We thank the reviewers for their careful criticism of the text and inciteful comments that have helped us to clarify and improve the manuscript. We think that these comments have helped us to improve the manuscript, and we include the revised version with this comment. We give detailed responses to specific comments below. For reviewers 1 and 2, the responses are only slightly expanded versions of the original replies to comments. For reviewer 3, we include a new detailed response here, which we did not have time to complete before the close of open discussion. Reviewer comments are in bold, and responses in normal font.

Reviewer 1

p2 L28: what does "steady state form of a landscape" mean here? You've just convinced me it doesn't exist in these settings...this is a bit more clear that you mean something like a flux steady state after reading the rest of the paper, but it seems that there is no steady landscape form except in the vertical-contacts case.

We changed the wording in this sentence, and added an additional sentence to clarify that we are talking about a flux steady state rather than topographic steady state.

p3 L19 A change in process (e.g. away from stream-power erosion) under steep conditions breaks this relationship, as noted above on L10 or so. This is discussed to some extent but could bear more emphasis. These boundaries are the very places where erosion processes are changing. For example, some of the same authors have published on how blocky debris from strong lithologies locally alters the erosion by streams in these settings. The change to effectively a transport-limited system may necessitate at least a change in the exponents, if not the form, of the erosion law. It is clear from the later discussion that the authors appreciate this; it would be useful at this point perhaps to point out that the formulation in Eq. 3 is effectively a reference case, deviations from which may reflect the process variability present in any particular landscape.

We agree with the reviewer on this point and have added a couple of sentences to make this assumption explicit.

p4 L15 What is considered "subhorizontal" here? How close to horizontal can the contact be before this singularity becomes important? It is rare in nature (but common in LEMs) to have a perfectly uniform, mathematically horizontal dip over a significant distance. I suggest adding an extra set of lines (or two) to Fig. 3 with some dip cases close to horizontal, perhaps 5 and 10 dip, in addition to the vertical and pure horizontal cases.

Subhorizontal is defined on Lines 1-2 of page 4. It is whenever rock dip is small compared to channel slope. Therefore, the cases shown actually span a wide range of possible contact and channel slopes. There is not a simple way that we can think of to show specific other choices of dip angle. We have modified the main text and figure caption to make it clearer that these two limits are not explicitly a function of rock dip, but rather a comparison between rock dip and channel slope.

p4 L18 "solely a function of erodibility." In this framework. I would argue that process variation is critical here. There is certainly field support for a retreat rate that is independent of slope but a function of drainage area in relevant landscapes, a la Crosby and Whipple 2006 (cited) and Berlin and Anderson 2007 JGR (not cited but quite relevant). But another way to view this singularity is that perhaps n=1 works well away from contacts in sub-horizontal rocks but the stream power erosion law itself is not a good model in these situations. As noted, this is also where numerical inaccuracies may become very important in LEMs. I appreciate the authors pointing out where numerical models may diverge from reality when considering this continuity framework.

We agree. For n = 1 the horizontal retreat rate is a function of erodibility AND drainage area and independent of slope. This is a direct consequence of stream power erosion law. (In chi space, for n = 1 the horizontal retreat rate is a function of erodibility and independent of slope AND recharge area.) We have corrected the text to include drainage area as a factor influencing retreat rate. We also agree that the singularity precludes validity of the stream power erosion law in these situations because it causes the predicted slopes to not be small enough. We have also added a few lines to the discussion concerning the cited field work and implications for the n=1 case.

p6 L13 "time-averaged incision rate through both rock types..." This needs some clarification. Do you mean vertical incision rate in both rocks is identical to the uplift rate? That doesn't seem quite right. Averaged over what time period?
p6 L17-18 "continuity state is a type of flux steady state" Here this is presented as if it follows from the above analysis, but it was stated on line 13 above that the analysis is based on assuming flux steady state. It reads as being a circular argument, but perhaps the phrasing just needs some clarification.

This section was not very clear and did appear circular. We have edited it to make it clearer. From the results of the simulations, and specifically the fact that the landscape is periodic in chi space, you can argue that the system must be in a flux steady state. Using this conclusion, we can derive full profiles. Finally, that the whole story holds together is further confirmed by the fact that we can match the simulated profiles using the equation derived from flux steady state.

p7 L27 "two cycles through the rock layers" not clear what this means - what cycles? The perturbation has traversed two sets of contacts?

Not exactly - it means the knickpoint caused by the perturbation has travelled so far upstream that two sets of contacts now separate it from the downstream end of the channel that is being perturbed. The number of contacts it traversed on its way (if any) depends on the ratio between horizontal retreat rates and knickpoint celerity. As a side note, knickpoints pass from one lithology to another unobstructed. They get damped through formation of stretch zones as in Royden and Perron (2013) and through interfering with one another. We have edited the text to try to clarify this point.

p7 L30 how does layer thickness affect this result? Presumably it affects the distances across which a profile is developed in each rock type. A common geological scenario is thinner layers of hard rock between thick layers of soft rock. Will thin layers of hard rock slow down knickpoints for less time than thick ones, reducing the damping lengthscale? The analytical expressions and 1D modeling here stick to equal thicknesses of each type. I suspect the general result is the same, but pointing out the effect would be useful, and how to account for it in the framework described on p7. I see this issue is addressed to some extent in the 2D model setup, but its effect is not then discussed, and the 200 and 300 m alternating thicknesses are similar enough that I wouldn't expect a big impact. What about 100 m of weak rock alternating with 10 m strong-rock interbeds?

Only the thickness of the stronger layer influences this length scale. This results because the problem is asymmetric with respect to the two rocks. The strong rock knickpoints are always slower. The time for the weak knickpoint to catch up depends on only three things: 1) how big of a head start the strong knickpoint has, 2) the velocity of the weak knickpoint, 3) the velocity of the strong knickpoint. The velocities of the two knickpoints are independent of layer thickness. The head start of the strong knickpoint is only dependent on the thickness of the strong rock. Therefore, the thinner the strong rock layer, the quicker the knickpoints should decay.

We agree that it would be interesting to simulate some cases with thin, hard layers, both to test the predictions of our theory for unequal thickness, and because this is a common situation in nature. We now include new simulation results with thin strong layers and have expanded the discussion to examine the implications of layer thickness on steady state form.

p10 L4-5 It's pretty hard to call the reach corresponding to a caprock waterfall a "channel", especially once flow is detached from the face. I think eSurf gives you the space to elaborate a bit more on how processes might commonly change in these settings (see my notes above) and how in general one would incorporate this into the continuity framework (without detailed exploration of such a case).

We agree that processes dramatically change in this setting. Our speculation in the manuscript is that stream power erosion, specifically in subhorizontal rocks with n<1 is one possible mechanism to drive the system toward the caprock waterfall state.

Once the system reaches this state, stream power erosion has certainly broken down. We have slightly expanded this part of the discussion, and make it clearer that the material on caprock waterfalls is a speculation about a possibility rather than something we have definitively shown.

Minor notes p2 L17: "responce" change to "response" Fig 7 caption is missing punctuation at the end.

These typos were corrected.

Reviewer 2

This manuscript on the influence of horizontally (or close to it) layered rocks and their influence on landscape evolution is very interesting. Some of the results are extremely counter-intuitive, and that always makes for a fun read. The math and modeling seem sound to me, and I'm generally supportive of this paper. The paper is timely, as another paper on a similar topic recently came out – Forte et al., which is cited here. Forte et al., also discussed that steady state is not reached with horizontal layers. Where this paper falls a bit short, in my opinion, is a lack of much discussion and also some lack in details of the modeling. As for the discussion, I thought they might tie in more with the Forte paper at some point, but that never happened. But in general I did not find the discussion to be very deep. As for the modeling, it was not always clear to me why the models were set-up as they were.

My general comment is just to give a bit more detail, including around the figures, and some suggestions for this are laid out in my line-by-line comments.

We thank the reviewer for their careful reading of the manuscript, and pointing out a number of items that were unclear or deserved further elaboration. Detailed responses to comments are given below.

Line by line comments: After reading the abstract I'm still not sure what channel continuity means. Notably, the sentence starting on line 5 made no sense to me, and I think that made me stumble through the rest of the abstract. I went back and read it after reading the manuscript and then it made sense to me. I think it was hard for me to envision what retreat in the direction parallel to a contact meant without the schematics, but after seeing the schematics it seems obvious. I don't have a great suggestion for improving this sentence.

We agree that it is difficult to understand without a figure. We have expanded this part of the abstract in an attempt to more carefully explain what we mean by continuity. The abstract is now a bit long, but we hope it is clearer.

The caption in Figure 2 and main text around it confuse me. In A, is the upper layer steeper, or is it simply that the upper layer is overhanging the lower layer, creating an instability? Similarly, in B, isn't the problem that there was a dam created? Equation 2: Is this vertical incision rate?

We have attemped to make this clearer. In case A, the upper layer can become steeper or create an overhang, it depends on the sign of the dip of the contact. In case B, the lower layer can create a dam or a low slope zone, depending on the sign of the dip. We have adjusted the text accordingly. Equation 2 does contain the vertical erosion rate, which we now specify in the text.

Page 4, first paragraph. I see the math, but this is confusing. A few things. I wonder if it would be helpful to remind people the relationship between K_w and K_s? As for equation 5 with n<1, the prediction is so counter to my 'gut', that I wonder if some discussion about whether n<1 is realistic, or about whether this counter intuitive relationship has been observed, would be useful. The n=1 case is also difficult for me to wrap my head around. Maybe more discussion is coming later.

It is definitely counter to the common intuition based on prior work. One of the main points of this manuscript is that the assumption behind prior work actually can break down in subhorizontal rocks. We try to explain this in lines 6-8 of this page.

