features eroded during large floods by macroturbulent flow (e.g. Sjorgen et al., 2002), although these are typically associated with bedrock channels (Baker, 2009 and references therein). Moreover, we see little evidence for other characteristic features, such as irregular anabranching channels (although they are observed elsewhere, e.g. Boyd, 1988; Brennand and Shaw, 1994), inner channels, furrows and large bars (e.g. Channelled Scablands: Bretz, 1923), while residual hills are not typically streamlined.

The alternative to the catastrophic hypothesis is that variations in width are strongly controlled by local basal and hydrological conditions. Indeed, there is greater similarity between tunnel valleys from the same network (e.g. in form, size and association with other landforms) compared to tunnels valleys from different networks, which hints at the importance of local conditions. Although there is no clear association with bed slope (Fig. 9) or geology, the strength and therefore stability of tunnel valleys sides would have been strongly modulated by variations in basal thermal regime, substrate properties and water flow during glaciation. Using this idea, we propose three theories that could produce these variations in width, and which we hope will motivate physical modelling studies (Fig. 15). Firstly, the variations in tunnel valley width may be a consequence of the very flat beds on which they form (Fig. 14). Water flow in such a landscape will be very sensitive to small changes in bed relief and variations in discharge. Coupled with sluggish water flow due to the low hydraulic gradients, we therefore envisage the tunnel valleys as a series of interconnected swampy regions (Fig. 15a). This is analogous to lakes and or swampy ground connected by overspill channels, or wide flood plains comprising dynamic river channels observed in fluvial systems flowing across similarly flat landscapes. Secondly, a basal thermal regime consisting of a mosaic of cold- and warm-based sediment patches (e.g. Kleman & Glasser, 2007) would locally influence how easily widening could happen (Fig. 15b). Frozen patches would inhibit channel formation and may even result in ponding of meltwater, while warm based patches would be more susceptible to erosion. Thirdly, as discharge increases the conduit can enlarge, either by eroding into the bed (forming tunnel valleys), melting up into the ice (R-channel) or both together (see Fowler, 2011) (Fig. 15c). What happens will vary depending upon, for example, the effective pressure, ice viscosity and sediment stiffness. Consequently, the manifestation of an increase in discharge on the bed imprint is likely to vary spatially and temporally depending on the competition between sediment erosion and the melting of



ice (e.g. Livingstone et al., 2016). This theory may therefore explain the fragmentation of some tunnel valleys into multiple segments (Fig. 7e,f).

5.3 *Landform associations*

5.3.1 Relative timing of tunnel valley formation

Cross-cutting relationships between moraines, outwash fans, and tunnel valleys have enabled their relative timing of formation to be used to build a history of formation (Figs. 10, 16). If a tunnel valley cuts through moraine positions, formation must have occurred during or after the moraine was deposited. These tunnel valleys, and those interrupted by outwash fans mid-way along their length, must therefore have been used as a drainage route either repeatedly or over a long duration during retreat (see Fig. 16b). Conversely, tunnel valleys that are cross-cut by recessional moraines were abandoned as ice retreated. In Fig. 16b these tunnel valleys correspond to the age of a single moraine position, and may have been eroded during a singular ‘event’ (i.e. outburst of a sub- or supra-glacial lake) or been abandoned due to a switch in drainage configuration or supply.

5.3.2 Moraines

The close link between tunnel valley networks and moraines (Figs. 3, 10; and see also Attig et al., 1989; Mooers, 1989; Patterson, 1997; Smed, 1988; Johnson, 1999; Cutler et al., 2002; Jørgensen and Sandersen, 2006) suggests formation and growth is intimately associated with pauses or slow-downs in ice retreat or ice advances and that meltwater drained to the ice margin. The implication is that tunnel valley formation requires a relatively stable ice-sheet configuration to allow headward growth or recharge of source storage areas. It also provides further support for the role of permafrost in tunnel valley formation given that rapid retreat will reduce the width of the frozen toe and consequently reduce the efficacy for water storage. However, whether a reconfiguration of the subglacial hydrological regime via the development of tunnel valleys behind ice margins (moraines) can influence ice retreat, for example causing the observed staccato jumps between still-stands (Fig. 16a), remains an open question.

Regularly spaced, low relief transverse ridges (e.g. Fig. 10b), termed washboard or corrugation moraine, have been interpreted as both (annual) end moraine deposits and as subglacial crevasse fill (Kemmis et al., 1981; Stewart et al., 1988; Patterson, 1997; Jennings, 2006; Cline et al., 2015; Ankersjerne et al., 2015). The deflection of transverse ridges towards the long axis of tunnel valleys (e.g. Fig. 10b), and buried sand and gravel deposits (see Stewart et al., 1988; Cline et al., 2015), indicates a temporal and possibly genetic relationship. One interpretation is that lower water and pore water pressures in tunnel valley and glaciofluvial deposits respectively, result in slower local ice velocities that cause the pattern of crevasses and thus ridges to be deflected (see Cline et al., 2015).



507 However, high pressure discharges have also been inferred from coarse-grained outwash fans
 508 deposited in front of tunnel valleys (Section 5.3.4, e.g. Cutler et al., 2002; Jørgensen and Sandersen,
 509 2006). There may therefore have been multiple modes of meltwater drainage down tunnel valleys;
 510 predominantly low pressure drainage interrupted by episodic high pressure outbursts.

511 5.3.3 Hill-hole pairs

512 The formation of tunnel valleys up-glacier from hill-hole-pairs of similar width (Fig. 12) suggests a
 513 temporal relationship. Hill-hole-pair formation is believed to require the ice to be strongly coupled to
 514 the bed so that it can exert sufficient shear stress to produce failure (Bluemle and Clayton, 1984; Aber
 515 et al., 1989). Thus, either the hill-hole pair was produced first, and the tunnel valley grew headward
 516 out of the ‘hole’, or once drainage through the tunnel valley had waned, ice re-coupled strongly to the
 517 bed and the downstream termination of the valley became the focus of large shear stresses that
 518 resulted in failure and formation of the hill-hole pair. We suggest the former is more likely as the
 519 tunnel valleys do not terminate at moraine positions as is typical elsewhere, while small channels and
 520 eskers emanating from and diverging around the hills appear to record the down-glacier leakage of
 521 pressurised water around the obstruction (Fig. 12b). If true, these tunnel valleys appear to be unique in
 522 having initiated up-glacier from the margin. The formation of a hill-hole pair may therefore have
 523 facilitated tunnel valley erosion by providing a pathway for water through a frozen toe.

524 5.3.4 Outwash Fans

525 Outwash fans occur at the down-glacier end of at least 10% of the tunnel valleys in our study area
 526 (e.g. Fig. 10b), and are particularly common along the margins of the Green Bay, Michigan and
 527 Langlade lobes (Attig et al., 1989; Clayton et al., 1999; Cutler et al., 2002; Fisher and Taylor, 2002).
 528 The fan sediments at the margin of the Green Bay Lobe include well-rounded pebbles and boulders up
 529 to 2 m diameter (Cutler et al., 2002), similar to accumulations documented in-front of European
 530 tunnel valleys (Piotrowski, 1994; Jørgensen and Sandersen, 2006; Lesemann et al., 2014). The coarse-
 531 grained sediments indicate high-energy discharges and or highly pressured subglacial meltwater flow
 532 through the tunnel valleys. Cutler et al. (2002) suggested there was at least one large outburst flood
 533 just before the termination of glaciofluvial activity through each tunnel valley. These high-energy
 534 floods may have been responsible for cutting the valley itself, or the valley could have acted as a
 535 preferential drainage route upon tapping into a water reservoir.

536 5.3.5 Giant Current Ripples

537 The occurrence of giant current ripples stretching across the whole width of a tunnel valley implies a
 538 large sub- or supra-glacial lake outburst event (e.g. Bretz et al., 1956; Carling, 1996; Rudoy, 2005).
 539 This is further supported by the circular incision at the southern end of the tunnel valley (Fig. 13),



which is similar in form to large potholes generated by macroturbulent eddies. The flood could have cut this particular tunnel valley or the valley pre-existed and became the route of a subglacial flood which completely filled it, further modifying and enlarging the valley (Bretz et al., 1956; Carling, 1996; Rudoy, 2005). The unique occurrence of this landform suggests that large floods were rare or the landform signature rarely preserved.

5.4 Implications for the formation of tunnel valleys

Based on our large-scale analysis of the morphological properties of tunnel valleys and associated bedform along the southern portion of the Laurentide Ice Sheet we are able to provide some new insights into their formation. The importance of ice geometry (Fig. 3) and the semi-regular spacing of individual tunnel valley networks (Fig. 4), implies a stable, self-organising basal hydrological system modulated by bed transmissivity, meltwater discharge and the hydraulic potential gradient (Piotrowski, 1997; Boulton et al., 2007a,b, 2009; Hewitt, 2011). While some tunnel valleys appear to have been short-lived, either as the preserved signature of a single event or because they were abandoned due to changes in melt delivery or ice retreat, it is inconceivable that an entire network pattern was formed during one catastrophic flood (e.g. Shaw, 2002) as many of the valleys are found to have formed incrementally (also see Mooers, 1989), remaining active and relatively stable over a long period of time (Fig. 16).

Recurrent outburst of stored water responsible for incremental incision of whole networks is appealing where tunnel valleys converge towards up-glacier basins (e.g. Superior and Langlade – Figs. 2a,c, 3) where one could infer that subglacial lakes periodically grew and drained (Evatt et al., 2006). However, many of the networks are very broad (>60 km across) and the tunnel valleys relatively parallel (e.g. Green Bay and eastern Superior – Fig. 3). To produce these networks would require lakes many tens or even hundreds of kms wide. This is difficult to reconcile with mean (<1km²) and maximum supraglacial lake areas (up to ~150 km² – which equates to a diameter of ~14 km if a circular lake is assumed) on the surface of the present-day Greenland Ice Sheet (e.g. Leeson et al., 2013). Moreover, while very large subglacial lakes do exist beneath the Antarctic Ice Sheet (Wright and Siegert, 2011, e.g. Lake Vostok, >250 km long by ~80 km wide) and are theorised to have existed in Hudson Bay and the Great Lake Basins (e.g. Shoemaker, 1991, 1999), they have neither been predicted by modelling or identified in the geological record (e.g. Livingstone et al., 2013).

Despite the lack of support for a mega-flood genesis of whole tunnel valley networks, drainage of stored water down individual valleys almost certainly did happen. Not all tunnel valleys formed in networks or were incised time-transgressively up-glacier (Figs. 3, 16), and the simplest explanation for the formation of fans containing boulders (Figs. 2, 10b) (e.g. Piotrowski, 1994; Cutler et al., 2002; Lesemann et al., 2014) and for giant current-ripples (Fig. 12) is high discharge (possibly bank-full)



575 events. Indeed, periodic higher energy or pressurised meltwater events (e.g. during penetration of
576 surface meltwater to the bed during summer months) were probably necessary to prevent armouring
577 of the valley sides by coarse sediment, while bedrock tunnel valleys are difficult to reconcile solely by
578 gradual formation. We therefore contend that large drainage events from sub- and supra-glacial lakes,
579 and by injections of surface meltwater down moulins did occur, contributing to the formation of
580 tunnel valleys either by eroding new valleys or enlarging existing ones. However, our data suggests
581 they were probably not the primary mechanism by which tunnel valleys formed. Firstly, the decline in
582 tunnel valley incidence away from LGM margin positions (Fig. 14) is inconsistent with increasing
583 contributions of surface melt in an ameliorating climate. Secondly, their typical length distribution
584 (Fig. 5a) is an order of magnitude less than the distance up-glacier (tens to hundreds of km) that
585 supraglacial and subglacial lakes are commonly documented in Greenland and Antarctica (e.g. Selmes
586 et al., 2011; Wright and Siegert, 2011).

