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A reduced-complexity model for sediment transport and step-pool morphology

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Abstract. A new particle-based reduced-complexity model, $CAST$, to simulate sediment transport and channel morphology in steep streams is presented. $CAST$ contains phenomenological parameterizations, deterministic or stochastic, of sediment supply, bed load transport, particle entrainment and deposition in a cellular-automaton space with uniform grain size. The model can reproduce a realistic bed morphology and typical fluctuations in transport rates observed in steep channels. Particle hop distances, from entrainment to deposition, are well-fitted by exponential distributions, in agreement with field data. The effect of stochasticity both in the entrainment and in the input rate is shown. A stochastic parameterization of the entrainment is essential to create and maintain a realistic channel morphology, while sediment transport in $CAST$ shreds the input signal and its stochastic variability. A jamming routine has been added to $CAST$ to simulate the grain-grain and grain-bed interactions that lead to particle jamming and step formation in a step-pool stream. The results show that jamming is effective in generating steps in unsteady conditions. Steps are created during high-flow periods and they survive during low flows only in sediment-starved conditions, in agreement with the jammed-state hypothesis of Church and Zimmermann (2007). Reduced-complexity models such as $CAST$ can give new insight into the dynamics of complex phenomena (such as sediment transport and bedform stability) and be useful to test research hypotheses, being an effective complement to fully physically-based models.

1 Introduction

The morphodynamics of steep gravel-bed rivers is characterized by complex feedbacks between sediment supply and storage (e.g Hassan et al., 2008; Hassan and Zimmermann, 2012; Recking, 2012; Recking et al., 2012), bed load transport and flow resistance (e.g. Yager et al., 2007; Recking et al., 2008) and a rather stable bed morphology with a variety of bed surface structures (see reviews by Comiti and Mao, 2012; Rickenmann, 2012; Church and Ferguson, 2015). The traditional sediment transport capacity approach (Wainwright et al., 2015), which has been developed for low-land streams, performs poorly in steep fluvial systems. Among other reasons, in steep channels the threshold of motion varies with slope, local bed structures and antecedent flood events (e.g. Lamb et al., 2008; Turowski et al., 2011; Scheingross et al., 2013; Prancevic and Lamb, 2015), a power-law relation between fluid shear stress and sediment transport yields orders of magnitude differences between measurements and predictions (e.g. Rickenmann, 2001), and the presence of macro-roughness elements (such as boulders and

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log-jams), whose size is comparable with the water depth, makes calculations of flow resistance extremely complex (Yager et al., 2007; Schneider et al., 2015).

The step-pool morphology is commonly encountered in mountain catchments at slopes grater than 3% (Montgomery and Buffington, 1997; Comiti and Mao, 2012), where large boulders and often logs create channel-spanning structures called steps, with pools immediately downstream formed by the scouring effect of the tumbling water flow (see reviews by Chin and Wohl, 2005; Church and Zimmermann, 2007). Step-pool channels have been studied extensively in the last decades, in order to understand under which conditions they are formed, remain stable and are eventually destabilized (e.g. Abrahams et al., 1995; Curran, 2007; Zimmermann et al., 2010). This topic is still an open issue and, despite observations of a certain degree of regularity in step-pool geometry and metrics (e.g. Chartrand et al., 2011), it has been recognized both in the field (e.g Zimmermann and Church, 2001; Molnar et al., 2010) and with lab experiments (e.g. Curran and Wilcock, 2005; Zimmermann et al., 2010) that there is no single mechanism behind step formation and collapse (Curran, 2007). In fact, we are of the opinion that step location and stability should be treated as stochastic processes, highly dependent on the random location of the step-forming boulders usually referred to as keystones (Church and Zimmermann, 2007; Zimmermann et al., 2010). A hypothesis on step stability has been proposed by Church and Zimmermann (2007) and tested experimentally by Zimmermann et al. (2010). The authors suggested a similarity between step structures and granular phenomena by postulating that steps are inherently more stable than predicted by the Shields diagram because they are arranged in a jammed state, which occurs in granular flows (for a review on jamming phenomenon see Liu and Nagel, 2010). They proposed a diagram where the likelihood of finding stable steps in a channel is shown to be dependent on 3 parameters: (1) the jamming ratio (the ratio between the channel width W and the d_{84} of the surface), (2) the transport stage (the ratio between the applied shear stress τ and the critical shear stress τ_{cr}), and (3) the sediment concentration (the ratio between sediment supply Q_S and water discharge Q). So far this theory has been tested against lab experiments and field data (Zimmermann et al., 2010) but to our knowledge step formation and collapse have not yet been explicitly modelled. Understanding the conditions under which step-pool sequences are stable is of major practical importance because step collapses and consequent boulder mobilization can severely impact human infrastructures causing natural hazards (e.g. Badoux et al., 2014). Moreover, artificial step structures are often built in alpine rivers as energy dissipators and their stability needs to be carefully assessed.

Physically-based modelling of flow and sediment transport in steep mountain streams in mobile bed conditions is impractical because (a) the flow field over the rough bed is very complex; (b) single-grain mobility is impossible to solve; (c) long-term simulations are required to develop a dynamically changing channel bed. An alternative to fully physically-based modelling is that of reduced-complexity models. The reduced-complexity (RC) modelling approach has been applied successfully in fluvial geomorphology (Nicholas, 2005) as a learning tool to gain new insight into the temporal and spatial dynamics of complex systems with a simplified phenomenological approach (Goldenfeld and Kadanoff, 1999; Paola and Leeder, 2011). Since the classical cellular model of Murray and Paola (1994) which effectively captured the main patterns of river braiding, RC models have been used to describe geomorphic phenomena, such as riverbank failure (Fonstad and Marcus, 2003), bedrock cover (Hodge and Hoey, 2012), river avulsion (Jerolmack and Paola, 2007), sand dunes (Narteau et al., 2009), river deltas (Seybold et al., 2009; Liang et al., 2015), patterns of erosion-sedimentation (Crave and Davy, 2001; Chiari and Scheidl, 2015), transport

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in gravel bed-rivers (MacVicar et al., 2006) and landscape evolution (Coulthard and Wiel, 2007; Van De Wiel and Coulthard, 2010). Instead of solving differential equations of flow and sediment transport, RC models formulate physically-meaningful local-interaction rules with very few parameters in a cellular automaton space. They have been shown to be a flexible and powerful tool to model many geomorphic and physical processes (e.g. Rozier and Narteau, 2014; Tucker et al., 2015).

In this paper we present a new reduced-complexity stochastic model for step-pool streams based on grain-grain interactions: $CAST$ (Cellular Automaton Sediment Transport). $CAST$ simulates a generic fluvial channel on a cellular-automaton domain, where the bed is formed by an arrangement of particles like in a sandpile model (e.g. Bak et al., 1988; Kadanoff et al., 1989). The basic processes (e.g. bed load, particle entrainment and deposition) are treated at the grain scale, as suggested by recent studies identifying many analogies between bed load transport and granular phenomena (e.g. Frey and Church, 2009, 2011; Houssais et al., 2015). The stochastic framework of $CAST$ has two main reasons. First, the goal of the model is not to predict deterministically the morphology of a specific river reach but rather to capture new features and feedbacks related to its dynamics and, more specifically, to test research hypotheses on step formation and stability. Second, both bed stability (Zimmermann et al., 2010) and bed load transport (Einstein, 1937, 1950) have been recognized to be stochastic processes, and recent approaches to sediment transport have successfully followed this framework (e.g. Turowski, 2010; Furbish et al., 2012; Heyman et al., 2013; Ancey and Heymann, 2014; Armanini et al., 2015).

The paper objectives are: (a) to present a new reduced-complexity model, $CAST$, that simulates bed load transport and channel morphology at the grain scale and to test the effect of different parameters and stochastic forcing on the model outcomes; (b) to explore the effect of jamming on sediment transport and step formation, in comparison with the framework of the the jammed-state hypothesis of Church and Zimmermann (2007). The paper is organized as follows. First, the model rationale and parameters are presented. Second, the effect of different parameter sets on the model outcomes are explored in steady-state simulations, and the effect of the stochasticity on different variables is shown. Then, jamming is parameterized and its role on step formation and stability is explored in unsteady simulations. Finally the results are discussed and compared to the jammed-state hypothesis.

2 The Model

2.1 Model Rationale

$CAST$ operates in a 2-D cellular-automaton space, which is a rectangular grid of constant longitudinal length (X) and transversal width (Y) corresponding to a generic river reach (see Fig. 1). The domain is discretized such that all the dimensional quantities are expressed as multipliers of particle size d. The model developed in this paper works with uniform-size particles, so d is also equal to the dimension of a cell. For example, a simulation domain having $X = 300d$ and $Y = 20d$ represents a river reach with an average length equal to 300 median diameters and an average width equal to 20 median diameters.