Basically, it results because horizontal retreat rate (or knickpoint celerity) is lower for steeper channels in the case where n<1. This is because, for the same rate of vertical erosion, horizontal retreat rates are less for steeper slopes. In cases where n<1, the increased erosion from the channel becoming steeper isn't sufficient to offset the slope effect. At n=1, these two effects are totally balanced, so slope has no effect on horizontal retreat rate. We have expanded this section to try to make it a bit clearer, and have reminded the reader that $K_w > K_s$. We do also discuss later cases where n<1 might be reasonable.

Page 4, line 28: What does it mean that experiments with resolution suggest that the conclusions are not affected by numerics? Does that mean you changed the resolution and ran with different numerical schemes, and got the same answer? Or that your results are not dependent on the resolution for a given implementation of stream power? Please clarify.

Our original statement was a bit too vague. We have edited this to clarify that we ran some higher resolution simulations for some cases that produced the same result.

Page 5, line 6, 7: Is layer thickness thought to vary with uplift? I don't think so. Why do you do this?

We are specifically examining cases where there are many different rock layers, such that the influence of base level perturbations dies out and we can see the continuity equilibrium form. This was just a practical way of generating a similar number of contacts in both the high and low uplift cases (albeit both with parameter ranges that are within the range of natural landscapes).

Page 5, L 14: What do you mean it holds if 'slope is replaced with slope'?

Slope is replaced with "slope in χ-elevation space." We agree that the repeated word obscures the meaning a bit. We now use the word "steepness" instead, and then note in a parenthetical that by steepness we mean slope in χ-elevation space.

Figure 4 caption: What is meant by the 'steady state profile predicted by the theory'? Just the elev-chi plot for a channel with that erodibility in vertical layers? Or is it the theory that you present in this paper. I'm confused.

We mean the theory presented in this paper. We have edited the caption to clarify this and have added the relevant equation references.

Page 7, summary in paragraph on line 25: I got a bit lost. I think a bit more description/hand holding for the reader would help. I recognize that λ^* is a way to show how large $\chi_{s,+}$ is. But in the description with respect to figure 6, the damping is described in terms of cycles through rock layers. I don't understand what this means, or how to get that from the equations. I must be missing something easy. How does $\chi_{s,+}$ related to the depth of the rock layers? How do I know from λ^* how many layers the knickpoint has propagated through?

$\chi_{s,0}$ is the χ length of the strong layer reach near base level at the moment that the weak layer becomes exposed at base level. Consequently, this distance is less than the profile distance spanned by a pair of weak and strong rocks, but is also on the same order of magnitude. The dimensionless damping length scale, $\lambda^* = \lambda/\chi_{s,0}$, therefore provides a rough (conservative) estimate of the number of strong/weak pairs that the knickpoint will pass before significant damping. We have expanded this text to clarify this point.

Figure 7 is difficult for me to interpret. I think I can see the knickpoint that is propagating up in elevation, but I can't really make out the knickpoint that it is 'catching'. Can you tell us how you determined that there was a knickpoint at the red line that was caught? If I look at the dashed line (intermediate time) in C, it does not look like there is any significant change in the chi-elev relationship at the red line, but I think that there is supposed to be a knickpoint there, right? Or at least one close to it that will soon catch up? I only see one knickpoint downstream from there, but maybe I am interpreting incorrectly? Actually, after watching the movies, I may understand this. But I still think it is

worthwhile to point out to readers exactly what you are calling knickpoints.

Figure 7 was the best way we could think of to show this statically, though it is much clearer in the animations, as we state in the text. We think that the point of confusion here is that the knickpoints we are talking about are just sudden changes in slope, and can correspond to increases or decreases in slope (depending on n). We have expanded this explanation in the text.

Fastscape runs: It is a bit unsatisfying that the n=2/3, 3/2 runs have channels that extend through 4+ layers of each rock type, but the n=1 run only just barely taps three week layers. I know this is a lot to ask, but it'd be more satisfying to see more of the n=1 profiles, i.e. just make the K values in this run smaller. I'm not adamant about this, as the 2D runs appear to be very similar to the 1D runs.

We agree that the choice of parameters for the $n = 1$ case was suboptimal. We have rerun the simulation with a lower K value, and now the simulation has a similar number of weak and strong layers.

In the beginning of Section 4, the authors mention that they include hillslope processes. It seems like this needs a bit more description. How are the different rock types treated with the hillslope model? How do they model hillslopes?

We are using the standard hillslope diffusion approach employed in Fastscape, which does not have the capability to adjust the diffusion coefficient with rock type. We now clarify this in the text.

Discussion and Conclusions: I liked that the authors brought in a real world example. However, this example confused me. I may be wrong, but my impression of Niagara Falls and the Niagara river is that the soft rocks underneath the hard caprock are indeed basically vertical at the waterfalls. But if you move any length downstream the channel is not so steep anymore. I might be wrong as I haven't studied the Niagara River, just visited it. But does the whole length of profile have the 'inverted' relationship (steeper in weak rocks) suggested by Figure 4D, or is it just 'inverted' around the waterfall? This may seem a picky point, but I would guess, as the authors brought up elsewhere, that the processes going on right at the knickpoint are not adequately modeled by stream power. So in some ways this comparison feels a bit odd to me. I felt as though the discussion could be expanded a bit.

If the stream power erosion law continued to hold as the channel steepened, then in theory one would expect the entire channel to remain steep in the weak rocks (for n<1)). However, the stream power erosion law breaks down for such steep channels as erosion processes take over that are not well-described by the stream power law. Therefore, we are only speculating that continuity can push the system toward this state (by first making the channel steep in the weak rocks). From a more standard assumption of topographic equilibrium, one would never expect steepening in weak rocks, so it is not clear how you would approach such a state to begin with. There are potentially other explanations, such as non-locality in erosion processes near the contact, that cannot be entirely ruled out. We have expanded this discussion slightly and tried to make it clearer that it is speculative. In the specific case of Niagara Falls, which is one of the most famous of many possible examples, the flattening below the waterfall in part occurs because of the nearby base level imposed by Lake Ontario.

The parameter n turns out to be extremely important in this study. Any thoughts beyond Niagara Falls on how your contribution plays in to the n debate? Have many studies suggested that n<1? Are there any other landscapes to call upon to illustrate the modeled behavior besides Niagara Falls? I also generally prefer a separate conclusions section. I think it is better for authors because often times the only sections of the paper that get read are the abstract and conclusions. But this is stylistic.

We do not attempt to constrain what realistic values of n should be. There are theoretical arguments that some incision processes will produce n<1 (e.g. Whipple et al. [2000] cited in this work, or Covington et al. [2015], GRL). While there are likely good field sites where a natural experiment could be used to test the ideas developed here, we think that finding and studying such a site is beyond the scope of this manuscript. Our main goal here is to solidify our theoretical understanding of the equilibrium behavior of the stream power erosion law in layered rocks.

We have subtantially expanded the discussion and written a separate conclusions section. The expanded discussion begins with a comparison to the work of Forte et al. (2016), and uses this as a framework to discuss implications that were not fully developed in the previous version. We have also tried to clarify our ideas concerning caprock waterfalls.

Reviewer 3 (Whipple)

This manuscript is well written and entails an important step forward in understanding the influence of rock strength variations in landscape evolution. The novel focus is on the influence of the slope exponent (n) in the stream power river incision model on landscape evolution in areas where sub-horizontal layered rocks with varying rock strength are exposed – extending beyond a recent treatment from my group (Forte et al., 2016, Earth Surface Processes and Landforms) that considered only the n = 1 case. It is remarkable that the venerable stream power model still holds surprises! Though of course it is always important to consider the degree to which processes and effects not encapsulated in the stream power model will alter the behavior of natural landscapes.

There is much value in the analysis and discussion presented. Reading and carefully reviewing this paper has notably advanced my own understanding of how landscapes described by the stream power model will evolve in the presence of layered rocks as a function of the relative strength between stronger and weaker layers, the relative thickness of strong and weak layers, and the dip of the contacts (only simple planar dip panels considered thus far) in cases with $n < 1$ or $n > 1$. As part of the process of reviewing this paper I re-derived most of the key relationships and updated an existing 1d finite-difference solver to handle a series of dipping layers with variable erodibility (K in the stream power model) and variable thickness so I could test both the author's initially counter-intuitive results (such as the formation of cliffs in the weak units, not the strong units, if $n < 1$) and my own derivations. I find complete agreement with Figures 3, 4, 5, and 8. Similarly, though I would word some aspects differently (reflecting differences in my derivations described below), I agree with the points made in the discussion and conclusions. Thus I agree with all the findings in a qualitative sense. Likewise I see no problems with the numerical simulation results – both in 1d and 2d using FastScape.

We thank the reviewer for this thorough review that has helped us to better understand the results that we present in this manuscript and to rethink and expand aspects of our approach.

However, I do not agree with some of the derivations and prefer a different approach to solving the problems discussed and explaining the interesting results of the 1d profile evolution models. As the only way I felt I could evaluate the derivations was to redo them following my own intuition for how to pose the problem, I present alternative solutions below. Rather than working the derivations here, I outline the logic the present the solution. Hopefully this will prove an effective and constructive approach. The alternate derivation given below results in an identical solution for horizontal bedding (Eqn 5), which is good, but suggests differing sensitivities to the dip of contacts and the relative thicknesses of strong and weak units.

Our disagreement centers around the general approach and conceptual model used to explain the observed behavior of the stream power model. Here we summarize our understanding of these differences, and provide a general response. More detailed responses to individual points are included below. Our approach uses a concept we called continuity. Continuity is a natural generalization of topographic equilibrium (where erosion rates are constant everywhere, with steepness adjusting to rock strength to accommodate those equal erosion rates). We define continuity at a contact as a condition where erosion rates in both rock types are equal in the direction parallel to the contact surface. In the case of vertical contacts, this produces equal erosion as is found in the case of topographic equilibrium. Continuity can also be applied along an entire profile. We argue that negative feedback between topography and erosion will tend to drive the system toward a state of continuity, in analogy to the negative feedback that results in topographic equilibrium. However, rather than assumming that profiles will approach a state of continuity, we have used this as a seemingly reasonable hypothesis to test against simulation results. In Section 2, we now more carefully explain that erosional continuity is a hypothesis to be tested.