587 We suggest that the majority of tunnel valleys along the southern sector of the Laurentide Ice Sheet
588 were initiated at the ice margin and then typically (although not exclusively) eroded gradually up-
589 glacier. Tunnel valley length and width display log-normal distributions and are positively correlated,
590 indicative of a growing phenomenon (cf. Fowler et al., 2013; Hillier et al., 2013). Their strong
591 association with moraine positions (Fig. 3) suggests that formation is time dependent (i.e. they require
592 time to grow), while cross-cutting relationships (Fig. 16) demonstrates that many of the features
593 remained active for extended periods. Thus, when retreat is slow or a stable position is reached
594 (allowing formation of a moraine), tunnel valleys have time to grow up-glacier and to widen and
595 deepen as more water is discharged through them (Fig. 17a). A more unstable/rapid ice-retreat will
596 limit the time for growth (headward and lateral) or may even produce a segmented tunnel valley if
597 retreat overtakes headwards incision (Fig. 17b). Indeed, the James and Des Moines ice lobes that are
598 thought to have rapidly surged to and then retreated from their maximum positions (Clayton and
599 Moran, 1982; Clayton et al., 1985) are relatively devoid of well-organised tunnel valley networks
600 compared to other ice lobes, such as Superior, that retreated more slowly (Dyke, 2004). We argue that
601 growth was not a function of conditions associated with the size of a stored water body and the
602 magnitude and frequency of its drainage because immature (smaller) tunnel valleys are also found to
603 preferentially terminate at ice-margin positions (e.g. southern margin of Green Bay Lobe, Fig. 3).
604 Hence, growth likely initiated and proceeded up-glacier from the ice margin rather than down-glacier
605 from a stored water body, and there is some evidence for this, including the presence of amphitheatre-
606 shaped tunnel valley heads (e.g. Onda, 1994; Abrams et al., 2009; Petroff, 2011) and the growth of
607 valleys out of hill-hole-pairs (Fig. 11).

608 6. Summary and Conclusions



609 There have traditionally been two main paradigms to explain the formation of tunnel valleys: (1)
 610 outburst formation by rapid drainage of sub- and/or supraglacially stored meltwater; and (2) gradual
 611 formation by headward sapping in low pressure subglacial channels (Fig. 1) (cf. Ó Cofaigh, 1996;
 612 Kehew et al., 2012; van der Vegt et al., 2012). To investigate these two models we undertook a large-
 613 scale mapping campaign to characterise the distribution and morphology of >1900 tunnel valleys and
 614 associated bedforms on the bed of the former Laurentide Ice Sheet.

615 Given our previous work on subglacial lakes beneath the Laurentide Ice Sheet (Livingstone et al.,
 616 2013), we specifically explored tunnel valleys with an expectation that they might link with predicted
 617 lake locations and be the geomorphological record of outburst floods. However, to the contrary the
 618 morphological evidence suggests gradual formation, with some contributions from large drainages of
 619 stored water (Fig. 18). In particular, our findings indicate that tunnel valleys comprise well-organised
 620 networks of semi-regularly spaced (1.9-9.1 km) valleys that formed incrementally during ice retreat.
 621 This pattern is strongly controlled by ice geometry and basal properties (e.g. permafrost, flat bed and
 622 conduit erosion), and this is a strong argument for a self-organising hydrological network influenced
 623 by local conditions. Second, tunnel valleys preferentially terminate at moraines (irrespective of their
 624 size), which suggests that growth was initiated at and then progressed headwards from stable ice-
 625 margin positions. The concept of a growing phenomenon is further supported by log-normally
 626 distributed valley morphologies, the positive correlation between length and width, their initiation and
 627 growth out of hill-hole-pairs and the existence of amphitheatre-shaped valley heads. Although we
 628 favour gradual headward formation as the primary process, our results also show examples where
 629 outburst of supraglacial and or subglacial lakes have incised and/or drained down valleys. Evidence
 630 includes, giant current ripples and outwash fans with large boulders (Cutler et al., 2002), and some
 631 valleys were only occupied for brief periods during deglaciation suggestive perhaps of a short-lived
 632 event. Indeed, cross-cutting relationships demonstrate a time-transgressive origin for many tunnel
 633 valleys, with individual networks forming within the same time frame but individual valleys evolving
 634 over different spans involving multiple discrete flow events.

635 Many of our observations are consistent with previous findings (e.g. Kehew et al., 2012 and
 636 references therein) and we are not the first to suggest a polygenetic origin (e.g. Hooke and Jennings,
 637 2006). However, whilst geomorphological and sedimentological investigations in certain areas have
 638 generally advocated *either* an outburst or gradual genesis for tunnel valleys (Fig. 1), when their
 639 morphology, distribution and association with other glacial bedforms are considered at a regional-
 640 scale it suggests that both processes occurred (Fig. 18).

641 At the ice-sheet scale, we find most tunnel valleys occur on the flat portions of palaeo-ice sheet beds,
 642 where subglacial water flow would have been largely unconstrained by topography. It is on these
 643 portions of the bed, where ice-geometry is the main control, that subglacial water becomes organised



644 down relatively stable and regularly-spaced drainage corridors (tunnel valleys). Once a tunnel valley
 645 has been initiated, it could provide a low pressure ‘release valve’ (i.e. generate a local hydraulic
 646 gradient) to evacuate basal water flowing slowly through water saturated sediments and swampy
 647 ground (after Kyrke-Smith and Fowler, 2014) in areas of the bed characterised by low hydraulic
 648 gradients, and also as a routeway for large injections of surface or stored water. These drainage
 649 corridors provide an effective means of transporting sediment and water from under the ice sheet and
 650 may thus have acted to increase basal traction across the bed and slow-down ice flow during
 651 deglaciation.

652 **Author contributions**

653 SJL and CDC designed the project. SJL generated the data on the tunnel valleys and other glacial
 654 bedforms. Both authors contributed to the analyses and interpretations of the data. SJL wrote the
 655 manuscript with input from CDC.

656 **Acknowledgments**

657 This work was supported by a NERC Early Career Research Fellowship awarded to SJL
 658 (NE/H015256/1). Underlying data are available by request to Livingstone. We are grateful to Andrew
 659 Sole for help writing the MATLAB algorithms, and Jeremy Ely for contributing shapefiles of
 660 mapped drumlins and eskers. We also thank Roger Hooke for helpful comments on an earlier version
 661 of the paper.

662 **References**

- 663 Aber, J.S., Croot, D.G., Fenton, M.M., 1989. Hill-Hole Pair, in: Glaciotectonic Landforms and
 664 Structures, Glaciology and Quaternary Geology. Springer Netherlands, pp. 13–28.
- 665 Abrams, D.M., Lobkovsky, A.E., Petroff, A.P., Straub, K.M., McElroy, B., Mohrig, D.C., Kudrolli,
 666 A., Rothman, D.H., 2009. Growth laws for channel networks incised by groundwater flow. *Nat.*
 667 *Geosci.* 2, 193–196.
- 668 Ankerstjerne, S., Iverson, N.R., Lacroix, F. 2015. Origin of washboard moraine of the Des Moines
 669 Lobe inferred from sediment properties. *Geomorphology*, 248, 452–463.
- 670 Attig, J.W., Mickelson, D.M., Clayton, L., 1989. Late Wisconsin landform distribution and glacier-
 671 bed conditions in Wisconsin. *Sediment. Geol.* 62, 399–405.
- 672 Baker, V.R., 1979. Erosional processes in channelized water flows on Mars. *J. Geophys. Res.* 84,
 673 7985–7993.
- 674 Boulton, G.S., Hindmarsh, R.C. a, 1987. Sediment deformation beneath glaciers: Rheology and
 675 geological consequences. *J. Geophys. Res.* 92, 9059.



- 676 Boulton, G.S., Lunn, R., Vidstrand, P., Zatsepin, S., 2007a. Subglacial drainage by groundwater-
 677 channel coupling, and the origin of esker systems: Part I—glaciological observations. *Quat. Sci.*
 678 *Rev.* 26, 1067–1090.
- 679 Boulton, G.S., Lunn, R., Vidstrand, P., Zatsepin, S., 2007b. Subglacial drainage by groundwater–
 680 channel coupling, and the origin of esker systems: part II—theory and simulation of a modern
 681 system. *Quat. Sci. Rev.* 26, 1091–1105.
- 682 Boulton, G.S., Hagdorn, M., Maillot, P.B., Zatsepin, S., 2009. Drainage beneath ice sheets :
 683 groundwater – channel coupling , and the origin of esker systems from former ice sheets. *Quat.*
 684 *Sci. Rev.* 28, 621–638.
- 685 Boyd, R., Scott, D.B., Douma, M., 1988. Glacial tunnel valleys and Quaternary history of the outer
 686 Scotian Shelf. *Nature* 333(6168), 61–64.
- 687 Breemer, C.W., Clark, P.U., Haggerty, R., 2002. Modeling the subglacial hydrology of the late
 688 Pleistocene Lake Michigan Lobe, Laurentide Ice Sheet. *Geol. Soc. Am. Bull.* 114, 665–674.
- 689 Brennand, T.A., Shaw, J., 1994. Tunnel channels and associated landforms, south-central Ontario:
 690 their implications for ice-sheet hydrology. *Can. J. Earth Sci.* 31, 505–522.
- 691 Bretz, J.H., 1923. The channeled scablands of the Columbia Plateau. *J. Geol.* 31, 617–649.
- 692 Bretz, J.H., Smith, H.T.U., Neff, G.E., 1956. Channeled Scablands of Washington: new data and
 693 interpretations. *Geol. Soc. Am. Bull.* 67, 957–1049.
- 694 Carling, P.A., 1996. Morphology, sedimentology and palaeohydraulic significance of large gravel
 695 dunes, Altai Mountains, Siberia. *Sedimentology* 43, 647–664.
- 696 Carling, P.A., Herget, J., Lanz, J.K., Richardson, K., Pacifici, A., 2009. Channel-scale erosional
 697 bedforms in bedrock and in loose granular material: character, processes and implications.
 698 *Megaflooding on Earth and Mars: Morphology, Modelling, and Water Release Mechanisms.*
 699 Cambridge University Press, Cambridge 13–32.
- 700 Carlson, A.E., Jenson, J.W., Clark, P.U., 2007. Modeling the subglacial hydrology of the James Lobe
 701 of the Laurentide Ice Sheet. *Quat. Sci. Rev.* 26(9), 1384–1397.
- 702 Clarke, G., 2005. Subglacial processes. *Annu. Rev. Earth Planet. Sci.* 33, 247–276.
 703 doi:10.1146/annurev.earth.33.092203.122621
- 704 Clark, P.U. 1992. Surface form of the southern Laurentide Ice Sheet and its implications to ice-sheet
 705 dynamics. *Geological Society of America Bulletin*, 104(5), 595–605.
- 706 Clark, P.U., Walder, J.S. 1994. Subglacial drainage, eskers, and deformable beds beneath the
 707 Laurentide and Eurasian ice sheets. *Geological Society of America Bulletin*, 106, 304–314.



