Response to referees for: Accurate simulation of transient landscape evolution by eliminating numerical diffusion: the TTLEM 1.0 model

We thank referee 1 for her/his comments, which helped improving the quality of the manuscript. Our responses the comments are in blue.

This paper is an extension of a recent contribution by Campforts and Govers (2015) that demonstrated the efficacy of using a higher-order flux limiting total volume method (TVD-FVM) for modeling the advective (i.e., stream power law) component of a coupled hillslope-fluvial landscape evolution model. The authors have extended the TVD-FVM method to 2D and they are making the new LEM available to the community as TTLEM. The main point of this paper is absolutely correct: that upwind differencing with no correction introduces significant numerical diffusion into LEMs. The conclusion that upwind differencing without correction is unacceptably diffusive can be found in every numerical modeling textbook of the last few decades. I don't point this out to minimize the important contribution that the authors have made. Rather, I agree with them that upwind differencing is overly utilized in the LEM community, often without scrutiny. About this there should be no debate.
It should be noted that the numerical diffusion introduced by upwind differencing can be computed and may, in some cases, be mitigated by reducing the diffusivity coefficient D by the same amount introduced by upwind differencing, but this work-around is not commonly performed and is only possible if the prescribed value of D is sufficiently large. I applaud the authors for highlighting the problem of numerical diffusion (first in Campforts and Govers (2015), and again here) and for proposing a robust solution to the problem.

We are grateful for the appreciation of the reviewer regarding our work.

1. That said, I think the tests employed by the authors do not always allow for a clear assessment of the advantages of TVD-FVM. The authors make comparisons between a first-order upwind method and a higher-order TVD method for computing fluxes. However, unless I have misunderstood something, the time steps used are variable within the models, making it difficult to clearly compare the errors associated with temporal discretization and clearly separate them from errors associated with spatial discretization.

   It is indeed true that time steps vary between the TVD-FVM and the implicit method on the one hand and the implicit method without a control on the time step on the other. The latter was done on purpose to illustrate how the main advantage of an implicit scheme, i.e. being stable at time steps exceeding the CFL criterion, is counterbalanced by numerical smearing once the CFL criterion is exceeded. If we only compared simulations where the time step obeys the CFL criterion, it would make no sense to use the implicit scheme as the explicit FDM would be as fast or faster (due to the possibility of vectorization).

2. Before I discuss this issue further, I think it is important to note that LEMs, like solutions to any other PDE or set of PDEs, should converge as the pixel size goes to zero, or at least be relatively insensitive to the grid resolution over the range of resolutions to which the model is applied. Without this, there is no unique solution for a given set of parameter values, making it impossible to know, in the absence of an analytic solution, if one has achieved the correct solution or to objectively compare results obtained with different schemes (the focus of this paper).

   We completely follow the argumentation that numerical models should converge at small resolutions. We applied an analytical solution, which per definition gives the 'the true solution' to illustrate that the different numerical methods applied in our paper indeed converge at small resolutions. Our approach to prove this is further clarified in detail under point 4.

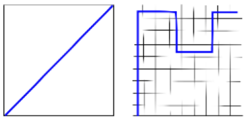
3. Moreover, if a LEM is grid-resolution dependent then the same numerical model operating at different resolutions has to be separately calibrated to data, rendering parameter values such as D and K that should be solely functions of natural processes and material properties also functions of grid resolution. Pelletier, Geomorphology, (2010) has provided some guidance on how to make coupled hillslope-fluvial LEMs grid-resolution independent. His approach involves reframing the stream power as unit stream power (following all sediment trans- port formulae ever proposed, which is not a trivial rescaling since the contributing area generally scales with the pixel size on planar hillslopes but is relatively independent of the pixel size in convergent portions of the landscape) and modifying the strength of the diffusion term to account for the fact that changes in cross-sectional slope at valley bottoms occur over a distance equal to the valley bottom width (a property of nature), not the pixel size (not a property of nature). The random component of the model used by Campforts et al. poses a special challenge to achieving grid-resolution independence. However, one can maintain grid-resolution independence in a model with spatial random variability by generating random field(s) sampled at a resolution that represents the largest resolution the model will be applied to, then bilinearly interpolating these fields for use in versions of the model run at higher resolution. I am not suggesting that the authors adopt all (or any) of these suggestions, but I do suggest that this issue needs to be addressed in some way. The error calculation (equation (22)) simply assumes that the solution with TVD-FVM is exactly correct and any difference from this solution is an error. Without establishing grid-resolution independence it is really impossible to tell whether outputs such as Figures 8A and 8B are even unique solutions for a given set of parameter values, much less which one is more accurate.

   Again, we agree with the reviewer that there is a need for a grid resolution independent solution in order to verify and compare the robustness of the different numerical schemes applied in TTLEM. We also appreciate the elegant suggestion to obtain grid independency as proposed in Pelletier 2010 and have modified the discussion of the manuscript to highlight the influence of grid resolutions. The implementation of the proposed methodology to make a numerical model grid resolution independent is however beyond the scope of our paper where we mainly want to illustrate the importance of numerical diffusion when using most frequently applied first order FDM to solve the SPL. The second message we want to bring with this paper is the suitability of a 2D variant of the TVD-FVM to simulate tectonic shortening. Although grid resolution dependency could most surely be investigated in a future release of TTLEM, we follow referee 2 in trying to present our main messages as clear as possible without drawing too much attention to the technicalities of the numerical model. For similar reasons, we decided to remove the part on grid symmetry from the manuscript and no longer discuss the different hillslope diffusion schemes implemented in TTLEM.

4. The different methods are only evaluated for a small number of cases (two grid resolutions and cases with and without a maximum time step). Error in a first order method will decrease linearly as you decrease dx and it will decrease with (dx)ˆ2 for a second order method. In moving from a grid with dx=500m to dx=100m, there is a large difference in the computed values of E depending on whether or not a first-order or higher-order method is used. This is expected, but this doesn't indicate a fundamental problem with any of the numerical methods. The error associated with each of the methods is dependent on the grid resolution. So, it is a given that there will be some range of grid resolutions where the differences between a 1st order and 2^{nd} order method appear unacceptable (i.e. numerical diffusion is excessive relative to the prescribed diffusivity). However, what really matters in judging method accuracy is the computational time required to reach a given level of accuracy relative to an exact/converged solution. What would be most helpful is to demonstrate that TVD-FVM saves considerable computational time by providing an acceptable solution at a much higher grid resolution and/or is robust for a much wider range of grid resolutions than first order methods. I suggest the following: First, for one method, perform the simulation for a range of grid resolutions (400m, 200m, 100m, 50m, 25m,12.5m) until the solution converges, i.e. becomes essentially grid-resolution independent. Use a time step that is small

enough so that the solution does not depend on the time step (this probably means using a time step that yields a very low Courant number for the coarser grids, but the magnitude of the time step is likely to be similar to the magnitude of the time step needed to keep the model stable on finer grids). Then, it is easy to argue that most of the error introduced into the solution is associated with the spatial component of the problem. Second, repeat step 1 for each of the numerical methods. Assuming all simulations are run on the same machine, keep track of the time required to perform the simulations. This would allow for a more robust comparison of the different methods and would give readers a better idea of the true differences between the methods. For instance, the TVD method should converge to an grid-resolution-independent solution more quickly than the lower order methods. But how much faster? How does this depend on uplift rate or other commonly varied parameters? What are the practical implications in terms of computing time? This would give readers more guidance on the necessity of using one method over the other.

We consider this remark as very essential and would like to thank the reviewer for his suggestion on developing a grid independent ‘true’ solution for the SPL and TTLEM in general. We decided that such an approach is indeed most essential and would offer the reader much more guidance in the performance of the algorithms and provides a robust method to compare the different numerical schemes. Moreover, also reviewer 2 requested a robust framework to illustrate the performance of the numerical schemes. However, carrying out the analysis as suggested by the reviewer introduced some complexities and uncertainties which are summarized below. Therefore, we performed an alternative test, also covering a wide range of resolutions and we compared our numerical solution with an analytical one so that resolution effects could be analyzed.
Complications which arise when performing the analysis as outlined above mainly come down to the fact that comparing model runs with similar parameter values at different resolutions is a very tricky business. First, interpolation from the ‘starting initial image’ to the other resolutions (e.g. from 10 m to 400 m) will change the initial location of the drainage network to a certain extent, depending on the interpolation method used. Hence, catchments and rivers might shift in location which complicates comparison between results. Second, and this one seemed to be very important while doing the exercise, changing the resolution from e.g. 400 to 10 m results in much more possible river paths. This is illustrated in the figure below where it is shown that river distance in higher resolution images might be much longer and can take many different shapes compared to the main resolution (where river length is 400 m or 400 m $\times$ sqrt(2)).



For these reasons, when comparing models, executed at different resolutions, one is rather evaluating the effect of raster resolution and the way it is reflected in topography than comparing the performance of numerical schemes. Although the latter is of utmost importance and has been elegantly illustrated in literature (Pelletier, 2010), this is not what is required to evaluate the performance of a numerical scheme.
In order to overcome these problems, we developed the following strategy to evaluate both the computational performance and accuracy of the numerical methods:

- We only consider river cells to quantify the performance of the different numerical schemes. These rivers cells set the base level for the hillslope cells and the way these hillslope cells respond to differences in numerical schemes is illustrated by the erosion rates calculated over several catchments and illustrated in the current figure 7 of the manuscript. We agree however, that our previous approach to document the difference between the TVD scheme and the implicit schemes using a RMSE is misleading. We will therefore no longer refer to the term RMSE to document the difference between two numerical schemes but simply report the difference between the schemes as an offset. E.g. the $O_{TVD\text{-}imp}$ represents the offset between the TVD-FVM and the implicit FDM.

- To document real RMSE values as a consequence of numerical diffusion we performed the following analysis:
    1. We initiate the analysis from the standard DEM, also used to calculate differences in erosion rates plotted in the current figure 7-9.
    2. All river heads with a contributing drainage area exceeding a threshold value are selected (in our case 10^6 m^2)
    3. The drainage network connecting these river heads with the outlet of the catchment is calculated. Very short river profiles <10km are not retained in the analysis to improve computational performance.
    4. For this initial drainage network the initial river elevations are extracted from the standard DEM.

       Steps 1-4 are illustrated in Figure 1.
    5. Next, landscape evolution is simulated for the three numerical models using the same model parameter values and uplift rates (current Fig. 6) as those reported in the paper in order to calculate erosion rates.
    6. At the end of the model runs, river elevations are extracted from the numerically simulated DEMs and compared with the analytical solution described below.
    7. Given that we consider the linear case where n=1 and keep the river network fixed for this analysis, there exists an analytical solution which is calculated with the slope patch method outlined by Royden and Perron (2013). This method will be further detailed in the revised version of the manuscript.
    8. The advantage of this analytical solution is that it is truly grid size independent and is giving the correct solution for elevations along the river profiles.
    9. To illustrate steps 5-8, we plotted the resulting numerical and analytical solutions for 4 selected resolutions in

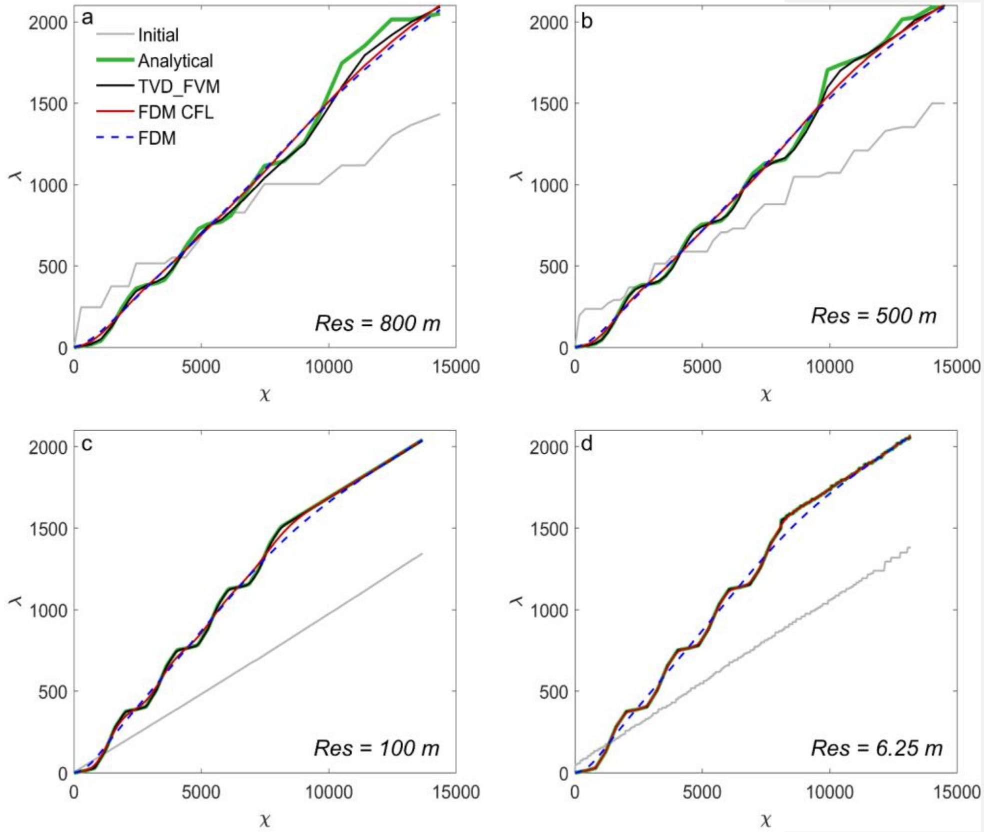


Figure .

10. The previous steps are repeated for a range of resolutions going from 950 m to 6.25 m. For each model run, the CPU time required to perform the analysis is stored.
11. Given that we have an analytical solution for all the cells of the drainage network, the numerical accuracy of the methods can be evaluated by calculating the RMSE between the three numerical methods and this analytical solution. The result of this exercise is plotted in Figure which is in fact reporting the data required by the reviewer.

    We will discuss these findings in detail in the revised manuscript but note that from this analysis, one can see that it would take for example 12 times longer to obtain the accuracy of the river processes obtained with a TVD-FVM at 500 m (RMSE = 18.17, 2.89 sec) with an implicit method (cfl<1, at 150 m, 36 sec). Such an analysis of course only holds for the river cells as higher model resolutions will also improve model performance in terms of hillslope processes.

- Note that we developed an updated, vectorized, version of the TVD algorithm to perform this analysis which will be released soon on GitHub.

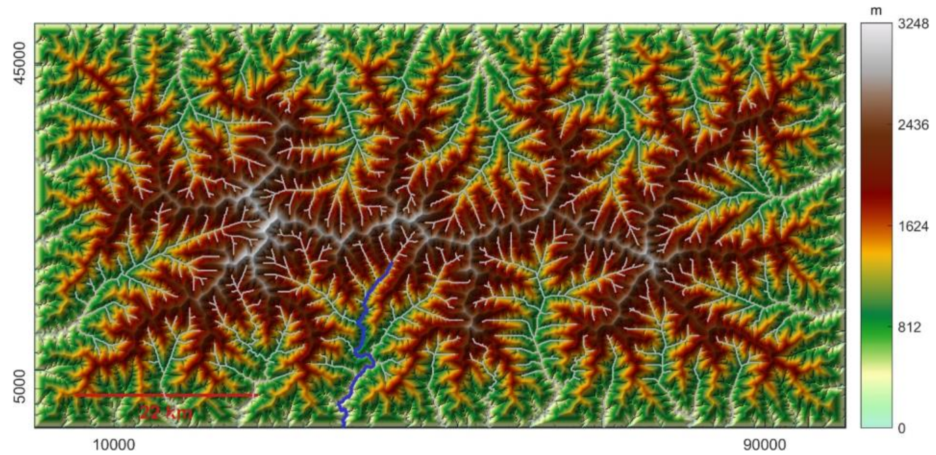


Figure 1: DEM of standard run used in the current version of the paper to calculate catchment wide erosion rates and here used as an initial DEM to run the performance analysis outlined in the comments of the reply. The grey lines indicate the drainage network for which the solution has been calculated analytically. The blue line indicates the river profile for which model results at different resolutions are plotted in figure 2.

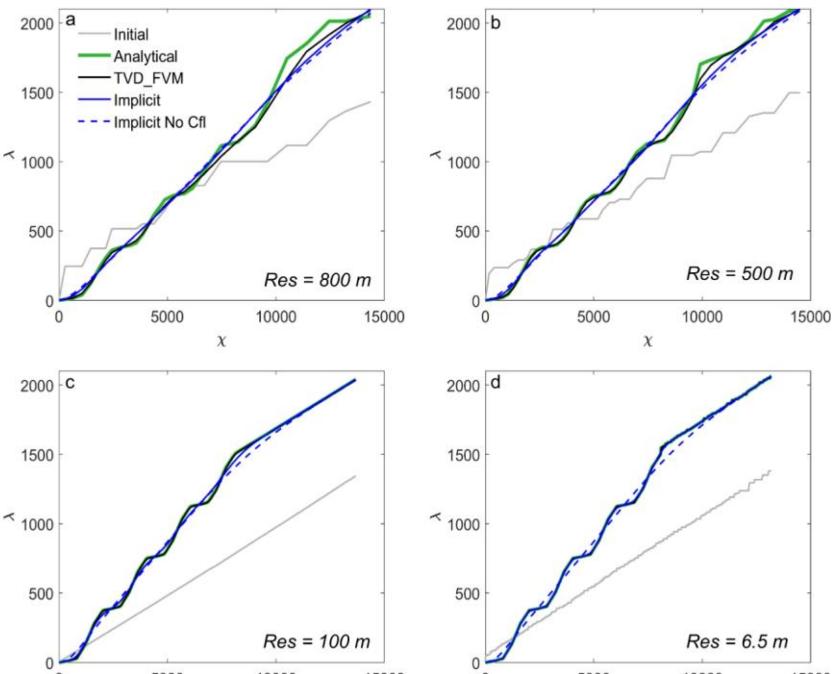


Figure 2: Comparison between different modelled resolutions for the river profile indicated in blue in figure 1. The green line is the 'true' analytical solution, obtained with the slope patch method of Royden and Perron (2013). The solid blue line presents the implicit solution when the CFL<1 and the dashed blue line represents the implicit solution when the time step is left free.

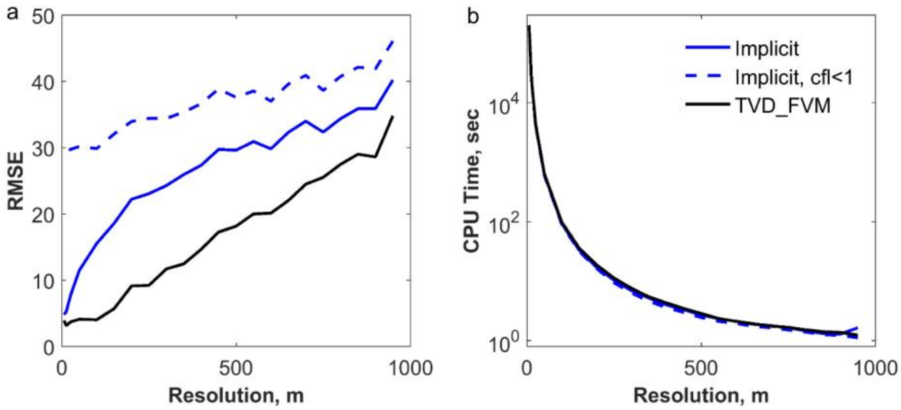


Figure 3: a. Performance of the different numerical schemes calculated with the RMSE between the analytical and numerical methods. b. CPU time required to perform the model runs at the indicated resolutions.