Whipple did not find our approach using continuity intuitive, which perhaps means that it can be improved or at least more clearly stated. He begins by examining knickpoint celerity in the different rock layers. Knickpoint celerity is mathematically identical to horizontal retreat rate. For n not equal to 1, he suggests that either the weak (if $n > 1$) or strong (if $n < 1$) layer controls the horizontal retreat rate of the contact (i.e. knickpoint celerity). The controlling layer maintains the steepness that it would have if it were the only rock layer and were in equilibrium with uplift rate. The non-controlling layer adjusts its horizontal retreat rate (knickpoint celerity) to match the retreat of the controlling layer. Since celerity and horizontal retreat are identical, the above conceptual framework is the same as our statement of continuity in the case of horizontal rocks. If the dip of the contact is non-zero, then the two approaches differ, in that the celerity approach is matching horizontal retreat rates at the contact, and the continuity approach is matching retreat rates in the direction parallel to the contact.

We may be missing something here, but we do not see an a priori reason why one rock layer should control the other, why this control should differ in cases with $n < 1$ and $n > 1$, or why explicitly celerity, rather than retreat rate, should be matched at the contact. On the last point, it is at least clear that celerities do not match in the case of vertical contacts, which is why the proposed celerity approach only applies to the horizontal limit. However, whether or not the proposed set of assumptions can be justified in advance, they do provide testable predictions about profile shapes that can be compared against the simulations. These predictions are also different than those produced by the continuity model, enabling a comparison of the two possible conceptual approaches with the numerical results.

The easiest prediction to test from the proposed celerity model is that the controlling rock layer maintains an equilibrium slope. If this were true, then it would certainly invalidate our conclusion that a flux steady state is reached, where long-term average vertical incision rates at any x position in the profile are equal to the uplift rate. This invalidation would result from the fact that incision rates in the controlling layer are equal to uplift. If that were true, then in order to match average incision to uplift, the incision rates in the non-controlling layer would also have to be equal to uplift. However, as pointed out by both us and Forte et al. (2016), the vertical incision rates are not equal in the two rock layers.

The above reasoning, along with our conclusion in the manuscript that the simulated profiles are in a flux steady state, leads us to believe that the proposed celerity model is not precisely correct. However, it can also be tested more explicitly using simulation results. For $n < 1$, the controlling rock layer is hypothesized to be the strong layer. If we examine a snapshot from a simultion just as the base level encounters a weak rock layer, then we can clearly see the equilibrium slope produced at base level within the strong rock (where uplift and erosion are equal). This slope can be compared with the slope attained far from base level, and they are not, in general, the same (Fig. 1). In his response to our short comment, Whipple also notes he observed some disagreement between his model and the slopes observed in the $n < 1$ case.

For the $n > 1$ case, the controlling rock layer is the weak layer. We can again compare the slope at base level with the slope observed far from base level (Fig 2). Here the case is not as clear, in part because of oscillations in slope that are produced by the base level perturbations. The first weak layer above the base level layer actually overshoots the continuity equilibrium slope (and goes to even lower slopes). Layers further up oscillate back toward the slope observed at base level, but ultimately settle on a slope that is slightly less than that at base level (see upper two weak layers). This oscillation is more easily observed in the animations. Since the difference in slope is not large, it is not surprising that Whipple comes to the conclusion that the simulations satisfactorily confirm his hypothesis. However, this is in part a result of the parameter choice and can also be predicted by our theoretical relations. Using the continuity theory, one can explicitly predict the ratio of continuity steady state slope ($S_{1,cont}$) to the slope at base level in a given rock layer ($S_{1,topo}$). Combining equations 1, 5, and 8 gives

$$\frac{S_1, cont}{S_{1,topo}} = \left(\frac{H_1/H_2 + (K_1/K_2)^{1/(1-n)}}{1 + H_1/H_2} \right)^{1/n} \qquad (1)$$

This relation is depicted (Fig. 3) as a function of n for $H_1 = H_2$ and $K_w = 2K_s$. (*Note: Since this figure and equation actually make an interesting and useful point, and help to clarify the difference between the two types of equilibrium, we have added this to the manuscript along with some additional discussion.*) For $n < 1$, the ratio of continuity steady state slope to topographic steady state slope is always substantially different from one, for both the strong and weak rocks, which is why the difference is more visible in these simulations. For $n > 1$, the adjustment of the weak rock, which is suggested to be the controlling layer, is always less than a factor of 2. For our simulated value of $n = 1.5$ it is only a difference of 10-20%. The continuity theory predicts slightly more difference in slope as n approaches 1. We simulate a case with $n = 1.2$ and, as predicted, observe a slightly larger difference between base level and far from base level slopes in the weak rock (Fig 4).

To summarize our current conclusions about cases with $n \neq 1$, we can see why one might come to the conclusion that one rock layer is controlling the retreat of the other, and derive a slope relationship based on knickpoint celerity matching. In fact, our theory predicts this to be an approximate solution for a variety of parameter choices (Fig 3). However, we do not think that this celerity model provides a precise description of the far from base level equilibrium state, as we have demonstrated by comparison with simulations. Rather than one rock layer controlling the other, the celerities of both rock layers adjust to meet in the middle. Is is true, however, that in most cases one rock layer is adjusting more than the other. We think that the approximate success of the knickpoint celerity model in horizontal rocks results exactly because knickpoint celerity is identical to horizontal retreat rate. The more general statement of continuity matches the retreat rate in the direction of the contact. As a result, the theory works for arbitrary dip, not just in the horizontal limit.

For $n = 1$, we think that the model presented by Whipple is essentially correct. Horizontal retreat rate is independent of slope, and therefore adjustments in slope cannot produce a match in horizontal retreat in the two rock layers. We think that this should ultimately lead to a state where the entire profile is retreating at the rate of the weak rock (at least if overhangs are not allowed to form). We have not tested whether this is precisely true in the numerical simulations, but we suspect that the retreat rate in simulations may be sensitive to the discretization scheme, because of numerical dispersion in the vicinity of the sharp features that form in the profile. Though the numerical schemes typically used in landscape evolution models enforce continuity at the contact, the continuity theory breaks down for the rest of the profile when $n = 1$ in horizontal rocks. Since continuity breaks down, we were having difficulty understanding the $n = 1$ case, and here we think that Whipple's explanation makes dynamics of $n = 1$ channels much clearer. We have adjusted our discussion accordingly.

First, I don't much like the conceptual model in Figures 1 and 2. Most important, a problem only arises in the strong-over-weak case: overhangs cannot be sustained, as illustrated in Figure 2a. Conversely, as illustrated by Forte et al. (2016) and commonly seen in nature, weak rocks can readily be stripped off the top of strong rocks, leaving a tapering wedge of weak rock in the case of an upstream-dipping contact like that shown in Figure 2b. I also don't like the use of the word "continuity" for this, since in much of the geomorphic and fluid flow literature "continuity" means conservation of mass, though I appreciate that you are imposing a continuous profile with no overhangs.

We do not think that continuity, as we defined it, applies only to the case of strong-over-weak rock. We think that negative feedback, as described by these figures, will in general tend to drive the system toward continuity. We only consider this line of reasoning to be suggestive that continuity is a reasonable hypothesis, and we test this hypothesis against the simulations. Relationships derived from continuity do predict stream profile shapes for stream power erosion with $n \neq 1$. In the case of $n = 1$, continuity cannot be preserved. In the simulations it is preserved at the contact because of the numerical scheme (e.g. overhangs are not allowed because the channel can only have one z position for every x position) rather than for physical reasons.

In our simulations where $n \neq 1$ we do not see stripping occur when weak layers are on the top (see animations in supp. material). We do, however, see this kind of stripping for $n = 1$, as noted by Forte et al. (2016). We agree that this form of plateau topped with weak layers is commonly found in nature, which could be an argument for $n = 1$ in those cases (or perhaps for the inapplicability of the stream power model in the steep topography). Alternatively, we do often see a flat plateau preserved at the top of the topography that results from the initial condition of flat topography. Our previous simulations had weak layers on top, but we have run additional cases with strong layers on top. This plateau is preserved independent of which layer is on top. There is some dependence on n, with stonger preservation of the flat initial condition when $n > 1$.

We agree that the choice of the word "continuity" is somewhat unfortunate. We were never completely satisfied with this word, though we also had not thought of the conflict with the more common meaning of "continuity" associated with conservation relationships. So far, we have not been able to think of a more appropriate word, though we are open to any suggestions. We will change it to "erosional continuity" to try to distinguish it from conservation of mass.

I find it most useful to think about this problem in terms of the controls on the kinematic wave speed that characterizes the evolution of river profiles governed by the stream power incision model (Rosenbloom and Anderson, 1994):

$Ce = KA^mS^{(n-1)}$. Key elements are (1) all else equal the kinematic wave speed is higher in weak rocks than strong, and (2) wave speed decreases with Slope for $n < 1$, is independent of Slope for $n = 1$, and increases with Slope for $n > 1$. The surprising results in this paper all stem from the curious effect that wave speed decreases with Slope for $n < 1$.

Kinematic wave speed happens to be numerically equal to horizontal retreat rate, so we agree with these statements when horizontal rocks are considred. While it may be more intuitive to think about celerity, we do not see a way to satisfactorily predict the simulation results from a celerity based approach. If such an approach could be found that successfully predicted the simulation results, and was more intuitive to other workers in the field, we would certainly consider using it instead. However, after some thought, we have not yet found such an approach.