Particles in the model domain can be either on the bed or in motion, i.e. part of the bed matrix or of the transport matrix (Fig. 1). The bed matrix Z is composed of particles piled one above the other like in a sandpile model (e.g. Bak et al., 1988). The local bed elevation at a generic location (i, j), where $i[1 : X]$ is the index for the longitudinal coordinate and $j[1 : Y]$ is

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that of the transversal coordinate, is given as the total number of particles $Z(i,j)$. Particles can leave the bed matrix as a result of entrainment and can enter the bed matrix as a result of deposition. These two processes are presented in detail in the next sections.

Particles in motion are allocated to the transport matrix TR, which consists of two layers, and they move as bed load, i.e. they are in contact with the bed and interact with it. They also interact with each other by collisions. Particles move with a constant velocity, one cross-section downstream for every time step. In this way particle velocity v_p, particle size d and time step Δt are connected:

$$v_p = \frac{d}{\Delta t} \tag{1}$$

Although in simulation Δt is a unitless time, Eq. 1 together with the grain and domain size give it a physical meaning connected to particle velocity.

Particles enter the system with a specified input rate I_R which is the number of particles entering the system for every time step, and they leave the system as output rate O_R, which is the number of particles leaving the system for every time step. In analyzing the spatial output of the model we consider only a reduced part of the domain which we will call hereafter the control volume, excluding the first 10 cross-sections upstream and the last 10 downstream, to avoid the influence of the upstream and downstream boundary conditions (see next sections).

2.2 Model Components

2.2.1 Sediment Input

The first parameter of $CAST$ is the particle input rate I_R or the specific input rate i_R, defined as the dimensional input rate I_R divided by the channel width Y; i_R can assume values in the interval $[0:1]$ because no more than one particle can enter a single cell at the upstream boundary in Δt. The supply of particles to the system can be treated as constant input by specifying a value of i_R for the entire simulation or as variable input by specifying a mean value $\overline{i_R}$ with a given random variability in time $i'_R(t)$. In $CAST$, i_R represents a generic amount of sediment which is delivered to the channel from all the possible sources (alluvial transport, colluvium activity, bank erosion, etc) rather than a specific transport rate in a given cross-section. The actual input to the system can be considered to be the transport rate measured in the first cross-section of the control volume.

2.2.2 Sediment Transport

Particles are transported as bed load along the channel with a constant velocity (see Equation 1). A particle can join the transport matrix TR when it enters the system as input or once it has been entrained from the bed. A particle can also leave the transport matrix TR when it moves beyond the last cross-section, becoming part of the sediment output O_R, or when it deposits on the bed surface. The maximum number of particles being transported from one cross-section to the next one is equal to $2Y$, i.e. 2 times the channel width, because the TR matrix has two layers.

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Particles move preferentially directly to the downstream cell (90% probability), with a small chance for lateral displacements (5% on the left and 5% on the right). This aims to represent a grain having a transport vector aligned with the dominant flow direction. Along its path, a particle can collide with another particle in transport or collide with one of the two boundaries (left and right banks). In both cases the collision leads to the cessation of motion and the particle deposits immediately on the bed. When a particle deposits on the bed, it changes the local roughness but without directly displacing other particles, i.e. the $CAST$ does not account for collective entrainment as described by Ancey and Heymann (2014).

Sediment flux in the model, q_S, is computed at the total number of particles in TR in the control volume divided by the domain size, i.e. $Y \cdot (X-20)$. This specific rate $q_S(t)$ is computed for every time step.

2.2.3 Particle Entrainment

The key process in $CAST$ is the particle entrainment which is considered to be dependent on the local topography and on the flow conditions. The degree of exposure of particles on the bed has been shown to strongly influence sediment entrainment and transport especially in steep streams (e.g Kirchner et al., 1990; Malmaeus and Hassan, 2002; Yager et al., 2012; Prancevic and Lamb, 2015). Moreover many feedbacks exist between bed roughness, flow resistance and particle mobility and transport (e.g. Recking et al., 2008; Wilcox et al., 2011) which makes it reasonable to consider particle entrainment as a stochastic process.

The effect of local topography is accounted by calculating the local degree of exposure of a particle on the bed, a proxy for the roughness of the bed surface. For a generic cell (i,j) in the domain, the local roughness $R(i,j)$ is given by the difference between the elevation of the cell $Z(i,j)$ and the average elevation of the neighboring cells (the 2 in the same cross-section and the 3 downstream):

$$R_{i,j} = Z_{i,j} - \frac{Z_{i,j-1} + Z_{i,j+1} + Z_{i+1,j-1} + Z_{i+1,j} + Z_{i+1,j+1}}{5} \tag{2}$$

In the case of a cell located close to one of the banks (i.e. when $j=1$ or $j=Y$), the local roughness $R_{i,j}$ is evaluated considering the cell in the same cross-section and the 2 cells downstream.

Entrainment is based on R exceeding a threshold R^* for entrainment. The probability of entrainment p_E is then defined as $p_E = P_r[R^* \leq R]$. $CAST$ can model the entrainment process as deterministic or stochastic (see Fig. 2). In the deterministic case, the threshold is a constant $R^* = E$ and therefore the probability of entrainment is:

$$p_E = P_r[R \geq E] = \begin{cases} 0 & R < E \\ 1 & R \geq E \end{cases} \tag{3}$$

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In the stochastic case, the threshold R^* is modelled as a random variable with a logistic distribution, with a probability density function $f(R^*)$ and a cumulative distribution function $F(R^*)$:

$$\begin{aligned} f(R^*) &= \frac{e^{-\frac{R^*-E}{S}}}{S\left[1+e^{-\frac{R^*-E}{S}}\right]^2} \\ F(R^*) &= \frac{1}{1+e^{-\frac{R^*-E}{S}}} \end{aligned} \quad (4)$$

The distribution has a mean $\mu_{R^*} = E$ and a variance $\sigma^2_{R^*} = \frac{\pi^2 S^2}{3}$. In this way the entrainment probability p_E depends on two parameters: the mean value of the threshold distribution E and the variability in the threshold R^* proportional to S:

$$p_E = P_r[R \geq R^*] = F(R) = \frac{1}{1+e^{-\frac{R-E}{S}}} \quad (5)$$

with R being the value of bed roughness of a given cell. The different cases are shown in Figure 2.

Conceptually, the value of E is inversely related to the magnitude of the flow. Large E means low probability of entrainment, typical of low flow conditions, while small E means high p_E for the same roughness values and so high flow conditions.

2.2.4 Particle Deposition

Particles in transport (i.e. belonging to the TR matrix) can deposit because of their interaction with the bed surface. The relation between particle deposition and bed surface is modeled using the roughness matrix R: particles in motion will deposit in areas of local depressions (i.e., cells with negative values of R). The deposition process is treated as deterministic, i.e. with a threshold function having a fixed value of $D = -0.5$ below which the probability of deposition $p_{dep} = 1$. This simplification allows from one hand to avoid redundant parameters poorly connected to physical processes, and on the other hand to shift the variability in sediment transport to the entrainment process which has a more straightforward relation with hydraulics and local channel bed topography.

2.2.5 Boundary Conditions

$CAST$ needs one boundary condition for the lateral banks and one for the downstream boundary at the channel outlet. The boundary condition for the banks is deposition when a moving particle in the TR layer collides with one of the two lateral boundaries. The boundary condition for the last cross-section at the downstream boundary is given by fixing its elevation $Z(X,j) = 0$. This is equivalent to a control section with any fixed elevation (e.g. a check dam or a weir) somewhere downstream. To minimize the effect of this boundary condition on the model outcomes, all spatial variables are computed only over the control volume, which is a reduced portion of the entire channel without the 10 cross-sections of the domain closer to upstream and downstream ends of the channel.

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2.3 Rough-Bed and Jamming Options is $CAST$

$CAST$ can operate in two conditions, by considering or not the effect of dynamic jamming. The model without a jamming threshold ($CAST_{RBM}$) simulates a generic rough-bed channel where processes of transport, entrainment and deposition are considered regardless of any additional granular effect (except for particle collisions and deposition after collisions with the channel banks). The model with the jamming threshold ($CAST_{JM}$) simulates explicitly the process of jamming (blocking) when too many particles are traveling together (i.e. in the same cross-section). In this case particle interactions lead to deposition of all grains on the bed. This blocking process is considered permanent, i.e. the jammed particles are locked into channel width spanning structures which cannot be entrained anymore. Intuitively and similarly to other phenomena where jamming is common (e.g. in hoppers) we set up the jamming threshold equal to the channel width Y. In a one grain-size model like $CAST$ this implies that jamming is happening when the transport layer is full of particles (one entire cross-section full of transported particles).

2.4 Model Operation

For every time step, $CAST$ operates as follows: (1) Sediment input enters the system in the first cross-section. (2) Particles in transport move one cell downstream (straight, left or right), if they collide with other particles or with one of the banks they deposit, otherwise they remain in transport. In the case of $CAST_{JM}$ the jamming condition is checked for every cross-section. When the number of particles traveling in the same cross-section exceeds the jamming threshold all grains are deposited on the bed and frozen. (3) For every particle in motion the condition for deposition is checked: if it is satisfied the particle leaves the transport matrix. (4) For every particle in the bed matrix (except for those jammed in stage 2 and deposited in stage 3) the condition for entrainment is checked: if it is satisfied the particle leaves the bed matrix and joins the transport matrix. (5) The boundary condition at the channel outlet is applied.