- 708 Clayton, L., Attig, J.W., Mickelson, D.M., 1999. Tunnel channels formed in Wisconsin during the last
 709 glaciation. *Special Papers-Geological Society of America* 69–82.
- 710 Clayton, L., Moran, S.R., 1982. Chronology of late wisconsinan glaciation in middle North America.
 711 *Quat. Sci. Rev.* 1, 55–82.
- 712 Clayton, L., Teller, J.T., Attig, J.W., 1985. Surging of the southwestern part of the Laurentide Ice
 713 Sheet. *Boreas* 14, 235–241.
- 714 Cline, M.D., Iverson, N.R., Harding, C. 2015. Origin of washboard moraines of the Des Moines Lobe:
 715 Spatial analyses with LiDAR data. *Geomorphology*, 246, 570–578.
- 716 Cofaigh, C.Ó., 1996. Tunnel valley genesis. *Prog. Phys. Geogr.* 20, 1–19.
- 717 Cutler, P.M., Colgan, P.M., Mickelson, D.M., 2002. Sedimentologic evidence for outburst floods
 718 from the Laurentide Ice Sheet margin in Wisconsin, USA: Implications for tunnel-channel
 719 formation. *Quat. Int.* 90(1), 23–40. doi:10.1016/S1040-6182(01)00090-8.
- 720 Das, S.B., Joughin, I., Behn, M.D., Howat, I.M., King, M.A., Lizarralde, D., Bhatia, M.P., 2008.
 721 Fracture propagation to the base of the Greenland Ice Sheet during supraglacial lake drainage.
 722 *Science* 320, 778–781.
- 723 Derouin, S.A., 2008. Deglaciation in the Upper Peninsula of Michigan since the Last Glacial
 724 Maximum and its relationship to tunnel valleys found in the Lake Superior basin (PhD
 725 dissertation). University of Cincinnati.
- 726 Dunlop, P., Clark, C.D. 2006. The morphological characteristics of ribbed moraine. *Quaternary*
 727 *Science Reviews*, 25, 1668–1691.
- 728 Dyke, A.S., 2004. An outline of North American deglaciation with emphasis on central and northern
 729 Canada. *Quaternary glaciations: extent and chronology* 2, 373–424.
- 730 Einstein, H.A., Li, H., 1958. Secondary currents in straight channels. *Eos Trans. AGU* 39, 1085–1088.
- 731 Finnegan, N.J., Roe, G., Montgomery, D.R., Hallet, B., 2005. Controls on the channel width of rivers:
 732 Implications for modeling fluvial incision of bedrock. *Geology* 33, 229–232.
- 733 Fisher, T.G., Jol, H.M., Boudreau, A.M., 2005. Saginaw Lobe tunnel channels (Laurentide Ice Sheet)
 734 and their significance in south-central Michigan, USA. *Quat. Sci. Rev.* 24, 2375–2391.
- 735 Fisher, T.G., Taylor, L.D., 2002. Sedimentary and stratigraphic evidence for subglacial flooding,
 736 south-central Michigan, USA. *Quat. Int.* 90, 87–115.
- 737 Fowler, A.C. 2011. *Mathematical geoscience*. Springer-Verlag, London.
- 738 Fowler, A.C., Spagnolo, M., Clark, C.D., Stokes, C.R., Hughes, A., Dunlop, P., 2013. On the size and
 739 shape of drumlins. *GEM-International Journal on Geomathematics* 4, 155–165.



- 740 Fricker, H.A., Scambos, T., Bindschadler, R., Padman, L., 2007. An active subglacial water system in
 741 West Antarctica mapped from space. *Science* 315, 1544–1548.
- 742 Fullerton, D.S., Bush, C.A., Pennell, J.N., 2003. Surficial deposits and materials in the eastern and
 743 central United States (east of 102 west longitude): US Geological Survey Geologic
 744 Investigations Series I-2789. US Geological Survey, Denver, Colorado, USA.
- 745 Gao, C. 2011. Buried bedrock valleys and glacial and subglacial meltwater erosion in southern
 746 Ontario, Canada. *Canadian Journal of Earth Sciences*, 48, 801–818.
- 747 Gibling, M.R., 2006. Width and Thickness of Fluvial Channel Bodies and Valley Fills in the
 748 Geological Record: A Literature Compilation and Classification. *J. Sediment. Res.* 76, 731–770.
- 749 Gilbert, R., Shaw, J. 1994. Inferred subglacial meltwater origin of lakes on the southern border of the
 750 Canadian Shield. *Canadian Journal of Earth Sciences*, 31, 1630–1637.
- 751 Greenwood, S.L., Clark, C.D., Hughes, A.L.C., 2007. Formalising an inversion methodology for
 752 reconstructing ice-sheet retreat patterns from meltwater channels: application to the British Ice
 753 Sheet. *J. Quat. Sci.* 22, 637–645.
- 754 Hewitt, I.J., 2011. Modelling distributed and channelized subglacial drainage: the spacing of channels.
 755 *J. Glaciol.* 57, 302–314.
- 756 Hillier, J.K., Smith, M.J., Clark, C.D., Stokes, C.R., Spagnolo, M., 2013. Subglacial bedforms reveal
 757 an exponential size–frequency distribution. *Geomorphology* 190, 82–91.
- 758 Hooke, R.L., Jennings, C.E., 2006. On the formation of the tunnel valleys of the southern Laurentide
 759 ice sheet. *Quat. Sci. Rev.* 25, 1364–1372.
- 760 Hovius, N. 1996. Regular spacing of drainage outlets from linear mountain belts. *Basin Research*, 8,
 761 29–44.
- 762 Hughes, T.J., 1995. Ice sheet modelling and the reconstruction of former ice sheets from
 763 Geo(morpho)logical field data. In: Menzies, J. (Ed.), *Modern Glacial Environments—Processes,*
 764 *Dynamics and Sediments.* Butterworth-Heinemann Ltd, Oxford.
- 765 Huuse, M., Lykke-Andersen, H., 2000. Overdeepened Quaternary valleys in the eastern Danish North
 766 Sea: Morphology and origin. *Quat. Sci. Rev.* 19, 1233–1253.
- 767 Janszen, A., Moreau, J., Moscariello, A., Ehlers, J., Kröger, J., 2013. Time-transgressive tunnel-valley
 768 infill revealed by a three-dimensional sedimentary model, Hamburg, north-west Germany.
 769 *Sedimentology* 60, 693–719.
- 770 Jennings, C.E., 2006. Terrestrial ice streams—a view from the lobe. *Geomorphology* 75, 100–124.



- 771 Johnson, M.D., 1999. Spooner Hills, northwest Wisconsin: High-relief hills carved by subglacial
 772 meltwater of the Superior Lobe. Geological Society of America Special Papers 83-92.
- 773 Jørgensen, F., Sandersen, P.B.E., 2006. Buried and open tunnel valleys in Denmark—erosion beneath
 774 multiple ice sheets. *Quat. Sci. Rev.* 25, 1339–1363.
- 775 Karcz, I., 1967. Harrow Marks, Current-Aligned Sedimentary Structures. *J. Geol.* 75, 113–121.
- 776 Kehew, A.E., Ewald, S.K., Esch, J.M., Kozłowski, A.L., 2013. On the origin of tunnel valleys of the
 777 Saginaw Lobe of the Laurentide Ice Sheet; Michigan, USA. *Boreas* 42, 442–462.
- 778 Kehew, A.E., Kozłowski, A.L., 2007. Tunnel channels of the Saginaw lobe, Michigan, USA. *Applied*
 779 *Quaternary Research in the Central Part of Glaciated Terrain*, Johansson P, Sarala P (eds).
 780 Special Paper 69–78.
- 781 Kehew, A.E., Nicks, L.P., Straw, W.T., 1999. Palimpsest tunnel valleys: evidence for relative timing
 782 of advances in an interlobate area of the Laurentide ice sheet. *Ann. Glaciol.* 28, 47–52.
- 783 Kehew, A.E., Piotrowski, J. a, Jørgensen, F., 2012. Tunnel valleys: Concepts and controversies — A
 784 review. *Earth-Sci. Rev.* 113, 33–58.
- 785 Kemmis, T.J., Hallberg, G.R., Luttenegger, A.J., 1981. Depositional environments of glacial sediments
 786 and landforms on the Des Moines Lobe. Iowa: Iowa Geological Survey Guidebook.
- 787 Kleman, J., Glasser, N. F. 2007. The subglacial thermal organisation (STO) of ice sheets. *Quaternary*
 788 *Science Reviews*, 26(5), 585-597.
- 789 Kozłowski, A.L., Kehew, A.E., Bird, B.C., 2005. Outburst flood origin of the Central Kalamazoo
 790 River Valley, Michigan, USA. *Quat. Sci. Rev.* 24, 2354–2374.
- 791 Kristensen, T.B., Huuse, M., Piotrowski, J.A., Clausen, O.R., 2007. A morphometric analysis of
 792 tunnel valleys in the eastern North Sea based on 3D seismic data. *J. Quat. Sci.* 22, 801–815.
- 793 Kristensen, T.B., Piotrowski, J.A., Huuse, M., Clausen, O.R., Hamberg, L., 2008. Time-transgressive
 794 tunnel valley formation indicated by infill sediment structure, North Sea – the role of
 795 glaciohydraulic supercooling. *Earth Surf. Processes Landforms* 33, 546–559.
- 796 Lamb, M.P., Fonstad, M.A., 2010. Rapid formation of a modern bedrock canyon by a single flood
 797 event. *Nat. Geosci.* 3, 477–481.
- 798 Lampkin, D.J., VanderBerg, J., 2011. A preliminary investigation of the influence of basal and
 799 surface topography on supraglacial lake distribution near Jakobshavn Isbrae, western Greenland.
 800 *Hydrol. Process.* 25, 3347–3355.