From study of the evolution of 1d river profiles cutting through layered rocks for cases $n < 1$, $n > 1$, and $n = 1$ revealed in numerical simulations (as in Figures 4 and 5), I suggest below a set of fundamental controls on the development of profile shape (cliffs formed in the weak rock ($n < 1$), the strong rock ($n > 1$), or through each strong-over-weak couplet ($n = 1$)), and the retreat rate of the slope-break knickpoint at the strong-over-weak contact.

The authors come close to stating what I believe is happening in the case of horizontal contacts: (1) fundamentally cliffs are forming because all-else held equal the kinematic wave speed of profile segments within the weak unit exceeds that of segments within the strong unit, so there is a tendency to undermine, or to form consuming knickpoints at strong-over-weak contacts, but as described by the authors and illustrated in the numerical simulations, the river profile will evolve toward an equilibrium where the upstream migration rate of the strong unit matches that of the weak unit at the contact; (2) for $n < 1$ wave speed decreases with increasing slope, so in response to the tendency to undermine, the profile steepens in the weak unit until the wave speed of the weak unit at the contact has slowed to equal the wave speed of the strong unit at the contact (the strong unit maintains an equilibrium slope at the contact and the knickpoint at the contact migrates at the rate set by the wave speed of the strong unit on an equilibrium slope – this best describes the basal strong-over-weak contact, see below); (3) for n=1 wave speed is independent of slope, so the river profile has no way to respond, and a vertical cliff forms (50% in weak, 50% in strong unit) with retreat rate = wave speed of the weak unit – the cliff grows in height until the full strong-over-weak doublet is incorporated and the retreat rate is set by the wave speed of the weak unit; (4) for $n > 1$ wave speed increases with increasing slope, so in response to the tendency to undermine, the profile steepens in the strong unit until the wave speed of the strong unit at the contact has increased to equal the wave speed of the weak unit at the contact (the weak unit maintains an equilibrium slope at the contact and the knickpoint at the contact migrates at the rate set by the weak unit on an equilibrium slope).

Once this realization is made, it is easy to write equations for the wave speed in each unit at the contact, set them equal, and solve for the ratio of the slope of the weak unit (S_w) to the slope of the strong unit (S_s). For horizontal beds, Equation 5 in the paper is recovered. So the derivation given is exact in the limit of horizontal contacts (also satisfies expectation for vertical contacts). However, in my analysis the derivation for the case of non-horizontal contacts (Eqn 2) appears to be incorrect. First, the solution only applies for strong-over-weak scenarios, so E1, S1 (downstream) could only be the weak unit. Second, if the derivation described above for horizontal contacts is generalized to account for planar dipping beds, a different solution is found.

In the case of dipping beds, the migration rate of the knickpoint at the strong-over weak contact is not set only by the kinematic wave speed ($Ce = E/S = KA^mS^{(n-1)}$) as it is for horizontal contacts, but must account for the slope of the contact (S_c). For example, if the contact were exactly parallel to the river bed, then the migration rate would approach infinity over that reach. Geometrically one can readily show that the local knickpoint migration velocity (Ce_{kp}) will be: $Ce_{kp} = E/(S - S_c)$ (which correctly reduces to the kinematic wave speed for $S_c = 0$).

(Note here that the caption to Fig. A1 indicates that Sc in the paper is defined positive for an upstream dip, while channel slope S is positive downstream. I worry that this could prove very confusing. Here I instead define Sc as positive for downstream dip).

We have changed this sign convention to avoid confusion.

As noted above, the migration rate of the slope-break knickpoint at the contact is set by the equilibrium wave speed within the strong unit for $n < 1$ (at least for the basal strong-over-weak contact), and within the weak unit for $n > 1$ - the problem then is how the dip of the contact amplifies or reduces knickpoint velocity relative to the kinematic wave speed. Solving for the equivalent of Eqn 5 in the presence of dipping beds, I find (derivations available on request):

$$S_w/S_s = (K_w/K_s)^{(1/(1-n))} * (1 - S_c/S_s)^{(1/(1-n))}; for\, n < 1 \tag{2}$$

and

$$S_w/S_s = (K_w/K_s)^{(1/(1-n))} * (1 - S_c/S_w)^{(1/(n-1))}; for\, n > 1. \tag{3}$$

Note that S_c/S_s appears in the $n < 1$ case because the retreat rate is set by the wave speed in the strong unit, and S_c/S_w appears in the $n > 1$ case because the retreat rate is set by the wave speed in the weak unit.

These solutions, however, only obtain over a range of $S_c/S_s \lesssim 1$ or $S_c/S_w \lesssim 1$ - basically restricted to sub-horizontal conditions - as outlined below. In addition, as mentioned above, the $n < 1$ solution applies best to the basal strong-over-weak contact: the oversteepening of the weak unit is damped up-section because the slope-break knickpoints at the strong-over-weak contacts act as a local baselevel, reducing local incision rate within the overlying strong unit (as happens in weak-over-strong contacts with $n = 1$). This causes a decrease in the slope within the strong unit, which increases the kinematic wave speed and thus decreases the degree of over-steepening of the weak unit. This complicating phenomenon is restricted to the $n < 1$ case.

I have tested these revised equations, and the limits on their applicability outlined below, against numerical simulations with satisfying results.

Above we outlined both why we think this solution does not adequately explain the simulation results, as well as why it should provide an approximate solution in some parts of the parameter space (e.g. $n \gg 1$), which is actually predicted by the continuity theory.

For $n < 1$ and downstream-dipping beds (S_c is positive), the solution only applies for $S_c/S_s < 1\breve{}(K_s/K_w)\hat{}(1/n)$: for larger (more positive) downstream dips, an equilibrium profile results (S_s and S_w have equilibrium values equal to the vertical contact case even though knickpoints are slowly migrating upstream over time). For $n < 1$ and upstreamdipping beds (S_c is negative), preliminary comparison with numerical simulations indicates the solution is only valid for $|(S_c/S_s)| \lesssim 1$. For steeper upstream dips, the profile transitions toward an equilibrium form (I have not studied this in detail).

For $n > 1$ and downstream-dipping beds (S_c is positive), the solution only applies for $Sc < Sw$. At $Sc = Sw$, knickpoint velocity is infinite. For $Sc > Sw$, the strong-over-weak contact propagates downstream, invalidating the analysis. For $n > 1$ and upstream-dipping beds (S_c is negative), the solution only applies for $S_c/S_w > 1 - (K_w/K_s)^{(1/n)}$: for larger (more negative) upstream dips, an equilibrium profile results (S_s and S_w have equilibrium values equal to the vertical contact case even though knickpoints are slowly migrating upstream over time).

We would like to point out that in addition to working in the subhorizontal case, for $abs(S_c/S_s) \gtrsim 1$, our Eqn 3 correctly predicts slopes similar to topographic equilibrium slopes for steep or vertical dips.

These solutions can be re-cast into the form of Eqn 2 (note I have inverted the relation here):

$$E_2/E_1 = E_s/E_w = (S_s - S_c)/S_w; for\, n < 1 \tag{4}$$

$$E_2/E_1 = E_s/E_w = S_s/(S_w - S_c); for\, n > 1 \tag{5}$$

Thus Eqn 2 should have two forms, one for $n < 1$ and one for $n > 1$. (remember that S_c is defined here as positive downstream).

Section 3.2. I did not attempt to reproduce or critically evaluate Equation 8, but found no dependence of erosion rate patterns on H_1/H_2 in my numerical simulations. For horizontal beds, Equation 5 is exactly satisfied for a very wide range of H_1/H_2. I did not investigate whether a greater sensitivity to layer thickness emerges with dipping contacts.

Equation 5 only predicts slope ratios, and we agree that it is independent of the thickness ratio (H_1/H_2). However, the absolute slopes, and consequently erosion rates, do depend on this relative thickness. Actually, this is an interesting prediction of our model that we had not fully tested or explored. In the revised version of the manuscript we present plots that compare simulations with non-equal thicknesses to some of our previous simulations. Keeping uplift, erodibility, and n the same, an increase in the percentage of the thickness that is occupied by weak layers results in a decrease in the slopes in both weak and strong units. In the limit where one rock layer is much thicker than the other, then this thick layer has erosion rates that match uplift, and it has a slope that would be the same as its slope in the case of topographic equilibrium. This also has interesting implications for natural systems, where thicknesses will not typically be regular. We have expanded the discussion to consider these results.

Section 3.3. I don't see the profile as being "perturbed" at baselevel because, as the authors note on page 6, line 27, new river segments formed at baselevel always begin in equilibrium ($E = U$, and equilibrium slopes). The perturbations develop above as the differential wave speeds near contacts begin to manifest in deviations from equilibrium slopes and erosion rates. Thus I'm not enthused about the "damping length scale" terminology. However, the result appears robust – differential wave speeds are rapidly accommodated at the first strong-over-weak contact, with knickpoints at contacts quickly converging on a migration velocity set by the equilibrium wave speed of either the weak unit ($n >= 1$) or the strong unit ($n < 1$).

We think this is a language issue: perturbed from which equilibrium? The newly forming segment does have an erosion rate equal to uplift. Segments at distances from base level that are farther than the damping length scale have erosion rates that depend only on the rock type, are not equal to uplift rate, and are collectively in flux steady state. That is what we mean by "damping length scale" (the scale over which erosion rates converge toward continuity steady state). Given that the manuscript is about continuity steady state, and the base level reach that is at topographic steady state produces disturbances that travel up the profile and decrease in size as they go, we think that it is reasonable to consider these as perturbations that are damped. This is certainly the visual impression we get when watching animations of the simulations. However, we have also edited the text throughout to try to make it clear which equilibrium we are referring to at any given time.