In the discussion the authors imply that their method is really the only acceptable method for the stream-power component of LEMs. Techniques that are widely used to prevent artificial numerical diffusion in many fields of science, including MPDATA and semi-Lagrangian techniques, are implied to be inferior or less robust with no evidence. For example, semi-Lagrangian methods are deemed to be potentially of higher accuracy, but then simply dismissed as inferior to TVD-FVM because "simulation of horizontal topographic shortening would require large amounts of incremental markers to prevent numerical diffusion when interpolating the solution." This sentence confuses two different methods (semi-Lagrangian and particle-in-cell methods are not the same) and is not based on any evidence. I don't see any point in discouraging the community from trying alternative methods until they are clearly tested and shown to be inferior for a wide range of potential applications.

We do accept that our considerations were worded somewhat too strongly. We have therefore adjusted this in the new version of the manuscript. That being said, and without the intention to discourage the community from testing other numerical methods, we are confident in stating that the TVD-FVM is a relatively easy to implement numerical solution which does minimize the amount of numerical smearing in the solution. I did implement an adapted version of the MPDATA scheme which ultimately leads to a similar performance compared to the TVD-DVM but only after applying the limiters as pointed out in the manuscript. That makes the scheme heavier and more complex compared to the TVD-FVM and so we concluded that in this particular case, there is no need for using an MPDATA scheme. Regarding the Lagrangian schemes, we agree with the referee that the current text was confusing and we have rewritten the paragraph as follows:

The numerical methods discussed so far are solved on an Eulerian grid. Eulerian grids represent immobile observations points, for which the solution of the variable, in our case topography, is calculated through time. Alternatively, Lagrangian points such as markers or particles are directly connected to the variable (topography) and evolve together with the variable over time (Gerya, 2010). An approach that has previously been shown to be successful in preventing numerical diffusion is the Marker In Cell method. Here, the solution of the system is simulated by interpolating independently propagating Lagrangian advection markers to fixed Eulerian grid points during each time step of the simulation (Harlow and Welch, 1965). In a 1D configuration, this method would produce very accurate results when applied to solve an advection equation such as the SPL. However, simulation of horizontal topographic shortening would require large amounts of incremental markers to prevent numerical diffusion when interpolating the solution to the Eulerian grid (Gerya, 2010).

Some of the weaknesses of the tested numerical solutions can be reduced by LEMs that rely on irregular grid geometries. Irregular grids do, for example, allow to simulate tectonic shortening using a fully Lagrangian approach where grid nodes are advected with the tectonically imposed velocity field (e.g. Herman and Braun, 2006). …

Minor issues:

1) The variable x is used for two different things (in eqn. (1) it represents one of the cardinal horizontal directions but in eqn. (2) is represents the along-channel distance).
We will fix this in the revised version of the manuscript.

2) There is some repetition and inconsistency in the equations. For example, there are 6 different equations for one variable (dz/dt). It would be better to use a notation that differentiates among different aspects of dz/dt (tectonic advection versus diffusive erosion/aggradation versus stream-power-driven erosion) and make it clear that dz/dt is the sum of these different components. As written, equations (1) and (6) and (9) are repetitive and incompatible, because they are almost the same equation, yet the left hand side of all of the equations is the same while the right hand side includes uplift in one of the equations but not in the other two.
We agree that our notation is currently not fully consistent and follow the suggestion of the reviewer to use different notations for the different sub components of the solution (eg. Eq. 6 and 9)

3) It would be helpful for the authors to address whether the method could be applied to the nonlinear stream power law (n not equal to 1), spatially variable K (e.g., strong over weak layers in sedimentary or metamorphic rocks), transport-limited fluvial processes, landscapes with a finite soil layer over bedrock or intact regolith, and other common LEM variants.
We thank the reviewer for these suggestions. For the moment the model supports (i) non-linear river incision (n~=1), variable K values, different precipitation input. Transport limited fluvial processes as well as a bedrock/regolith interface are currently not supported but are planned to incorporate in future versions of TTLEM.

4) The paper is comprehensively referenced, which I appreciate, but some of the references do not support the points being made. To take one example, McGuire and Pelletier (2016) is used to defend the use of a detachment-limited model on the basis that unconsolidated sediment can be easily evacuated from the fluvial network. This is simply untrue. Unconsolidated sediments obviously do get stored in fluvial systems. Whether a detachment-limited model is a reasonable approximation depends on the application (including details such as mean grain size), and I don't think a paper that deals with small channels forming on alluvial terraces is an appropriate basis for defending the use of a detachment limited model in an LEM designed to model the large-scale evolution of mountain belts.
We agree with RC1. We will change the referencing and wording in this sentence.

5) The structure of the paper is good but the sections/subsections could be slightly improved. For example, the issue of artificial symmetry that can arise with rectangular grids is first introduced on line 206 with no prior mention or subsection break. I think this issue should be addressed in its own subsection of section 3 (as it is in section 4.2).
We will no longer discuss the issue of artificial symmetry in this paper as suggested by referee 2.

6) The stream power model is introduced using its nonlinear form (the exponent n is general) but the remainder of the paper, including the CFL condition (eqn. (19)), applies only to the linear case.
All the simulations could be easily performed for non-linear cases. However, we preferred linear examples when demonstrating the impact of numerical smearing on the results to enhance clarity in general. How non-linear slope dependency affects river incision is discussed in Campforts and Govers (2015) in due detail, including the way in which the CFL criterion should be adapted.

7) The use of D8 routing seems unsubstantiated. Dinfinity is the choice of nearly every modern LEM, because it more faithfully represents flow on hillslopes.

Dinf (or D∞) is certainly the flow routing scheme of choice to represent flow on hillslopes. However, in TTLEM fluvial erosion is limited to the channelized domain of the landscape and thus the flow routing scheme on hillslopes of minor significance. Nevertheless, even in the channelized domain Dinf has advantages over D8 since it enables diverging flows on landforms such as alluvial fans and braidplains. The current implementation of TTLEM, however, focuses on the modelling of detachment-limited systems or bedrock rivers where divergent flows are usually confined by valley walls. This is also consistent with other models such as Fastscape (Braun and Willett, 2013) and DAC (Goren et al., 2014) models that use the D8 flow routing scheme. We thus disagree that Dinf is the choice of the majority of modern LEMs. Still, we like to stress that we do not exclude to implement Dinf or other multiple flow direction algorithms in a future version of TTLEM, in particular since the topological sorting algorithm (Braun and Willett, 2013; Heckmann et al., 2015) is equally suitable for the efficient computation of flows on thus derived networks.

8) Please use lat/lon or UTM coordinates in Fig. 2. If these are UTM coordinates, please specify.
We will fix this in the updated version of the manuscript.

9) The method of the paper is referred to as TVD-TVM throughout the abstract but TVD-FVM in the paper. If this is not a typo, please explain the difference between these abbreviations.
We will fix this in the updated version of the manuscript.

10) w_A and w_k are introduced in the equation but then (unless I missed it) never discussed again (not even in the table of parameter values).
These parameters are weighting parameters used to scale for changes in precipitation and lithology. We will clarify this.

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Interactive comment on "Accurate simulation of transient landscape evolution by eliminating numerical diffusion: the TTLEM 1.0 model" by Benjamin Campforts et al.

Anonymous Referee #2

Received and published: 17 August 2016

We thank referee 2 for her/his comments, which helped us to improve the quality of the manuscript. Our replies are in blue. Throughout this reply, we will also refer to the answers formulated in the author comments on referee 1 (further referred to as RC1) where we also added some figures for clarification.

Campforts et al. addresses an important problem for fluvial landscape evolution models: numerical diffusion of the solution to the stream power advection equation. The authors first of all present a solution to the problem based on a higher-order flux-limiting method (TVD-TVM), and secondly, they outline a new modeling platform (TTLEM), which makes use of TVD-TVM and is available to everyone as part of the TopoToolbox.

Overall, my opinion is that numerical accuracy of fluvial landscape evolution models has received too little attention in the past, and it is therefore good to see the authors address it here. The method proposed to reduce numerical diffusion is convincing, and the damping of numerical diffusion in stream-power advection as well as in tracking horizontal tectonic displacements is significant. I hope that this contribution gets published in Esurf, although I do have some concerns and suggestions, which I list below:

We are grateful for RC2's appreciation of our work. We also appreciate the constructive comments which will help us to enhance the overall quality and readability of the manuscript.

General comments:

First of all, I think the dual purpose of the manuscript: 1) discussing numerical diffusion and presenting TVD-TVM, and 2) presenting TTLEM as a more general landscape evolution model leads to a rather diffuse and ackward structure of the text. The main strength of this text is in my opinion the focus on numerical diffusion and the presentation of TVD-TVM, but the TTLEM presentation calls for many details that are not needed to address this issue (see for example Fig. 1). For example, because the introduction focuses mostly on the influence of numerical diffusion, it is hard to understand the motivation for the first couple of experiments focusing on drainage networks and the influence of different hillslope models. I would strongly recommend simplifying the flow of the manuscript focusing more exclusively on the issue of numerical diffusion. Likewise the authors should consider skipping the first two experiments and in stead perform more like the one shown in Fig. 7. I think that it would increase the impact of the contribution, and the presentation of TTLEM could perhaps be saved for another manuscript in a more software-oriented journal.

We follow the advice of the reviewer to focus the entire manuscript on the role of numerical diffusion in landscape evolution modelling. We will therefore remove the two first experiments (e.g. the role of hillslope diffusion and the presence of artificial symmetry) from the paper. Nonetheless, we consider this paper as the first description of the new TTLEM simulation software. Therefore, we will move the flow chart illustrating the different modules of the model to the appendix of the paper along with the picture illustrating the functionality of the different hillslope response schemes. We consider TTLEM as a tool for the community which can be used to reconstruct landscape evolution as well as to test hypotheses. The latter might require a combination of insights in the different existing modules as well as a guidance on how to add new modules. We feel that both objectives, require an overview of the software in its present shape.

Secondly, I suggest the authors give a short introduction to basic knowledge about numerical diffusion in advection problems. This could be inspired by simple textbook material and use linear advection as a starting point. By this the authors could avoid some awkward reflections, like in line 378: it is not at all counterintuitive that time steps smaller than the CFL criterion leads to more numerical diffusion. Most numerical analysis textbooks I know of give very simple explanations for why numerical diffusion is minimized exactly at the CFL criterion. Overall, I think the authors can make better use of basic textbook wisdom to prepare the reader for the main points of the manuscript.

In the revised manuscript, the readers are introduced into the issue of numerical diffusion when solving hyperbolic partial differential equations (section 3.2). We also updated some references to excellent textbooks on this matter (Harten, 1983; Toro, 2009). An extended discussion on numerical diffusion when solving the stream power law can be found in Campforts and Govers (2015). We will also rephrase the sentence in line 378 although we find it important to document these findings which are indeed well discussed in numerical textbooks but less well known/introduced in the earth surface community.

Finally, while I fully appreciate the comparison experiments between the different numerical methods, I suspect that it is not completely fair.

Part of this answer is addressed in the reply to RC1 where we illustrate how the analytical slope patch method (Royden and Taylor Perron, 2013) is used to evaluate the performance of the different numerical schemes.

The main advantage of the implicit method (as FastScape by Braun and Willett) is that it becomes more compute efficient at high spatial resolution than the explicit methods, simply because it is not similarly constrained by the CFL condition. Thus, if explicit and implicit methods were compared in experiments with similar compute time (which I think they should be), would the implicit method not allow for finer spatial resolution than the explicit method? If so, would the finer spatial resolution in combination with the larger time steps not reduce the numerical diffusion of the implicit method? I am not questioning the advantages of TVD-TVM here. I just feel that the authors are not appreciating the real strength of the implicit method, which is how the compute time scales with spatial resolution.

This is an interesting remark that we address in a revised version of the manuscript. We hope that the additional analysis outlined in our comment to RC1 will provide more insight into the trade-offs between numerical accuracy and computational efficiency. The answer to the referee's question comes in multiple points.

- An essential characteristic of an implicit scheme like that of Braun and Willett is that it fails to allow for 'vectorization' which is in contrast to explicit methods (like TVD). By vectorization, we mean ways to exploit single-instruction multiple-data parallelism. Hence, the fact that TVD requires more operations per execution and requires a time step which obeys the CFL criterion may partly compensate for sequential looping through all stream network nodes required by the implicit scheme. From the analysis presented in our discussion of the comments of RC1, we show that both schemes end up running in almost the same time. We will address this point in the new version of the manuscript.
- It is important to note that rivers only occupy part of the landscape. Although TTLEM indeed allows to simulate all cells as rivers cells (as suggested in comment on line 206), we do not test this configuration as we consider it of little use in real world landscape evolution where hillslope processes may dominate where drainage area drops below a threshold value. Hence, while refining the resolution does indeed result in more accurately simulated river elevations, the computational overhead related to hillslopes processes which comes with refining the grid resolution is unacceptably large at the spatial scales and resolutions that we consider. Also notice that even at very high spatial resolutions (6.25 m), the TVD method is still more accurate compared to the implicit (cfl<1) method.
- We appreciate the remark of the reviewer that the higher spatial resolution, which is in principle allowed by the implicit method for similar timescales, is the real strength of the implicit method. This argument is exactly the reason why we simulated the landscape using both an implicit method which is free of any time criterion (and where dt is set by the main model time step, e.g. 2e4 yr) and one simulation where a CFL is applied to the implicit method. The latter was done on purpose to illustrate how the main advantage of an implicit scheme,

i.e. being stable at time steps exceeding the CFL criterion, is counterbalanced by numerical smearing once the CFL criterion is exceeded. If we only compared simulations where the time step obeys the CFL criterion, it would make no sense to use the implicit scheme as the explicit FDM would be as fast or even faster (due to the possibility of vectorization). Furthermore, it is not only the inherent nature of an implicit scheme which is not suited to properly simulate propagating knickpoints. If very large timescales are applied in landscape evolution models, uplift is inserted very suddenly at the beginning of the time step. This results in unrealistic simulations where uplift is a discrete stepwise function rather than a continuous function (e.g. the sine waves used in this paper). In Fig. 2 of this file, we have shown two extremes, i.e. a configuration where CFL<1 and one where CFL >>1. One could argue that intermediate solutions (e.g with CFL closer to 1) would result in more desirable results than the one shown with the dotted lines in Fig. 1-3 of RC1. This is true but, given that computational gains are marginal and numerical accuracy will never be higher than the implicit method simulated at CFL< 1 (solid blue lines), we see little reason to follow such an approach when simulating transient landscape evolution.

- To summarize, a first order implicit scheme is not suited to properly simulate propagating knickpoints in detachment limited erosional basins. First order implicit methods are therefore only suited to simulate configurations where transiency, caused by local base level falls, tectonic faults or lithological contacts can be considered to be minor.

More specific comments:
Line 30: “availability of potential energy”
Line 85: delete “most”
Eqn 1: Why are vx and vy bold?
Because they are representing velocity fields being variable in space.
Eqn 2: Are wk and wa used for anything here? If not flush them out.
They are used as weighing factors to introduce the impact of variable lithological strength an precipitation in the model. We will further clarify this in the updated manuscript. We now refer to them as well in the discussion section.
Line 102: what is “eroding settings”?
Where the detachment limited assumption holds.
Eqn 3: The diverge operator should include a dot between nabla and qs
OK
Line 113: hillslope erosivity and erodibility. What is the difference?
Should be simple erodibility. Erosivity can be removed
Eqn 7: Again, is the variability on m really needed to demonstrate the points of numerical diffusion? If not skip it to clean the text. More complicated means less convincing.
Point taken. Section will be removed in the updated manuscript.
Eqn 8: I do not understand the effect of densities here. Is U not simply uplift of the surface? If so, I guess the densities should be on the second term, right?
The way it was written in the previous version of the manuscript was actually correct. The correction for the bedrock versus soil bulk densities is required on hillslopes where erosion and deposition in governed by soil fluxes (Perron, 2011). Nonetheless, we agree that the way in which this was presented was somewhat confusing and we adapted the presentation of the mass balance equation in the new version of the manuscript.
Line 153: “. . .transforms returns. . .”
Eqns 11-17: The use of subscripts seems inconsistent.
We only solve one component of the differential equation. The full deriviation can be found in the textbook we refer to or online in are GitHub Code.
Line 192: “.. is similar than the one. . .”
Eqn: 19: I guess A varies by several orders of magnitude in the grid. Please discuss

the CFL criterion in the light of this. Is max(A) used here?

Fixed

Line 199: Description of the inner time step is confusing, and I do not understand why it is needed. Again I suspect that it is the general presentation of TTLEM that stands in the way for a clear and concise presentation of the numerical experiments.

We will clarify this further in a revised version of the manuscript. An inner time step is needed because hillslope processes which are diffusive in nature allow the use of semi-implicit methods used to solve them. Here, the implicit nature of the schemes can be fully exploited and large time steps can be used to solve the equations (Perron, 2011). The TVD method which is explicit, on the other hand does not allow such big time steps and does require the main model time step to be split up in so called 'inner time steps'.

Lines 206-215: This kind of randomness should be avoided here. The authors are documenting the level of numerical diffusion in different numerical techniques, and in this process it is very important that we know what advection equation is solved. m seems to be varied in order to make the drainage networks look more realistic. But that is not important here. And by the way: varying m randomly does not remove the grid dependency (which is inherent to stream-power advection and D8 drainage), it just obscures the close links between the grid, the (random) variability of m, and the drainage network. Please keep m fixed and the equations as simple as possible!

Section removed

Section 3.4 is not well written. In spite of carefully reading the text I am still confused about how hillslope processes are implemented. But more importantly: Can the experiments documenting numerical diffusion not be run without hillslope processes? This would require that Ac=0 in Eqn 8, but why not? It seems a bit silly to deliberately add physical diffusion to an experiment were one wants to measure numerical diffusion? The authors should consider if the experiments can be made simpler (see first general comment above). Skipping hillslope processes and deleting this section could be a quick fix.

As outlined above, we agree with the reviewer that the experiments on hillslope diffusion and varying values for m are distracting for the main message of the paper. We will also further motivate our choice for the D8 algorithm in the updated manuscript (see also RC1). However, for reasons also discussed above, we did not remove the hillslope processes from our model to explicitly address how numerical diffusion in channel incision affects hillslope diffusion and ultimately basin wide erosion rates.

Section 4: I recommend skipping the first two experiments on hillslope processes and drainage networks (or save them for another paper). This would free up space to dig deeper into advection and numerical diffusion.

Fixed, section removed from the manuscript

Line 276: I am not impressed by this strategy. I agree that the artificial symmetry is a problem, but at least we know where it comes from. Fixing this by introducing variability in the exponent m obscures the link between model input and model output, which is otherwise critical for use of computational experiments. Variability on K is better, because the linear scaling does not alter the form of the equation.

Fixed, section removed from the manuscript

Line 344: So, what happens if the grid resolution is lowered to 10 m?

See RC 1

Line 391: overcomes -> reduces

Fixed

Line 403: A small time step is not the essential factor here. The implicit method first of all offers a fine spatial resolution in combination with a large time step. The advantage of this combination should be explored more.

This issue is discussed in the reply to the major comments above.

Line 464-474: All of this seems rather irrelevant to the main points of this study. See first general comment.

We will consider moving part of the paragraph to the appendix in the revised version of the manuscript.

Line 481: ". . . the current debate. . ." calls for references.

Fixed

Fig. 1: I almost get dizzy by looking at this. What is the point of showing this level of complexity in the first figure?

We will skip this figure and add it to the appendix

Fig. 2: While this is interesting I do not understand the motivation. The introduction spins me up to read about numerical diffusion, not this.

We will skip this figure and add it to the appendix

Fig. 3: Same comments as for Fig. 2.