- 801 Leeson, A.A., Shepherd, A., Sundal, A.V., Johansson, A.M., Selmes, N., Briggs, K., Hogg, A.E.,
802 Fettweis, X., 2013. A comparison of supraglacial lake observations derived from MODIS
803 imagery at the western margin of the Greenland ice sheet. *J. Glaciol.* 59, 1179–1188.
- 804 Leopold, L.B., 1964. Wolman, MG, and Miller, JP, *Fluvial Processes in Geomorphology*.
- 805 Leopold, L.B., Maddock, T., Jr, 1953. The hydraulic geometry of stream channels and some
806 physiographic implications.
- 807 Lesemann, J.-E., Piotrowski, J. a, Wysota, W., 2014. Genesis of the “glacial curvilineation” landscape
808 by meltwater processes under the former Scandinavian Ice Sheet, Poland. *Sediment. Geol.* 312,
809 1–18.
- 810 Limpert, E., Stahel, W.A., Abbt, M., 2001. Log-normal Distributions across the Sciences: Keys and
811 Clues on the charms of statistics, and how mechanical models resembling gambling machines
812 offer a link to a handy way to characterize log-normal distributions, which can provide deeper
813 insight into variability and probability—normal or log-normal: That is the question. *Bioscience*
814 51(5), 341–352.
- 815 Livingstone, S.J., Clark, C.D., Piotrowski, J.A, Tranter, M., Bentley, M.J., Hodson, A., Swift, D.A,
816 Woodward, J., 2012. Theoretical framework and diagnostic criteria for the identification of
817 palaeo-subglacial lakes. *Quat. Sci. Rev.* 53, 88–110.
- 818 Livingstone, S.J., Clark, C.D., Tarasov, L., 2013. Modelling North American palaeo-subglacial lakes
819 and their meltwater drainage pathways. *Earth Planet. Sci. Lett.* 1–16.
- 820 Livingstone, S.J., Utting, D., Ruffell, A., Clark, C.D., Pawley, S., Atkinson, N., Fowler, A.C. 2016,
821 Sub. Discovery of relict subglacial lakes: their geometry and mechanism of drainage. *Nature*
822 *Communications*.
- 823 Lowe, A.L., Anderson, J.B. 2003. Evidence for abundant meltwater beneath the paleo-ice sheet in
824 Pine Island Bay, Antarctica. *Journal of Glaciology*, 49, 125–138.
- 825 Margold, M., Stokes, C.R., Clark, C.D., 2015. Ice streams in the Laurentide Ice Sheet: Identification,
826 characteristics and comparison to modern ice sheets. *Earth-Sci. Rev.* 143, 117–146.
- 827 Mathews, W.H. 1974. Surface profiles of the Laurentide Ice Sheet in its marginal areas. *Journal of*
828 *Glaciology*, 10, 146–163.
- 829 Mickelson, D.M., Colgan, P.M., 2003. The southern Laurentide Ice Sheet, in: Gillespie, A.R., Porter,
830 S.C., Atwater, A.B.F. (Eds.), *The Quaternary Period in the United States*. Elsevier, pp. 1–16.
- 831 Moores, H.D., 1989. On the formation of the tunnel valleys of the Superior lobe, central Minnesota.
832 *Quat. Res.* 32, 24–35.



- 833 Moreau, J., Huuse, M., Janszen, A., van der Vegt, P., Gibbard, P.L., Moscariello, A., 2012. The
 834 glaciogenic unconformity of the southern North Sea. Geological Society, London, Special
 835 Publications 368, 99–110.
- 836 Mullins, H.T., Hinchey, E.J., 1989. Erosion and infill of New York Finger Lakes: Implications for
 837 Laurentide ice sheet deglaciation. *Geology* 17, 622–625.
- 838 Onda, Y., 1994. Seepage erosion and its implication to the formation of amphitheatre valley heads: A
 839 case study at Obara, Japan. *Earth Surf. Processes Landforms* 19, 627–640.
- 840 Palmer, S., Shepherd, A., Nienow, P., Joughin, I., 2011. Seasonal speedup of the Greenland Ice Sheet
 841 linked to routing of surface water. *Earth Planet. Sci. Lett.* 302, 423–428.
- 842 Patterson, C.J., 1997. Southern Laurentide ice lobes were created by ice streams: Des Moines Lobe in
 843 Minnesota, USA. *Sediment. Geol.* 111, 249–261.
- 844 Patterson, C.J., 1994. Tunnel-valley fans of the St. Croix moraine, east-central Minnesota, USA.
 845 Formation and deformation of glacial deposits 69–87.
- 846 Petroff, A.P., Devauchelle, O., Abrams, D.M., Lobkovsky, A.E., Kudrolli, A., Rothman, D.H., 2011.
 847 Geometry of valley growth. *J. Fluid Mech.* 673, 245–254.
- 848 Piotrowski, J.A., 1994. Tunnel-valley formation in northwest Germany—geology, mechanisms of
 849 formation and subglacial bed conditions for the Bornhöved tunnel valley. *Sediment. Geol.* 89,
 850 107–141.
- 851 Piotrowski, J.A., 1997. Subglacial hydrology in north-western Germany during the last glaciation:
 852 groundwater flow, tunnel valleys and hydrological cycles. *Quat. Sci. Rev.* 16, 169–185.
- 853 Piotrowski, J.A., Hermanowski, P., Piechota, A.M., 2009. Meltwater discharge through the subglacial
 854 bed and its land-forming consequences from numerical experiments in the Polish lowland during
 855 the last glaciation. *Earth Surf. Processes Landforms* 34, 481–492.
- 856 Praeg, D., 2003. Seismic imaging of mid-Pleistocene tunnel-valleys in the North Sea Basin—high
 857 resolution from low frequencies. *J. Appl. Geophys.* 53, 273–298.
- 858 Ravier, E., Buoncristiani, J.-F., Guiraud, M., Menzies, J., Clerc, S., Goupy, B., Portier, E., 2014.
 859 Porewater pressure control on subglacial soft sediment remobilization and tunnel valley
 860 formation: A case study from the Alnif tunnel valley (Morocco). *Sediment. Geol.* 304, 71–95.
- 861 Regis, R.S., Patterson, C.J., Wattrus, N., Rausch, D., 2003. Relationship of deep troughs in the eastern
 862 Lake Superior basin and large-scale glaciofluvial landforms in the central Upper Peninsula of
 863 Michigan. *North Central Geological Society of America Abstracts with Program*, Paper 19–10.



- 864 Rose, K.C., Ross, N., Bingham, R.G., Corr, H.F.J., Ferraccioli, F., Jordan, T. a, Le Brocq, A., Rippin,
 865 D.M., Siegert, M.J., 2014. A temperate former West Antarctic ice sheet suggested by an
 866 extensive zone of subglacial meltwater channels. *Geology* 42, 971-974. doi:10.1130/G35980.1
- 867 Rudoy, A.N., 2005. Giant Current Ripples: History of Research, Their Diagnostics, and
 868 Paleogeographical Significance.
- 869 Sandersen, P.B.E., Jørgensen, F., 2012. Substratum control on tunnel-valley formation in Denmark.
 870 Geological Society, London, Special Publications 368, 145–157.
- 871 Selmes, N., Murray, T., James, T.D., 2011. Fast draining lakes on the Greenland Ice Sheet. *Geophys.*
 872 *Res. Lett.* 38. doi:10.1029/2011GL047872
- 873 Shaw, J., 2002. The meltwater hypothesis for subglacial bedforms. *Quat. Int.* 90, 5–22.
- 874 Shaw, J., Gilbert, R., 1990. Evidence for large-scale subglacial meltwater flood events in southern
 875 Ontario and northern New York State. *Geology* 18, 1169-1172. doi:10.1130/0091-
 876 7613(1990)018<1169
- 877 Shoemaker, E.M., 1999. Subglacial water-sheet floods, drumlins and ice-sheet lobes. *J. Glaciol.* 45,
 878 201-213.
- 879 Shoemaker, E.M., 1991. On the formation of large subglacial lakes. *Can. J. Earth Sci.* 28(12), 1975-
 880 1981.
- 881 Shoemaker, E.M., 1986. Subglacial hydrology for an ice sheet resting on a deformable aquifer. *J.*
 882 *Glaciol.* 32, 20–30.
- 883 Sjögren, D., Fisher, T.G., Taylor, L.D., Jol, H.M., Munro-Stasiuk, M.J., 2002. Incipient tunnel
 884 channels. *Quaternary International* 90, 41–56.
- 885 Smed, P. 1998. Die Entstehung der dänischen und norddeutschen Rinnentäler (Tunneltäler) —
 886 Glaziologische Gesichtspunkte. *Eiszeitalter und Gegenwart* 48, 1–18.
- 887 Smith, L.N., 2004. Late Pleistocene stratigraphy and implications for deglaciation and subglacial
 888 processes of the Flathead Lobe of the Cordilleran Ice Sheet, Flathead Valley, Montana, USA.
 889 *Sediment. Geol.* 165, 295–332.
- 890 Soller, D.R., PH Garrity, C.P., 2011. Database for USGS Map I-1970—Map Showing the Thickness
 891 and Character of Quaternary Sediments in the Glaciated United States East of the Rocky
 892 Mountains.
- 893 Spagnolo, M., Clark, C.D., Ely, J.C., Stokes, C.R., Anderson, J.B., Andreassen, K., Graham, A.G.C.,
 894 King, E.C., 2014. Size, shape and spatial arrangement of mega-scale glacial lineations from a
 895 large and diverse dataset. *Earth Surf. Processes Landforms* 39, 1432–1448.



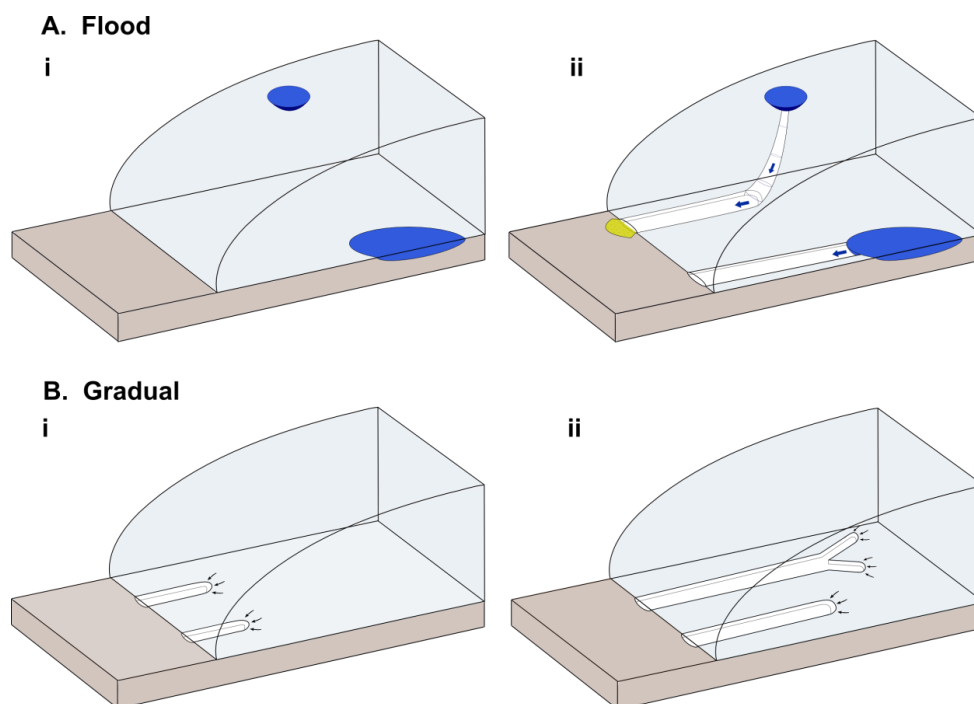
- 896 Stackebrandt, W., 2009. Subglacial channels of Northern Germany--a brief review [Die subglazialen
 897 Rinnen Norddeutschlands--ein kurzer Überblick]. Zeitschrift der deutschen Gesellschaft für
 898 Geowissenschaften 160, 203–210.
- 899 Stewart, M.A., Lonergan, L., 2011. Seven glacial cycles in the middle-late Pleistocene of northwest
 900 Europe: Geomorphic evidence from buried tunnel valleys. *Geology* 39, 283–286.
- 901 Stewart, R.A., Bryant, D., Sweat, M.J., 1988. Nature and origin of corrugated ground moraine of the
 902 Des Moines Lobe, Story County, Iowa. *Geomorphology* 1(2), 111–130.
- 903 Storrar, R.D., Stokes, C.R., Evans, D.J.A., 2014a. Morphometry and pattern of a large sample
 904 (>20,000) of Canadian eskers and implications for subglacial drainage beneath ice sheets. *Quat.*
 905 *Sci. Rev.* 105, 1–25.
- 906 Storrar, R.D., Stokes, C.R., Evans, D.J.A., 2014b. Increased channelization of subglacial drainage
 907 during deglaciation of the Laurentide Ice Sheet. *Geology*, 42, 239–242.
- 908 Talling, P.J., Stewart, M.D., Stark, C.P., Gupta, S., Vincent. 1997. Regular spacing of drainage outlets
 909 from linear fault blocks. *Basin Research* 9, 275–302.
- 910 Tooth, S., 2000. Process, form and change in dryland rivers: a review of recent research. *Earth-Sci.*
 911 *Rev.* 51, 67–107.
- 912 Van der Vegt, P., Janszen, A., Moscariello, A., 2012. Tunnel valleys: current knowledge and future
 913 perspectives. Geological Society, London, Special Publications 368, 75–97.
- 914 Wingfield, R., 1990. The origin of major incisions within the Pleistocene deposits of the North Sea.
 915 *Mar. Geol.* 91, 31–52.
- 916 Wingham, D.J., Siegert, M.J., Shepherd, A., Muir, A.S., 2006. Rapid discharge connects Antarctic
 917 subglacial lakes. *Nature* 440, 1033–1036.
- 918 Winsborrow, M.C.M., Clark, C.D., Stokes, C.R., 2010. What controls the location of ice streams?
 919 *Earth-Sci. Rev.* 103, 45–59.
- 920 Wright, A., Siegert, M.J., 2011. The identification and physiographical setting of Antarctic subglacial
 921 lakes: An update based on recent discoveries. *Geophysical Monograph Series* 1–30.
- 922 Wright, H.E., 1973. Tunnel Valleys, Glacial Surges, and Subglacial Hydrology of the Superior Lobe,
 923 Minnesota. Geological Society of America Memoirs 136, 251–276.
- 924 Zwally, H.J., Abdalati, W., Herring, T., Larson, K., Saba, J., Steffen, K., 2002. Surface melt-induced
 925 acceleration of Greenland ice-sheet flow. *Science* 297, 218–222.