That said, I am confused by Eqn 10. First, there appears to be a typo in Eqn (10): as derived, the last term should be $A_0\hat{}(m/n)$ not $A\hat{}(m/n)$. Further, Ce = kinematic celerity = horizontal migration rate of river "patch" (patch as used by Royden and Perron, 2012) = $KA\hat{}mS\hat{}(n-1)$. For a steady-state river patch, $S = (U/K)\hat{}(1/n)A\hat{}(-m/n)$. Combining these, Whipple and Tucker (1999) showed that the horizontal migration rate of a steadystate river patch is $Ce = U\hat{}(n-1)/nK\hat{}(1/n)A\hat{}(m/n)$ – this is the relation given for Eqn 10, so the equation appears to be correct, but the derivation (and the apparent typo) implies it is incorrect.

There was a typo in this equation, A should have been A_0, as noted. The celerity we are deriving is also celerity in χ space, which may have been another source of confusion. We now clarify this in the text.

Finally, although widely appreciated, it seems worth stating that readers should beware the difference between the mathematics of the stream power model (SPM), insightful though they can be, and the physical reality of nature. Many processes are not represented in the SPM and therefore predictions may fail. Despite this, I am very supportive of publishing papers like this that explore model predictions because this allows one to: (1) generate testable hypotheses, constrain parameters, or recognize where models fail and why; (2) use any failures to improve the model; and (3) know what will happen in landscape evolution simulations based on the SPM under different conditions.

We are aware of the limitations of SPM and hope that we express this clearly in the manuscript. One of the central points of the manuscript is to clarify that common (implict) assumptions about equilibrium landscapes do not always hold for sub-horizontal layered rocks. So, to the extent that we are using the SPM and these assumptions to interpret real landscapes, we should be aware of these limitations. However, beyond this, we provide a framework that can extend the standard topographic equilibrium model and account for subhorizontal rocks. Whether evidence of continuity steady state is to be seen in nature is still uncertain. However, our theoretical work does generate testable hypotheses, and help us to understand what is occuring in landscape evolution simulations.

I have a few additional comments listed below with reference to page and line number.

1. Title: I suggest revising title to remove "continuity" as this will mean "conservation of mass" to many. Also I suggest emphasizing your key finding about the dependence on n, if you can find an effective wording.

We have changed this to "erosional continuity" to try to clarify this difference. We have not been able to think of a better word to describe this concept than "continuity."

2. Page 1, Line 21-22: This is not true. Many studies of bedrock channel morphology are expressly seeking information about the history of climate, tectonics, or drainage divide migration recorded in non-steady state profiles (as you note on page 2, line 4).

Here our text was unclear. Of course bedrock channel profiles are often used to explore transience, but an understanding of steady state is used to identify the transience. We have edited these sentences to try to make our meaning clearer.

3. Page 2, line 9: better to not call the stream power model (SPM) a "law".

We agree and have modified the text accordingly.

4. Page 2, line 11-12: the SPM is widely used in modeling studies, but is not required as a basis of profile analysis – channel steepness and concavity can be measured and interpreted in terms of relative uplift rate, climate, or rock strengths independent of the river incision rule.

We agree that profile analysis doesn't require specific use of a river incision rule, so we have edited this sentence to remove the "stream power model" phrase.

5. Page 3, line 3-4: as you show in your analysis, this is not true for n<1.

The wording here was not precisely correct. We have edited it to note that we are considering retreat rates specifically in the direction of the contact plane and vertical erosion rates. We also now make it clearer that the idea of continuity is treated as a hypothesis, which is tested using the simulations.

6. Page 3, line 5-6: I disagree. Where a weak layer overlies a strong layer, there is no constraint on the relative stream segment migration speed – the weak layer can be stripped off, leaving a bench on the underlying strong layer or a tapering wedge of the weak layer. Such forms are very common in nature.

We do think that creation of a low or reverse slope segment will reduce erosion rates as a result of reduced slope in the contact zone. However, as discussed above, we treat this as a hypothesis to be tested by simulations. The models do seem to display this behavior, except for $n = 1$.

7. Page 9, line 9: This sentence is confusing since channel segments formed at baselevel are always initially at equilibrium (E=U, steady state form) in systems described by the SPM.

It is true that these segments are in topographic equilibrium at baselevel. Topographic equilibrium assumes conditions stay constant. So if conditions, including K, change with time / uplift, topographic equilibrium is not actually an equilibrium for the system. Our system has an equilibrium different from topographic, so we can (and successfully do) treat the events at the baselevel as perturbations. However, we agree that it is potentially confusing to use the phrase “disequilibrium at base level,” and we have slightly reworded to clarify our meaning.

8. Page 9, line 16: “channel steepness” here would be better written as “channel slope” or “channel gradient”, since “steepness” commonly refers to the steepness index.

Agreed.

Steady state, erosional continuity, and the ~~erosion~~ topography of landscapes developed in layered rocks

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Abstract. ~~Considerations of landscape steady state have~~

The concept of topographic steady state has substantially informed our understanding of the relationships between landscapes, tectonics, climate, and lithology. ~~Topographic~~ In topographic steady state, ~~where topography is fixed in time, is a particularly important tool in the interpretation of landscape features, such as bedrock channel profiles, within a context of uplift patterns and rock strength. However, topographic steady state cannot strictly be attained in a landscape with layered rocks with non-vertical contacts. Using a combination of analytical solutions, stream erosion simulations, and full landscape evolution simulations, we show that an assumption of channel~~ erosion rates are equal everywhere, and steepness adjusts to enable equal erosion rates in rocks of different stengths. This conceptual model makes an implicit assumption of vertical contacts between different rock types. Here we hypothesize that landscapes in layered rocks will be driven toward a state of erosional continuity, where ~~channel retreat rates in the direction parallel to~~ retreat rates on either side of a contact are equal ~~above and below the contact , provides a more general description of~~ in a direction parallel to the contact rather than in the vertical direction. For vertical contacts, erosional continuity is the same as topographic steady state, whereas for horizontal contacts it is equivalent to equal rates of horizontal retreat on either side of a rock contact. Using analytical solutions and numerical simulations, we show that erosional continuity predicts the form of flux steady state landscapes ~~in~~ that develop in simulations with horizontally layered rocks. ~~Topographic steady state is a special case of the steady state derived from continuity. Contrary to prior work,~~ For stream power erosion, the nature of continuity steady state depends on the exponent, n, in the erosion model. For $n = 1$, the landscape cannot maintain continuity. For cases where $n \neq 1$, continuity is maintained, and steepness is a function of erodibility that is predicted by the theory. The landscape in continuity steady state can be quite different from that predicted by topographic steady state. For $n < 1$ continuity predicts that channels incising subhorizontal layers will be steeper in ~~weaker rocks in the case of subhorizontal rock layerswhen the stream power erosion exponent~~ ~~$n < 1$~~the weaker rock layers. For subhorizontal layered rocks with different erodibilities, continuity also predicts larger slope ~~differences than would be predicted by~~ contrasts than in topographic steady state. ~~Continuity steady state is a type of flux steady state, where uplift is balanced on average by erosion. Under conditions of constantuplift rate~~If uplift rate is constant, continuity steady state ~~cannot be attained~~ is perturbed near base level~~. This occurs because continuity steady state requires different rates of vertical incision in rocks with different erodibility. However, perturbations introduced by disequilibrium at base level rapidly decay for cases with strong~~

~~contrasts~~, but these perturbations decay rapidly if there is a substantial contrast in erodibility. Though examples explored here utilze the stream power erosion model, continuity steady state provides a general mathematical tool that may also be useful to understand landscapes that develop by other erosion processes.

1 Introduction

The formation of landscapes is driven by tectonics and climate, and often profoundly influenced by lithology, the substrate on which tectonic and climate forces act to sculpt Earth's surface. Much of our interpretation of landscapes, and their relationship to climatic and tectonic forces, employs concepts of landscape equilibrium, or steady state. Though there are a variety of types of landscape steady state (Willett and Brandon, 2002), topographic steady state, in which topography is constant over time, is perhaps most often used in the interpretation of landscapes. Understanding of steady state also enables identification of transience within the landscape. In particular, concepts of topographic steady state ~~is used ubiquitously~~ and transient response to changes in climate or tectonics are frequently used within studies of bedrock channel morphology.

Bedrock channels are of particular geomorphic interest because they span most of the topographic relief of mountainous terrains (Whipple and Tucker, 1999; Whipple, 2004), providing the pathways through which eroded material is routed to lowlands and a primary means by which the landscape is dissected and eroded. Therefore, bedrock channels exert important controls on the relief of mountain ranges and set the pace at which mountainous landscapes respond to changes in climate or tectonic forcing. Research on bedrock channels has driven new understanding concerning the coupling between mountain building, climate, and erosion (Molnar and England, 1990; Anderson, 1994; Whipple et al., 1999; Willett, 1999).

The elevation profiles of bedrock channels ~~provide a primary means for analyzing~~ enable analysis of landscapes for evidence of transience, contrasts in rates of tectonic uplift, or the influence of climate (Stock and Montgomery, 1999; Snyder et al., 2000; Lavé and Avouac, 2001; Kirby and Whipple, 2001; Lague, 2003; Duvall et al., 2004; Wobus et al., 2006; Crosby and Whipple, 2006; Bishop and Goldrick, 2010; DiBiase et al., 2010; Whittaker and Boulton, 2012; Schildgen et al., 2012; Allen et al., 2013; Prince and Spotila, 2013). Within this analysis, erosion rates are typically assumed to scale as power law relations ~~with~~ of drainage area and slope, as given by the stream power erosion ~~law~~ model (Howard and Kerby, 1983; Whipple and Tucker, 1999),

$$E = KA^m S^n, \tag{1}$$

where E is erosion rate, K is erodibility, A is upstream drainage area, S is channel slope, and m and n are constant exponents. While the stream power ~~erosion law~~ model has known limitations (Lague, 2014), it remains the most frequently used tool for channel profile analysis and landscape evolution modeling. Under steady climatic and tectonic forcing, channels are typically assumed to adjust toward topographic steady state (Hack, 1960; Howard, 1965; Willett and Brandon, 2002; Yanites and Tucker, 2010; Willett et al., 2014), where uplift and erosion are balanced and topography is constant with time. This ~~assumption, in conjunction with the stream power erosion law,~~ framework enables interpretation and comparison of stream profiles to identify spatial contrasts in uplift rates or transient responses to changes in tectonic or climatic forcing.