We will skip this figure

Fig. 4: This is a nice, simple figure and to me the extension of this existing result to 2D simulations is the essential contribution of this study. This figure could be a great opening figure.

Point taken

Fig. 7: If the authors choose to follow my advice and skip the first experiments, then more like this could be performed. It would be useful to see experiments with different setting of m and n (linear vs. nonlinear). Also to have experiments at finer spatial resolution where the advantages of the implicit method should start to kick in.

See discussion above and figures in RC1. We will remove the first three figures from the manuscript.

Fig. 9: It is good to see the difference between methods here, but it would also be great to see pictures of the two separate erosion rates. I wonder if knickpoints can be recognized in both?

Fig. 9 illustrates the difference between erosion rates for the two numerical methods. In our opinion the addition of another figure showing the erosion rates for each method is not very meaningful as the differences in erosion patterns and rates would be less clear. With respect to the knickpoints it is important to consider that the use of a different numerical method does not change the average speed of knickpoint advection (see Campforts and Govers, 2015), but it does strongly affect the evolution of the gradient of the knickpoint: we will add this clarification in the revised version of the manuscript. Hence, it is not meaningful to compare maps of knickpoint locations.

Fig. 10: great figure

Thanks

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Accurate simulation of transient landscape evolution by eliminating numerical diffusion: the TTLEM 1.0 model

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Abstract. Landscape evolution models (LEM) allow studying how ~~the~~ the earth surface responses to ~~a~~ changing climatic and tectonic forcings. While much effort has been devoted to the development of LEMs that simulate a wide range of processes, the numerical accuracy of these models has received much less attention. Most LEMs use first order accurate numerical methods that suffer from substantial numerical diffusion. Numerical diffusion particularly affects the solution of the advection equation and thus the simulation of retreating landforms such as cliffs and river knickpoints with potential ~~unquantified~~ consequences for the integrated response of the simulated landscape. Here we test a higher order flux limiting finite volume method that is total variation diminishing (TVD-FVM) ~~and solves~~to solve the partial differential equations of river incision and tectonic displacement.~~–~~ We show that the choice of the TVD-FVM to simulate river incision significantly influences the evolution of simulated landscapes and the spatial and temporal variability of catchment wide erosion rates. Furthermore, a 2D TVD-FVM accurately simulates the evolution of landscapes affected by lateral tectonic displacement, a process whose simulation is hitherto largely limited to LEMs with flexible spatial discretization. We implement the scheme in TTLEM,~~Here we present TTLEM,~~ a spatially explicit, raster based LEM for the study of fluvially eroding landscapes in TopoToolbox 2. ~~TTLEM prevents limits numerical diffusion by implementing a higher order flux limiting total finite volume method that is total variation diminishing (TVD-TVMTVD-FVM) and solves the partial differential equations of river incision and tectonic displacement. We show that the choice of the TVD-TVMTVD-FVM to simulate river incision significantly influences the evolution of simulated landscapes and the spatial and temporal variability of catchment wide erosion rates. Furthermore, a 2D TVD-TVMTVD-FVM accurately simulates the evolution of landscapes affected by lateral tectonic displacement, a process whose simulation is hitherto largely limited to LEMs with flexible spatial discretization. By providing accurate numerical schemes on rectangular grids, TTLEM is a widely accessible LEM that is compatible with GIS analysis functions from the TopoToolbox interface.~~

~~https://github.com/wschwanghart/topotoolbox~~

1. Introduction

Landscape evolution models (LEMs) simulate how the earth surface evolves in response to different driving forces including tectonics, climatic variability and human activity. LEMs are integrative as they amalgamate empirical data and conceptual models into a set of mathematical equations that can be used to reconstruct or predict terrestrial landscape evolution and corresponding sediment fluxes (Glotzbach, 2015; Howard, 1994). Studies that address how climate variability and land use changes will affect landscapes on the long term increasingly rely on LEMs (Gasparini and Whipple, 2014).

~~A large number of geophysical processes act on the earth surface, mostly driven by gravity and modulated by the presence of water, ice and organisms (Braun and Willett, 2013). These processes critically depend on the availability potential energy, brought into or withdrawn from the landscape by tectonic forces (Wang et al., 2014). Weathering and erosion respond to~~

~~tectonic uplift, shaping the landscape through the lateral transport of sediments and, to a certain degree, also through feedback on regional uplift patterns (Whipple and Meade, 2004).~~

~~LEMs allow to integrate growing field evidence covering different spatial and temporal timescales (Glotzbach, 2015), thereby accommodating a broad range of applications with fundamental importance in the development of geosciences (Bishop, 2007). LEMs are key to understanding landscape evolution both over time scales of millions of years (van der Beek and Braun, 1998; Tucker and Slingerland, 1994; Willett et al., 2014; Willgoose et al., 1991b) and much shorter, centennial and millennial, timescales (Coulthard et al., 2012). LEMs simulate the interaction between different processes and provide insights into how these interactions result in different landforms. Moreover, visualizing LEM output in intuitive animations stimulates the development of new theories and hypotheses (Tucker and Hancock, 2010). LEMs have also successfully been used for higher education in geomorphology and geology, improving students' understanding of geophysical processes (Luo et al., 2016).~~

Landscape evolution is not always smooth and gradual. Instead, sudden tectonic displacements along tectonic faults can create distinct landforms with sharp geometries (Whittaker et al., 2007). These topographic discontinuities ~~are~~ not necessarily smooth~~ed~~ out over time, but may persist over long time scales in transient landscapes (Mudd, 2016; Vanacker et al., 2015) ~~(Mudd, 2016)~~. For example, faults may spawn knickpoints along river profiles. These knickpoints will propagate upstream as rapids or water falls (Hoke et al., 2007), thereby maintaining their geometry through time (Campforts and Govers, 2015). After an uplift pulse, the river will only regain a steady state when the knickpoint finally arrives in the uppermost river reaches. Transiency is not limited to individual rivers but also affects larger systems such as the Southern Alps of New Zealand where the landscape may never reach a condition of steady state due to the permanent asymmetry in vertical uplift, climatically driven denudation and horizontal tectonic advection (Herman and Braun, 2006).

Transient 'shocks' and ~~Topographic~~ topographic discontinuities ~~that result from transient 'shocks'~~ are inherently difficult to model accurately. Most of the widely applied LEMs ~~(Valters, 2016),~~ use first order accurate explicit or implicit finite difference methods to solve the partial differential equations (PDE) that are used to simulate river incision (Valters, 2016). These schemes suffer from numerical diffusion (Campforts and Govers, 2015; Royden and Perron, 2013). Numerical diffusion will inevitably lead to the gradual disappearance of knickpoints and~~: the inherent inaccuracy of (implicit) first order accurate methods~~ will result in ~~ever~~ ever smoother shapes. It has already ~~already~~ been shown that ~~this~~ ~~While t~~~~This topographic~~numerical smearing ~~has already been shown to have implications for~~decreases the accuracy of modelled longitudinal river profiles (Campforts and Govers, 2015). Here, ~~(Reference here),~~ ~~, yet~~ we hypothesize that it is also relevant for the simulation of hillslope processes: hillslopes respond to river incision and, thus, inaccuracies in river incision modelling will propagate to the hillslope domain. Whether and to what extent this occurs, is yet unexplored.

Tectonic displacement is similar to river knickpoint propagation; in both cases, sharp landscape forms are laterally moving. Numerical diffusion may therefore significantly alter landscape features when tectonic shortening or extension is~~f~~ simulated using first order accurate methods. In principle, flexible gridding overcomes ~~This~~ this problem through dynamically adapting ~~can in principle be overcome with flexible gridding, whereby~~ the density of nodes on the modelling domain ~~is dynamically adapted~~ to the local rate of topographic change ~~in topography~~. However, models using flexible gridding have other constraints. They are much more complex to implement ~~and hence less easy to adapt, require permanent mesh grid updates~~ and impose the structure of the numerical grid to the natural drainage network as rivers are forced to follow the ~~numerically composed~~ grid structure. Furthermore, the output of flexible grid models is not directly compatible ~~for~~ with most software that is available for topographic analysis ~~(Schwanghart and Kuhn, 2010)~~.

Here we present TTLEM, a spatially explicit raster based LEM, which is based on the object-oriented function library TopoToolbox 2 (Schwanghart and Scherler, 2014). Contrary to previously published LEMs we solve the stream power river incision model using a flux limiting total volume method (TVM) which is total variation diminishing (TVD) in order to ~~prevent~~ avoid numerical diffusion ~~when solving the stream power law~~. Our numerical scheme expands on previous work (Campforts and Govers, 2015) by extending the mathematical formulation of the TVD method from 1D to entire river networks. Moreover, we develop~~ed~~ a 2D TVD-FVM scheme to simulate horizontal tectonic displacement on regular~~ly~~ grids, thus allowing to account~~ing~~ for three dimensional variations in tectonic deformation. The objective of this paper is to evaluate TTLEM and assess the performance of the numerical methods ~~to~~ for a variety of real-world and synthetic situations. ~~We show that the use of this updated numerical method has implications for the simulation of both catchment wide erosion rates and landscape topography over geological time scales.~~

~~TTLEM provides the geoscientific community with an easily accessible and adaptable tool. TTLEM is therefore a fully open source software package, written in MATLAB and based on the TopoToolbox platform. Users should be able to run TTLEM using both real data and synthetic landscapes. Moreover, the integration of TTLEM in TopoToolbox allows direct digital terrain analysis using the TopoToolbox library (Schwanghart and Scherler, 2014). In its current form, TTLEM is limited to uplifting, fluvially eroding landscapes: further development will allow to integrate other processes (e.g. glacial erosion) as well as the explicit routing of sediment through the landscape.~~

2. Theory and geomorphic transport laws

~~2.~~2.1. Tectonic deformation

In its ~~most~~ simplest form, tectonic deformation is represented by vertical rock uplift, $U(x,y,t)$ [L t^{-1}]. However, many tectonic configurations imply that displacements have both a vertical (uplift or subsidence) and a lateral (extension or shortening) component (Willett, 1999; Willett et al., 2001). The change in elevation of the earth surface ~~(*z*)~~ over time due to lateral tectonic displacement (thus not including vertical rock uplift)~~eformation~~ $(\partial z/\partial t)_{td}$ ~~dect~~ is then :

$$\left(\frac{\partial z}{\partial t}\right)_{td} = \mathbf{u}\frac{\partial z}{\partial x} + \mathbf{v}\frac{\partial z}{\partial y} \qquad (1)$$

where ~~v_x~~ **u** and ~~v_y~~ **v** [L T^{-1}] are the tectonic displacement velocities in the cardinal ~~*x* and *y*~~ directions (horizontal **u** and vertical **v**)~~, respectively~~.

2.2. River incision

Detachment limited fluvial erosion $(\partial z/\partial t)_{fluv}$ is calculated ~~based on the well-knownestablished relation between the channel gradient and the contributing drainage area (*A*), also referred to as the Stream~~with the ~~-~~stream ~~Power~~ power ~~Law~~ law (SPL) (Howard and Kerby, 1983):

$$\left(\frac{\partial z}{\partial t}\right)_{fluv} = -K(A)^m\left(\frac{\partial z}{\partial x_\Gamma}\right)^n \qquad (2)$$

The equation is solved on a dendritic stream network domain Γ where x_Γ refers to the distance from the outlet. A [L^2] is catchment area and K [L^{1-2m} t^{-1}] is an erodibility parameter that depends on local climate, hydraulic roughness, lithology and sediment load. ~~*K* can be adapted to reflect local variations in erodibility by using a scaling coefficient w_K [dimensionless]. In case of uniform erodibility, w_K is set to one.~~ *A* is the drainage area, which is used as a proxy for the local discharge. ~~Similar to *K*, *A* can be corrected for regional precipitation variabilities through a scaling coefficient w_A [dimensionless].~~

m and *n* ~~represent~~ are the area and slope exponents: their values reflect hydrological conditions, channel width, as well as the dominant erosion mechanism. *K*, *m* and *n* are interdependent and it is usually impractical to constrain any of their values alone (Croissant and Braun, 2014; Lague, 2014). Thus, many studies provide estimates for the *m/n* ratio. For *m/n* ratios between 0.35

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and 0.8, *K* values span several orders of magnitude between 10^{-10} - 10^{-3} $m^{(1-2m)}$ yr^{-1} (Kirby and Whipple, 2001; Seidl and Dietrich, 1992; Stock and Montgomery, 1999). ~~In order to represent fluvial sediment transport, it has previously been proposed to add a diffusion component (Rosenbloom and Anderson, 1994). However, we follow others in assuming that in eroding settings, detachment limited erosion is controlling landscape evolution and is represented by the advection equation represented in Eq. (2(2) (Attal et al., 2008; Goren et al., 2014; Howard and Kerby, 1983; Whipple and Tucker, 1999).~~

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2.3. Hillslope processes

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River incision drives the development of erosional landscapes by changing the base level for hillslope processes. Steepening of hillslopes subsequently leads to increased sediment fluxes from hillslopes to the river system. Hillslope ~~erosion~~ denudation $(\partial z/\partial t)_{hill}$ is equal to the divergence of the flux of soil/regolith material ($\mathbf{q_s}$, [L^3 L^{-1} T^{-1}]):

$$\left(\frac{\partial z}{\partial t}\right)_{hill} = -\nabla \cdot \mathbf{q_s} \qquad (3)$$

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Different geomorphological laws describe hillslope response to lowering base levels. The model of linear diffusion assumes that the soil/regolith flux is proportional to ~~the~~ hillslope gradient (Culling, 1963):

$$\mathbf{q_s} = -D\nabla z \qquad (4)$$

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where *D* is the diffusivity [L^2 t^{-1}] that parameterizes hillslope ~~erosivity and~~ erodibility and determines rate of soil/regolith creep. ~~Linear hillslope diffusion produces convex upward slopes.~~ Main controls on variations of *D* include substrate, lithology, soil depth, climate and biological activity~~, amongst others~~. Values of *D* vary widely and range between 10^{-3} and 10^{-1} m^2 yr^{-1} for slopes under natural land use (Campforts et al., 2016; DiBiase and Whipple, 2011; Jungers et al., 2009; Roering et al., 1999; West et al., 2013). Linear hillslope diffusion produces convex upward slopes. Field evidence~~,~~ however, suggests that ~~this~~ the linear ~~form~~diffusion model is only rarely appropriate (Dietrich et al., 2013). Instead, hillslopes often tend to have convex-planar profiles because rapid, ballistic particle transport and shallow landsliding dominate as soon as slopes approach or exceed a critical angle (DiBiase et al., 2010; Larsen and Montgomery, 2012). To account for this rapid increase of flux rates with increasing slopes, Andrews and Bucknam (1987) and Roering et al. (1999) proposed a nonlinear formulation of diffusive hillslope transport, assuming that flux rates increase to infinity if slope values approach a critical slope S_c:

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$$\mathbf{q_s} = -\frac{D\nabla z}{1-\left(|\nabla z|/S_c\right)^2} \qquad (5)$$

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~~Main controls on variations of *D* include substrate, lithology, soil depth, climate and biological activity, amongst others. Values of *D* vary widely and range between 10^{-3} and 10^{-1} m^2 yr^{-1} for slopes under natural land use (Campforts et al., 2016; DiBiase and Whipple, 2011; Jungers et al., 2009; Roering et al., 1999; West et al., 2013).~~

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2.4. Overall landscape evolution

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In summary, TTLEM solves the following ~~partial differential equation~~PDE, where~~by~~ an explicit distinction is made between river and hillslope cells, based on a threshold contributing area, A_c ~~cells sculpted by fluvial versus hillslope processes is made.~~:

~~s: First, it simulates the horizontal tectonic displacements over the entire model domain:~~

$$\cancel{\frac{\partial z}{\partial t} = \mathbf{v_x}\frac{\partial z}{\partial x} + \mathbf{v_y}\frac{\partial z}{\partial y}} \qquad ~~(6)~~$$

~~Second, TTLEM simulates detachment limited river incision for the parts of the landscape that are predominantly sculpted by fluvial processes. We determine that domain where contributing drainage area (A) exceeds a critical drainage area (A_c):~~

$$\cancel{\frac{\partial z}{\partial t} = U + \left(v_x\frac{\partial z}{\partial x} + v_y\frac{\partial z}{\partial y}\right) - \left(Kw_K\,(w_A A)^{(m+\mathrm{var}(m))}\left(\frac{\partial z}{\partial x}\right)^n\right)} \qquad ~~(7)~~$$

~~where var(m) refers to the variability on m which is explained further (Eq. (20)).~~

~~Third, we define the hillslope domain where $A < A_c$. Topographic changes in this domain are calculated by:~~

$$\cancel{\frac{\partial z}{\partial t} = U - \nabla\mathbf{q_s}} \qquad ~~(8)~~$$

~~where ρ_r and ρ_s are the bulk densities of the bedrock and the regolith material, respectively [m L^3]. The formulation of Eq. (8) implies that we assume that hillslopes are generally covered by regolith and/or soil.~~

$$\frac{\partial z}{\partial t} = \left(\frac{\partial z}{\partial t}\right)_{td} + \begin{cases} U + \left(\dfrac{\partial z}{\partial t}\right)_{fluv} & \text{for } A \geq A_c \\ \dfrac{\rho_r}{\rho_s}U + \left(\dfrac{\partial z}{\partial t}\right)_{hill} & \text{for } A < A_c \end{cases} \qquad (6)$$

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~~3.~~ ~~where an explicit distinction between cells sculpted by fluvial versus hillslope processes is made.~~Rivers are assumed to incise directly into bedrock whereas material fluxes on slopes are assumed to mobilize ~~-~~either soil or regolith, having a different bulk density than the bedrock. This is accounted for by multiplying the rock uplift rate with the density ratio between ρ_r and ρ_s [m L^3] representing the bulk densities of the bedrock and the regolith material respectively ~~-~~(Perron, 2011). The fluvial domain is determined by the cells having a contributing drainage area (A) exceeding a critical drainage area (A_c).

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3. Implementation and numerical schemes of TTLEM

Our main motivation to develop TTLEM is to provide users with a multi-process landscape evolution model that has a good overall computational performance and high numerical accuracy. TTLEM is ~~predominantly~~ written in the MATLAB programming language; to reduce run times, however, TTLEM encompasses some C-code where this significantly improves performance (e.g. for the non-linear hillslope diffusion algorithm of Perron (2011)). Integrating TTLEM into TopoToolbox enables running the model, visualizing and analyzing its output in the same computational environment.

~~Figure 1 shows a schematic representation of the TTLEM workflow.~~ Users can configure the tectonic setting by providing (i) a 2D or 3D array that represents spatially and spatio-temporally variable vertical uplift patterns, respectively, and (ii) two matrices to represent horizontal velocity fields (~~$\mathbf{v_x}$~~$\mathbf{u}$ and ~~$\mathbf{v_y}$~~$\mathbf{v}$). TTLEM accepts synthetic topographies and real world DEMs and leaves users with full control on model parameter values. In the following sections, we will discuss the numerical methods ~~involved~~ used in TTLEM to solve the PDEs described in section 2. The section numbers correspond to the processes indicated in the ~~workflow~~ model flowchart in the appendix (~~in~~ Fig. A1).

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3.1. Drainage network development

~~3.1.~~ TopoToolbox provides a function library for deriving and updating the drainage network and terrain attributes in MATLAB (Schwanghart and Scherler, 2014). The calculation of flow-related terrain attributes, i.e., data derived from flow directions, relies on a set of highly efficient algorithms that exploit the directed and acyclic graph structure of the river flow network (Phillips et al., 2015). Nodes of the network represent grid cells and edges represent the directed flow connections between the cells in downstream direction. Topological sorting of this network of grid cells transforms ~~returns~~ an ordered list of cells in that upstream cells appear before their downstream neighbors. Based on this list, we calculate terrain attributes such as upslope area with a linear scaling thus enabling efficient calculation ($O(n)$) at each ~~time step~~time step ~~of the simulation~~ even for large grids (Braun and Willett, 2013).