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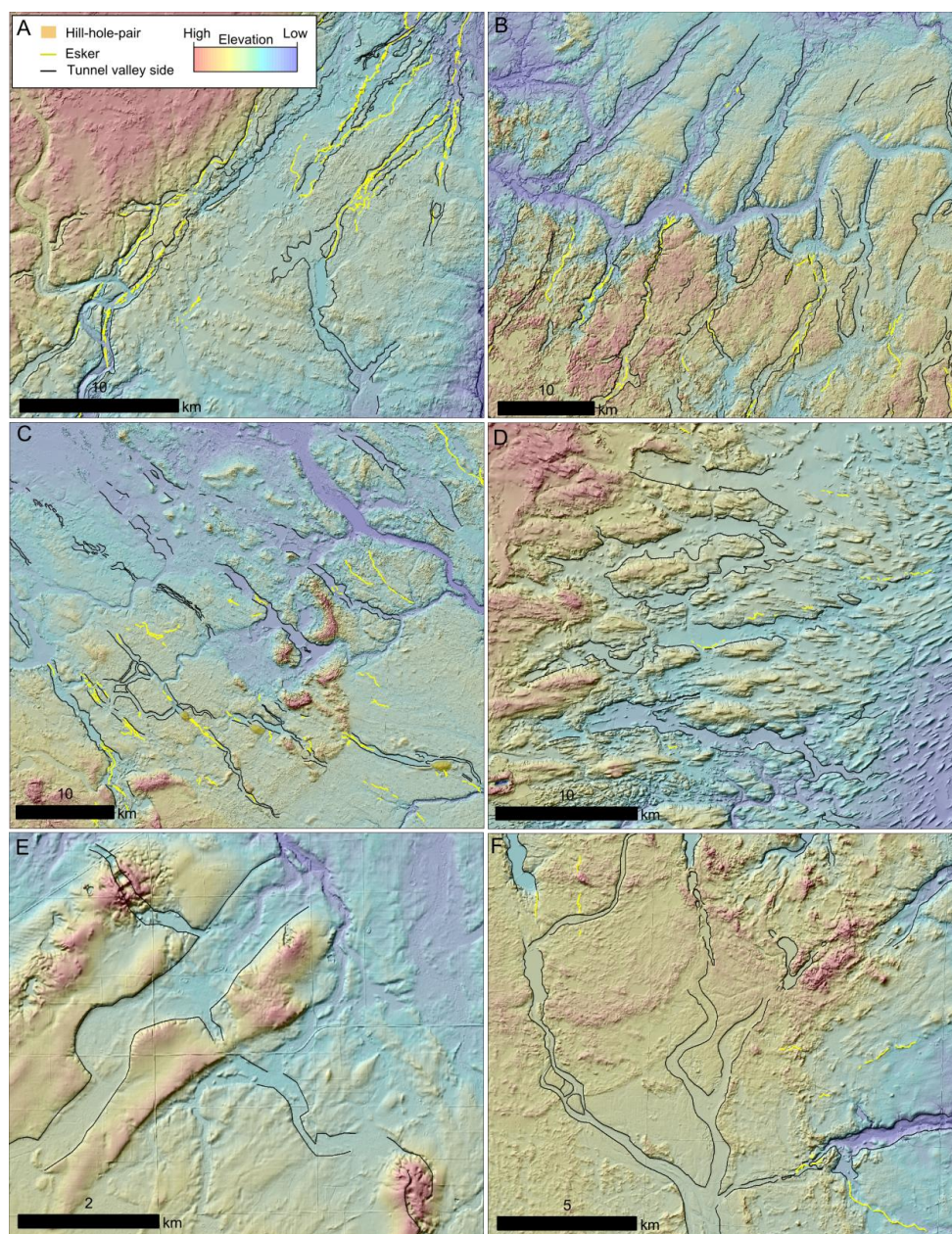


928 **Figures**



929

930 **Figure 1:** Cartoons depicting the two main models for tunnel valley formation: **A.** Outburst floods
 931 from supraglacial and/or subglacially stored water; and **B.** Gradual headward growth by sapping.

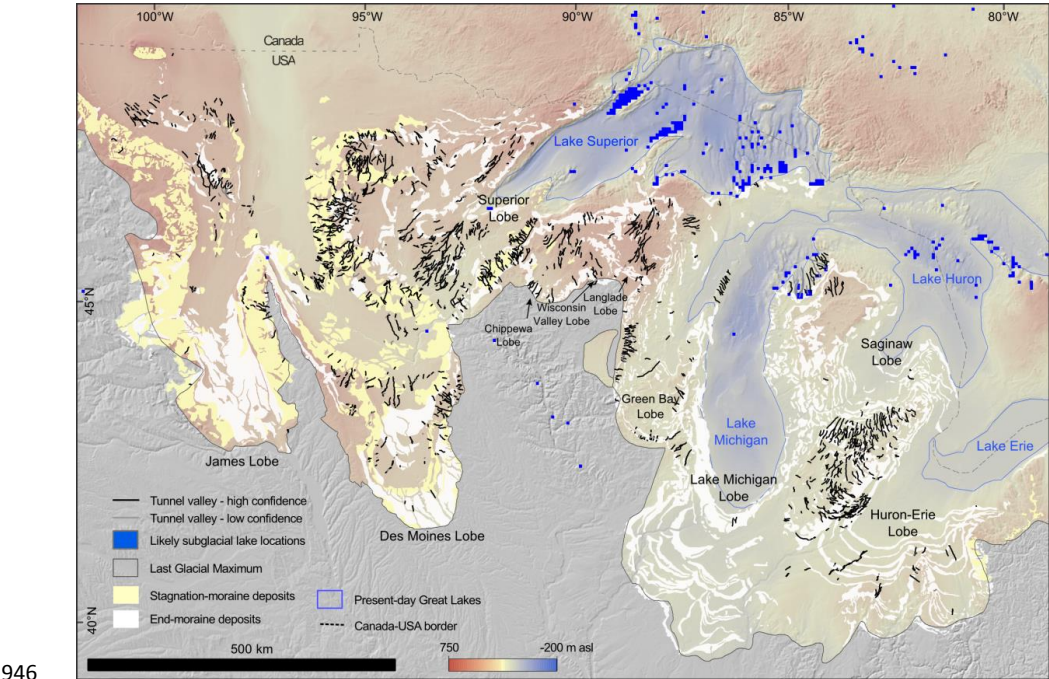


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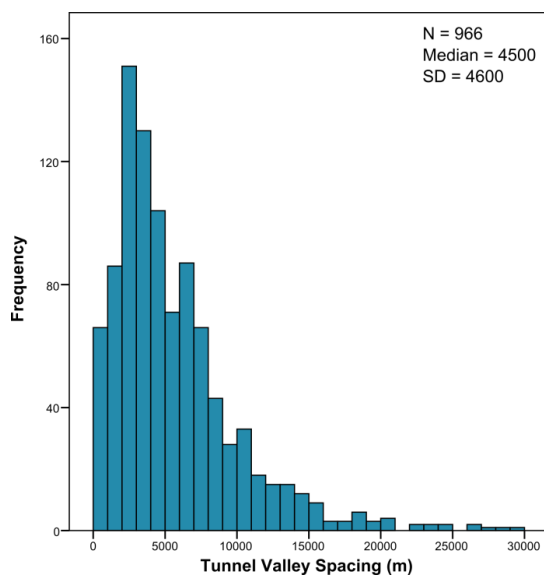
933 **Figure 2:** Examples of mapped valleys and the assignment of confidence levels (1 = high confidence
 934 to 3 = low confidence) along the southern sector of the former Laurentide Ice Sheet. Valleys in panels
 935 **A** (Superior), **B** (Saginaw), **C** (North Dakota) and **D** (Green Bay) are assigned a confidence of 1. The
 936 relict valleys contain eskers, are parallel and relatively straight, and do not trend along the regional
 937 slope. In panels **A**, **C** and **D** the tunnel valley networks terminates at a moraine position. The large



938 valley in panel E (Superior) is assigned a confidence level of 3 as it does not contain any subglacial
939 bedforms and exhibits a gradual and consistent change in bed slope consistent with a proglacial
940 spillway. However, the smaller NW-SE valleys that bisect it is given a confidence of 2 as they have
941 undulating thalwegs that cut across moraines. The dendritic valley network in panel F (North Dakota)
942 is given a confidence of 3 as it is not associated with any subglacial bedforms and has a consistent bed
943 slope indicating water flow towards the south. A braided channel morphology and a widening reach
944 towards the south allows us to interpret this valley system as a proglacial spillway (fed by tunnel
945 valleys emanating from under the ice to the north).