3 Results

3.1 Steady-State Simulations

First, we show a set of simulations using $CAST_{RBM}$ with (a) stochastic entrainment with constant E and S parameters, and (b) constant specific input rate i_R. The domain is $Y = 20d$ and $X = 300d$. The simulations were run until a steady state was reached. This condition is achieved when for a given combination of i_R and E the channel slope does not change, i.e. the point at which the particle count (stored sediment volume) in the channel reaches a steady state and the sediment output is on average equal to the input. In the same simulation both the input rate i_R and the entrainment parameter E were kept constant; however, these two parameters where changed in different simulations, to explore their effect on the final bed structure. Moreover, since $CAST$ is a stochastic model, we perform 20 realizations for every set of parameters: in this way it is possible to detect which parameters are more sensitive to the stochastic forcing. The set of parameters used in these steady-state simulations is shown in Tab. 1.

The results of the model are analyzed in terms of:

- storage volume V: the total number of particles in the bed matrix. The time series of this variable indicates if the channel is in a phase of aggradation (V increasing in time), degradation (V decreasing in time) or equilibrium (V constant in time). It also indicates massive sediment evacuation events;
- specific sediment flux q_S: the number of particles in motion per unit length and unit width in the control volume;
- mean bed roughness $< R >$: the spatially-averaged value of the bed roughness of all the cells in the control volume domain evaluated with Eq. 2;
- standard deviation of the bed roughness σ_R: standard deviation of all the roughness values evaluated with Eq. 2;
- particle hop distance HD: the step length of a single particle from the point it is entrained (or enters the channel) to the point it is deposited or exits the channel.

3.1.1 Storage Volume, Sediment Transport, Bed Morphology, and Hop Distances

The main outputs of $CAST_{RBM}$ in a steady-state simulation ($E = 1.5$; $i_R = 0.5$) at equilibrium are shown in Fig. 3. First, looking at the time series of the storage volume (Fig. 3a) it can be seen how the equilibrium is a dynamical condition rather than a static one: the volume oscillates, alternating phases of aggradation and degradation. Second, the times series of sediment transport (Fig. 3b) show that even with a constant input rate, sediment transport fluctuates as observed in the field and in the lab (e.g. Recking et al., 2009; Saletti et al., 2015). Moreover, $CAST_{RBM}$ produces a realistic rough-bed morphology (see for example Fig. 3c), with the average roughness and its standard deviation being a function of the input rate i_R and the entrainment parameter E.

One of the advantages of a RC models like $CAST$ is that it is possible to track the movement of every single particle in the system and so to compute all particle step lengths (measured from start to stop, from entrainment to deposition). This is an important quantity which, since Einstein's probabilistic theory on bed load transport (Einstein, 1937, 1950), needs to be reproduced by any reliable particle-based transport model. Since the word 'step' in this paper refers to the bed structures created by grain arrangements, for the sake of clarity we will call hereafter, following Furbish et al. (2012), particle hop distances (HD) what in literature is usually called step length, i.e. the distance travelled by a particle from entrainment to deposition (Fig. 3d) . For every simulation we computed values of HD for all the particles and we find they are well fitted by an exponential distribution. In Fig. 4 we show for 4 different combinations of i_R and E the probability density functions of the data and the exponential fit. The good fit given by the this distribution is in agreement with previous studies dealing with particle travel distances (e.g. Hill et al., 2010; Hassan et al., 2013; Schneider et al., 2014). Since in the model no specific HD distribution is specified a priori, the agreement between $CAST$ hop distance distributions and travel distances observed in the field and in flume experiments shows that the phenomenological rules of particle entrainment, transport, and deposition of $CAST$ are realistic.

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3.1.2 Effect of Input Rate and Entrainment Probability

With the steady-state simulations we explored the effect of changing input rate and entrainment parameter on the model outcomes. These two parameters are important because they can be linked to the jammed-state diagram parameters of Church and Zimmermann (2007). The input rate I_R is related to the sediment concentration $\frac{Q_S}{Q}$, which quantifies the effect of sediment supply on step stability. The entrainment parameter E determines the entrainment probability and therefore it is directly related to the transport stage $\frac{\tau}{\tau_{cr}}$, which quantifies the effect of the hydraulic forces on step stability.

Some of the 30 simulations, characterized by low input rate and high entrainment probability ($E = 1$, $i_R < 0.4$), yield what we call 'washed-out' case, i.e. the bed matrix remains empty. This represent a limiting case where hydraulic forces are too high and sediment supply is too low to be able to sustain a fluvial channel. Moreover, this constrains our parameter space. In the remaining 27 simulations a channel was formed and maintained around an equilibrium point.

The stochastic simulation of 20 realizations of each of the 27 parameter sets showed that the mean storage volume $\overline{V}$ and the mean bed roughness $< R >$ converged to the same values. Also the mean hop distances $< HD >$ and the standard deviations of the bed roughness σ_R did not change significantly.

Specific sediment flux $q_S(t)$ is on average equal to the input rate i_R at equilibrium, but fluctuating around its mean value, as shown in Fig. 3b. The degree of memory of these fluctuations is captured by the Hurst exponent H_{q_s}, whose mean value for the steady state simulations is shown in Fig. 5c. The values of H_{q_S} obtained in all the realizations of all the simulations are consistent with those obtained from flume experiments by Saletti et al. (2015), being in the interval $[0.5 : 1]$. This identifies a long-memory regime which is stronger in the model ($H \to 1$) when the entrainment probability is high (low E) and the input rate is low. H_{q_S} shows large variability in different realizations, although its mean value shows a clear trend with both i_R and E (see Fig. 5c).

The values of 4 variables for the 27 simulations (parameter combinations), averaged over the 20 realizations, are shown in Fig. 5. The mean bed roughness ($< R >$ in Fig. 5a) is obtained by a spatial average of all the values of bed roughness for a given time step, and then temporally averaged over the last 20000 time steps (in the equilibrium phase). $< R >$ is directly related to the slope of the channel and the storage volume. It increases with increasing input rate and increasing entrainment parameter: channels with larger $< R >$ and larger storage volumes are those with large sediment supply and low entrainment probability. The same trend can be inferred by looking at the mean standard deviation of bed roughness (σ_R Fig. 5b), also obtained as a spatial average over the equilibrium phase for every time step.

The mean particle hop distances ($< HD >$ in Fig. 5d) display two contrasting trends. For values of $E \leq 1.5$ (high entrainment probability) $< HD >$ is decreasing consistently for increasing input rates i_R, as a consequence of large particle activity (collisions between particles become very frequent). For larger values of E (low entrainment probability) the maximum of $< HD >$ is for average values of i_R (around 0.4). For $i_R < 0.4$ there is a stronger interaction with the bed (which is rougher for larger E, as shown in Fig. 5a,b) and so more likelihood of particle deposition, whereas, for large i_R, $< HD >$ collisions again dominate because of large particle activity. In both cases this leads to a reduction of $< HD >$.

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3.2 $CAST_{JM}$: Particle Jamming

$CAST_{RBM}$ is able to reproduce fluvial channels with a spatially variable rough bed and to capture basic and important physical phenomena, such as the variability in sediment flux and the exponential distribution of particle hop distances. However, to simulate the formation and test the stability of steps, we need to take into account the effect of particle jamming. This is a well-studied phenomenon in granular physics that has been advocated to be essential in the step stability process and considered through the jamming ratio (the ratio between the channel width and d_{84} of the surface) in the diagram proposed by Church and Zimmermann (2007). In our reduced-complexity model $CAST_{JM}$ we account for the jamming effect by imposing that when too many particles are being transported together in the same cross-section (i.e. when local sediment concentration exceeds the jamming threshold) they are instantly deposited and frozen in that cross-section. This aims to represent the chain forces that originate at the granular scale being responsible for the enhanced stability of step structures observed in the field. In $CAST_{JM}$, since particles all have the same dimension, the jamming threshold is set equal to the channel width for geometric reasons.

Simulations were run with the same parameter sets of the steady-state case. Three different situations occur:

1. When particle activity is too low (low sediment transport) the jamming threshold is rarely (often never) exceeded.

2. When particle activity is too high (high sediment transport) jamming is occurring too often in time and space, and the storage volume of the system keeps increasing because of the large amount of particles depositing upstream of the step structures. As a result, an equilibrium channel is never reached.

3. When particle activity is in between the two previous situations, jamming is occurring at a rate which allows the formation of steps and maintains an approximately equilibrium channel.

While the first situation is trivial per se, the second one represents a case which is very unlikely to happen in river systems where fluvial sediment transport is rarely going to exceed the jamming threshold and certainly not for very long periods of time (e.g. only during large flood events). For the purpose of this study we focus on the last situation, where jamming is effectively creating steps.