~~3.2.~~ DEMs of real landscapes frequently contain data artifacts that generate topographic sinks. Topographic sinks can also occur as a result of diffusion on hillslopes by creating "colluvial wedges" damming the sections of the river network. By adopting algorithms of flow network derivation from TopoToolbox, TTLEM makes use of an efficient and accurate technique for drainage enforcement based on auxiliary topography to derive non-divergent (D8) flow networks (Schwanghart et al., 2013; Soille et al., 2003). Based on the thus derived flow network, TTLEM uses downstream minima imposition (Soille et al., 2003) that ensures that downstream pixels in the network have lower or equal elevations than their upstream neighbors.

~~3.3.~~3.2. Tectonic displacement

We implement a 2D version of a flux limiting total volume method to reduce numerical diffusion when simulating tectonic displacements on a regular grid. Equation (1) can be written as a scalar conservation law:

$$z_t + f(z)_u + f(z)_v = 0 \qquad (7)$$

where $f(z)_u$ = ~~$\mathbf{v}$~~$\mathbf{u}$~~$_x$~~z and $f(z)_v$ = $\mathbf{v}$~~$_y$~~z are the flux functions of the conserved variable z. We refer to the supplementary material of Campforts and Govers (2015) for a derivation of the differential form of Eq. (7) which can be converted to a numerical semi-conservative flux scheme:

$$z_{i,j}^{k+1} = z_{i,j}^{k} + \frac{\Delta t}{\Delta x}\left[f_{i-\frac{1}{2},j} - f_{i+\frac{1}{2},j} \right] + \frac{\Delta t}{\Delta y}\left[f_{i,j-\frac{1}{2}} - f_{i,j+\frac{1}{2}} \right] \qquad (8)$$

where $z_{i,j}^{k}$ is the elevation of the cell at row i and column j at time $k \times \Delta t$. f represents the numerical approximation of the physical fluxes from Eq. (7). The in- and out coming fluxes are subsequently approximated with a flux limiting upwind method which is TVD. A TVD scheme prevents the total variation of the solution to increase in time and hence prevents spurious oscillations that are associated with higher order numerical methods (Toro, 2009). The use of a flux limiter allows the method to have a hybrid order of accuracy being second order accurate in most cases but shifting to first order accuracy near discontinuities. Hence the TVD-FVM method establishes a compromise between two desirable properties of a numerical method: it achieves a higher order of accuracy than first order schemes while ensuring numerical stability (Harten, 1983). TTLEM uses a staggered Cartesian grid for numerical discretization. The data grid points, or elevations from the DEM (z), are considered to represent the center of the computational cells, whereas the velocity fields ($\mathbf{u}$~~$\mathbf{v}_x$~~ and $\mathbf{v}$~~$_y$~~) are located at the cell faces.

The numerical TVD fluxes are calculated following Toro (2009). In the following, we illustrate how ~~:~~the flux over one cell boundary can be derived:

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$$f^{TVD}_{i+\frac{1}{2},j} = f^{LO}_{i+\frac{1}{2},j} + \varphi_{i+\frac{1}{2},j}\left[f^{HI}_{i-\frac{1}{2},j} - f^{LO}_{i+\frac{1}{2},j} \right] \tag{9}$$

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where f^{HI} and f^{LO} represent the high and low order fluxes respectively:

$$\begin{aligned} f^{LO}_{i+\frac{1}{2},j} &= \alpha_0 v_{i+\frac{1}{2},j} z^k_{i,j} + \alpha_1 v_{i+\frac{1}{2},j} z^k_{i+1,j} \\ f^{HI}_{i+\frac{1}{2},j} &= \beta_0 v_{i+\frac{1}{2},j} z^k_{i,j} + \beta_1 v_{i+\frac{1}{2},j} z^k_{i+1,j} \end{aligned} \tag{10}$$

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The low order fluxes are solved with a first order explicit upwind Godunov scheme (1959):

$$\alpha_0 = \frac{1}{2}\left(1 + sign\left(\mathbf{v}\right)\right) \quad and \quad \alpha_1 = \frac{1}{2}\left(1 - sign\left(\mathbf{v}\right)\right) \tag{11}$$

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The high order fluxes are solved with a Lax-Wendroff scheme (1960):

$$\beta_0 = \frac{1}{2}\left(1 + \mathbf{v}\frac{\Delta t}{\Delta x}\right) \quad and \quad \beta_1 = \frac{1}{2}\left(1 - \mathbf{v}\frac{\Delta t}{\Delta x}\right) \tag{12}$$

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From Eq. (10), Eq. (11) and Eq. (12) it follows that:

$$\begin{aligned} f^{LO}_{i+\frac{1}{2}} &= v_{i+\frac{1}{2},j} z^k_{i+1} \\ f^{HI}_{i+\frac{1}{2}} &= \frac{1}{2} v_{i+\frac{1}{2},j}\left(z^k_i + z^k_{i+1}\right) - \frac{\left(v_{i+\frac{1}{2},j}\right)^2 \Delta t}{2\Delta x}\left(z^k_{i+1} - z^k_i\right) \end{aligned} \tag{13}$$

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$\varphi_{i+\frac{1}{2},j}$ represents the flux limiter, which is solved with the van Leer scheme (1997):

$$\varphi_{i+\frac{1}{2},j} = \frac{r_{i+\frac{1}{2},j} + abs\left(r_{i+\frac{1}{2},j}\right)}{1 + abs\left(r_{i+\frac{1}{2},j}\right)} \tag{14}$$

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where r is a smoothness index calculated as:

$$r_{i+\frac{1}{2},j} = \frac{z^k_{i+2,j} - z^k_{i+1,j}}{z^k_{i+1,j} - z^k_{i,j}} \tag{15}$$

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The overall performance of the TVD-FVM is evaluated by comparing it with the first order accurate upwind Godunov scheme which is not flux limiting Eq. (11). In the remaining part of the ~~text~~text, we refer to this scheme as the first order Godunov Method (GM).

3.3. River network updating

~~3.4.~~3.3.1. Numerical solution

TTLEM features a 1D version of the flux limiting TVD-FVM to solve for river incision (Eq. (2) ~~(7)~~) which can be written as a scalar conservation law ~~is~~:

$$z_t + f(z)_x = 0 \qquad (16)$$

where *f(z)* represents the flux function of the conserved variable *z*, representing the channel elevation. The method is similar ~~than~~ to the one described in section 3.2 although fluxes are only calculated in one direction. We refer to the Supplementary Information provided by Campforts and Govers (2015) for a full derivation of this scheme.

In addition, we implement a first order ~~explicit and~~ implicit FDM for the solution of the stream power law detailed in Braun and Willett (2013). Implicit schemes provide stable solutions regardless of the ~~time step~~time step ~~considered~~ length, a property desired when simulating landscape evolution over long timescales and large spatial domains. An explicit scheme (both FDM and TVD-FDM), in turn, requires ~~time steps~~time steps that satisfy the Courant Friedrich Lewy condition (CFL):

$$\frac{K \max(A)^m \Delta t}{\Delta x} \le 1 \qquad (17)$$

Hillslope processes allow for the use of ~~a fairly~~ long~~big~~ time steps due to the diffusive nature of the processes and the implementation of implicit methods to solve them. River incision however requires the use of smaller time steps ~~in case of~~ (i) because an explicit scheme~~s~~ is used, requiring CFL≤1 and (ii) to avoid that a sudden input of vertical uplift in the solution ~~resulting~~would result in the generation of an artificial shockwave~~s~~. To optimize model performance ~~and allowing different time steps for different model modules such as hillslope processes and river incisions,~~ we therefore introduce the use of ~~We introduce a~~a~~n~~ smaller, inner ~~time step~~time step (Δt_{inner}) ~~for the simulation of river uplift and incision~~ for river incision simulation ~~to achieve a sufficiently small time step~~~~time step~~ ~~while maintaining an acceptable runtime~~ (Fig. 1). TTLEM also allows using an ~~for~~ inner ~~timesteps~~time step~~s~~ and satisfying the CFL criterion if an ~~the~~ implicit solution is used for river incision. Although this is not strictly necessary as such schemes are unconditionally stable it ~~. While the implicit solution is unconditionally stable, an inner time~~ allows us to investigate the impact of the length of the ~~timestep~~time step on model outcomes (see section 5.1.2). At low spatial resolutions ~~Even when~~ the Courant criterion is satisfied, even for~~,~~ ~~model runs at low spatial resolutions can potentially allow~~ very large ~~timesteps~~time steps. ~~Large timesteps could imply a sudden input of vertical uplift in the solution resulting in the generation of artificial shockwaves.~~ Therefore, TTLEM also allows ~~to user to~~users to set a maximum length of the inner ~~timestep~~time step (Δt_{max}) ~~which we set by default to 3000 yr~~. alscheme

~~*Regular grids introduce artefacts in the planform geometry of river networks because local drainage directions are restricted to eight directions (Braun and Sambridge, 1997). Moreover, as the process formulations are deterministic and flow direction algorithms follow a predefined order, LEMs tend to produce landscapes that are too uniform with respect to slope morphology and river planform patterns. To overcome this issue, we apply the method of Grimaldi et al. (2005) to explicitly integrate*~~

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~~*some randomness in the calculation of the value of the drainage area exponent (m) by attributing a variance to m :*~~

$$\mathrm{var}(m) = \frac{\ln\left(1+\frac{k_1}{k^2}\right)}{\left(\ln(A)\right)^2} \qquad (20)$$

~~*where k1 and k are proportionality coefficients. We update at each time step a new value of m for each grid cell randomly drawing an error value from the distribution described by Eq. 13 and adding it to the mean value of m.*~~

3.3.2. ~~*Another way to add variability in evolving landscapes is to allow the erodibility parameter K, to vary in space, thereby mimicking local, semi-random variations in rock strength. Here, variability on K is simulated by introducing a normally distributed random deviation with a zero mean.*~~ *Analytical solution*

While the comparison of different numerical methods can provide valuable insights with respect to their relative accuracy and performance, the ultimate test is the comparison of numerical results with an analytical solution of the PDE.

~~An a~~Analytical solutions are fully correct and are evidently grid resolution independent, contrary to numerical solutions where model parameter values might depend on the grid resolution (Pelletier, 2010). However, they are not universally applicable. We implemented an analytical solution for the stream power law ~~was implemented to test the overall model performance and to obtain as an~~as an independent benchmark to compare the performance of the different numerical schemes implemented in TTLEM under conditions where an analytical solution can indeed be obtained. ~~Moreover, analytical solutions are grid resolution independent, contrary to numerical solutions where model parameter values might dependent on the model resolution (Pelletier, 2010).~~

~~In the following, the strategy to investigate the performance the model is outlined. The analysis is initiated from~~ First we created an artificial DEM where a steady state between uplift and erosion ~~is~~is reached. From this DEM, the drainage network and corresponding river elevations ~~are~~are extracted by selecting all cells exceeding a threshold value (in our case 10^6 m²). Very short river profiles (<10 km) ~~are~~are not retained. ~~in the analysis to improve the performance.~~ Subsequently, landscape evolution ~~is~~is simulated using the numerical models documented in the previous sections assuming spatially invariant uplift rates. At the end of the model runs, river elevations ~~are~~are again extracted from the numerically simulated DEMs and compared with ~~analytically calculated~~ river elevations that were analytically calculated using the pre-uplift, steady state river profiles as input. Analytical solutions for the stream power law ~~can be~~are obtained using the slope patch method of Royden and Perron (2013). This~~se~~ method is based on a non-dimensionalisation of the stream power law. ~~Therefore,~~ Longitudinal river profiles ~~are~~are converted to a dimensionless height (λ) and distance (χ):

$$\lambda = \frac{z_x}{h_0} \qquad (18)$$

$$\chi = \frac{A_0^{\,m/n}}{h_0}\int_0^x \frac{dx}{A_x^{\,m/n}} \qquad (19)$$

where z_x represents the dimensionless elevation along the river profile, h_0 is a reference length scale (~~typically~~ set to 1 m) and A_0 is a reference value for the drainage area (typically set to 1×10^6 m²). To properly integrate over abrupt changes in the drainage area along the rivers, Eq. (19) is solved using the rectangle rule (Mudd et al., 2014). Steady state river profiles (equilibrium between erosion and uplift) are represented as straight lines in this non-dimensional coordinate system. ~~To~~

~~properly integrate over abrupt changes in the drainage area along the rivers, Eq. (19)(19) is solved using the rectangle rule (Mudd et al., 2014). In case of a Steady state between erosion and uplift, non-dimensional river profiles are represented as a straight line in non-dimensional coordinates. From these non-dimensional river profiles The slope patch solution can subsequently be applied for temporally variant but spatially invariant uplift rates assuming an initial elevation of the river profile.~~ The analytical slope patch solution ~~method~~ ~~developed by Royden and Perron (2013)~~ then calculates the evolution of a dimensionless river profile in response to uplift. ~~by is based on the tracing of~~~~tracing~~The method is detailed in the appendix of Royden and Perron (2013) and based on tracing individual patches which are initiated at the outlet of the drainage network and propagate upstream with a velocity depending on the uplift rate and the parameters of the SPL (Eq.(2)).

We applied the slope patch solution to the steady-state pre-uplift river profiles extracted from the DEM using the simulated uplift rates as input. ~~Given the analytical solution,~~ The accuracy of the numerical methods can then be assessed using a Root Mean Squared Error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(z_{i,analytical} - z_{i,numerical}\right)^2}{nb_{riv}}} \qquad (20)$$

where $z_{i,analytical}$ and $z_{i,TVD}$ refer to the analytically and numerically calculated elevation of ~~the~~a river cell~~s~~ respectively and nb_{riv} is the total number of ~~considered~~ river cells.

3.4. Hillslope processes

We implemented linear hillslope diffusion using ~~an efficient~~the implicit Crank-Nicolson scheme (Pelletier, 2008) that is unconditionally stable at large time steps~~. This scheme is implicit and therefore allows large time steps~~time steps. Implicit solutions are well suited since the linear diffusion equation is a parabolic PDE and ~~much less sensitive~~relatively insensitive to numerical diffusion in comparison to ~~the stream power law,~~hyperbolic advection equation of the stream power incision law ~~which is a hyperbolic PDE~~.

A numerical solution of the nonlinear hillslope equation, however, is ~~yet~~ more demanding. The maximum time step length ~~The of an~~ explicit FDM sharply decreases as slopes approach the threshold gradient~~is limited by the maximum length of the time step~~~~time step at which numerical stability is maintained~~. To overcome this restriction, Perron (2011) developed Q-imp, an implicit solver that allows to increase the time step length~~s of the time step~~~~time step~~ by several orders of magnitude. Whereas the per-operation computational cost of this algorithm is higher in comparison to the explicit solution, the overall performance of this method is better than ~~hitherto~~ alternative solutions (Perron, 2011). Q-imp efficiently calculates hillslope diffusion, even for high-resolution simulations. However, rapid incision during one time step may generate slopes along rivers that are greater than the threshold slope, a situation that cannot be addressed using Q-imp~~, but is restricted to hillslopes below the threshold slope~~. What is thus needed is an approach that adjust hillslopes to the threshold slope to warrant that the nonlinear diffusion equation can be solved.

~~Our approach to adjust hillslopes to the threshold slope Therefore, Q-imp must be combined with a hillslope adjustment algorithm.~~

~~We a~~We assume~~s~~ that hillslopes instantaneously adjust to oversteepening by mobilising the amount of material required to reduce the slope gradient to the threshold value S_c ~~along fault scarps and due to river undercutting~~ (Burbank et al., 1996). We refrain from simulating individual landslides although we acknowledge that single high magnitude low frequency events may be relevant at the time scales of our simulations (Korup, 2006). Instead, our approach implicitly accounts for the combined effects of a large number and variety of landslides that effectively adjust slopes to a threshold slope. ~~S_c.~~ ~~The~~ This threshold

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slope can be thought of~~ –~~ "an average effective angle of internal friction which controls hillslope stability" (Burbank et al., 1996). We implement this hillslope adjustment using a modified version of the excess topography algorithm (Blöthe et al., 2015). In this algorithm, elevations z at ~~time step~~time step $t + 1$ are ~~derived~~ calculated ~~in a way~~so that ~~entails that~~ the absolute local gradient at each grid cell ~~is~~ becomes less or equal than S_c. This is achieved by decreasing elevations at location~~s~~ i to the minimum elevation of all other locations j to which we add an offset calculated as the product of ~~by~~ the Euclidean distance $\|i,j\|$ and S_c:

$$z_i^{t+1} = \min\left\{z_i^t, \min\left[z_j^t + S_c \cdot \|i,j\|\right]\right\} \qquad (21)$$

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The above equation entails that z_i^{t+1} at one location depends on all other grid cells and that the algorithm has a time complexity of $O(N^2)$, which would render it unsuitable for frequent updating during LEM simulations. To avoid an ~~overtly~~ excessively high computational load, we implement the algorithm using morphological erosion with a gray-scale structuring element (see MATLAB function ordfilt2), which is a minimum sliding window with additive offsets calculated from the window size and Sc. This significantly reduces run times as we calculate elevations at one location from the sliding window. Yet, this approach not necessarily removes all gradients greater than S_c. We solve this by calling the algorithm repeatedly until all slope values are equal or less than S_c.

~~We assume that albeit sediment might be temporarily redeposited in the system, it will be easily evacuated within a relatively short time span due to the unconsolidated nature of the deposits (Mcguire and Pelletier, 2016). This assumption is reasonable for rapidly uplifting and eroding mountain belts, but may not be applicable in other environments where mass wasting occurs (Vanmaercke et al., 2014).~~

3.5. Boundary conditions

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TTLEM allows the use of Dirichlet or Neumann boundaries conditions. Alternatively, one can opt for a random disturbance at one or more boundaries of the modelled domain. The latter may be desirable~~is especially of useful when~~ when simulating strong lateral displacements which may otherwise generate artificially straight river profiles in the direction of the shortening.