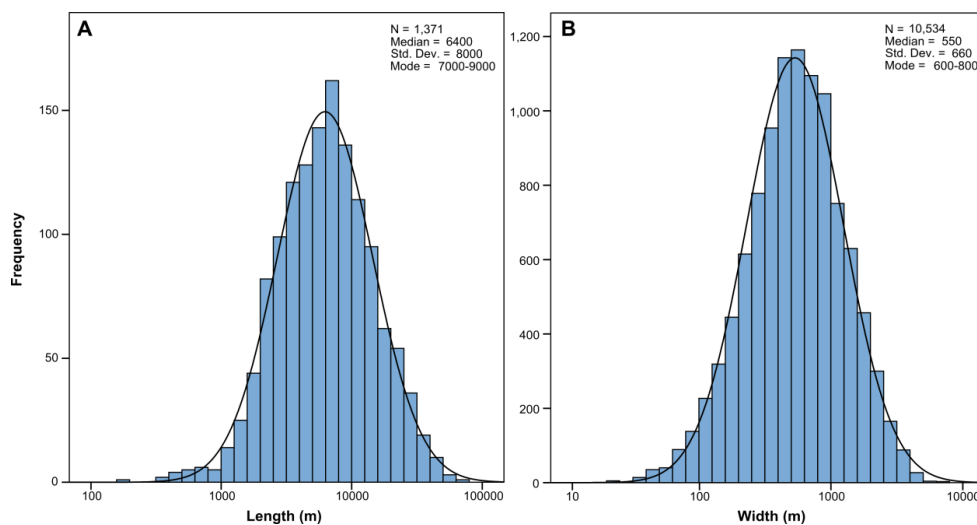


947 **Figure 3:** Distribution of mapped tunnel valleys and moraines along the southern sector of the
948 Laurentide Ice Sheet. Likely subglacial lake locations are predictions from Livingstone et al., (2013).
949 The Last Glacial Maximum extent is from Dyke et al., (2004) and moraines are from Fullerton et al.
950 (2003).



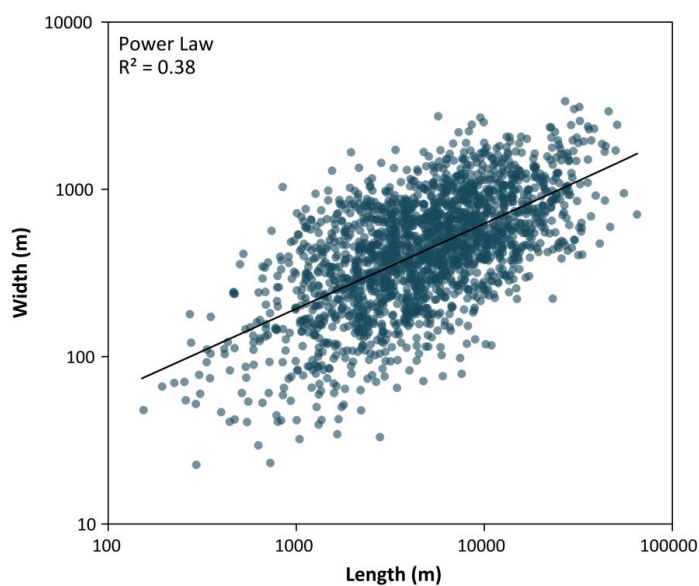
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952 **Figure 4:** Frequency histogram of the spacing of 966 tunnel valleys from 24 discrete networks across
 953 the southern sector of the former Laurentide Ice Sheet.



954

955 **Figure 5:** Frequency histogram of tunnel valley length and width (for confidence levels 1 and 2). Line
 956 is the log-normal distribution for comparison. Width values were extracted at 1 km intervals along the
 957 centre-line of each tunnel valley.



958

959 **Figure 6:** Relationship between tunnel valley length and average width (for single thread valleys with
 960 a confidence level of 1 and 2, N=1135). Note, there is a tendency for longer tunnel valleys to be
 961 wider.

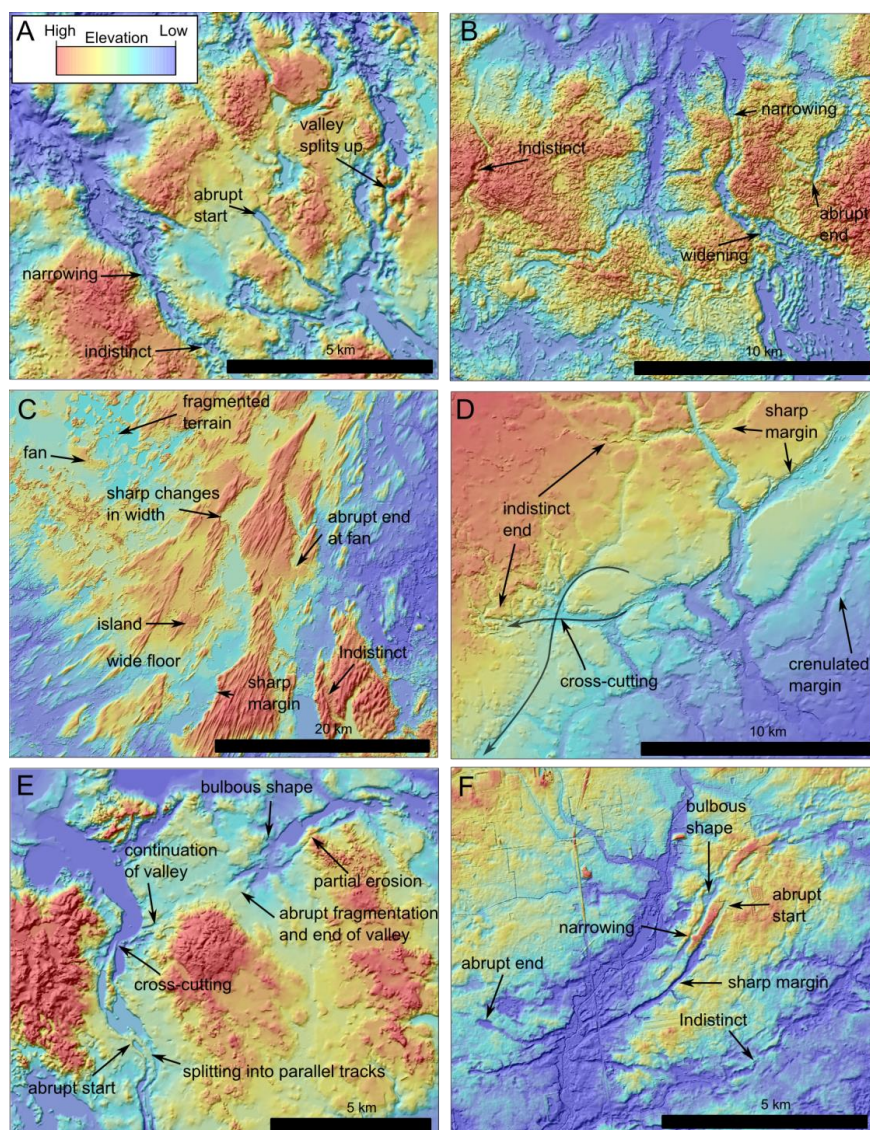
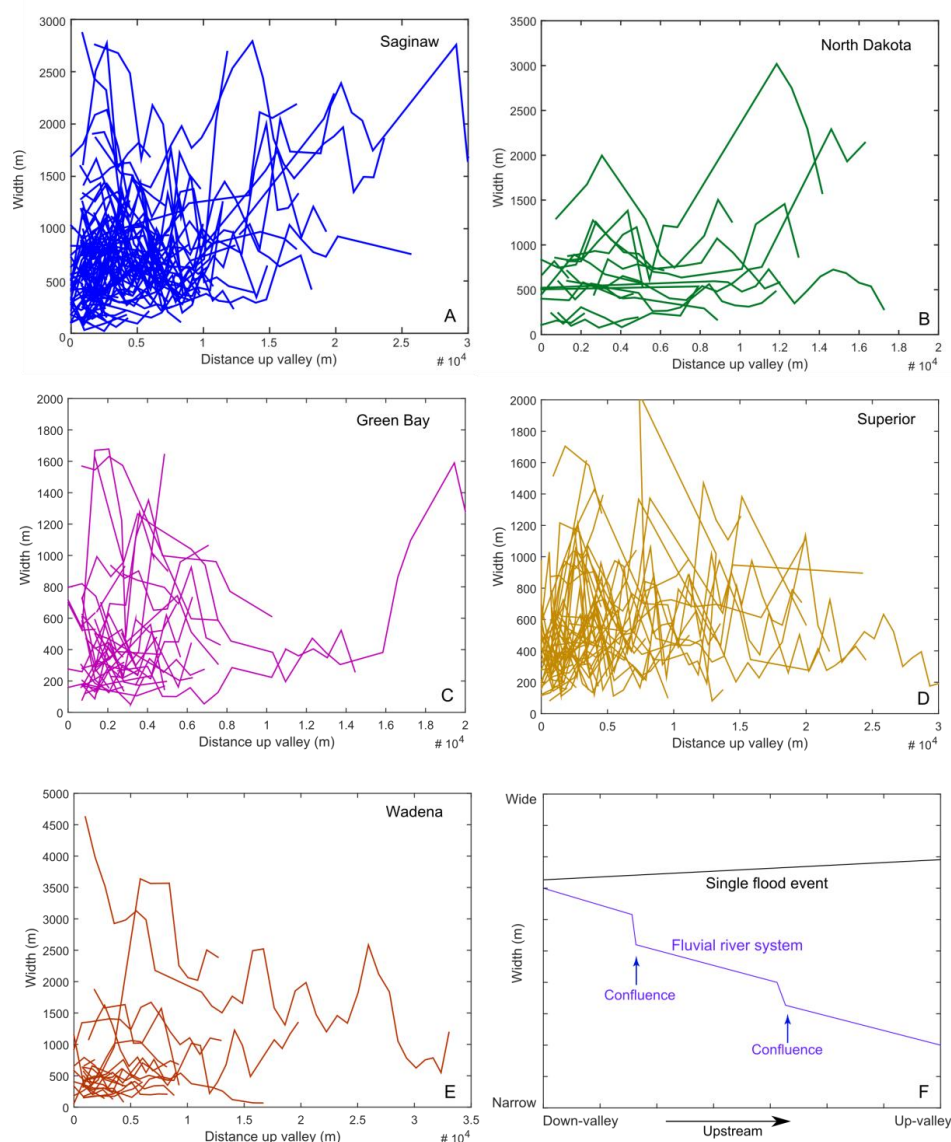
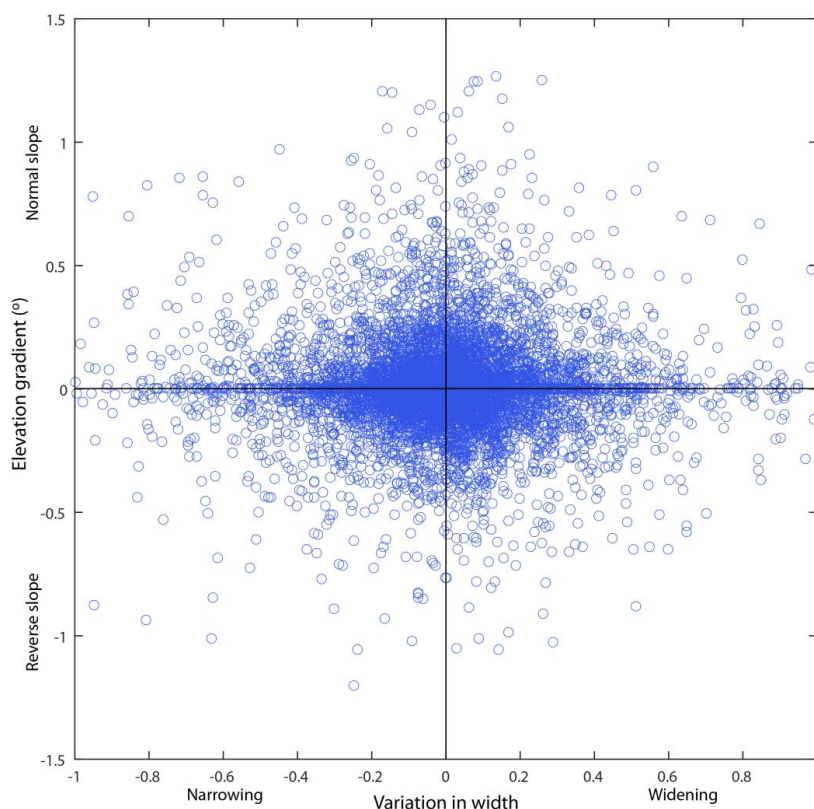


Figure 7: Examples of tunnel valley morphology of tunnel valleys with a confidence of 1 or 2. **A.** Superior Lobe (note the amphitheatre heads of some valleys); **B.** Wadena Lobe (note the large downstream changes in tunnel valley width); **C.** Langlade Lobe; **D.** Saginaw Lobe; and **E.** Wadena (note the parallel valleys) and **F** Huron-Erie (note the abrupt start and end points of the tunnel valleys and parallel organisation).