Topographic steady state has also been used to explain channel ~~responce~~ response to substrate resistance, generally leading to a conclusion that channels are steeper within more erosion resistant bedrock and less steep within more erodible rocks (Hack, 1957; Moglen and Bras, 1995; Pazzaglia et al., 1998; Duvall et al., 2004). However, this result depends on an implicit assumption of vertical contacts between strata as in Fig. 1A. Strictly speaking, topographic equilibrium does not exist when channels incise layered rocks with different erodibilities and non-vertical contacts (Howard, 1988; Forte et al., 2016). In the case of non-vertical contacts, the contact positions shift horizontally as the channel incises, resulting in topographic changes as shown in Fig. 1B,C. Studies of bedrock channel morphology have primarily focused on regions with active uplift, where rock layers are often deformed and tilted from horizontal. However, a substantial percentage of Earth's surface contains subhorizontal strata. Many of these settings also contain bedrock channels, with examples including the Colorado Plateau, the Ozark Plateaus, and the Cumberland and Allegheny Plateaus. In such settings, intuition developed from assumptions of topographic equilibrium does not necessarily apply. ~~Here we explore the limitations of topographic equilibrium~~

Forte et al. (2016) used landscape evolution models to demonstrate that erosion rates vary in space and time in potentially complex ways as landscapes incise through layered rocks with different erodibilities. These simulations also suggest that deviation from topographic equilibrium is strongest for rock layers that are horizontal. While topographic equilibrium does not hold in general in ~~landscapes formed in~~ layered rocks~~and develop a generalized mathematical framework to predict the~~ , here we explore whether landscapes incising layered rocks develop any kind of steady state form~~of a landscape developed within layered rocks that have an arbitrary dip.~~ , and whether there are regular relationships between steepness and rock erodibility. We show that such a form does exist in some cases, and that it is a type of flux steady state that can be derived from an assumption of erosional continuity across the rock contacts. We further examine how this steady state depends on the erosion model employed and on the contact dip angle, focusing on the case of subhorizontal layers.

2 ~~Landscape continuity and steady state~~

2 Erosional continuity and steady state

~~To understand erosion and morphology near a lithologic contact , we begin with an ansatz that the land surface will strive to maintain topographic continuity at the contact. This is equivalent to an assumption that retreat rates~~

Conceptual models of land surface response to changing rock type typically employ the concept of topographic steady state, which makes an implicit assumption of vertical contacts between the different rock types. In topographic steady state, vertical incision rates are matched in the two rock types (Fig. 1A). Considering the opposite limit, with horizontal contacts between rocks, it seems natural to think about horizontal retreat rates rather than vertical incision rates (Fig. 1B). It is plausible that a similar steady state exists where steepness in each rock type is fixed, and horizontal retreat rates are equal at the contact. This would not be a topographic steady state, but steepness would maintain a one-to-one correspondence with rock erodibility. The land surface would retreat horizontally at a fixed rate above and below the contact ~~will be equal in the direction parallel to~~ while undergoing continued uplift. Generalizing between these two limiting cases, we consider a possible steady state for arbitrary rock contact dip where surface erosion rates are equal in a direction paralleling the contact plane (Fig. 1C). We refer to equal retreat in the direction of the contact plane as erosional continuity. Mathematically speaking, it means that retreat rate in the direction of a contact is a continuous function across the contact.

Physical reasoning supports the idea that landscapes in layered rocks would tend toward erosional continuity. ~~This assumption is also identical to topographic equilibrium in the case of vertical contacts. The ansatz of continuity is supported by physical reasoning .~~ If the upper layer ~~erodes~~ retreats slower than the lower layer in the direction of the contact, this produces a steep, or possibly overhanging, land surface at the contact (Fig. 2A). This steepening or undercutting will lead to faster vertical erosion in the upper layer and drive the system towards continuity (Fig. 2~~B~~C). Similarly, if the upper layer retreats faster in the direction of the contact, this produces a ~~flattened~~ low slope or reversed slope zone near the contact (Fig. 2B) that will ~~reduce erosion rates in the upper layer and~~ also push the system toward continuity. Therefore, the same types of negative feedback mechanisms between topography and erosion that drive landscapes to topographic steady state (Willett and Brandon, 2002) will also plausibly drive landforms near a contact into a state that maintains continuity. We refer to this hypothesized type of equilibrium as continuity steady state. ~~Localized discontinuities are sometimes produced~~

There are cases in natural systems where continuity is not maintained at all times. For example, caprock waterfalls are similar to the case in Fig. 2A. However, even in this case the discontinuity cannot grow indefinitely. If the waterfall reaches a steady size then the system has once again obtained a state where continuity is maintained in a neighborhood near the contact. Numerical landscape evolution models do not typically allow cases such as Fig. 2A-B. Therefore, numerical models are likely to maintain continuity even more rigidly than natural landscapes. While these lines of reasoning suggest that both natural systems and landscape evolution models may be driven toward erosional continuity, here we consider continuity steady state to be a hypothesis that we test against landscape evolution models. Erosional continuity makes quantitative predictions about steady state landscapes that are elucidated below and then tested against numerical landscape evolution models.

Using the constraint of erosional continuity, one can write a very general relationship between surface erosion rates and slopes at a contact between two rock types,

$$\frac{E_1}{E_2} = \cancel{\frac{S_1+S_c}{S_2+S_c}} \frac{S_1-S_c}{S_2-S_c}, \tag{2}$$

where E_i and S_i are vertical erosion rates and slopes, respectively, and the index refers to rock types 1 and 2. S_c is the slope of the rock contact and is defined as positive in the downstream direction. This relationship results from an assumption of equal retreat rate at the contact within both rock layers in a direction parallel to the rock contact plane, ~~illustrated in Fig~~as illustrated in Figs. 1C and ~~Fig.~~ A1. A similar relationship is used by Imaizumi et al. (2015) to examine the parallel retreat of rock slopes. If we consider the more specific case of stream power erosion through a pair of weak and strong rocks, this leads to

$$\frac{K_w {S_w}^n}{K_s {S_s}^n} = \cancel{\frac{S_w+S_c}{S_s+S_c}} \frac{S_w-S_c}{S_s-S_c}, \tag{3}$$

where K_w is the erodibility of the weaker rock, K_s is the erodibility of the stronger rock, $S_w = \tan\theta_w$ and $S_s = \tan\theta_s$ are the slopes of the channel bed in each rock type, and the contact slope is $\cancel{S_c = \tan\phi}$ $S_c = -\tan\phi$ (derivation in Appendix A). Here we have assumed that erosion processes in both rock types can be expressed with the same exponent, n. While n may vary with rock type if erosion processes are different (Whipple et al., 2000) , fixed n provides a useful starting point to understand erosion of layered rocks and is also the most common choice used in landscape evolution models.

The implications of the relationship in Equation 3 are most easily understood by examining two limiting cases, a vertical contact limit, which applies whenever contact dip is large compared to channel slope, and a subhorizontal limit, which applies when contact dip is small compared to channel slope. When the contact slope is much larger than the channel slopes ($\cancel{S_c \gg S_w, S_s)}$ ~~,~~ $|S_c| \gg S_w, S_s$) the right hand side of Eq. (3) is approximately one, and vertical erosion rates in both lithologies are roughly equal. Rock uplift can thus be balanced by erosion in both segments, and the standard relationship between channel ~~steepness~~ slopes in the two lithologies, normally derived from topographic equilibrium, is recovered, with

$$K_w {S_w}^n = K_s {S_s}^n. \tag{4}$$

If the contact slope is in this steep limit, but not vertical, the contact position and topography will gradually shift horizontally with erosion and vertically with uplift, while still obeying this relation derived from topographic equilibrium.

For the ~~case~~ subhorizontal limit, where channel slopes are much greater than the slope of the contact ($\cancel{S_w, S_s \gg S_c}$), ~~as would be common in subhorizontal rock layers,~~ $S_w, S_s \gg |S_c|$), Eq. (3) simplifies to

$$K_w {S_w}^{n-1} = K_s {S_s}^{\cancel{n-1, \mathrm{or}}\; n-1} \qquad \text{or} \qquad \frac{S_w}{S_s} = \left(\frac{K_w}{K_s}\right)^{\frac{1}{1-n}}. \tag{5}$$

In this case, continuity results in roughly the same rate of horizontal retreat in both rocks at the contact, as in Fig. 1B. This contrasts with the standard assumption of equal rates of vertical erosion, and leads to unexpected behavior. Specifically, if $n < 1$~~then~~, since $K_w > K_s$, higher slopes are predicted in weaker rocks, which is in strong contrast to intuition developed from the perspective of topographic equilibrium. This results because the rate of horizontal retreat within a given rock layer ($dx/dt \propto K_i S_i^{n-1}$) is a decreasing function of slope if $n < 1$. Steeper slopes can retreat more slowly horizontally because a given increment of vertical incision produces less horizontal retreat on a steeper slope than a shallower slope. For $n < 1$ vertical erosion does not increase quickly enough with slope to offset this effect. Since horizontal retreat rate is an increasing function of erodibility, continuity requires that increases in erodibility are offset by increases in slope. For subhorizontal contacts with $n > 1$, higher slopes are once again predicted in stronger rocks.