~~1. Experiments~~

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~~In order to demonstrate possible applications of TTLEM we carry out two series of numerical experiments. We first illustrate the impact of different hillslope process models on simulated landscape evolution, using a 30 m resolution DEM of the Big Tujunga region in California as an example. Second, we investigate the amount of bias and artificial symmetry introduced in the landscape through the use of regular grids.~~

~~1.1. Hillslope processes~~

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~~TTLEM allows to simulate hillslope processes assuming (non)-linear slope dependent diffusion with the consideration of a threshold hillslope. Figure 2 illustrates how different hillslope process algorithms affect the evolution of hillslopes in the Big Tujunga region, California (Fig. 2a). We assume no tectonic displacement and use standard parameter values for river incision and hillslope diffusion (Table 1) and a threshold slope (S_c) of 1.2 (m/m) when applicable (Fig. 2b). We illustrate model results after 500 ky in Fig. 2c-d using the current topography as the starting condition. Linear diffusion (Eq. (4)) is not capable to keep up with river incision, which results in strongly oversteepened hillslopes near the river channels (Fig. 1c and 1g). While higher values for the diffusion coefficient D will eliminate this problem (e.g. Braun and Sambridge, 1997) they are incompatible with~~

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~~experimental findings (Roering et al., 1999) and will restrict hillslopes to convex upward shapes. The use of non-linear diffusion in combination with a threshold slope results in hillslopes similar to those simulated with linear diffusion in combination with a threshold slope. However, for a similar value of *D*, hilltops become more smoothed assuming non-linear diffusion as sediment fluxes due to diffusive processes now reach higher values when hillslopes approach the threshold slope.~~

~~Artificial symmetry~~

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~~Regular gridded LEMs may introduce artificial symmetry in evolving landscapes (Braun and Sambridge, 1997). We perform simulations with an entirely flat initial surface as well as with a random initial surface with uniformly distributed elevations between 0 and 50 m to investigate how random perturbations of the values of *m* or *K* affect drainage network evolution (Movie S1 and Movie S2). We consider four different scenarios for each initial surface (Fig. 3). Scenario 1 is the reference simulation, with a low spatial resolution of 1000 m, a large time step of 5×10^4 years and a *K* value of 6×10^{-6} m$^{-0.1}$ yr^{-1}. In scenario 2, the mean erodibility *K* is halved. In scenario 3 the time step is set to 1×10^4 years while in scenario 4, the spatial resolution is set to 200 m.~~

~~At low spatial and temporal resolutions, the use of uniform parameter values results in clear artificial symmetry (Fig. 3). Introduction of random variability on *m* mainly decreases similarity close to the river heads where the drainage areas are the smallest (scenario 1). This is a consequence of the formulation of Eq. (20): the introduced variability is relatively larger for small catchments. Variability in *K* slightly decreases overall artificial symmetry at low spatial resolutions (scenario 1). The use of a lower (mean) *K* value, representing slower river incision also decreases overall artificial symmetry (scenario 2). Decreasing the time step (scenario 3) results in slightly different drainage networks in comparison to simulations with larger time steps but fails to reduce the symmetry in the result. At a high spatial resolution (scenario 4), artificial symmetry is still present when constant parameter values are used. However, inserting variability on the *m* and *K* parameters is much more effective in reducing symmetry at this resolution.~~

~~Drainage networks simulated using an initial surface with elevations that randomly vary between 0 and 50 m are almost free of artificial symmetry and the final geometry of the drainage network is now less dependent of parameter variability. The latter underscores the importance of initial DEM conditions for the final results of a simulation (Perron and Fagherazzi, 2011). Nonetheless, even with a randomly varying initial surface, the perturbation on parameter values clearly affects the drainage network that is produced. Parameter value perturbation generally results in drainage networks which are less rectilinear than those simulated without perturbation.~~

4. Impact of numerical methods

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~~In a next step w~~We investigate ~~to what extent the~~how numerical schemes implemented in TTLEM affect simulated landscape evolution. ~~We distinguish between the effects on simulated river incision on the one hand and on simulated tectonic displacement on the other.~~ Because we focus on evaluating the model's performance~~All~~ all simulations are run~~We use a~~ with synthetically generated landscapes ~~for all simulations~~ as ~~a starting~~ initial ~~condition~~ surfaces~~because we are interested in~~~~focus on evaluating the model's performance~~~~the evaluation of the functionality of the model and not on the correct simulation of the evolution of a particular landscape or region~~. Hence, our simulations are uncalibrated and results remain untested~~were not compared~~ ~~with~~ against an actual ~~'true'~~ landscape: however, the chosen parameter values are realistic ~~(~~(e.g. Gasparini and Whipple, 2014; Whipple and Tucker, 1999)~~maybe provide one-two refs for these parameter values~~). We distinguish between the effects on simulated river incision on the one hand and on simulated tectonic displacement on the other. To investigate the accuracy and implications of river incision methods, we compare ~~two different~~the explicit TVD-FVM– with the first-order implicit FDM ~~numerical schemes~~ and further differentiate between the ~~and~~ implicit FDM ~~solution~~ where no limitation is set on the time step and the implicit ~~solution~~FDM where the CFL criterion limits the time step length~~is limited by the CFL~~

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~~criterion~~. To investigate the accuracy and implications of river incision methods we compare an explicit first order Godunov method (GM) with the 2D TVD-FVM.

4.1. River incision

~~3.5.~~ *4.1.1.* *1D river incision*

~~Ideally,~~ ~~The~~ The impact of numerical diffusion on propagating river profile knickpoints can most clearly be ~~is~~ demonstrated~~most obvious~~ in situations where an analytical solution is available. The first simulation illustrates such a situation, with an artificial river profile characterized by a major knickzone between 8 and 12 km from the river head (Fig. ~~4~~1). We assume that the drainage area is increasing in proportion to the square of the distance and uplift equals zero. For this simplifiede configuration, an analytical solution for the SPL ~~can be found using~~relies on the method of characteristics (Luke, 1972). Notwithstanding the relatively high spatial resolution of 100 m, ~~both implicit and explicit~~the first order implicit ~~Finite Difference Methods (~~FDM~~s)~~ suffers from ~~clear~~ considerable numerical diffusion when river incision is calculated over a time span of 1 Myr (Fig. ~~4~~1). The TVD-FVM systematically achieves a much higher accuracy~~, a finding that is systematic, occurring~~ over a wide range of spatial resolutions and parameter values (Campforts and Govers, 2015).

4.1.2. Drainage network

The ~~second simulation~~we assess ~~overall~~ numerical accuracy of the entire drainage network is assessed using spatially and temporally constant values for all model parameter values (Table 1) and assuming a fixed drainage networks (see~~is assessed using the approach outlined in~~ section 3.4). ~~Model performance is evaluated using a simple model set-up with spatially and temporally constant values for all model parameter values and assuming fixed drainage networks.~~ We first create a steady-state artificial landscape that we initialize with uniformly distributed random elevation values between 0 and 50 m on a 50 km × 100 km grid with a spatial resolution of 100 m (Movie S3). Landscape evolution is simulated using Dirichlet boundary conditions and by inserting spatially and temporally uniform vertical uplift of 1 km Myr^{-1} over a period of 150 Myr. Outer model time steps are set to 5 × 10^{4} yr. ~~Parameter values for river incision and hillslope response are constant in space and time and are reported in Table 1.~~ Figure 2 shows the resulting steady state landscape.

We impose four consecutive sinusoidal uplift pulses of equal magnitude to this artificial landscape over 1 Myr. Uplift pulses have a wavelength of 0.25 Myr and an amplitude of 3 × 10^{-3} m yr^{-1} (Fig. 3). ~~This~~~~is~~We repeat ~~All~~ the simulations ~~are performed using~~with three different numerical schemes to simulate river incision (implicit FDM without time step limitation, implicit FDM with time step limitation (CFL condition applied) and TVD-FVM), each at ~~and~~ 22 different spatial resolutions (6.25, 12.5, 25, 50, 100, 150, …950 m). Hillslopes are simulated using linear hillslope diffusion in combination with threshold slopes, a configuration typically used to simulate landscape evolution at ~~the~~ geological timescales (e.g. Goren et al., 2014). The threshold slope is set to a value of 0.8 m m^{-1} and hillslope diffusivity is set to a value of 0.01 m^{2}y^{-1}. ~~Model~~The computational performance is assessed by calculating the CPU time required to perform a 1 Myr simulation. In order to facilitate the high resolution run (at 6.25 m where the spatial domain covers 7950 × 15950 cells) all model runs were executed on one computational node of the Flemish Super Cluster (VSC) using a single core (Broadwell, E5-2680v4) and featuring 128 Gi~~i~~B RAM. We evaluate the numerical performance of the schemes and the impact of spatial resolution against an analytical solution (slope patch method) for the entire drainage network ~~Independent from the numerical simulations, river evolution is calculated using the slope patch method for the entire drainage network,~~ represented by all cells exceeding 1 km^{2} (~~indicated in grey on~~ Fig. 2).

Figure 4 displays the comparison between the numerical methods and the analytical solution. ~~Note that t~~The initial river profile (grey line) slightly differs depending on ~~the~~spatial resolution ~~considered~~ due to interpolation of the steady-state artificial landscape with a spatial resolution of 100 m. The results show~~se figures illustrate how the~~that TVD-FVM and implicit

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Formatted: Not Superscript/ Subscript numerical solutions converge when model resolution is increased. In case no CFL criterion is imposed on the solution, however, ~~T~~the implicit solution deviates from those adhering to the CFL criterion. This ~~does not converge in case no CFL criterion is imposed to the solution. The latter was done on purpose to~~ illustrates that there is trade-off between ~~how increased~~ numerical ~~smearing~~accuracy ~~countervails the~~ and ~~gain in~~ numerical ~~main advantage~~stability ~~of~~ for an implicit scheme at long time steps~~, i.e. being stable at time steps exceeding the CFL criterion, is counterbalanced by numerical smearing once the CFL criterion is exceeded~~. In addition, ~~The fact that the~~an implicit scheme at high spatial resolution fails to converge to an analytical solution ~~with~~if time steps are large ~~time steps is not converging at high resolutions, is however only partly explained by the first order spatial accuracy of the scheme. If very large timescales are applied in landscape evolution models,~~since uplift is inserted discretely~~very suddenly~~ at the beginning of ~~the~~each time step. This results in unrealistic simulations where uplift is a discrete stepwise ~~function~~ rather than a continuous function (e.g. the sinoidal uplift history~~e waves used in this paper~~ used here) ~~and~~that inserts artificial shocks in the solution.

Figure 5 illustrates that the TVD-FVM is more accurate than the implicit methods at all spatial resolutions although the implicit FDM (CFL<1) approaches the high accuracy of the TVD-FVM~~. Only~~ at very high resolutions (6.25 m)~~, the implicit FDM method is approaching the accuracy obtained with the TVD-FVM~~. At lower spatial resolutions (>10 m) the numerical accuracy of the TVD-FVM is significantly higher compared to the accuracy obtained with the implicit methods at the cost of ~~only~~ a slightly increased~~, without requiring~~ additional computation time. ~~that we optimized due by a to the vectorized implementation of the TVD-FVM.~~ To achieve the same numerical accuracy as the TVD-FVM at 500 m spatial resolution (RMSE = 18.17, model runtime = 2.89 seconds), the implicit method (CFL<1) would need to be evaluated at 150 m which would take 12 times longer (model runtime = 36 sec) (Fig. 5). ~~From Fig. 5 it can be derived that it would for example take for 12 times longer to obtain the accuracy of the river processes obtained with a TVD-FVM at 500 m (RMSE = 18.17, model runtime = 2.89 seconds) with an implicit method (cfl<1, at 150 m, model runtime = 36 sec).~~

4.1.3. River incision and catchment wide erosion rates

We hypothesize that ~~apart from river profile evolution,~~ the diffusive nature of commonly applied FDM is not restricted to the simulation of river longitudinal profiles but has systematic consequences for ~~accurate simulation of river knickpoints will influence~~ other measures derived from simulations with LEMs~~s~~~~landscape evolution as a whole~~. Such measures include catchment-wide erosion rates that ~~often~~ constitute the basis for model-field data comparison and model parametrization ~~(~~(Gasparini and Whipple, 2014; Moon et al., 2015)~~REFS)~~. In order to investigate the sensitivity of LEM-derived catchment wide erosion rates to different numerical schemes of the river incision model, w~~e first create~~e use the ~~a~~ steady-state artificial landscape ~~that we initialize with uniformly distributed random elevation values between 0 and 50 m on a 50 km × 100 km grid with a spatial resolution of 100 m (Movie S3)~~described in the previous experiments (section 4.1.2). ~~Landscape evolution is simulated using Dirichlet boundary conditions and by inserting spatially and temporally uniform vertical uplift of 1 km Myr^{-1} over a period of 150 Myr. Outer model timesteps are set to 5 × 10^{4} yr. Parameter values for river incision and hillslope response are constant in space and time and are reported in Table 1. Figure 5 shows the resulting steady state landscape.~~ Similar to these experiments~~ simulations outlined in section 4.1.2~~, we impose~~d~~ four consecutive uplift pulses of equal magnitude to this artificial landscape ~~(Fig. 5).~~ but here, ~~U~~uplift pulses have a wavelength of 1.25 Myr ~~and an amplitude of 1.5 × 10^{-3} m yr^{-1} (Fig. 6)~~ and~~.~~ TTLEM is run over 5 Myr ~~with main model time steps of 5 × 10^{4} yr~~, again with Dirichlet boundary conditions and a ~~pla~~planform fixed drainage network. We use two spatial resolutions (100 m and 500 m) and three different numerical methods (implicit FDM without ~~time step~~time step limitation, implicit FDM with ~~time step~~time step limitation (CFL condition applied) and TVD-FVM) to simulate river incision. ~~When applicable, t~~The maximum length of the inner ~~time step~~time step is set to 3

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× 10^3 yr for all schemes to ensure ~~make sure~~ that ~~also~~ the implicit method is converging at higher resolutions, too (see section 4.1.2). ~~Without inner time steps, river incision is calculated once for each main (outer) model time step (5 × 10^4 yr).~~

We compare differences in simulated erosion rates by randomly selecting a number of catchments with drainage areas ranging between 1 and 50 km^2 (221 and 202 catchments for runs at a spatial resolution of 100 m and 500 m respectively) (Fig. ~~8~~7). We calculate the erosion rates for each ~~time step~~time step by subtracting the elevation grid in the previous ~~time step~~time step from the updated, current, elevation grid. The difference between the results obtained with different numerical schemes is quantified by calculating ~~a Root Mean Square Error~~the offset between the TVD method and the first order implicit FDM schemes (~~*RMSE*~~$O_{TVD\text{-}FDM}$):

$$O_{TVD-FVM} = \sqrt{\frac{\sum_{i=1}^{n}\left(\varepsilon_{i,TVD} - \varepsilon_{i,FDM}\right)^2}{nb_{\Delta t}}} \qquad (22)$$

where $\varepsilon_{i,TVD}$ and $\varepsilon_{i,FDM}$ refer to the catchment wide erosion rates simulated with the TVD-FVM and FDM respectively to simulate river incision and $nb_{\Delta t}$ is the total number of discrete ~~time steps~~time steps of the simulated erosion record.

We rank the catchments ~~from low to high~~in increasing order of $O_{TVD\text{-}FDM}$ ~~RMSE~~ for each ~~comparison~~ simulation to investigate overall variations in catchment wide erosion rates. Figure ~~7~~ 6 shows the results for the catchments at 10%, 50% (median) and 90% percentile. ~~Note that t~~The ranking is performed separately for the models runs at 100 m and 500 m as different sub catchments are randomly generated for both simulation runs. The percentiles shown in Fig. ~~7~~ 6 therefore represent different catchments.

For most catchments, we observe significant differences in erosion response between the three numerical methods at a spatial resolution of 100 m. The amplitude of the response to a tectonic uplift pulse increases when reducing numerical diffusion: the use of a first order implicit FDM without ~~time step~~time step restriction results in a much smoother response in comparison to the TVD-FVM. The variations in response amplitude are significant: the majority of the catchments record amplitude reductions by more 50% when modelled with the implicit FDM without ~~time step~~time step restriction. ~~Time step~~Time step restriction (and thereby sacrificing the main advantage of the implicit FDM) significantly reduces numerical diffusion so that most catchments display an erosional response comparable to that simulated by the TVD-FVM. However, this ~~finding is supported only by the~~ is only true for simulations with a 100 m spatial resolution. The advantage of a ~~time step~~time step restricted implicit FDM over a non-restricted implicit FDM disappears almost completely for a coarser grid resolution of 500 m.

~~Catchment-wide erosion rates vary systematically with the use of different numerical methods.~~ Figure ~~8~~ 7 shows that erosion rates diverge between the different methods with increasing distance to the outlet of the main river while they are similar for larger catchments. A smaller effect of the numerical scheme on large catchment areas may partly arise from~~be partly due to~~ stronger averaging of local variations in catchment erosion rates~~s~~. In addition, catchments at a large distance from the outlet—and thus likely with smaller catchment areas—~~tend to~~will experience upstream migrating knickpoints only~~the uplift signal only~~ after several model ~~time steps~~time steps. ~~.~~ If catchments are far from the fault zone, knickpoints will then be significantly smoothed ~~if~~ by an implicit FDM ~~is used~~, which will ultimately affect the response of the catchment. ~~This smoothening is not apparent if the catchment is close to the border of the modelling domain.~~ Again, spatial resolution matters: a larger grid size not only results in larger differences on average but also in larger differences between small and large catchments (Fig. 7~~8~~).

The differences in catchment response relate to the differences in simulated erosion rates within the catchments. Figure ~~9~~ 8 illustrates the spatial difference in erosion rates calculated with the two numerical methods during the final step of the model run (after 5 Myr). This figure shows that spatial differences are significant and form a systematic banded pattern related to the upslope migration of the erosion waves of the individual uplift pulses.

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4.2. Tectonic displacement

We test the performance of the 2D version of the flux limiting TVD-FVM to simulate tectonic displacement. ~~using a simplified model setup.~~ ~~We use a~~A synthetic landscape is used as ~~an~~ initial ~~condition~~ surface and ~~impose~~ a constant lateral tectonic displacement is imposed while keeping erosion rates zero. Theoretically, this should result in a laterally displaced landscape that, apart from this displacement, remains unchanged in comparison to the initial state. We compare the flux limiting TVD-FVM with a first order accurate upwind Godunov Method (GM). Figure ~~10~~ 9 illustrates the results when applying a tectonic displacement in two directions (~~v_x~~**u** = ~~v_y~~**v** = 10 mm yr^{-1}) over a time span of 1 Myr. ~~The results show that~~ The explicit GM strongly smooths the resulting DEM whereas the 2D TVD-FVM scheme produces a DEM that is very similar to the initial DEM, with minimal amounts of numerical diffusion.

In order to quantify ~~and better understand~~ the amount of numerical diffusion (D_N [L^2 yr^{-1}]) introduced by the GM and the TVD-FVM method, we test a range of different model configurations and calculate the numerical diffusivity, D_N, corresponding to the observed smoothing. ~~The latter is done~~ D_N ~~by~~is ~~calculated~~ ~~calculating by~~ the diffusivity required to transform the initial DEM (DEM_{ini}) to the final DEMs produced at the end of the simulations (DEM_{fint}). The optimum amount of diffusion is determined by minimizing the misfit function H with a sequential quadratic programming method (Nocedal and Wright, 1999). H is given by:

$$H = \sqrt{\frac{\sum_{px=1}^{nb_{px}} \left(DEM_{ini} - DEM_{f\,\mathrm{int}}\right)^2}{nb_{px}}} \qquad (23)$$

where nb_{px} is the number of pixels in the DEM. ~~–~~ Figure ~~11~~10 a illustrates the relation between D_N and the spatial resolution of different numerical approximations. The 2D TVD-FVM decreases numerical diffusion by a factor of 5-60 compared to the GM (Fig. ~~11b~~10b). The accuracy increases for both schemes with increasing resolution and increasing CFL numbers. ~~Accuracy increases with The increase in accuracy with higher spatial resolution because is due to smaller spatial steps that result in better approximations of the spatial derivatives.~~ Yet, the gain in accuracy with increasing spatial resolution is higher for the TVD-FVM than for the GM. Our analysis shows that the explicit FDM performs best with a CFL criterion close to one~~. This may seem counterintuitive as one might expect smaller time steps (CFL = 0.5) to lead to higher accuracies. However, the accuracy gain from an increase in temporal resolution is reduced by additional~~ where ~~numerical diffusion that is introduced by more~~additional required iterations within a given time interval are at a minimum (Gulliver, 2007).