We show the effect of adding jamming to the model by comparing simulations having the same parameter sets and same initial conditions ($i_R = 0.5$ and $E = 1.25$) in $CAST_{RBM}$ and $CAST_{JM}$ runs. The cumulative number of jammed cross-sections shows that jamming is a rather intermittent phenomenon with many long periods of no jamming (Fig. 6a). At the end of this simulation 29 cross-sections were jammed (around 10% of the total). The longitudinal profiles of bed elevation (Fig. 6b) show how $CAST_{JM}$ is able to create step structures and this increases the total slope of the channel and its storage, even if the slope between steps is the same as in the case without jamming ($CAST_{RBM}$). The boxplots of the instantaneous (i.e. calculated for every time step) values of bed roughness, both the mean $< R >$ (Fig. 6c) and the standard deviation σ_R (Fig. 6d), show that the model with jamming yields a rougher and more variable bed. Instantaneous values of specific sediment flux q_s (Fig. 6e) show that jamming increases the variability in q_S and also that the equilibrium condition (which would imply $q_S \simeq i_R = 0.5$) is not yet reached because the system is still aggrading and increasing its storage. Finally the values of the Hurst exponent of sediment flux H_{q_S} (Fig. 6f) for the 20 realizations clearly plot separately in the case with and without jamming,

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with the latter having larger values. This longer-term memory is likely due to a combination of step collapse and the weak but present trend towards aggradation in the $CAST_{JM}$ simulations.

3.3 The Effect of Stochasticity

In $CAST$ the processes of particle entrainment and sediment supply can be parameterized as deterministic or stochastic. In the simulations presented in the previous sections we used a stochastic parameterization for particle entrainment and a constant sediment supply. To explore the effect of stochasticity on the model results, in the next two sections we quantify the effect of stochasticity in entrainment and sediment supply explicitely.

3.3.1 Stochasticity in the Entrainment

The entrainment probability in $CAST$ can be parameterized as a deterministic or stochastic process (Section 2.2.3). A stochastic parameterization allows a degree of variability in the entrainment threshold and can be controlled by few parameters (E and S as a fraction of E), while the deterministic parameterization has a unique entrainment threshold E (Fig 2).

The comparison for a simulation with $i_R = 0.5$ and $E = 1$ is shown in Figure 7. When the entrainment process is treated as stochastic, the variability of sediment flux is much larger both in case with and without jamming (Fig. 7a). This is due to the fact that when the channel has reached an equilibrium in the deterministic case the interaction between the bed and the transport is very low. All particles below the threshold stay on the bed and those above the threshold are entrained. The reduced particle activity can be inferred also by looking at the distribution of particle hop distances (Fig. 7b). In the deterministic case the distributions are shifted towards larger values because particles interact much less with the bed and travel further downstream. Interestingly, the effect of jamming seems to reduce sediment transport in the stochastic entrainment case. The effect of modeling the entrainment as a deterministic process on the bed morphology itself is that the final configuration of the channel in the threshold case is much steeper (cumulative distribution functions of R plot towards larger values in the T-case) and this is due to the fact that, since no entrainment is possible below the threshold, the channel can bear steeper slopes and store more sediment (Fig. 7c). However, this does not translate into a rougher surface; the standard deviation of R shows that channels where the entrainment is modeled with a threshold function have very low variability around the mean roughness: they tend to look more like steep and uniform ramps then like realistic fluvial channels (Fig. 7d).

This analysis support our choice of modeling the entrainment as a stochastic process. Despite the fact that this is more physically reasonable (the process of particle entrainment is random per se), in a reduced-complexity model like $CAST$ without a stochastic parameterization of particle entrainment it is not possible to obtain a realistic rough-bed morphology.

3.3.2 Stochasticity in the Input Rate

The effect of stochasticity in the input rate i_R is shown on simulations having $E = 1.5$ and constant $i_R = 0.5$, and with a variable input rate $i_R = < i_R > + i'_R$, where the mean value $< i_R > = 0.5$ and fluctuations $i'_R = \pm 0.5$, i.e. the input rate is a stochastic signal with values between 0 and 1 and a mean of 0.5.

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The effect of stochasticity in the input rate is much smaller than that of entrainment. The distributions of sediment flux (Fig. 8a) almost overlap both in the case with and without jamming. Like in the entrainment case, jamming causes a constant increase in sediment storage (and so the median transport rate is slightly below the equilibrium value of 0.5). An overlap of the 4 simulations is also observed when looking at the distributions of particle hop distances and final bed roughness (Fig. 8b and c). The simulations with and without jamming plot separately only in the case of the standard deviation of bed roughness (Fig. 8d). The larger values of σ_R in the case of variable i_R and jamming are due to the fact the variability in the input facilitates the jamming process, because even if $< i_R >= 0.5$, there are around 50% of time steps having $i_R > 0.5$ and this leads to more jamming and increases the bed roughness.

The results highlight how the variability and the fluctuations observed in the output variables of the model do not depend on the variability of the input, but are instead function of the internal dynamics of the system, given by the local grain-grain and grain-bed interactions. $CAST$ acts as a shredding filter of the input forcing (Jerolmack and Paola, 2010; Van De Wiel and Coulthard, 2010).

3.4 Unsteady Simulations

Although jamming is effective in generating a step-like morphology under certain input and entrainment conditions, we recognize that step formation is an intermittent process in which flow variability in time is important. Typically step-pool sequences are partially or totally destroyed during large flood events and then reworked and stabilized during the following low flows periods (e.g. Lenzi, 2001; Turowski et al., 2009). We show the effects of changing flow conditions by simulating 4 consecutive floods of equal magnitude (Fig. 9a). In $CAST$ the hydraulic conditions are represented by the entrainment parameter E. Therefore, to simulate a change in the flow, we modify the value of E to represent two extreme cases (Fig. 9b): low flow with $E = 2$ (low entrainment probability) and high flow with $E = 1$ (high entrainment probability). Moreover, we explore two different situations: (1) we keep the input rate i_R constant, incorporating all the effects of the unsteadiness in the entrainment parameter E (Case I in Fig. 9c), and (2) we change also the input rate i_R in response to changes in the flow conditions (i.e. Case II in Fig. 9d). To facilitate the comparison, the total sediment input over the entire simulation is the same in Case I and II. To mimic the rising and falling limb of an hydrograph, both E and i_R were increased and decreased gradually. The relevant parameters of these unsteady simulations are summarized in Tab. 2. Runs were performed both with and without jamming (i.e. with $CAST_{RBM}$ and $CAST_{JM}$), to check if and when steps are formed and how many of them remain stable.

3.4.1 Sediment Storage and Sediment Transport

The temporal pattern of storage volume and sediment flux in the unsteady simulations is shown in Fig. 10. The volume for the rough-bed case (in blue) clearly displays phases of degradation during high flow and aggradation during low flow. Without jamming the channel tends to empty during high flows when the entrainment probability is high and to gain sediment again during low flow when the entrainment probability decreases. This turnover is more evident when the input rate is constant (Fig. 10a). With the effect of jamming the picture changes. During high flows the mobile grains are trapped in the channel in steps, while during low flows the channel increases its storage because of grain deposition between steps and channel infilling. With

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a variable input (Case II), jamming creates more steps and increases the storage volume which then remains constant during the following low flow phases because of the reduced input rate and low entrainment probability.

The specific sediment flux when the input is constant (Fig. 10c) shows a large variability for the rough-bed case responding to changes in E during low and high flow conditions. In the jamming case, the response to the change in flow conditions is also present but the jamming process modulates the sediment flux towards the equilibrium conditions rapidly. When the input varies with flow conditions (Fig. 10d), the rough-bed model yields the same pattern of the case with constant input with the difference that here the equilibrium condition is different during low and high flow (0.3 and 0.6 respectively). In the jamming model instead, the sediment flux is almost instantly in equilibrium with the input rate during low flows, while during high flows, the large input rate, together with the high entrainment probability, causes so many jamming events that inhibit the system to reach the equilibrium point of 0.6 and the channel keeps increasing its storage. We show the statistical distributions of specific sediment flux for the 4 cases in Fig. 11. It can be seen that the distributions are centered around the equilibrium point of 0.4 (especially the jamming case in red for Case I), with the rough-bed model having a more spread function due to the more intense phases of aggradation and degradation. The distributions of Case II (dashed lines) are clearly bimodal because of the two equilibrium sediment input rates (0.3 and 0.6).

3.4.2 Bed Roughness and Step Formation

The unsteady flow also has impacts on the development of bed roughness in $CAST$. The time series of the standard deviation of bed roughness σ_R for the unsteady simulation with constant and variable input rate is shown in Fig. 12. In both cases jamming produces a rougher surface during high flow which is a indication that step structures, causing a larger departure from the mean bed roughness, are being formed. When the input is constant (Fig. 12a), σ_R goes back to value of low flow for all the 4 floods, because steps that were formed are being buried by sediment. When instead the input rate is reduced during low flow to simulate sediment-starved conditions (Fig. 12b), σ_R decreases but not to its pre-flood value because many of the steps created during high flow can survive and do not get buried in between floods.