The slope ratio (S_w/S_s) is depicted for the vertical and horizontal limits in Fig. 3A as a function of n for an erodibility contrast of $K_w = 2K_s$. In general, contrasts in the slopes within the two strata in the subhorizontal case ~~as in Eq. (~~(Eq. 5) are larger than would be predicted using the standard formulation for vertical contacts ~~in Eq. (~~(Eq. 4). In subhorizontal rocks (i.e. whenever rock dip is small compared to channel slope), channel slopes may become sufficiently high or low to be driven to values outside the range of validity of the stream power ~~erosion law~~model, particularly for cases of $n \approx 1$. Perhaps the most common value of n used within landscape evolution models is $n = 1$, therefore it is also notable that the continuity relation for subhorizontal strata contains a singularity at $n = 1$ (Fig. 3). The slope ratio (S_w/S_s) diverges for $n \to 1^-$ and approaches zero for $n \to 1^+$. This suggests strong dependence of channel behavior on n when n is close to 1. The singularity results because for $n = 1$ the horizontal retreat rate is independent of slope and solely a function of erodibility and drainage area. Therefore the channel cannot maintain continuity by adjusting steepness.

3 Continuity steady state and stream profiles

The channel continuity relations above apply to channels within the neighborhood of a contact. Though there are clear long-term constraints on the relative retreat rates of any two contacts, these are not sufficient to determine an entire profile. ~~Here, we examine whether~~ However, we hypothesize that the continuity relation applies along ~~an entire profile~~ entire profiles, and therefore that it can be used to describe a type of equilibrium state that develops in layered rocks. If this is correct then there is a one-to-one relationship between erodibility and steepness that is predicted by the continuity relations. Here we test this hypothesis using simulations of channel and landscape evolution in horizontally layered rock.

3.1 Methods for one-dimensional simulations and analysis

We solve the stream power ~~erosion law~~ model using a first order explicit upwind finite difference method. This method is conditionally stable, and the timestep was adjusted to produce a stable Courant-Friedrich-Lax number of $\mathrm{CFL} = 0.9$. The explicit upwind scheme has commonly been used for prior studies, though it is also known to produce smoothing of channel profiles near knickpoints (Campforts and Govers, 2015). ~~Experimentation with resolution suggests that the conclusions presented here are not dependent on the numerics.~~ The simulations employed 2000 spatial nodes, though we also ran a few cases with higher resolution that produced the same results. For simplicity, basin area was held fixed over time and was computed as a function of longitudinal distance, with

$$A = k_a x^h, \tag{6}$$

where $k_a = 6.69\,\mathrm{m}^{0.33}$ and $h = 1.67$. These parameter values are representative of natural drainage networks (Hack, 1957; Whipple and Tucker, 1999). Simulations were run with $n = 2/3$, $n = 1$, and $n = 3/2$. The value of m in the stream power ~~erosion law~~ model was adjusted according to the choice of n to assure that the concavity $m/n = 0.5$, which is typical of natural channels (Snyder et al., 2000). Both high uplift ($2.5\,\mathrm{mm\,y^{-1}}$) and low uplift ($0.25\,\mathrm{mm\,y^{-1}}$) cases were run. Simulation parameters were adjusted to provide a similar number of rock contacts in each case. For the high uplift cases, rock layers were 50 m thick, whereas for the low uplift cases rock layers were 10 m thick. Longitudinal distances were also adjusted with the high uplift cases simulating 50 km long profiles and the low uplift cases simulating 200 km long profiles. Specific parameter values are provided in Table 1.

Simulation results are most easily visualized in χ space (Perron and Royden, 2013; Royden and Taylor Perron, 2013), where the horizontal coordinate x is replaced with a transformed coordinate χ:

$$\chi = \int_{x_0}^{x} \left(\frac{A_0}{A(x)} \right)^{m/n} \mathrm{d}x. \tag{7}$$

One advantage of this transformation is that the effect of basin area is removed such that equilibrium channels that evolve according to the stream power ~~erosion law~~ model appear as straight lines in this transform space. The relation predicted by Eq. (5) is invariant under the transformation to χ space, and therefore the relation also holds if slope is replaced with ~~slope~~ steepness (gradient in χ-elevation space~~.~~). Throughout this work, we use a value of $A_0 = 1\,\mathrm{m}^2$ in the χ transforms.

3.2 Comparison of continuity steady state and simulated profiles

For simulations where $n \neq 1$, as hypothesized, channel profiles far from base level approach a steady configuration, in which channel slope in χ space is a unique function of rock erodibility, and the profiles exhibit straight line segments in each rock type (Figs. 4,5). For the horizontally layered case, channel profiles evolve towards a state in which they are maintaining the same shape in χ space while retreating horizontally into the bedrock. For small changes in basin area, this is equivalent to a channel maintaining constant horizontal retreat rates. For non-horizontal rocks, profile shapes will gradually change in χ space, as the slope of the contact plane in χ space changes with basin area. Animations of the simulations depicted in Figs. 4 and 5 are provided in the online supplementary material.

For $n = 1$ there is no one-to-one relation between erodibility and ~~slope~~steepness, and the profiles do not exhibit straight-line segments in each rock type. The $n = 1$ case produces this result because the horizontal retreat rates are independent of slope and purely a function of erodibility and basin area. Consequently, adjustments of slope cannot produce equal horizontal retreat rates along the channel. Instead, segments within weaker rocks will retreat more quickly than those within stronger rocks. This produces “stretch zones” as a channel crosses from weak to strong rocks and “consuming knickpoints” as a channel crosses from strong to weak rocks (Royden and Taylor Perron, 2013; Forte et al., 2016). The channels in the simulations ultimately reach a steady stepped shape (Figs. 4C,5C) in which weak rock layers retreat until they intercept and undermine the contact with strong layers~~; however, this state will contain near-vertical channelsthat~~ . Near-vertical cliffs, containing both strong and weak rocks, develop at the contact channels. These dynamics are described in more detail by Forte et al. (2016) . It is important to note that channels in the $n = 1$ subhorizontal case contain reaches that are sufficiently steep to negate ~~the applicability of the stream power erosion law. Within simulations~~assumptions behind the stream power model. Additionally, the nature of such profiles in simulations may be strongly dependent upon the numerical algorithm employed as a result of numerical diffusion of sharp features (Campforts and Govers, 2015).

The continuity relation (3) predicts a slope ratio rather than absolute values of slope in each rock type. The predicted slope ratio matches the slopes in the simulation at sufficient distances from base level. Notably, the counterintuitive prediction that profiles would be steeper in weaker rocks for $n < 1$ is confirmed by the simulations (Figs. 4A,5A). However, absolute slopes, and therefore entire profiles, can be predicted by realizing that continuity steady state is actually a type of flux steady state (Willett and Brandon, 2002), where the rate of uplift of rock into the domain is equal to the rate of removal of material by erosion. First, it must be noted that the weak and strong rocks experience different rates of vertical incision in the equilibrium state (Forte et al., 2016). ~~The rates of vertical incision within two alternating layersat equilibrium can be calculated from an assumption of flux steady state , or, equilvalently, an assumption that the time-averaged incision rate through both rock types~~ However, since the shape of the landscape in χ space repeats with each pair of rock layers, the long term average incision rate must be the same at all horizontal positions on the stream profile. Furthermore, the topography is not growing or decaying over time after continuity steady state is reached, which means that the average incision rate at all positions is equal to the uplift rate~~. This~~ , or, equivalently, that the system is in flux steady state. This conclusion that the long term average rate of vertical incision at each point along the profile is equal to the uplift rate leads to a relation ~~between vertical erosion rates in the two~~

~~layers given by~~ for the erosion rate in a given layer,

$$E_1 = U\frac{(H_1/H_2)+(K_1/K_2)(S_1/S_2)^n}{1+H_1/H_2}, \tag{8}$$

where ~~E_i~~ E_1 is the erosion rate of ~~the ith~~ one rock layer, H_i is the thickness of the ith layer measured in the vertical direction, and U is the uplift rate (see derivation in Appendix B). Entire theoretical profiles can be constructed using this relationship, in combination with the stream power ~~erosion law~~ model and the continuity relation (Eq. 5), which provides the slope ratio. These profiles closely match the simulations in cases where $n \neq 1$ at a sufficient distance from base level (Figs. 4A-B,5A-B)~~. Therefore,~~ , further confirming that continuity state is a type of flux steady state. In addition to describing behavior near contacts, ~~it~~ continuity steady state also describes portions of the profile that are distant from contacts. For ~~sub-horizontal~~ subhorizontal rocks this often produces a landscape that is quite different from that which would be predicted by topographic steady state (Fig. 3).

In continuity steady state the slopes in both rock types are different, in general, than the slopes that would be predicted by topographic steady state. Combining Eqns. 1, 5, and 8 gives

$$\frac{S_{1,\mathrm{cont}}}{S_{1,\mathrm{topo}}} = \left(\frac{H_1/H_2+(K_1/K_2)^{1/(1-n)}}{1+H_1/H_2}\right)^{1/n}, \tag{9}$$

where $S_{1,\mathrm{cont}}$ and $S_{1,\mathrm{topo}}$ are the slopes for rock layer 1 that would be obtained under continuity steady state and topographic steady state, respecitvely. Setting the thicknesses equal, $H_1 = H_2$, and using an example case of $K_w = 2K_s$, we plot the ratio of continuity and topographic steady state slopes for both the weak and strong layers (Fig. 6). For $n < 1$ there is always a strong difference between the continuity and topographic steady state slopes in both rocks. For $n > 1$ the weak rock in continuity steady state never has a slope more than a factor of two different than the slope that would be predicted by topographic steady state. For large n the continuity steady state slopes of both weak and strong rock layers obtain the same slope as they would in topographic steady state. Additionally, if one layer is much thicker than the other (e.g. $H_1 \to \infty$), then the slope of this layer approaches the slope that it would have under topographic steady state.