5. Discussion

~~There is a growing consensus that mMost eroding landscapes are in a transient state (Mudd, 2016; Vanacker et al., 2015) that can be assessed using . LEMs. The dynamics of drainage networks and divides (Willett et al., 2014) and the nonlinear models involved, however, entail that LEMs can hardly rely on analytical solutions (Fox et al., 2014), but require numerical solvers of the governing PDEs. The successful use of these simulation tools thus requires knowledge about their numerical accuracy with high numerical accuracy are thus needed to capture transiency correctly, yet . Despite the growing interest in the development and use of LEMs, the assessment of LEM numerical accuracy has fallen short, yet. We show that most commonly applied first order accurate numerical methods introduce numerical diffusion and smear discontinuities that are inherent in~~

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~~transient landscapes. To overcome this problem, we present a higher order flux limiting scheme referred to as TVD-FVM. We exemplify the use of this technique by simulating the upward migration of~~ Knickpoints ~~knickpoints and the evolution of river longitudinal profiles as well as horizontal tectonic movementsin river systems are of particular concern to geomorphologists as their analysis reveals insights into the tectonic and climatic controls on evolving landscapes. However, no analytical solution exists that allows to simulate river incision for changing drainage areas (Fox et al., 2014). Because drainage networks and drainage divides evolve in dynamic ways (Willett et al., 2014), the analysis of transient landscapes must thus rely on numerical methods, although analytical models can be applied in specific cases (Perron and Royden, 2013). Similarly, current grid-based models do not allow to accurately simulated the evolution of a landscape subject to tectonic shortening with a spatially variable velocity field. We present a higher order flux limiting scheme (referred to as TVD-FVM) that overcomes this problem of numerical diffusion.~~.

Our analysis of numerical solvers ~~focus~~focusseed s on three interrelated ~~numerical~~ issues: numerical accuracy, spatial resolution and computational efficiency. Adopting highly simplifying assumptions allow~~ed~~ us to benchmark the solvers against analytical solutions. Our focus ~~was~~is on testing an implicit FDM against TVD-FVM. The implicit FDM has ~~numerous~~several desirable properties ~~advantages~~. It is unconditionally stable and tolerates time step lengths exceeding those prescribed by the CFL criterion. LEMs are often run over time spans of millions of years and the CFL criterion is dictated by a few ~~DEM~~ grid cells with high upslope areas. ~~Thus,~~ Adopting an implicit scheme is therefore potentially interesting ~~tempting~~ as it allows to significantly decreas~~eing~~ the computation~~al~~ time while it enables~~ing~~ simulations at high spatial resolutions. Our results, however, show that this major advantage vanishes if the aim of a~~n~~ LEM simulation is to capture transiency correctly. For CFL > 1, the implicit FDM introduces significant numerical smearing, and for CFL >> 1, the approach tends to insert artificial shockwaves of uplift ~~as fault movements are modelled~~because gradual uplift is approximated by a step function if time steps are (very) large. ~~as stepwise functions rather than continuously.~~

For time step lengths approaching those prescribed by the CFL criterion, we show that computational gains by implicit FDM are marginal compared to TVD-FVM. The TVD-FVM code can be vectorized, i.e. it exploits single-instruction multiple-data parallelism to save CPU time. The ~~is performance gain may not be reached by the~~ implicit FDM requires a lower ~~despite the lower~~ number of numerical operations required~~, as this method must sequentially~~ but ~~loop through~~ all stream network nodes need to be treated sequentially. ~~However, we have not fully exploited ways to improve the computational performance of the implicit FDM such as processing individual drainage basins in parallel (Braun and Willett 2013). While unexplored in our study, we expect that separating the data by drainage basins will likely add significant computation and communication overhead.~~ Simulations at higher spatial resolutions increase the numerical accuracy and may balance the low accuracy of the implicit FDM. Our results indicate that there is indeed a strong gain in numerical accuracy for all methods (Fig. 4 and 5) with increasing spatial resolution. However, to achieve the same numerical accuracy as the TVD-FVM, the implicit method with a CFL<1 constraint ~~would~~ requires the use of spatial resolution that is ca. ~~3~~three~~x~~ times higher, resulting in a computation time that is ca. ~~spatial resolutions and 12~~twelve times ~~the CPU time~~higher (Fig. 5). In summary, while a first order implicit scheme is stable and accurate for long-term, steady-state solutions (Braun and Willett 2013), it has severe shortcomings when~~in accurately~~ simulating transient landscape evolution caused by knickpoint propagation in detachment limited erosional basins. These shortcomings can, to a large extent, be avoided by using a TVD-FVM~~, a finding that can also be transferred to the nonlinear river incision model (n≠1) (Campforts and Govers, 2015)~~. ~~(Campforts and Govers (2015)).~~

~~for such simulations~~.

~~As we focus on the numerical accuracy of landscape evolution models, we focused on relatively simple simulations considering only linear river incision (n=1), spatially and temporally constant parameter values, uplift and precipitation. Nonetheless, TTLEM supports temporal and spatially variable input values for all these parameters, e.g. by changing the erodibility weighting matrix (w_K) or contributing drainage area weighting matrix (w_A). The impact of non-linear river incision is discussed in detail in Campforts and Govers (2015). Currently TTLEM does not yet support transport limited river fluvial processes, neither glacial erosion or a bedrock/regolith interface to simulate soil evolution processes (Campforts et al., 2016). TTLEM uses D8 routing to update the drainage network during model simulations. Dinf (or D∞) is the flow routing scheme of choice to represent flow on hillslopes (Pelletier, 2008). However, in TTLEM fluvial erosion is limited to the channelized domain of the landscape and thus the flow routing scheme on hillslopes is of minor significance. Nevertheless, even in the channelized domain Dinf has advantages over D8 since it enables diverging flows on landforms such as alluvial fans and braidplains. The current implementation of TTLEM, however, focuses on the modelling of detachment-limited systems or bedrock rivers where divergent flows are usually confined by valley walls. This is also consistent with other models such as Fastscape (Braun and Willett, 2013) and DAC (Goren et al., 2014) models that use the D8 flow routing scheme. Nonetheless, we do not exclude to implement Dinf or other multiple flow direction algorithms in a future version of TTLEM, in particular since the topological sorting algorithm (Braun and Willett, 2013; Heckmann et al., 2015) is equally suitable for the efficient computation of flows on thus derived networks.~~

~~What are the implications of numerical diffusion of transient river profiles for LEMs in general? A performance analysis allowed to evaluate the computational efficiency and the numerical efficiency of the different schemes implemented in TTLEM. In order to perform this analysis we implemented an analytical slope patch method for the stream power law being resolution independent. The analytical solution functions as a robust benchmark to evaluate not only the numerical accuracy of the river incision methods but also offers a tool to evaluate model performance in general. The performance analysis demonstrates (i) that the numerical methods (the implicit FDM method and the TVD-FVM) converge at high resolutions. Moreover, the analysis shows how the implicit method is only marginally performing better in terms of computational timesperformance for similar resolutions which is due to the fact that implicit schemes cannot be vectorized (see section 4) and river cells only occupy part of the landscape. The performance analysis shows how implicit methods without a restriction on the time step does not converge, partly due to the increased amount of numerical smearing introduced in the solution for CFL>1 and partly due to the fact that uplift in inserted to abruptly in the model if CFL>>1. Therefore, the main advantage of an implicit scheme, i.e. being unconditionally stable against varying time steps vanishes as also implicit schemes require the definition of an inner time step in order to properly simulate river incision. In Fig. 4, two extremes are shown, i.e. a configuration where CFL<1 and one where CFL >>1. One could argue that intermediate solutions (e.g with CFL closer to 1) would result in more desirable results than the ones shown Fig. 4. This is true but, given that computational gains are marginal and numerical accuracy will never be higher than the implicit method simulated at CFL< 1 (solid blue lines), we see little reason to follow such an approach when simulating transient landscape evolution. In summary, we conclude that a first order implicit scheme is not suited to properly simulate propagating knickpoints in detachment limited erosional basins. First order implicit methods are therefore only suited to simulate configurations where transiency, caused by local base level falls, tectonic faults or lithological contacts can be considered to be minor.~~

~~Our simulations show that optimizing numerical schemes of LEMs is far from being only a numerical exercise. The~~~~W~~We also show that the impact of the numerical scheme used to simulate ~~detachment limited~~ river incision ~~on model outcomes is substantial and~~ is not limited to river profile development alone. Hillslopes adjust to local base level changes dictated by river incision. Hillslope denudation rates therefore must ~~thus~~ —at least partly— reflect the geometry and dynamics of a knickpoint and will respond differently to ~~, whether it is~~ a diffuse signal that is the result of relatively slow, continuous uplift on the one hand and ~~or~~ a sharp discontinuity ~~migrating upstream~~ caused by a rapid base level drop of major fault activity on the other hand. Our simulations show that, depending on the spatial and temporal resolution, catchment wide erosion rates are more responsive to uplift when fluvial incision is calculated by ~~derived from the~~ TVD-FVM rather than by~~in comparison to~~ the implicit FDM~~s~~. This is because ~~F~~first order (explicit and implicit) FDMs fail to properly reproduce transient incision waves

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(Campforts and Govers, 2015) due to knickpoint smoothing. This also ~~with the effect that the smoothing propagates to inferred rates of~~ affects hillslope denudation as the drop in hillslope base level due to the passage of a knickpoint is smeared out in time when smoothing occurs. The response of ~~and that C~~catchment wide erosion rates to uplift will therefore also be smoothed, resulting in significantly lower peak erosion rates. ~~are smeared over geological time.~~ ~~Our results show that this effect will not be strong in catchments in direct vicinity to faults, but is~~ This effect will be most significant in upstream catchments which are far away from the base level as smoothing increases with time and knickpoint migration distance.~~s further upstream.~~ ~~Empirical studies that aim to link their findings from e.g. detrital cosmogenic nuclide derived denudation rates to LEMs may consider that potential bias introduced by commonly used FDMs.Thus, the use of a shock preserving method such as TVD-FVM is strongly recommended for accurate simulations of transient landscapes.~~

O

~~It could be argued that TVD-FVMs are unnecessary as long as one applies an implicit method in combination with a sufficiently small time resolutionstep. Although small time resolutionssteps partly resolve the problem of smearing, their effect on numerical accuracy can hardly be generalized. Our simulations show that, for the selected parameter value combinations, results were only acceptable if a time step restriction is combined with a relatively high spatial resolution (100 m). In addition, it is well possible that, for other parameter value sets, numerical diffusion will be important, even if a fine grid is used. It would be infeasible for a model user to detect smearing problems in standard applications as comparable exact, analytically derived solutions, usually are nonexistent. Hence, we argue that the use of a shock capturing TVD-FVM numerical scheme is preferable since it avoids significant numerical diffusion under a wide range of parameter values and spatial resolutions. Moreover, by constraining the time step of a first order implicit method below the CFL criterion, the main advantage of an implicit scheme, i.e., the stability for any time step, disappears.~~

~~O~~One might ~~debate~~ question the significance and necessity of numerical schemes that avoid diffusion of retreating knickpoints. Given ~~In th~~the many assumptions and uncertainties that ~~underly~~underlie many LEMs, numerical accuracy may seem ~~the least~~a problem of lesser importance. We argue~~think~~ that the simulations presented in this paper show that this is not the case and that it is indeed critical to simulate knickpoint retreat as accurately as possible.~~—~~ ~~using a method that avoids numerical diffusion.~~However, our analysis does not cover all situations wherein the accurate simulation of knickzones is ~~necessary~~important. Simulation of sharp knickpoints is also required in geomorphological and lithological settings where knickpoint retreat is caused by rock toppling, possibly triggered during extreme flood events~~, where knickpoint diffusion through abrasion and plucking of small blocks is minor~~ (Baynes et al., 2015; Lamb et al., 2014; Mackey et al., 2014). Similarly, glacial incision often creates hanging valleys which are reshaped by migrating fluvial knickpoints after glacial retreat (Valla et al., 2010).

In all these cases simulation tools with a minimum of numerical diffusion are required to correctly quantify natural knickpoint diffusion and to study the underlying processes.

~~Even in bedrock-dominated landscapes knickzones are often smoothed, possibly due to flow acceleration above knickzone lips and subsequent localized higher erosion (Berlin and Anderson, 2007). The discrepancy between actual and simulated longitudinal profiles of hanging valleys has prompted (Valla et al., 2010) to prefer a transport-limited model (Willgoose et al., 1991) over a detachment-limited model (Howard, 1994; Whipple and Tucker, 1999b). The presence of significant sediment loads does not necessarily imply that transport limitations control river incision. Sediment flux dependent models, as first proposed by Sklar and Dietrich (1998) consider the hybrid role of sediment particles, acting as a tool to break and erode river beds in eroding regimes and as a covering armor in depositional regions (Gasparini and Brandon, 2011; Sklar et al., 1998). One-dimensional analytical simulations have shown that this process might generate over-steepened river reaches and explain~~

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~~the presence of permanent hanging fluvial valleys (Crosby et al., 2007). Numerical LEMs accounting for saltation-abrasion have so far not been able to reproduce such permanent hanging valleys: however this may be caused by the effects of numerical diffusion rather than by an inadequate process formulation (Crosby et al., 2007). Simulation of sharp knickpoints is also required in geomorphological and lithological settings where knickpoint retreat is caused by rock toppling, possibly triggered during extreme flood events, where knickpoint diffusion through abrasion and plucking of small blocks is minor (Baynes et al., 2015; Lamb et al., 2014; Mackey et al., 2014). Thus, various scenarios of knickpoint retreat exist, some~~ of ~~which are characterized by significant natural diffusion, while others are not. In both cases simulation tools with a minimum of numerical diffusion are required to correctly quantify the importance natural diffusion~~ and to study the underlying processes~~.~~

First order numerical methods also inadequately simulate lateral tectonic displacement on a regular grid. The amount of numerical diffusion that is introduced by these methods will, in many cases, far exceed natural diffusion rates, thus ~~rendering~~ making accurate simulation of hillslope development impossible. A 2D variant of the TVD-FVM~~, instead,~~ strongly reduces the amount of numerical diffusion (D_N) to values well below natural diffusivity values, an effect that is especially apparent at high spatial resolutions. ~~We thus implemented a scheme that~~The 2D TVD-FVM thus allows to accurately model ~~a~~ this process, that significantly impacts the evolution of topography and river networks ~~(Willet, 1999)~~ (Willett, 1999), using a fixed grid. ~~, but whose simulation was~~ This was hitherto only ~~mainly restricted to LEMs~~ possible with flexible spatial discretization schemes.

Although most LEMs use first order accurate discretization schemes (Valters, 2016), the problem of numerical diffusion has been ~~widely~~ discussed in the broader geophysical community (Durran, 2010; Gerya, 2010). An alternative family of shock capturing Eulerian methods ~~being frequently applied to avoid the problem~~ are ~~the~~ MPDATA advection schemes (Jaruga et al., 2015). These schemes are based on a two-step approach in which the solution is first approximated with a first order upwind numerical scheme and then corrected by adding an antidiffusion term (Pelletier, 2008). However, contrary to the TVD-FVM, the standard MPDATA scheme (Smolarkiewicz, 1983) is not monotonicity preserving (i.e. it is~~or is~~ not TVD). Instead, MPDATA introduces dispersive oscillations in the solution if combined with a source term (such as uplift) in the equation (Durran, 2010). Adding limiters to the solution of the antidiffusive step (Smolarkiewicz and Grabowski, 1990) renders the MPDATA scheme oscillation free (Jaruga et al., 2015). However, by adding this additional correction, the method approaches the numerical nature of the TVD-FVM which does not require further adjustments in any case.

~~Lagrangian schemes offer another alternative and are based on so called markers which evolve with the changing variable over time (Gerya, 2010). In the framework of a raster-based LEM, a fully Lagrangian tracing scheme is not desired and can be replaced by semi-Lagrangian methods that require interpolation between the propagating markers and the grid cells (Spiegelman and Katz, 2006). These methods could potentially achieve high accuracy. However, simulation of horizontal topographic shortening would require large amounts of incremental markers to prevent numerical diffusion when interpolating the solution to the grid used in TTLEM. Both memory requirements and interpolation processing times therefore legitimize the use of the TVD-FVM which is sufficiently accurate and avoids interpolation.~~

~~Some of the weaknesses of the tested numerical solutions can be reduced by LEMs that rely on irregular grid geometries.~~ Some of the weaknesses of the tested numerical solutions can be reduced by using LEMs that rely on irregular grid geometries. Irregular grids ~~do~~, for example, allow to simulate tectonic shortening using a Lagrangian approach where grid nodes are advected with the tectonically imposed velocity field (e.g. Herman and Braun, 2006). In TTLEM ~~We implemented~~~~Here,~~ the TVD-FVM solvers are implemented ~~in the simulation tool~~in ~~TTLEM that performs all calculations on~~using a fixed grid, ~~gridded datasets~~which has some advantages.~~. avoids these techniques but rather attempts to run on rectangular grids with a maximum of accuracy. We chose so for several the following reasons.~~: First, input data such as topography, climate, lithology or tectonic displacement fields are typically available as raster datasets and thus require only minor modifications before they

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can be used whereas irregular grids require substantial preprocessing. Second, TTLEM output can instantly be analyzed and visualized using the TopoToolbox library (Schwanghart and Kuhn, 2010; Schwanghart and Scherler, 2014) or any other geographic information system. Thus, while irregular grid geometries and flexible grids may have some advantages over rectangular grids~~ with respect to numerical accuracy~~, TTLEM's implementation of numerically ~~highly~~ accurate algorithms strongly ~~reduces~~reduce the shortcomings of rectangular grids while facilitating straightforward processing of model in- and output.~~ therefore enhancing the ease of modelling.~~

~~As we focus on the numerical accuracy of landscape evolution models, we focused on relatively simple simulations considering only linear river incision (n=1), spatially and temporally constant parameter values, uplift and precipitation. Nonetheless, TTLEM supports temporal and spatially variable input values for all these parameters, e.g. by changing the erodibility weighting matrix (w_K) or contributing drainage area weighting matrix (w_A). The impact of non-linear river incision is discussed in detail in. Currently TTLEM does not yet support transport limited river fluvial processes, neither glacial erosion or a bedrock/regolith interface to simulate soil evolution processes (Campforts et al., 2016). TTLEM uses D8 routing to update the drainage network during model simulations. Dinf (or D∞) is the flow routing scheme of choice to represent flow on hillslopes (Pelletier, 2008). However, in TTLEM fluvial erosion is limited to the channelized domain of the landscape and thus the flow routing scheme on hillslopes is of minor significance. Nevertheless, even in the channelized domain Dinf has advantages over D8 since it enables diverging flows on landforms such as alluvial fans and braidplains. The current implementation of TTLEM, however, focuses on the modelling of detachment-limited systems or bedrock rivers where divergent flows are usually confined by valley walls. This is also consistent with other models such as Fastscape (Braun and Willett, 2013) and DAC (Goren et al., 2014) models that use the D8 flow routing scheme. Nonetheless, we do not exclude to implement Dinf or other multiple flow direction algorithms in a future version of TTLEM, in particular since the topological sorting algorithm (Braun and Willett, 2013; Heckmann et al., 2015) is equally suitable for the efficient computation of flows on thus derived networks.~~

~~TTLEM offers users the flexibility to address a number of issues. It allows users to define different initial conditions such as a flat surface, a randomly disturbed surface or a DEM of a real landscape. TTLEM particularly benefits from the adoption of highly efficient drainage network algorithms that outscore GIS implementations in terms of computational efficiency while maintaining their ability to handle the artefacts (artificial topographic sinks) pertinent in real world DEMs (see Table 1 in Schwanghart and Scherler (Schwanghart and Scherler, 2014)). TTLEM provides access to different models of hillslope denudation, and allows to model tectonic displacement at any desirable level of detail. Finally, TTLEM provides different numerical schemes to solve the governing equations allowing users to trade-off between computational efficiency and accuracy. To our knowledge, such LEM versatility is hitherto inexistent and thus adds to the plethora of available LEMs (Valters, 2016). Its ability to be directly run on available DEMs renders TTLEM a simulation environment to explore trajectories of landscape evolution under different scenarios of geomorphological, climatological and tectonic controls.~~

6. Conclusion

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Most eroding landscapes are in a transient state characterized by dynamic river networks ~~that can be assessed using LEMs. The dynamics of drainage networks and divides and the nonlinear models involved, however, entail that LEMs can hardly rely on analytical solutions alone, but require~~requiring numerical methods to solve ~~solvers of~~ the governing PDEs. ~~The successful use of these simulation tools however requires knowledge about their numerical accuracy.~~ Despite the growing interest in the development and use of LEMs, ~~the~~ accuracy assessment of the numerical methods used has received little attention. ~~LEM numerical accuracy has fallen short, yet.~~ We show that the most commonly applied ~~first order accurate~~ numerical methods introduce numerical diffusion and artificially smoothen ~~ear~~ the discontinuities that are inherent ~~in~~to transient landscapes. To overcome this problem, we present a higher order Total Variation Diminishing Finite Volume Method referred to as TVD-FVM.~~TTLEM v1.0 is a raster based Landscape Evolution Model (LEM) contained within TopoToolbox. It allows using a flux~~

~~limiting Total Variation Diminishing Finite Volume Method (TVD-FVM) to solve the stream power law and to simulate lateral displacements.~~ The TVD-FVM solves river incision much more accurately than the traditional schemes: this does not only affect river development but also ~~which is reflected in~~ catchment wide erosion rates. The magnitude of the errors related to numerical smearing ~~D~~depend ~~ing~~ on the spatial and temporal resolution used as well as on the position of the catchment in the landscape. ~~during model runs, first order implicit methods to simulate river incision lead to catchment wide erosion rates which are smeared out over the simulated time span and does not allow to properly capture transient landscapes response.~~ The fact that the impact of numerical schemes is not only altering simulated topography but also simulated erosion ~~records~~ rates is of utmost importance in the light of ~~the~~ current ~~debate~~ research efforts which aim at using ~~ere~~ long term erosion histories ~~are increasingly used~~ to unravel the ~~coupling uplift~~climate~~,~~ erosion ~~and c climate~~uplift enigma~~: however, such long-term simulations are not the only ones for which an accurate representation of knickpoint dynamics is necessary~~. ~~dynamics.~~ The ~~A~~ 2D version of the TVD-FVM, ~~on the other hand,~~ allows to accurately simulate the impact of lateral tectonic displacement in a fixed grid environment~~, which facilitates the incorporation of this process in many existing LEMs that use such a structure~~.