968

969 **Figure 8:** Along-valley plots highlighting tunnel valley width variations. **A.** Saginaw Lobe; **B.** North
 970 Dakota; **C.** Green Bay Lobe; **D.** Superior Lobe; and **E.** Wadena Lobe. **F.** Cartoon showing the
 971 expected relationship between width and distance downstream for a fluvial river (Leopold and
 972 Maddock, 1953; Leopold et al., 1964) and single flood event (Lamb and Fonstad, 2010). Note that the
 973 measured tunnel valley width variations conform to neither of these expectations, but instead show
 974 variations in width greatly exceeding any possible systematic trends.



975

976 **Figure 9:** Scatter plot compiled to investigate if downstream variation in channel width was
 977 controlled by variations in downstream slope gradient (see text for details). That the data are centred
 978 on zero and spread fairly evenly around this demonstrates that there is no systematic relationship
 979 between elevation gradient (i.e., whether it is a normal or reverse gradient slope) and width (i.e.,
 980 whether the tunnel valley is narrowing or widening).

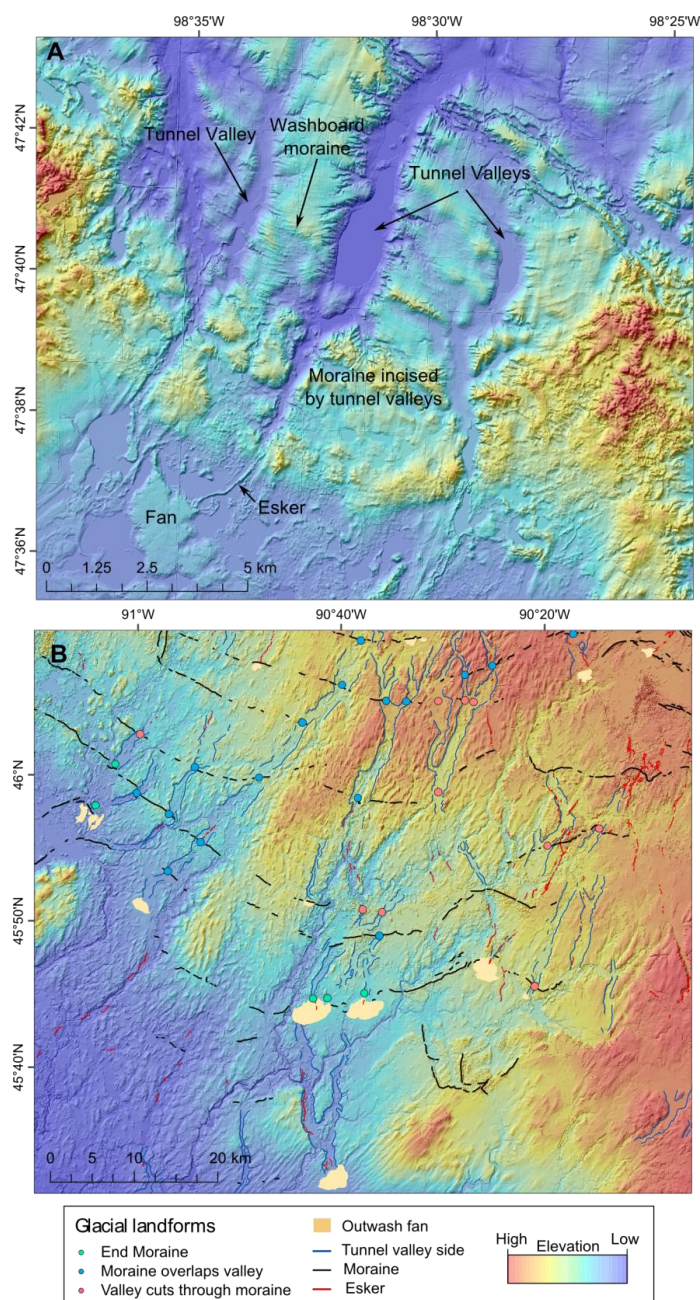
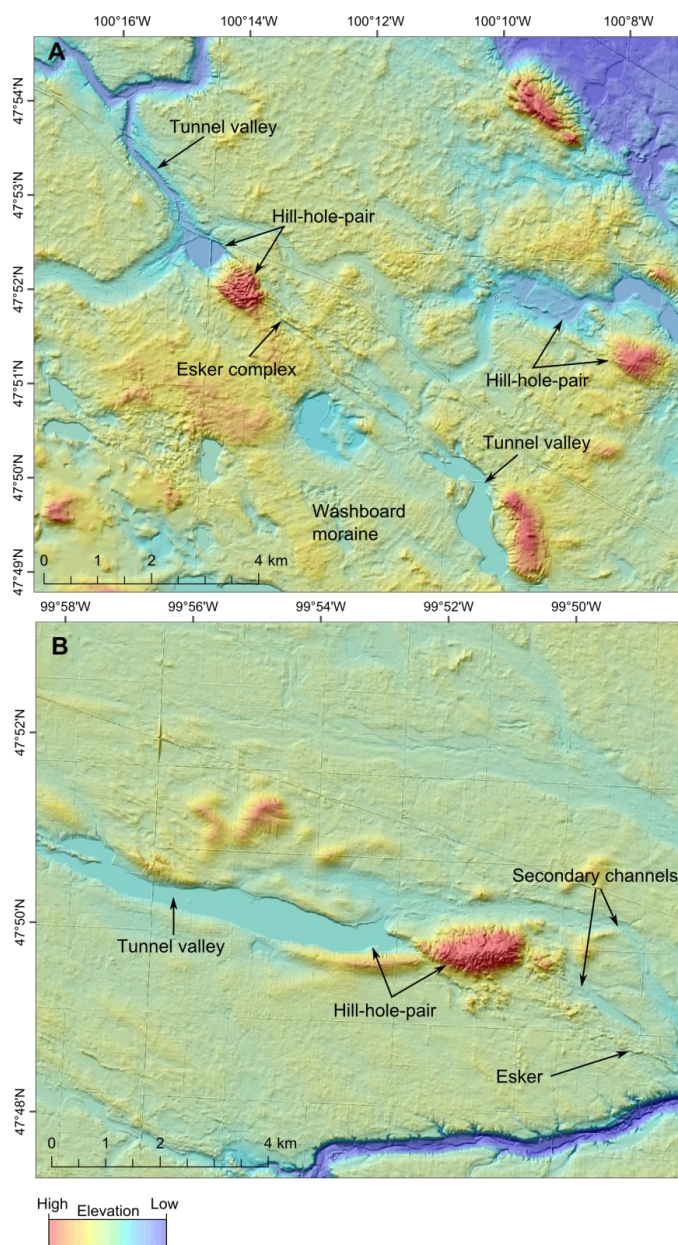
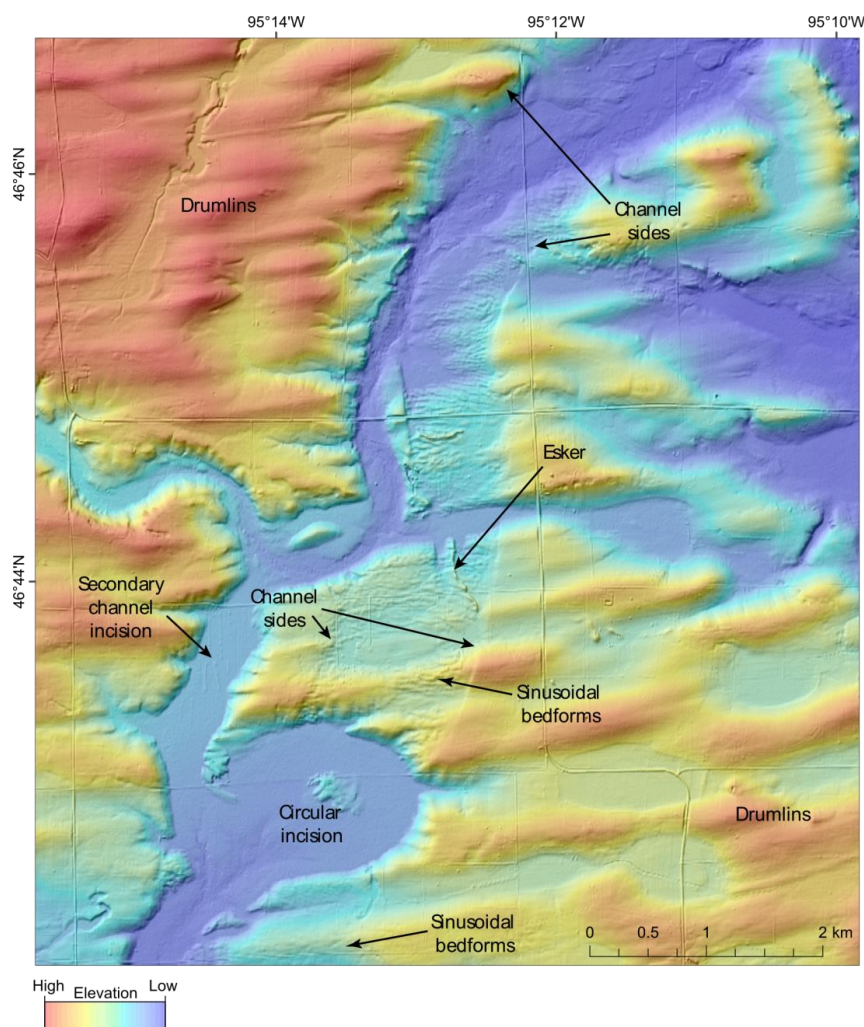


Figure 10: The varied cross-cutting associations between moraines, outwash fans and tunnel valleys in: **A.** North Dakota – note how the washboard moraines curve up-glacier towards the tunnel valleys; and **B.** Wisconsin (Chippewa Lobe).



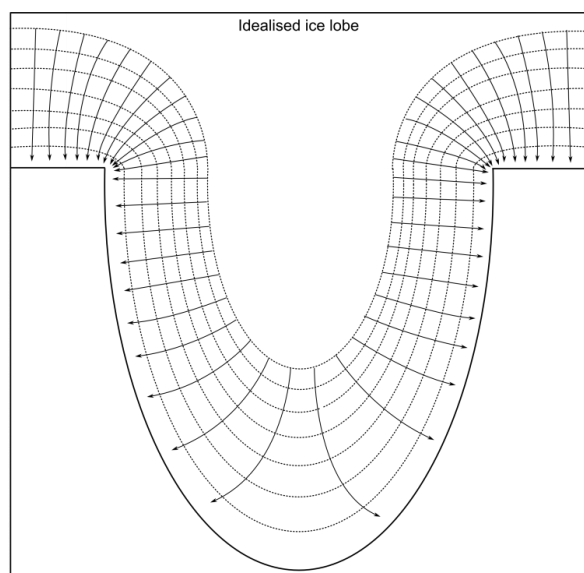
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986 **Figure 11:** Hill-hole-pairs in North Dakota and their association with tunnel valleys. **A.** Note the
 987 esker downstream of the hill, which trends into an aligned tunnel valley segment. **B.** Note the
 988 secondary meltwater channels and eskers that diverge around the hill.