The same can be inferred by looking at the longitudinal profiles of bed elevation of the simulations with jamming (Fig. 13). At the end of every high-flow period, the longitudinal profile shows a stepped morphology due to jamming. In the following low-flow periods the steps were buried in Case I (having input rate $i_R = 0.4$), while in Case II (having a lower input rate during low flow: $i_R = 0.3$) some of them survived because of the sediment-starved conditions.

To quantify this effect in a meaningful way and to compare the different simulations, we introduce step density d_S, defined as the ratio between the number of cross-sections with steps and the total number of cross-section of the channel. The variable d_S can vary between 0 when no steps are present in the channel, and 1 when all the channel morphology is made by steps. The definition of a step is not straightforward, even in the field and in the lab where many different identification algorithms have been proposed (e.g. Milzow et al., 2006; Zimmermann et al., 2008). Since our goal here is not to identify and count the number of steps or to test which step identification algorithm works best, we simply define a step in terms of local departure from the equilibrium channel slope, similarly to the method of Milzow et al. (2006). The steady-state simulations give us the value of the final slope at equilibrium for a given set of parameters (E and i_R). We define that a cross-section in $CAST$ has a

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step if its local slope is greater than the equilibrium slope by a factor β. The time-series of step density evaluated in this way is shown for different values of β in Fig. 14. The temporal pattern of step density variations is largely independent of β. Two conclusions can be drawn. First, there is a clear difference between simulations with and without jamming in that jamming is clearly responsible for step formation, and without it there are practically no steps formed in the channel (blue and red lines in Fig. 14). Second, after steps are generated during high-flow periods due to jamming, they only survive during low flow if the sediment supply decreases (yellow lines in Fig. 14), in sediment-starved conditions. This illustrates the temporal dynamics of step counts as observed in the field (e.g. Molnar et al., 2010) and in flume experiments (e.g. Curran and Wilcock, 2005).

4 Discussion

4.1 Bed load: a Stochastic, Granular and Shredding Phenomenon

The $CAST$ model without jamming, $CAST_{RBM}$, simulates bed load transport over a rough-bed at the grain scale, considering particle entrainment as a stochastic process driven by a local resistance field. Our model describes bed load from a granular prospective because local granular effects in particle mobility and transport are key for developing the morphology, especially in steep and well-structured streams. Kirchner et al. (1990) pointed out the role of granular interactions between gravel particles on a river bed and proved how the erodibility of a grain is controlled by its protrusion and friction angle; given the associated high variability, they also suggested that, instead of using one single value for the shear stress, a probabilistic approach should be applied. The role played by particle interlocking and partial burial in increasing measured friction angles in steep channels has been shown recently also by Prancevic and Lamb (2015). Many laboratory studies have also increased our knowledge of bed load transport exactly by looking at the granular scale (e.g. Lajeneusse et al., 2010; Houssais et al., 2015), suggesting that we might be more successful in describing this phenomenon when borrowing concepts from the granular physics community (e.g. Church and Zimmermann, 2007; Frey and Church, 2011).

$CAST$ assumes a stochastic description of sediment transport, following and corroborating recent research (Furbish et al., 2012; Roseberry et al., 2012; Heyman et al., 2014; Ancey and Heymann, 2014). Our model produces fluctuations in transport rates by the interaction with the bed through entrainment and deposition of individual particles (Ancey and Heymann, 2014) and this fluctuations are observed in our simulations even with a constant input forcing. What in our model is defined as specific sediment flux q_S is equivalent to the particle activity as defined in Furbish et al. (2012), and the finding of Roseberry et al. (2012) that changes in transport rates with changing stress are dominated by changes in activity and not velocity justify our choice of assuming constant particle velocity but varying entrainment threshold in our simulations.

The stochastic parameterization of $CAST$ does not assume a priori any probability distribution for particle hop distances, and yet they turn out to be well fitted by an exponential distribution, in agreement with previous theoretical and field studies (e.g. Hill et al., 2010; Hassan et al., 2013; Schneider et al., 2014). The fact that, despite its simplicity, the model can reproduce in a robust way this important feature, proves at least partially that the local grain-grain and grain-bed interaction rules in $CAST$ are appropriate and that the phenomenological descriptions of the simulated phenomena are going in the right direction.

Finally, $CAST$ reproduces also the shredding effect sometimes visible in sediment transport (Jerolmack and Paola, 2010). The measured variability of sediment flux and its fluctuations are dictated by the internal dynamics of the system and the degree of fluctuations in the input forcing does not always affect the sediment flux in a clear way (see Fig. 8). Our results then show the RCM potential of modeling of bed load transport as a stochastic phenomenon at the grain scale. Interactions between individual particles can give rise to, or at least strongly impact, the variability observed in natural fluvial systems. Reduced-complexity models like $CAST$ can serve to model these interactions and their effects and can be used to gain new insights into the complex dynamics of sediment transport and to test new research hypotheses.

4.2 Step Formation and Stability: a Granular Problem

We showed with $CAST_{JM}$ that dynamic jamming of particles in movement is effective in forming steps (see Fig. 13). In fact only by including the jamming process we did generate step-pool like morphologies in our numerical experiments. Moreover, once steps are formed, they remain stable if the flow conditions change (i.e. the entrainment probability decreases) and the supply of sediment is low enough to avoid that these steps are buried by particles (as shown in Fig. 14). These results are consistent with the main ideas of the jammed state hypothesis of Church and Zimmermann (2007) who theorized and showed experimentally (Zimmermann et al., 2010) that step stability needs (a) jamming, expressed as a low width to diameter ratio so to enhance granular forces, (b) low flow stage, in order to avoid the mobilization of keystones, and (c) sediment-starved conditions, because a too high sediment concentration would bury the steps. Despite its simplifications, especially uniform sediment and no explicit flow parametrization, $CAST_{JM}$ can reproduce these observations and support the jammed state hypothesis for step stability.

4.3 Outlook

Our modeling approach has by definition some simplifications and limitations which we think can be improved in future research.

First, the uniform size of the sediment prevents us to model specifically any grain-size effect that might indeed be very important in steep-channel dynamics. We partially incorporated these effects in the stochastic parameterization of entrainment: the fact for the same value of local roughness R and entrainment parameter E some particles are displaced and some are not accounts also for differences in their dimension and weight. Also the jamming process may be grain-size dependent, as well as the particle velocity.

Second, the parameterization of changing flow conditions is done indirectly, summarized entirely in the entrainment parameter E. This is done mainly because we are not aiming to model discharge, flow, shear stress on the bed, but rather transfer their effects onto the probability of entraining grains. However, future improvements of $CAST$ could include a more direct relation between hydraulic stresses on the bed and the E and S parameters in our model.

Third, the granular interactions (i.e. collisions) among particles always lead to deposition, which might not always be realistic, at least in the real case with particles having different sizes and shapes. The same can be said about interactions with the banks of the channel. However, in a uniform-size case this assumption does not seem to be too strong.

In conclusion, in our opinion the strongest limitation of the current model is the absence of sediment-sorting and other grain-size effects. All these phenomena will be incorporated in the next version of the model which will have different grain-size fractions.

5 Conclusions

We presented a new particle-based reduced-complexity model $CAST$, Cellular Automaton Sediment Transport, that simulates bed load transport and changes in channel morphology including the processes of jamming and step formation. The model works with uniform-size particles and can have stochastic or deterministic parameterizations for sediment input rate and particle entrainment. With only few parameters, it it possible to simulate channels with different sediment supply and flow conditions. At steady state, $CAST$ can reproduce a realistic bed morphology and typical fluctuations in transport rates, whose memory features are consistent with previous experimental data. Moreover, particle hop distances are well-fitted by exponential distributions, in agreement with field observations.

One of the main results is the role played by stochasticity both in the entrainment and in the input rate. A stochastic input rate does not change the final outcome of the model compared to a constant input having the same mean. However, if the entrainment is modeled deterministically, the resulting channel does not have the typical variable bed roughness encountered in real fluvial systems.

The dynamical effect of particle jamming was added, to test under which conditions steps are formed and remain stable in steep channels. This has been tested in unsteady simulations, where the entrainment probability and the input rate have been changed to simulate a sequence of high-flow and low-flow periods. $CAST$ generates step structures during high-flow periods and they survive during low flows in simulations with sediment-starved conditions, in agreement with the jammed-state hypothesis.

Our results support the jammed-state hypothesis as a framework to explain step formation and stability and, more in general, they show the potential of reduced complexity-models at a grain scale with stochastic parameterizations. Models such as $CAST$ can give new insight into the dynamics of complex phenomena like sediment transport and step formation and be useful to test research hypotheses in fluvial geomorphology.