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The TVD_FVMs are implemented in the open access raster based Landscape Evolution Model (TTLEM) contained within TopoToolbox and featuring ~~TTLEM features~~ a range of ~~hillslope~~ response schemes to simulate hillslope processes ~~and allows accurate simulation of lateral tectonic displacements, for example due to tectonic shortening~~. The combination of geomorphological laws to capture landscape response to changes in both internal (e.g. tectonic configurations) and external (e.g. climate changes) ~~forcings~~forcing provides the community with a novel tool to accurately ~~reconstruct,~~reconstruct, predict and explore landscape evolution scenarios over different spatial and temporal timescales. In its current form, TTLEM is limited to uplifting, fluvially eroding landscapes. Further development will allow to integrate other processes (e.g. glacial erosion) as well as the explicit routing of sediment through the landscape.

Code availability

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TTLEM 1.0 is embedded within TopoToolbox version 2.2. The source code and future updates can be downloaded from the GIT repository https://github.com/wschwanghart/topotoolbox. TTLEM is platform independent and requires MATLAB 2014b or higher and the Image Processing Toolbox. Documentation and user manuals for the most current release version of TopoToolbox and TTLEM can be found at the GIT repository in the help folders of the software. The user manual of TTLEM includes three tutorials which can be accessed from the command window in MATLAB. ~~To get started: download and extract the main TopoToolbox folder from the repository to a location of your choice. Add the folder to the Matlab search path by entering the following code in the command window *addpath(genpath('C:\path\...\TT_folder'))*. The software package comes with three examples which can be initiated from the command window by entering *TTLEM_usersguide_1_intro* ; *TTLEM_usersguide_2_Synthetic_model_run* or *TTLEM_usersguide_3_Synthetic_Geological_Configuration*. These tutorials are also documented in the Help folder of ttlem.~~ The source code for the solution of the one dimensional Stream Power Law (SPLM) can be downloaded from the GIT repository https://github.com/BCampforts/SPLM. SPLM contains the solution of the 1D river incision codes including four examples.

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This work was motivated by the meeting "Landscape evolution modelling - bridging the gap between field evidence and numerical models" in Hannover, 21-23, October 2015, that was organized by the *FACSIMILE* network and funded by the Volkswagen Foundation. Additional support comes from the Belgian Science Policy Office in the framework of the Interuniversity Attraction Pole project (P7/24): SOGLO - The soil system under global change. Numerical simulations were performed in the MATLAB environment (2015b) using numerical schemes as referred to in the text. Computational resources and services used to evaluate model performance were provided by the VSC (Flemish Supercomputer Center), managed by the Research Foundation - Flanders (FWO) in partnership with the five Flemish university associations. We are grateful to the IDYST group of the University of Lausanne and in particular Frédéric Herman and Aleksandar Licul for inspiring discussions on numerical methods and Nadja Stalder for the figure design. We further thank Taylor Perron for sharing his source code. We also thank two anonymous reviewers for constructive feedback that improved the manuscript.

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Appendix

Model structure

The model architecture of TTLEM is illustrated in Fig. A1.

Hillslope processes

~~TTLEM offers users the flexibility to address a number of issues. It allows users to define different initial conditions such as a flat surface, a randomly disturbed surface or a DEM of a real landscape. TTLEM particularly benefits from the adoption of highly efficient drainage network algorithms that outscore GIS implementations in terms of computational efficiency while maintaining their ability to handle the artefacts (artificial topographic sinks) pertinent in real world DEMs (see Table 1 in Schwanghart and Scherler (Schwanghart and Scherler, 2014)). TTLEM provides access to different models of hillslope denudation, and allows to model tectonic displacement at any desirable level of detail. Finally, TTLEM provides different numerical schemes to solve the governing equations allowing users to trade-off between computational efficiency and accuracy. To our knowledge, such LEM versatility is hitherto inexistent and thus adds to the plethora of available LEMs (Valters, 2016). Its ability to be directly run on available DEMs renders TTLEM a simulation environment to explore trajectories of landscape evolution under different scenarios of geomorphological, climatological and tectonic controls.~~

~~**Experiments**~~

~~In order to demonstrate possible applications of TTLEM we carry out two series of numerical experiments. We first~~ illustrate the impact of different hillslope process models on simulated landscape evolution, using a 30 m resolution DEM of the Big Tujunga region in California as an example (Fig A2). ~~Second, we investigate the amount of bias and artificial symmetry introduced in the landscape through the use of regular grids.~~

~~Hillslope processes~~

TTLEM allows to simulate hillslope processes assuming (non)-linear slope dependent diffusion with the consideration of a threshold hillslope. Figure A2 illustrates how different hillslope process algorithms affect the evolution of hillslopes in the Big Tujunga region, California (Fig. A2a). We assume no tectonic displacement and use standard parameter values for river incision and hillslope diffusion (Table 1) and a threshold slope (S_c) of 1.2 (m/m) when applicable (Fig. A2b). We illustrate model results after 500 ky in Fig. 2c-d using the current topography as the starting condition. Linear diffusion (Eq. (4)) is not capable to keep up with river incision, which results in strongly oversteepened hillslopes near the river channels (Fig. ~~1~~A2c and 1g). While higher values for the diffusion coefficient *D* will eliminate this problem (e.g. Braun and Sambridge, 1997) they are incompatible with experimental findings (Roering et al., 1999) and will restrict hillslopes to convex upward shapes. The use of non-linear diffusion in combination with a threshold slope results in hillslopes similar to those simulated with linear diffusion in combination with a threshold slope. However, for a similar value of *D*, hilltops become more smoothed assuming non-linear diffusion as sediment fluxes due to diffusive processes now reach higher values when hillslopes approach the threshold slope.

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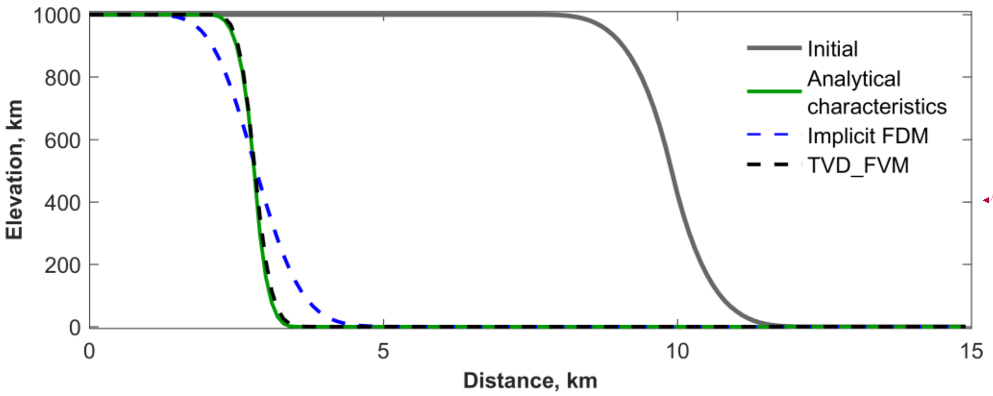

Figure 1. Solution of the linear 1D stream power law for a synthetic knickzone over a timespan of 1 Myr. The analytical solution is obtained with the method of characteristics. The spatial resolution equals 100 m. Other model parameter values are listed in Table 1.

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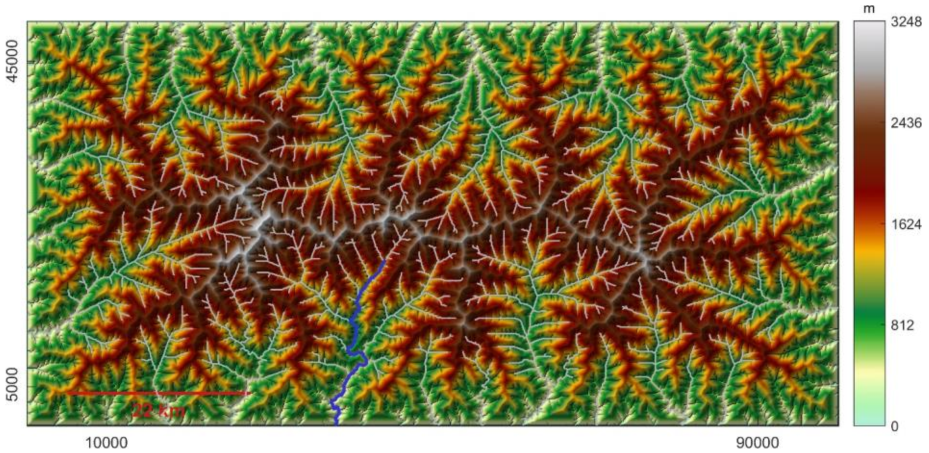

Figure 2. A synthetic steady state landscape produced as the testing environment to verify and compare the different numerical schemes implemented in TTLEM. Model runtime was 150 Myr, uplift rate was assumed to be spatially uniform over the area (block uplift) and fixed to ~~10^{-3} m~~1 km Myr^{-1}. Other model parameter values are listed in Table 1. Dynamic landscape evolution is presented in Movie ~~S3~~S1. The grey lines indicate the drainage network for which the solution has been calculated analytically as a benchmark solution. The blue line indicates the river profile for which model results at different resolutions are plotted in Fig. 4.

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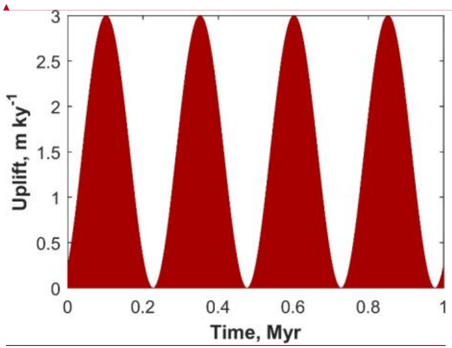

Figure 3. Uplift imposed to the steady state landscape show in Figure to investigate the impact of different numerical schemes.

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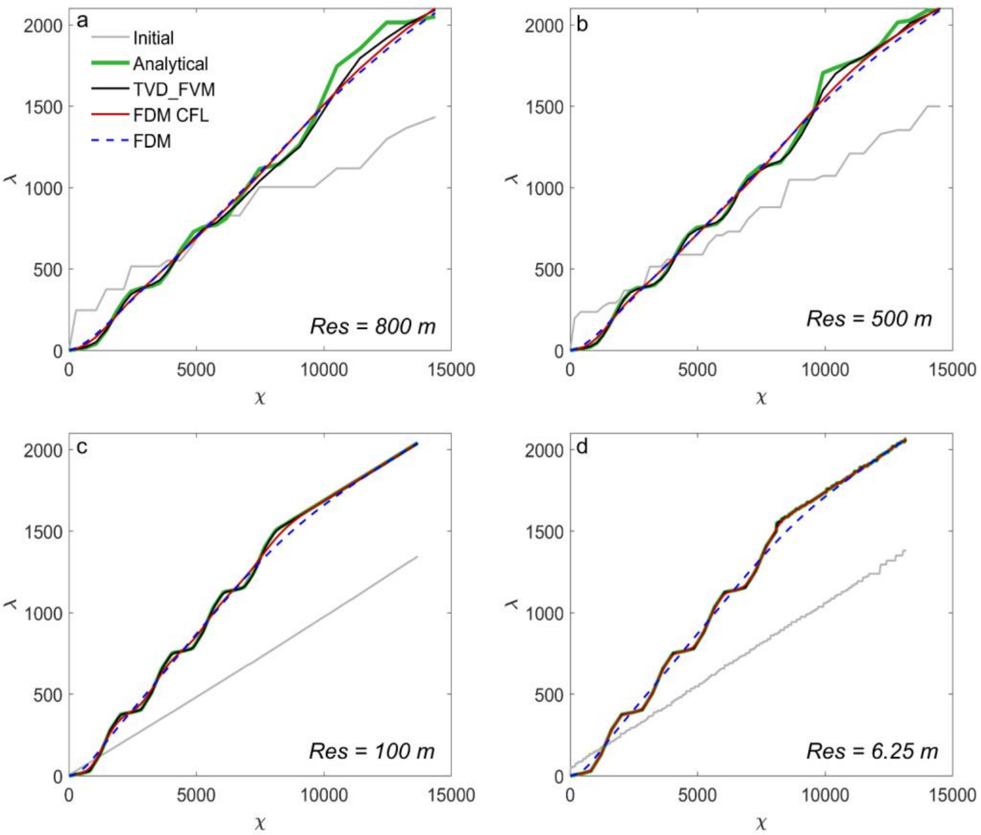

Figure 4. Comparison between different modelled resolutions for the river profile indicated in blue on figure ~~1~~2.~~–~~ The green line is the analytical, 'true' solution, obtained with the slope patch method of Royden and Perron (2013). The full red line represents the implicit solution when the CFL<1 and the dotted blue line represents the implicit solution when the ~~time step~~time step is left free. The implicit solutions where CFL<1 are simulated with a time step equal to the time step used for the TVD-FVM.

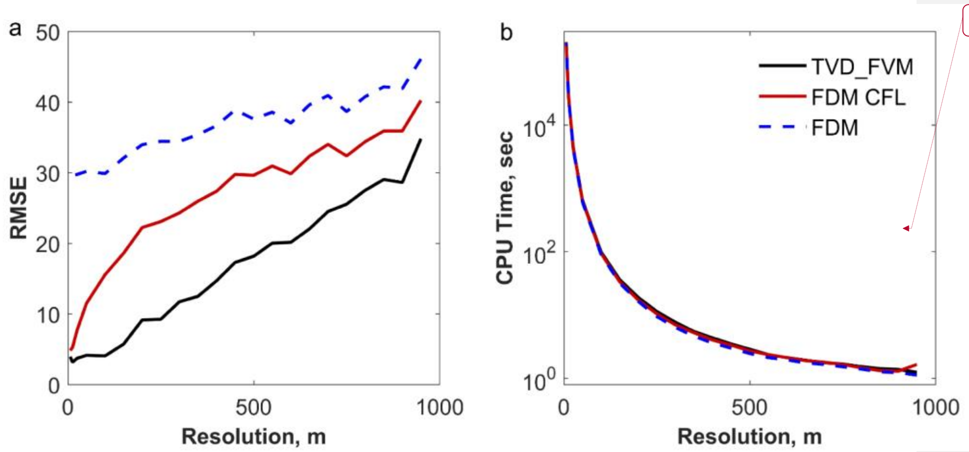

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Figure 5. a. Performance of the different numerical schemes ~~calculated with~~where the RMSE is calculated between the analytical and numerical methods.~~–~~ b. CPU time required to perform the model runs at the indicated resolutions.

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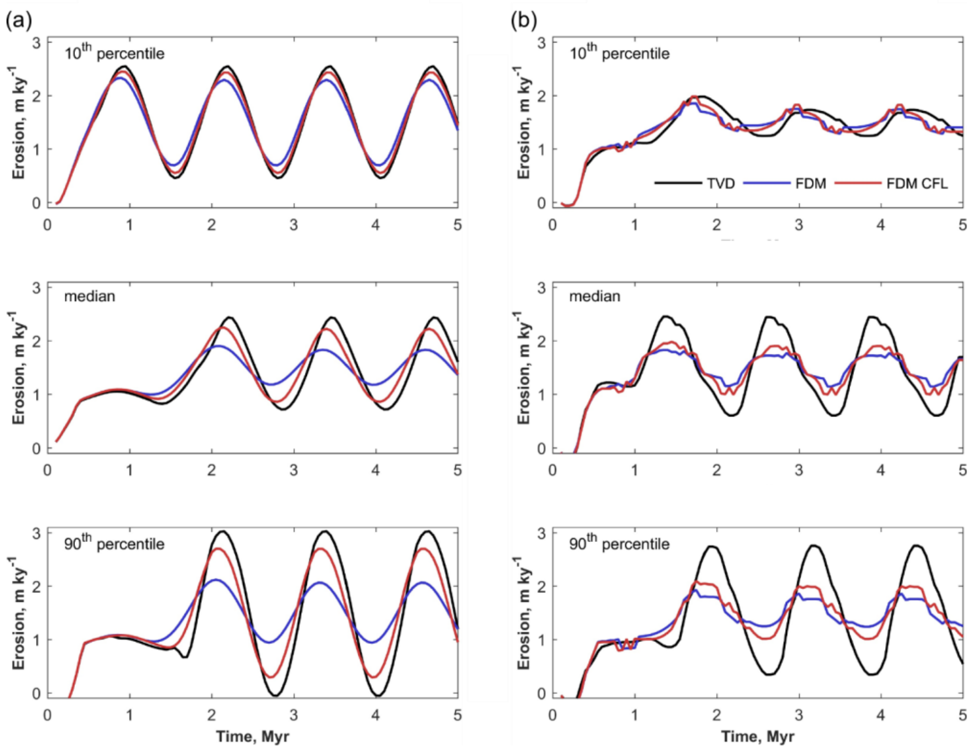

Figure 6. Temporal variation in simulated catchment wide erosion rates using different numerical methods to simulate river incision. The black lines represent simulations where a flux limiting TVD-FVM is used, the blue lines represent the implicit FDM without constraints on the ~~timesteps~~time steps and the red lines represent the FDM with an inner ~~timestep~~time step
1620 calculated with the CFL criterion. (a) Simulations performed at a spatial resolution of 100 m. (b) Simulations performed at a spatial resolution of 500 m. Here, a median filter with a window of 3 ~~timesteps~~time steps ~~was~~ is applied ~~on~~ to the simulated erosion rates to eliminate spikes which might occur at low resolutions.