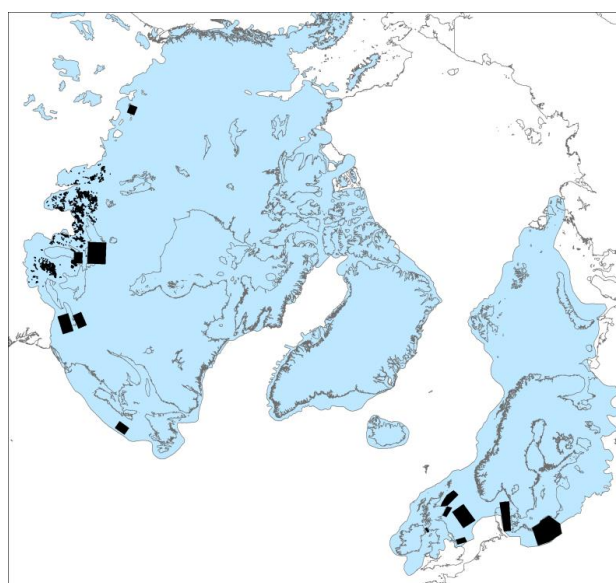
989

990 **Figure 12:** Giant Current Ripples spanning the width of a shallow tunnel valley that is cut into an
 991 obliquely-oriented drumlin field (water flow to the south). These sinusoidal bedforms are interpreted
 992 as giant current ripples, which formed during a large subglacial flood. Note the undulating thalwegs
 993 and esker in the valley that indicates subglacial deposition, and the circular incision (with a remnant
 994 island in its centre) in the south of the valley that may have formed by a large eddy during high-
 995 energy turbulent flow.



996

997 **Figure 13:** Idealised ice lobe, hydraulic potential contours (dotted lines) and drainage routes. Note
 998 how this predicts that with uniform upstream basal melting that the resultant water paths diverge
 999 down the lobe axis and away from the terminus (yielding low water delivery here) and converge in
 1000 interlobate regions (high water delivery).

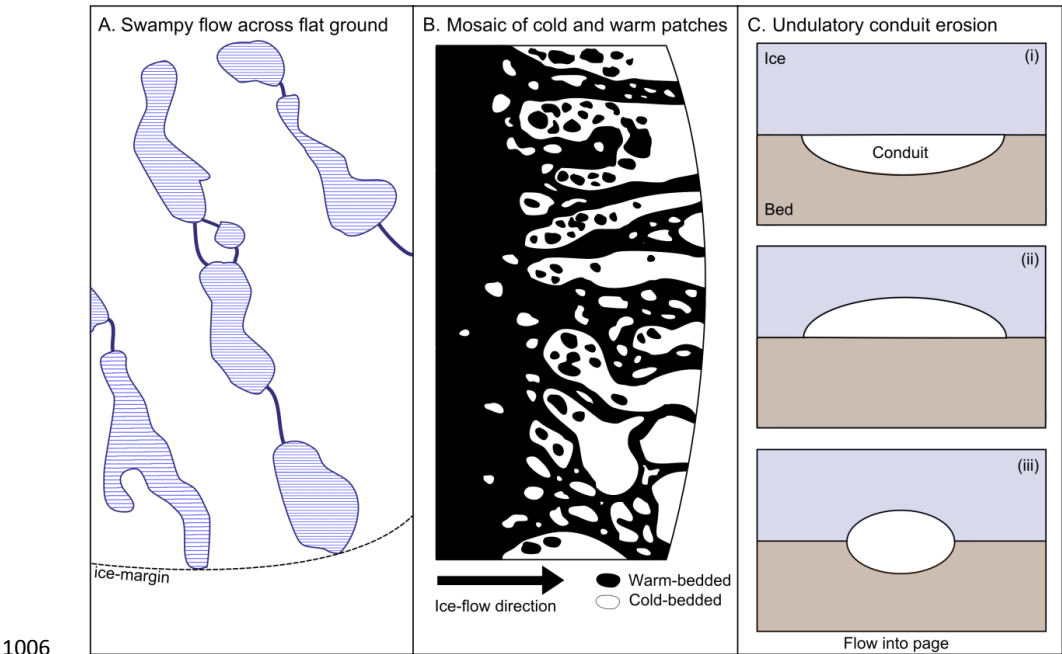


1001

1002 **Figure 14:** Currently known Northern Hemisphere distribution of tunnel valleys that have been
 1003 attributed to the last glaciation. The opaque blue shading is the Last Glacial Maximum ice sheet



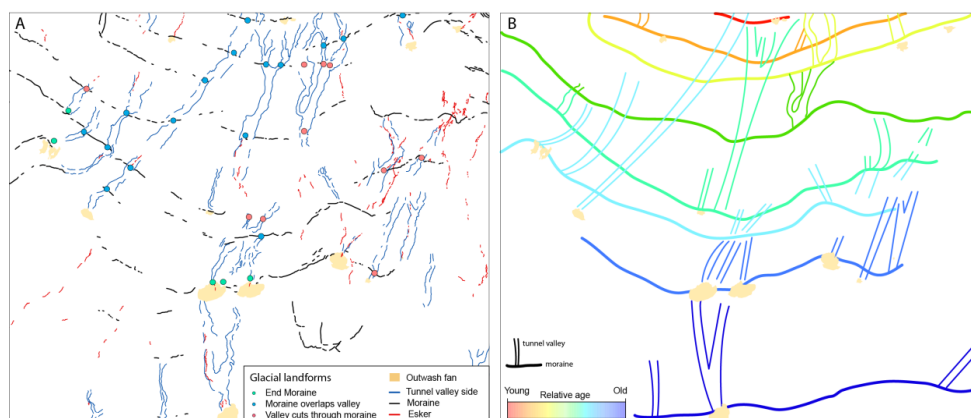
1004 distribution. Black lines are the mapped tunnel valleys from Fig. 3 and black boxes are where tunnel
1005 valleys have been identified.



1007 **Figure 15:** Cartoons showing three theories to explain the downstream variation in tunnel valley
1008 width. **A.** Swampy ground (blue stipples) and channels (blue lines) associated with water flow across
1009 very flat ground. In such a flat landscape tunnel valleys are able to easily expand laterally, in response
1010 to small changes in water flux, and there is little impetus for rapid vertical erosion due to shallow
1011 hydraulic gradients. **B.** Tunnel valley formation is modulated by the basal thermal regime (modified
1012 from Hughes, 1995). Channels are able to develop more easily across warm sediment patches, and the
1013 mosaic of cold and warm sediment patches results in variations in width. **C.** Undulatory conduit
1014 erosion. In this theory the width of the channel eroded into sediment depends upon the competition
1015 between erosion down into the sediment (canals) vs. melting up into the ice (R-channel) (see Fowler,
1016 2011; Livingstone et al. Sub). Note that each of the conduits (i-iii) have the roughly the same area,
1017 but that in (ii) no channel forms and in (iii) the channel width is roughly half that of (i).



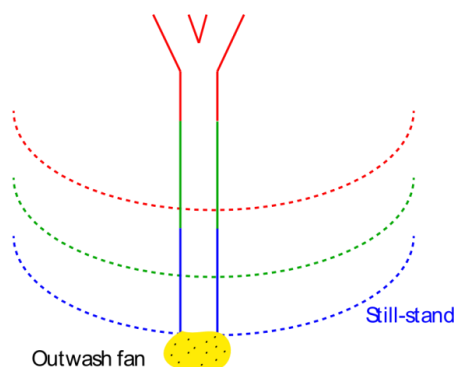
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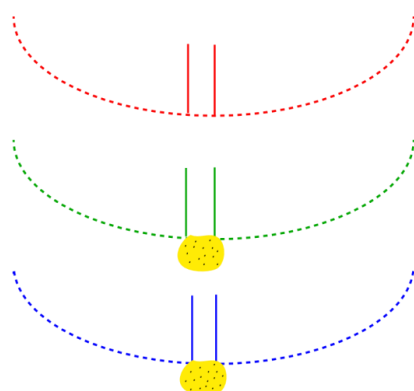
1019 **Figure 16:** Using cross-cutting relationships to reconstruct tunnel valley evolution during ice margin
 1020 retreat. **A.** Mapping of tunnel valleys and associated glacial bedforms in Wisconsin (Chippewa Lobe)
 1021 (from Fig. 11B). **B.** Reconstructed history of valley formation behind a back-stepping ice margin.
 1022 Note that some valleys were long-lived during deglaciation and some abandoned shortly after their
 1023 formation. The relative age relations help explain the variation in lengths between long continuous
 1024 tunnel valleys and those comprising short fragments.



A. Tunnel valley headward growth > ice retreat rate

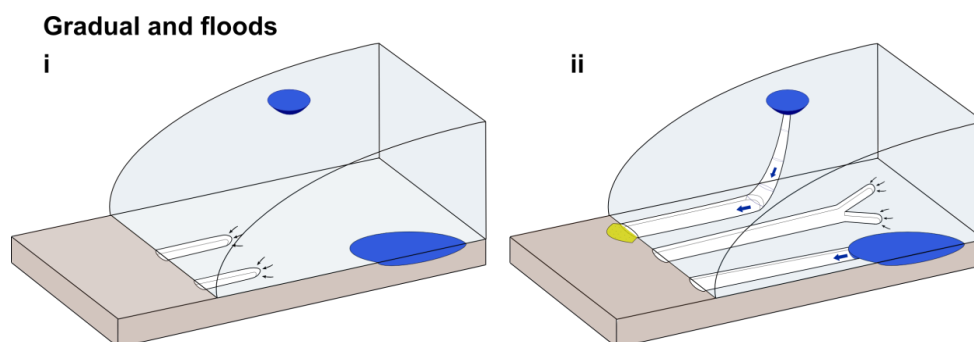


B. Tunnel valley headward growth < ice retreat rate



1025

1026 **Figure 17:** Cartoon demonstrating the dependence of tunnel valley evolution (by headward growth)
 1027 on ice margin retreat rate **A.** If headward growth of a tunnel valley is faster than the rate of ice retreat
 1028 the valley will be able to extend continuously up-glacier and its length will only be limited by water
 1029 supply and hydraulic properties of the bed. **B.** If however, headward growth of a tunnel valley is
 1030 slower than the rate of ice retreat the valley is likely to be discontinuous, only being able to form and
 1031 extend up-ice during slow-downs or pauses in retreat.



1032

1033 **Figure 18:** For the southern Laurentide region we consider gradual headward erosion as the usual
 1034 mechanism, but with some floods down selected valleys – note the potential for stored water to cut
 1035 their own valleys (e.g. supraglacial lake drainage example) or to drain along pre-existing corridors
 1036 that may have tapped into a reservoir (e.g. subglacial lake example).