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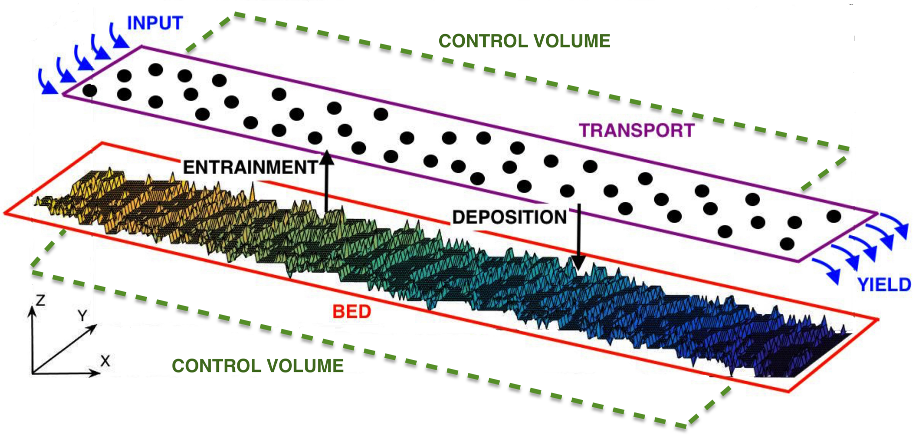

Figure 1. Sketch of the model. The space is discretized in a longitudinal dimension X and a transversal dimension Y. Bed elevation is accounted in the coordinate Z. Particles can be either in the bed matrix or in the transport matrix. They can enter the transport matrix as sediment input from the upper boundary or by entrainment from the bed, while they can leave the transport matrix as sediment output or by deposition on the bed. Sediment in input at the upstream boundary and simulated as sediment yield leaving the downstream boundary.

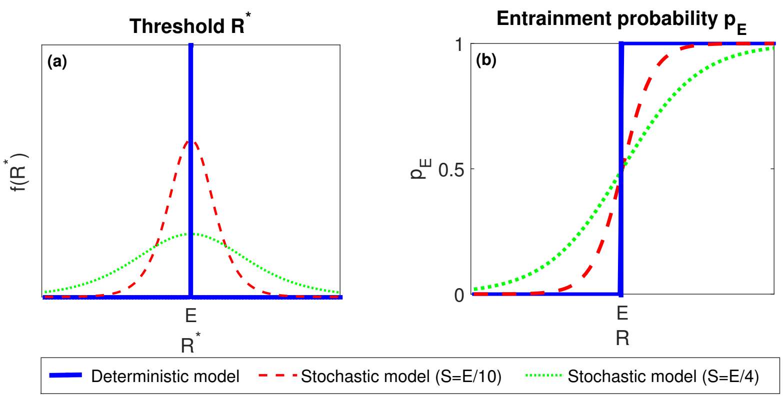

Figure 2. Entrainment deterministic and stochastic parameterization in $CAST$. (a) The probability density function of the threshold R^*. (b) Entrainment probability as a function of bed roughness R.

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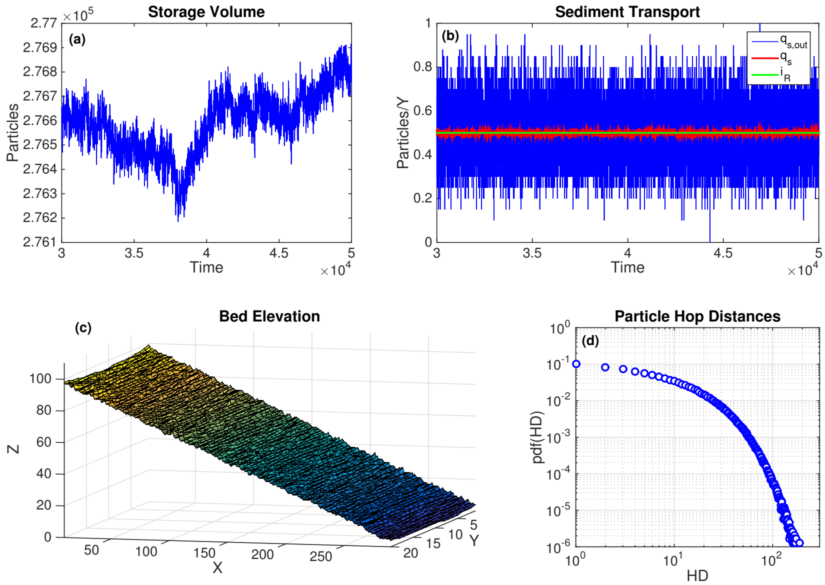

Figure 3. $CAST_{RBM}$ steady-state results of simulation with $E = 1.5$ and $i_R = 0.5$. (a) Time series of Storage Volume: adjustment phases of aggradation and degradation around the equilibrium condition. (b) Time series of Sediment Transport: even with a constant input rate (green line) the sediment flux fluctuates. Fluctuations are large if measured in a single cross section at the downstream end (blue line), but become smaller if averaged over the entire control volume of the channel (red line). (c) Bed elevation in the final configuration: the model produces a rough bed with particles having different exposures R. (d) Probability Density Function of particle hop distances with an exponential distribution estimated over more than 2 millions of values of simulated HD.

Table 1. Values of the parameters used in the steady-state simulations

Parameter	Name	Value(s)
Channel length	X	300 d
Channel width	Y	20 d
Simulation duration	T	20000
Deposition parameter	D	0.5
Shape parameter	S	$\frac{E}{5}$
Entrainment parameter	E	$1 \div 2$
Specific input rate	i_R	$0.2 \div 0.7$

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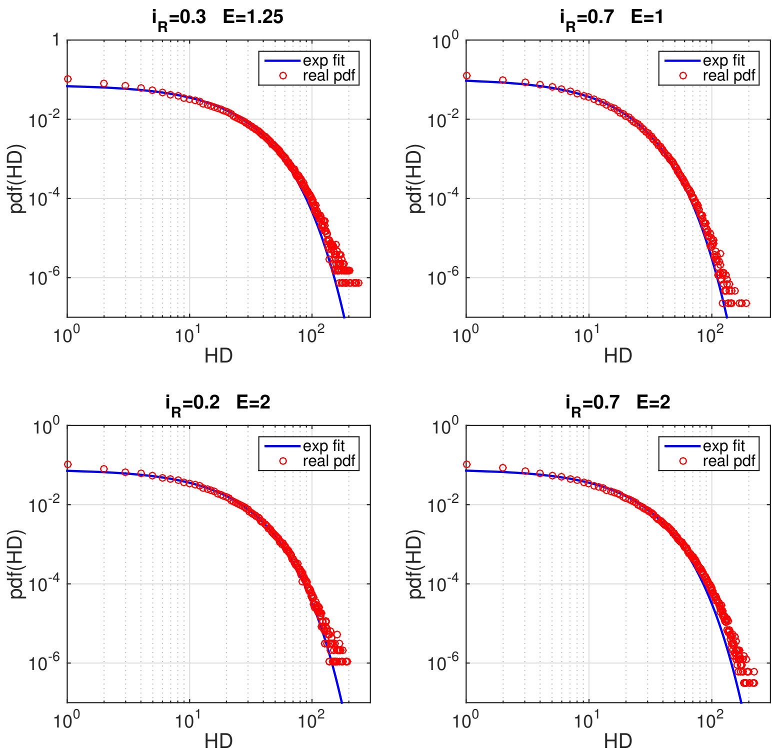

Figure 4. Probability density functions of simulated particles hop distances (red dots) fitted with an exponential distribution (blue line) for 4 different parameter sets.

Table 2. Values of the parameters used in the unsteady simulations.

Parameter	Name	Case I	Case II
Channel length	X	300 d	300 d
Channel width	Y	20 d	20 d
Simulation duration	T	550000	550000
Low flow duration	T_{low}	70000	70000
High flow duration	T_{high}	70000	70000
Rising limb duration	T_{ris}	2000	2000
Falling limb duration	T_{fal}	8000	8000
Entrainment parameter (low flow)	E_{low}	2	2
Entrainment parameter (high flow)	E_{high}	1	1
Specific input rate (low flow)	$i_{R,low}$	0.4	0.3
Specific input rate (high flow)	$i_{R,high}$	0.4	0.6

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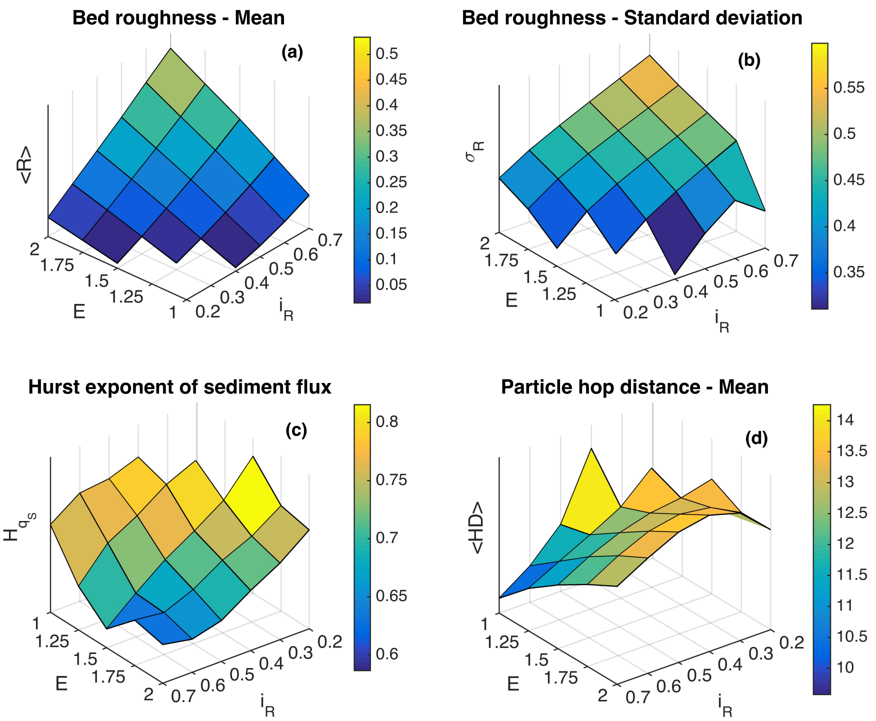