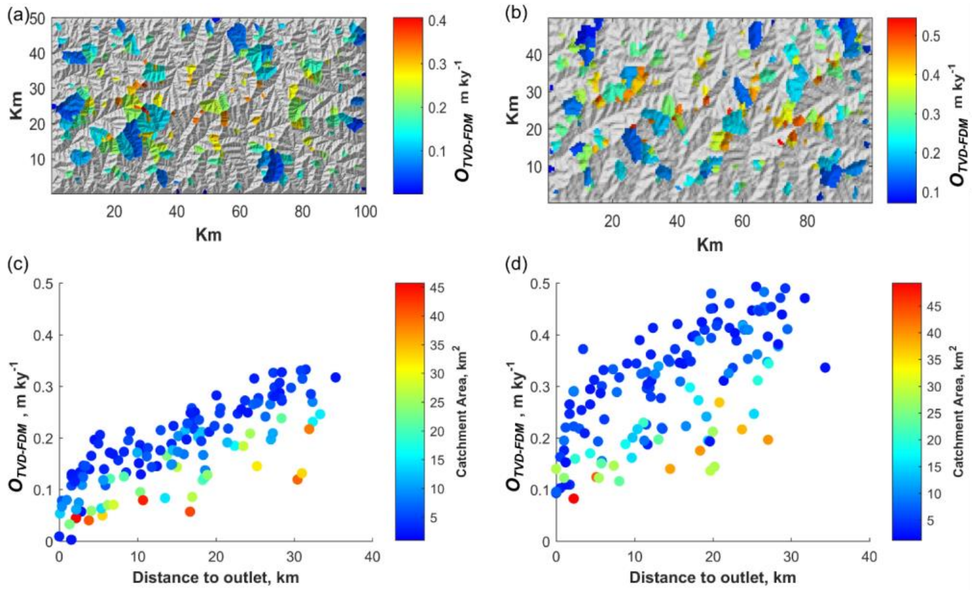

Figure 7. Spatial variation of differences between simulated erosion rates calculated with a flux limiting TVD-FVM for simulating river incision and an implicit FDM. Here, we compare methods both run with an inner ~~timestep~~time step constrained with the CFL criterion (see text). $O_{TVD-FDM}$ ~~RMSE~~ is thus calculated between the black and red lines from Figure . Left column represents simulations run at a spatial resolution of 100 m, right column at 500 m. (a and b) Location of the randomly selected catchments with an area > 1 km² and < 50 km². Colors refer to the $O_{TVD-FDM}$ ~~RMSE~~ between the two simulations. (c and d) Differences between the schemes increase with increasing distance from the river outlets and are inversely correlated with the catchment area.

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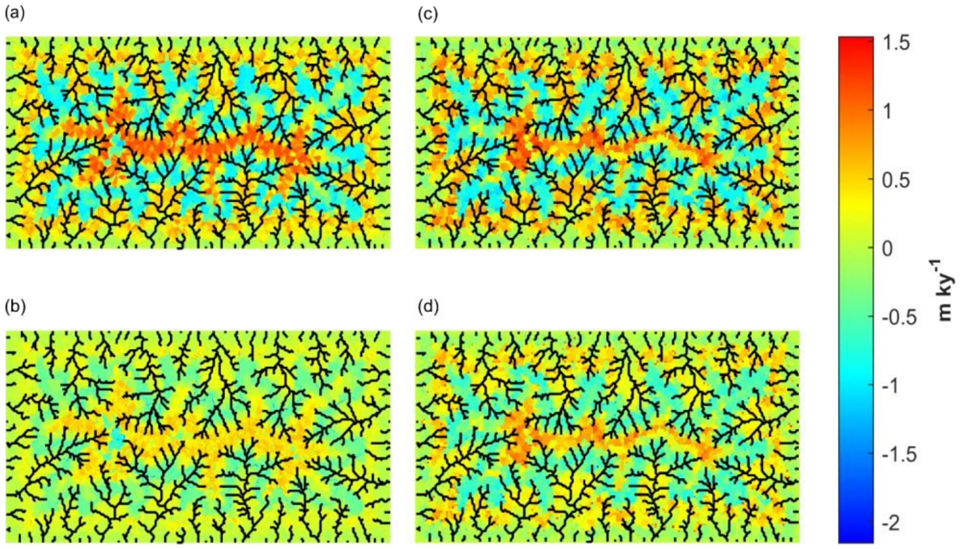

Figure 8. *Spatial pattern of erosion rates during one model ~~timestep~~time step when simulating landscape evolution with the flux limiting TVD-FVM versus the first order implicit FDM. (a) simulation at a resolution of 100 m where the ~~timestep~~time step of the implicit method is not constrained (b) simulation at a resolution of 100 m where the ~~timestep~~time step of the implicit method is constrained with the CFL criterion (c) simulation at a resolution of 500 m where the ~~timestep~~time step of the implicit method is not constrained (d) simulation at a resolution of 500 m where the ~~timestep~~time step of the implicit method is constrained with the CFL criterion.*

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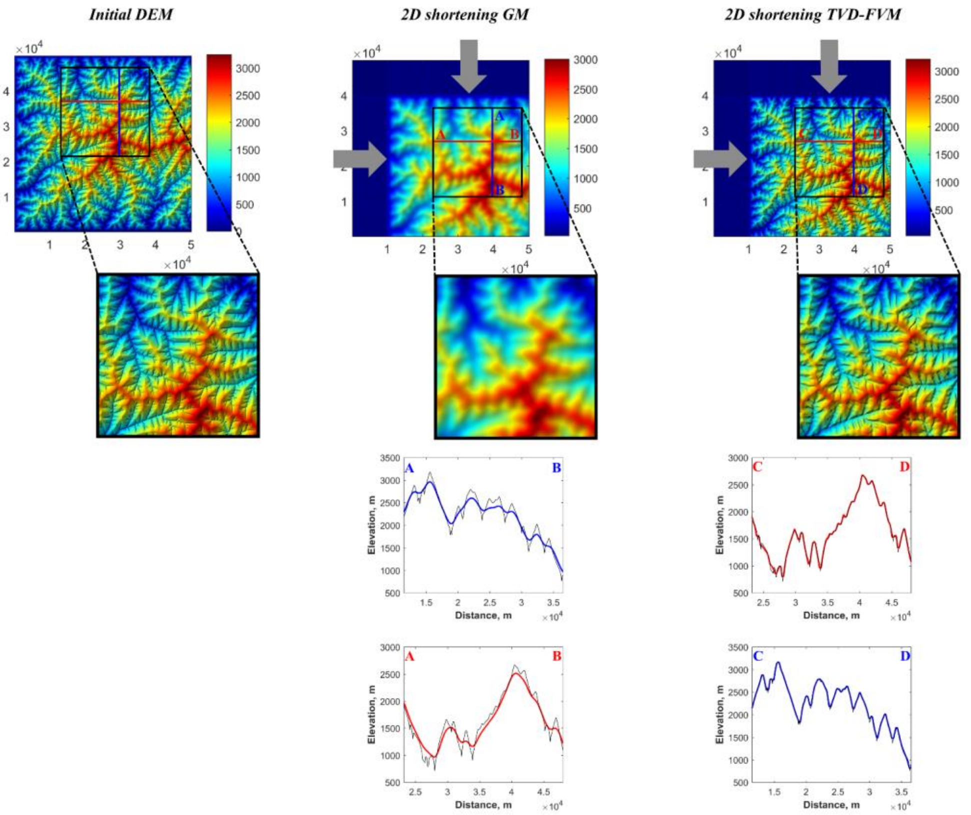

Figure 9. Impact of numerical schemes when simulating horizontal shortening on a fixed grid. Left: extract from synthetically produced DEM from Fig. ~~5~~2. Middle: horizontal shortening in two directions simulated with a 2D explicit first order Godunov Method (GM). Right: horizontal shortening in two directions simulated with a 2D explicit flux limiting TVD-FVM.

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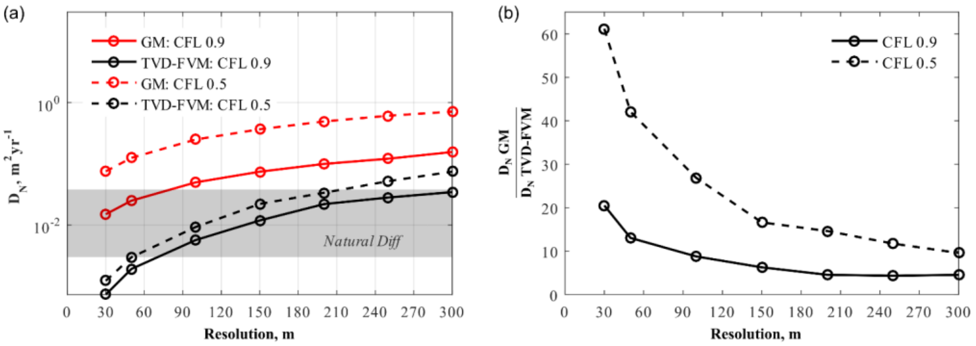

Figure 10. (a) Amount of numerical diffusion (D_N) introduced in the system when simulating lateral tectonic displacement in two directions as a function of raster resolution. The grey zone indicates the range of naturally observed diffusion rates. (b) The ratio between the amount of numerical diffusion for the first order Godunov Method (GM) versus the flux limiting TVD-
1650 FVM.

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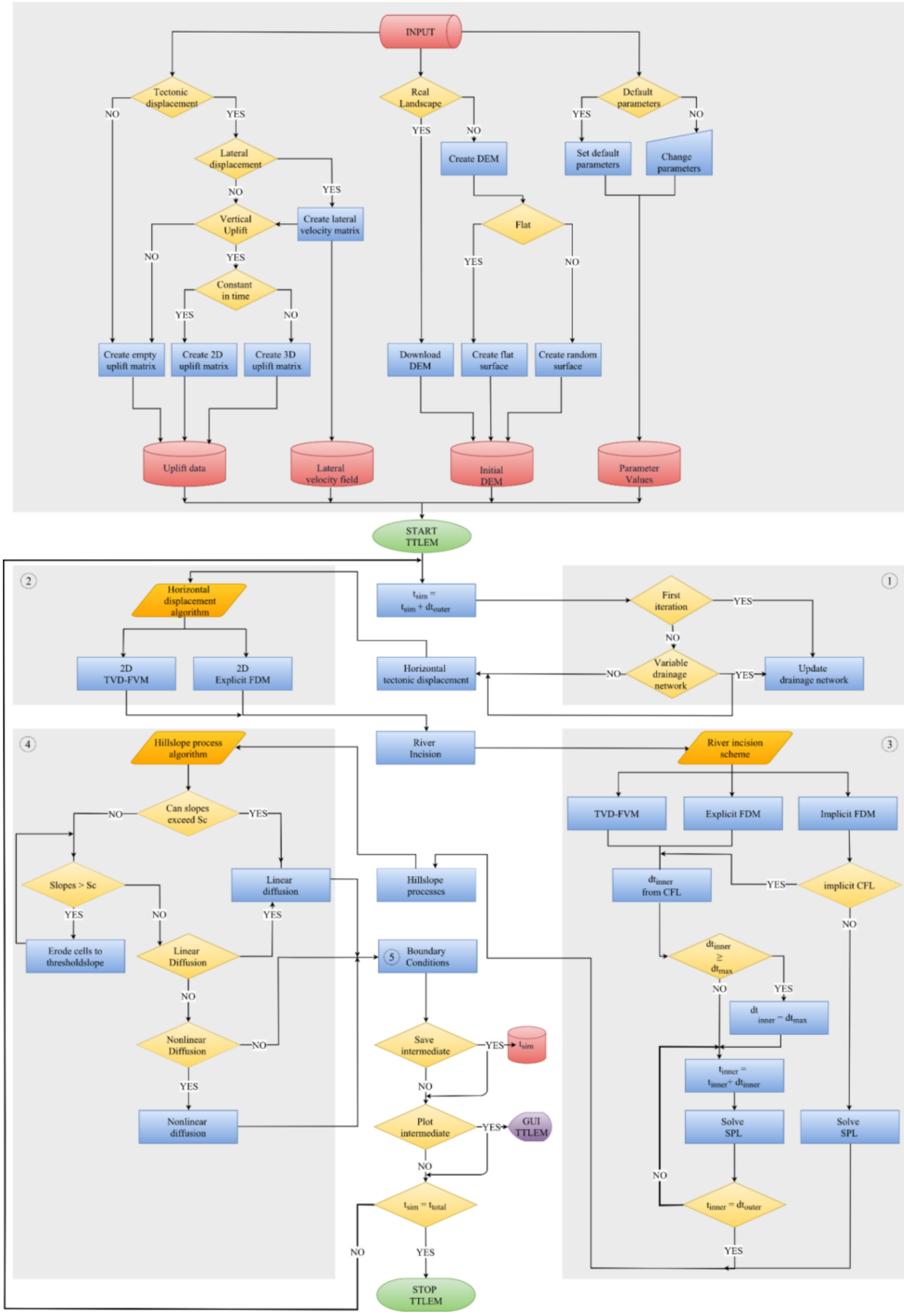
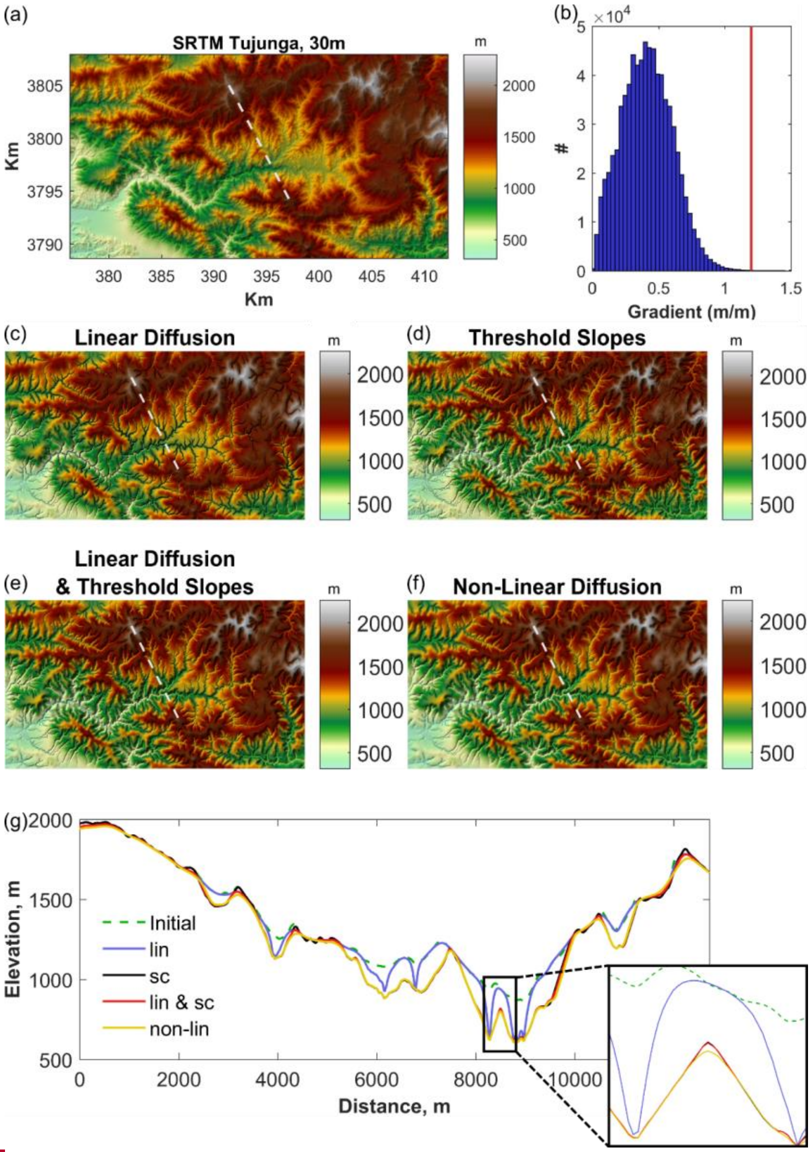
Table 1. Model parameters used for the TTLEM simulations.

Parameter	Units	Figure 1	Figure 2	Figure 4-5	Figure 6-8	Figure 9-10	Figure 2A
Initalization							
InitialSurface		flat, 1D	random	synthetically produced DEM shown in Fig. 2	synthetically produced DEM shown in Fig. 2	synthetically produced DEM shown in Fig. 2	Tujunga SRTM
UpliftPattern		no uplift	uniform	uniform	uniform	Lateral Displacement	-
UpliftRate	m yr^{-1}	0	1×10^{-3}	$0 - 3\times10^{-3}$	$0 - 3\times10^{-3}$	0	0
SpatialStep	m	100	100	varying	100 - 500	varying	30
Computational parameters							
TimeSpan	yr	1×10^{6}	150×10^{6}	1×10^{6}	5×10^{6}	1×10^{6}	5×10^{5}
TimeStep (outer)	yr	ca. 6×10^{3}	5×10^{4}	5×10^{4}	5×10^{4}	resolution dependent	1250
AreaThresh	m^2	-	5×10^{4}	5×10^{4}	5×10^{4}	-	5×10^{4}
DrainDir		-	variable	Fixed	Fixed	-	variable
SS_Value	m	-	0.5	-	-	-	-
Boundary conditions							
BC_Type		-		Dirichlet	Dirichlet	Neumann	Neumann
BC_dir_DistSites		-		-	-	-	-
BC_dir_Dist_Value		-		1	1	-	1
BC_dir_value		-		0	0	-	0
BC_nbGhost		-		1	1	-	1
FlowBC		-		-	-	-	-
River incision							
Kw	L^{1-2m} t^{-1}	5×10^{-6}		7×10^{-6}	7×10^{-6}	-	4×10^{-6}
m		0.42		0.42	0.42	-	0.45
n		1		1	1	-	1
Hillslope response							
D	m^2 yr^{-1}	-		0.01	0.036	-	0.015
ρ_r ρ_s^{-1}	-	-		1.3	1.3	-	1.3
DiffTol		-		1×10^{-4}	1×10^{-4}	-	1×10^{-4}
Sc	m m^{-1}	-		0.8	1	-	1.2
Sc_unit		-		tangent	-	-	tangent
Tectonic shortening							
u	m yr^{-1}			-	-	0.01	-
v	m yr^{-2}	-		-	-	0.01	-
Numerics							
riverInc		implicit_FDM TVD_FVM		implicit_FDM	implicit_FDM TVD_FVM	-	implicit_FDM
cfls		0.9		0.9	0.9	-	0.9
diffScheme		-		imp_lin_sc	imp_lin_sc	-	imp_lin only_sc imp_lin_sc imp_nonlin_sc
shortening_meth		-		-	-	Upwind_TVD Godunov Method	-



~~Figure 1.~~ Figure A1: Schematic representation of the TTLEM model flow. The numbered methods correspond with the paragraphs from section 3 in the main text.



7.

Figure A2: ~~**Figure 2.**~~ Hillslope response to river incision. (a) Standard SRTM DEM (30 m) included in TopoToolbox representing the Tujunga region. The dotted grey line indicates the location of the transect shown in subplot g. (b) Resulting topography after 500k years using four different descriptions for hillslope evolution. (c) Linear diffusion over all slope values (lin). (d) Threshold landscape where no slopes exceed the threshold slope (Sc). (e) Linear diffusion combined with immediate adjustment to a threshold slope (Sc). (f) Non-linear diffusion combined with immediate adjustment to a threshold slope (Sc). (g) Elevation profiles of the different model runs compared with the initial profile. Model parameter values are listed in Table 1.

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