Figure 5. $CAST\_RBM$, variables trends as a function of input rate and entrainment: (a) Mean bed roughness $< \Delta R >$, (b) Standard deviation of bed roughness $\sigma_{\Delta R}$, (c) Hurst exponent of the specific sediment flux H_{q_S}, and (d) Mean particle hop distance $< HD >$.

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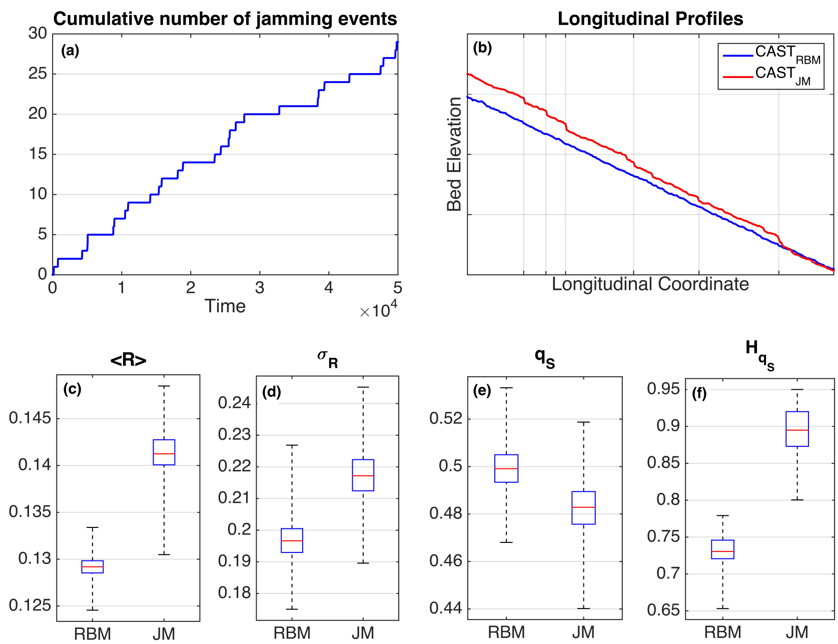

Figure 6. Comparison between simulations without jamming (RBM) and with jamming (JM). (a) Cumulative number of jammed cross-sections. (b) Longitudinal profiles at the end of the simulations. (c) Box-plots of the instantaneous values of mean bed roughness. (d) Boxplots of the instantaneous values of standard deviation of bed roughness. (e) Boxplots of the instantaneous values of specific sediment flux. (f) Box-plots of the values of Hurst exponent of specific sediment flux computed for the 20 realizations.

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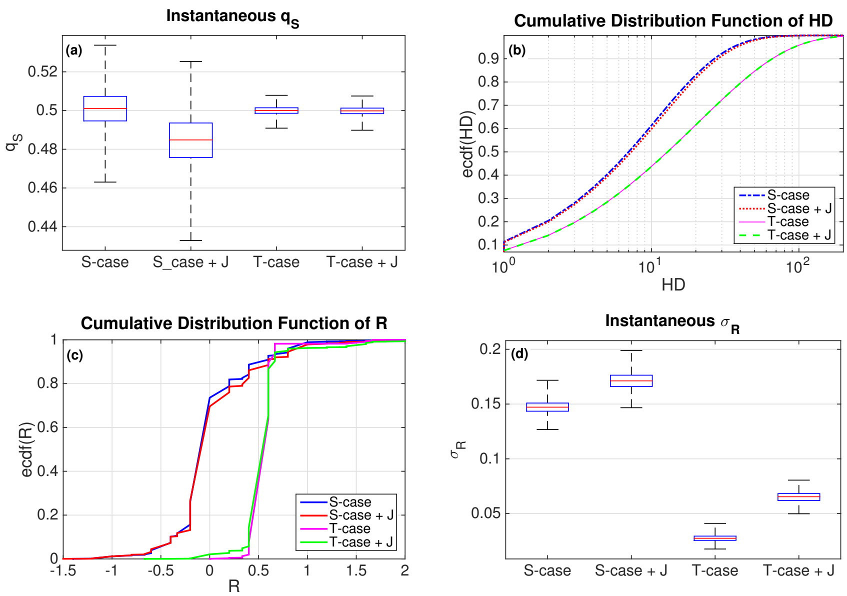

Figure 7. Comparison between stochastic entrainment ($S-case$ without jamming and $S-case+J$ with jamming) and threshold entrainment ($T-case$ without jamming and $T-case+J$ with jamming). (a) Boxplots of the instantaneous values of specific sediment flux q_S. (b) Empirical cumulative distribution function of particle hop distances HD. (c) Empirical cumulative distribution function of bed roughness R computed on the entire control volume at the end of the simulation. (d) Boxplots of the instantaneous values of the spatial standard deviation of bed roughness σ_R.

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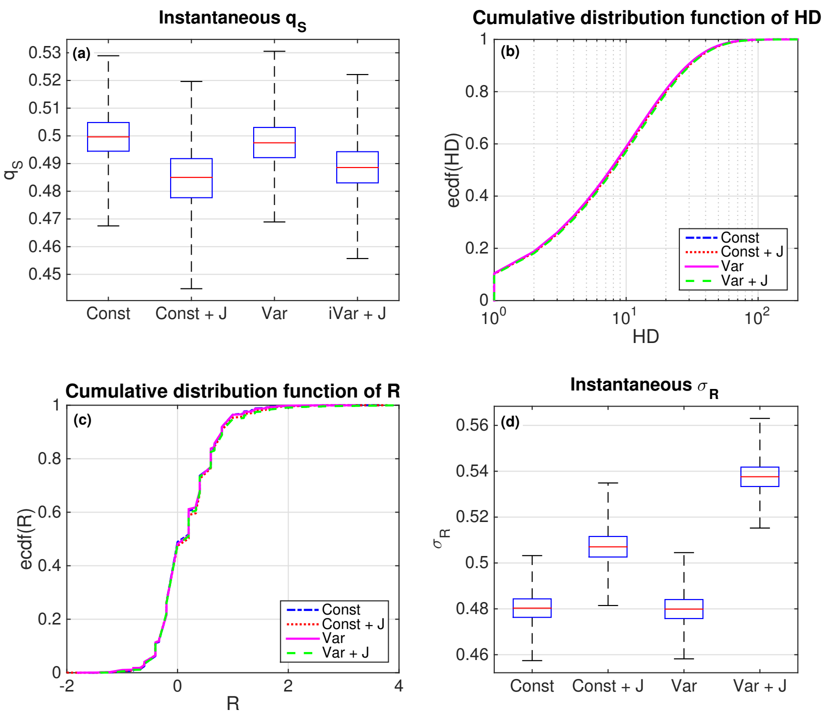

Figure 8. Comparison between constant input rate ('Const' without jamming and 'Const + J' with jamming) and variable input rate ('Var' without jamming and 'Var + J' with jamming). (a) Boxplots of the instantaneous values of specific sediment flux q_S. (b) Empirical cumulative distribution function of particle hop distances HD. (c) Empirical cumulative distribution function of bed roughness R computed on the entire control volume at the end of the simulation. (d) Boxplots of the instantaneous values of the spatial standard deviation of bed roughness σ_R.

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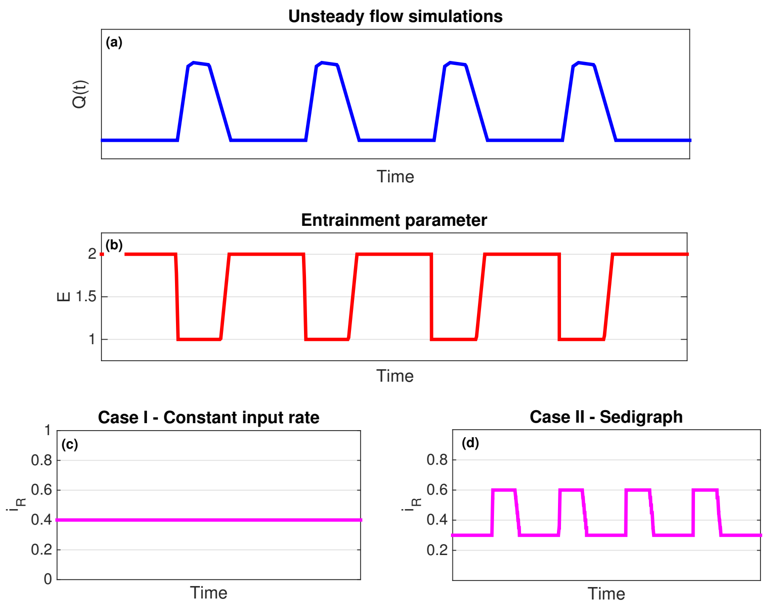

Figure 9. Unsteady simulations with 4 consecutive floods. (a) Generic hydrograph the model is simulating. (b) Variation of the entrainment parameter E to simulate the changing flow conditions. (c) Case I: simulations with varying E and constant input rate i_R. (d) Case II: simulations with varying E and varying input rate i_R.

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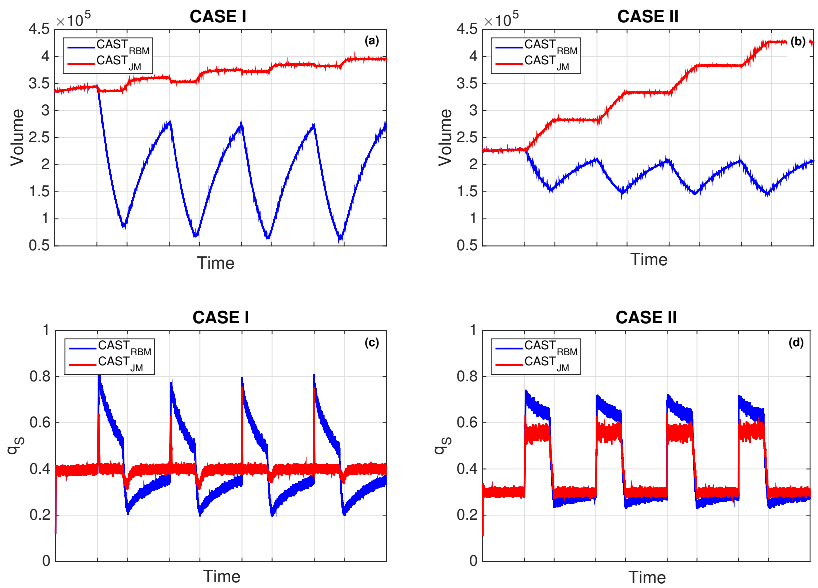

Figure 10. Unsteady simulations with 4 consecutive floods. (a) Time series of storage volume for the constant input case. (b) Time series of storage volume for the variable input case. (c) Time series of specific sediment flux for the constant input case. (d) Time series of specific sediment flux for the variable input case.

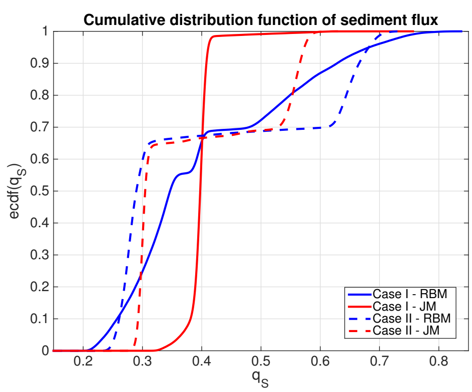

Figure 11. Unsteady simulations with 4 consecutive floods. Empirical cumulative distribution functions of specific sediment flux. Blue identify the rough-bed model, red the jamming model. Solid lines identify the constant input case (Case I), dashed lines the variable input case (Case II).

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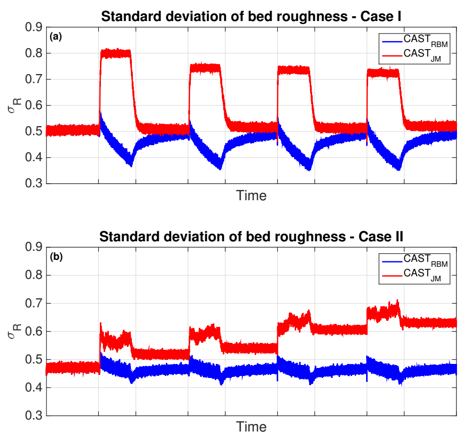

Figure 12. Unsteady simulations with 4 consecutive floods. Time series of the standard deviation of bed roughness for (a) Case I and (b) Case II, both for the rough-bed case (in blue) and the jamming case (in red).

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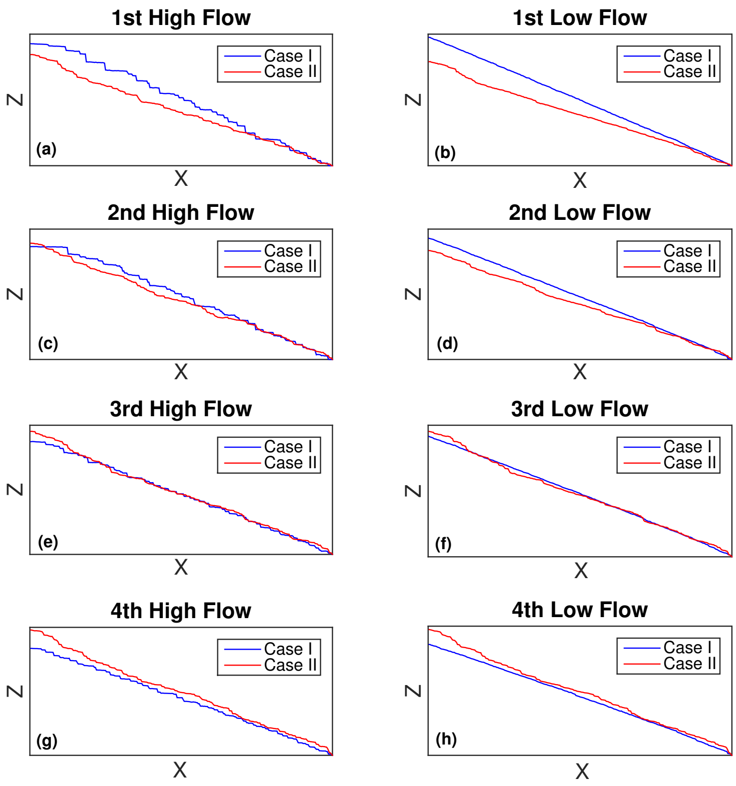

Figure 13. Unsteady simulations with 4 consecutive floods. Longitudinal profiles of bed elevation computed at the end of every high-flow period (left column: a, c, e and g) and at the end of each of the following low-flow period (right column: b, d, f and h).

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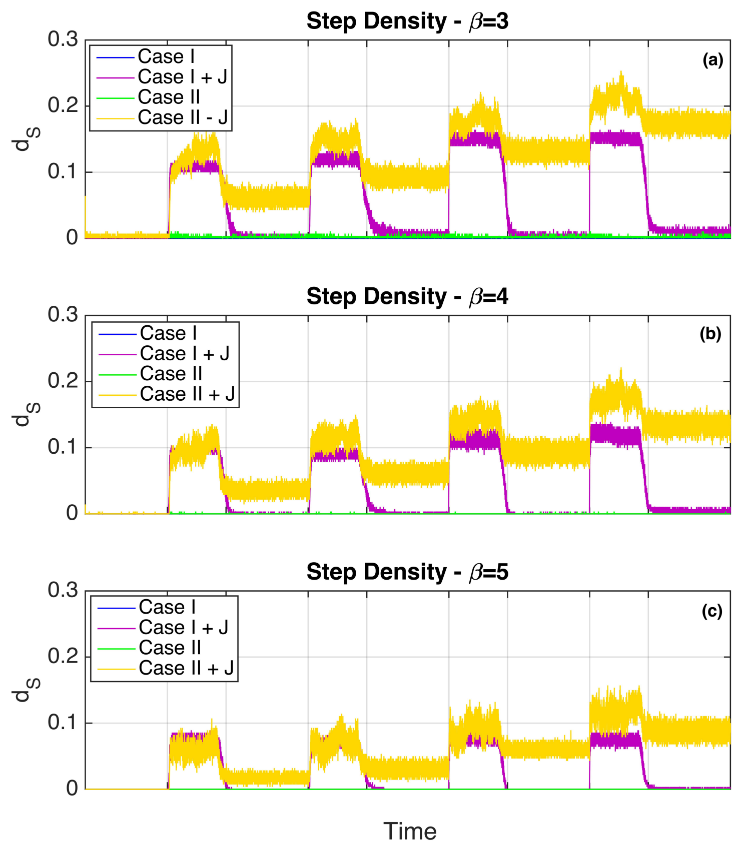

Figure 14. Unsteady simulations with 4 consecutive floods. Time series of step density with (a) $\beta = 3$, (b) $\beta = 4$ and (c) $\beta = 5$.