Continuity steady state predicts that the ratios of slopes in the weak and strong layers are independent of layer thickness (Eq. 5). However, it also predicts that erosion rates and absolute slope values in both rocks are dependent on the thickness of the layers (Eqs. 8 and 9). To test this prediction, we resimulated the high uplift cases above with $n = 2/3$ and $n = 3/2$ and changed the layer thickness. For ease of comparison, the total thickness of both layers was kept equal to 100 m, but the weak layer thickness was increased to 90 m. As predicted, the continuity steady state slopes vary with relative layer thickness (Fig. 7). The thicker of the two rock layers adjusts its slope toward the slope that it would have under topographic steady state. Increasing the percentage of weak rock adjusts both slopes in such a way that it reduces the total topography (Fig. 7).

3.3 Dynamics of base level perturbations

Continuity steady state is perturbed near base level, ~~where~~ because a constant rate of base level fall is imposed ~~. This results because vertical incision occurs~~ and continuity steady state requires vertical incision at different rates in each rock

~~in continuity steady state, whereas uplift is constant~~type. Despite this ~~disequilibrium near base level~~ discrepancy between base level topographic equilibrium and continuity steady state, theoretical profiles produced using Eq. (3) and Eq. (8) closely match the shapes of the profiles for the cases where n is not one. Therefore, these perturbations decay rapidly away from base level in the simulated cases. However, a question remains as to what controls this decay length scale, and how typical the cases are that we have simulated.

In a horizontally layered rock sequence, a segment of stream profile with erosion rate equal to uplift is continuously developing at base level. The slope of this base level segment in χ-space is given by

$$\frac{dz}{d\chi} = \left(\frac{U}{KA_0^m}\right)^{1/n}. \tag{10}$$

The difference between this slope ~~,~~ and the continuity steady state slope produces a knickpoint that propagates upstream with a celerity ~~given by~~ in χ space given by

$$C = \frac{U}{dz/d\chi} = U^{(n-1)/n} K^{1/n} A_0{}^{m/n}, \tag{11}$$

As the knickpoint crosses into the other rock type, continuity demands that C does not change, because C is identical to horizontal retreat rate and continuity requires this to be equal across a horizontal contact. Since celerity is a monotonic increasing function of erodibility, knickpoints formed at base level in the stronger rock are slower than those formed in the ~~soft~~ weak rock. Therefore, the ~~soft~~ weak rock knickpoints catch up to the ~~hard~~ strong rock knickpoints, and the profile damps toward equilibrium as the two interact. Consequently, we can estimate the damping length scale as the χ distance at which the knickpoints generated in ~~soft~~ weak rock at base level catch up to the knickpoints generated in ~~hard~~ strong rock at base level.

The strong rock knickpoint begins with a head start equal to the χ distance spanned by the strong rock segment, which we call ~~χ_s~~ $\chi_{s,0}$ and is given by

$$\chi_{s,0} = H_s\left(\frac{KA_0^m}{U}\right)^{1/n}. \tag{12}$$

The strong rock knickpoint will travel an additional distance $\chi_{s,+}$ before the ~~soft~~ weak rock knickpoint catches up, and these distances are related by

$$\frac{\chi_{s,0}+\chi_{s,+}}{C_w} = \frac{\chi_{s,+}}{C_s}, \tag{13}$$

where C_s and C_w are the knickpoint celerities in the strong and weak rocks, respectively. The damping length scale, $\lambda = \chi_{s,0}+\chi_{s,+}$, is the distance from base level over which the weak rock knickpoint catches the strong one and can be solved for by combining Eqs. 11, 12, and 13, leading to

$$\lambda = H_s\left(\frac{K_sA_0^m}{U}\right)^{1/n}\left[1+\left((K_w/K_s)^{1/n}-1\right)^{-1}\right]. \tag{14}$$

To generalize the damping behavior of the base level perturbations it is useful to analyze a dimensionless version of λ, which is normalized by $\chi_{s,0}$,

$$\lambda^* = \frac{\lambda}{\chi_{s,0}} = 1+\left[\left(\frac{K_w}{K_s}\right)-1\right]^{-1}. \tag{15}$$

It can be seen that the damping length scale is primarily a function of the relative erodibilities of the two rock types. When the contrast is large, damping occurs rapidly, whereas when the contrast is small the damping length scale is large. However, in this latter case there is also very little contrast in steepness, since the erodibilities are similar. Since $\chi_{s,0}$ is the χ length of the strong rock reach near base level at the moment that the weak layer becomes exposed, $\chi_{s,0}$ is less than but on the same order of magnitude as the profile distance spanned by a pair of weak and strong rock layers. Therefore, λ^* can be interpreted as a conservative order of magnitude estimate of the number of pairs of weak and strong rocks that are required to produce damping. That is, if $\lambda^* \sim 1$ then damping should occur within a single pair. We show λ^* as a function of the erodibility ratio for several choices of n in Figure 8. Here it can be seen that if the erodibility ratio is greater than about two or three ~~damping occurs within two cycles through the rock layers away~~ then $\lambda^* \lesssim 2$, or, equivalently, damping occurs for parts of the profile that are separated from base level by more than two sets of contacts between the two rock types. If the erodibility ratio is greater than about ten, then $\lambda^* \lesssim 1$, and damping occurs within a single ~~cycle~~pair of the two rock types.

To illustrate this damping behavior, we run two simulations with somewhat longer damping length scales. Both simulations have profile lengths of 500 km, uplift rates of 2.5 $\mathrm{mm\,y^{-1}}$, repeating rock layers with a 50 m thickness, and weak rock layers that have an erodibility of 1.5 times the ~~hard~~ strong rock layers. One case uses: $n = 1.2$, $m = 0.6$, and $K_s = 1.5 \times 10^{-5}$, whereas the second case uses:$n = 0.8$, $m = 0.4$, and $K_s = 1 \times 10^{-4}$. For the $n = 1.2$ case, $\lambda = 2.25$, and for the $n = 0.8$ case, $\lambda = 2.45$. Profiles are shown for these simulations in Figure 9. Fast knickpoints catch the slow knickpoints at roughly the calculated length scale (Fig. 9C,D). ~~This~~ Note that the knickpoints we are describing here are breaks in steepness, which can be downstream decreases or increases in steepness. The knickpoint interference can be seen as the gradual reduction in the size of a topographic equilibrium slope patch near base level that reaches zero size at approximately $\chi = \lambda$. This process is visualized more clearly in animations in the supplementary material that depict the damping length scale. Beyond this damping length scale, some minor perturbations remain, and one can see fast and slow knickpoints migrating through the upper parts of the profile as the system evolves. However, beyond λ the theoretical profiles derived from continuity and flux steady state are good approximations to profile shape.

4 Full landscape simulations

To determine whether continuity steady state is obtained within whole landscape models, or whether addition of hillslope processes might eliminate it, *FastScape* V5 (Braun and Willett, 2013) was used to simulate stream power erosion coupled to an entire landscape model. All simulated cases employ a constant rock uplift rate and horizontal rock layers with alternating high and low erodibility.

The stream power ~~erosion law~~ model used in *FastScape* has the form

$$E = K_f \Phi^m S^n, \tag{16}$$

where Φ is discharge, calculated as the product of the drainage area and the precipitation rate P. Each of the three presented model runs uses two different erodibility coefficients, K_{fw} for the weak rock and K_{fs} for the strong rock, in place of K_f. For each one of them, a grid of 3000×3000 pixels representing 100×100 km is simulated. The initial condition used is a slightly randomly perturbed flat surface at base level. The boundary condition is open on all sides. 15000 m of uplift is simulated in 60000 timesteps. The ~~softer~~ weaker rock is exposed for the first 10800 m of the uplift, allowing an initial drainage network to establish. Afterwards, a layered rock structure starts to be exposed, with alternating layers of 200 m of the ~~harder~~ stronger rock and 300 m of the ~~softer~~ weaker rock. The ~~different bed thicknesses enables testing of whether any of the previous theoretical results require the layered rocks to have equal thickness. The~~ main difference between the model runs is in the slope exponent n, with cases using $n = 2/3$, $n = 1$, and $n = 3/2$. A listing of numerical parameters is provided in Table 2. The necessary timestep was calculated from the uplift rate and the ratio of total uplift to the number of timesteps.

Floating point digital elevation models (DEMs) were produced for the final time step for each *FastScape* simulation. Using the *Landlab* landscape evolution model (Tucker et al., 2013) to calculate flow routing, channel profiles were extracted from the *FastScape* DEMs for each case of n. *Landlab* was extended to enable calculation of χ values for each channel. χ-plots were then generated for 50 channels in each simulation and are shown in Fig. 10. The continuity equilibrium state described above is also reflected within the full landscape evolution model, and plots of elevation versus χ for channels within each model demonstrate similar relationships as displayed in Figure 4~~A-C~~A,C,E.

5 ~~Discussion and conclusions~~

~~The standard concept of topographic equilibrium and its implications for channel form break down~~

5 Discussion

Topographic steady state is not attained within layered rocks ~~as the layers approach horizontal(Howard, 1988; Forte et al., 2016) . However, under constant forcing, the stream power erosion law drives channels to a different type of steady state , which we refer to as~~ with non-vertical contacts since the spatial distribution of erodibility changes in time (Howard, 1988; Forte et al., 2016) . Forte et al. (2016) show that departures from topographic steady state are greatest when the layers have contacts that are near horizontal. They use simulations of landscape evolution with a stream power erosion model with $n = 1$. These simulations demonstrate that erosion rates vary across the landscape in complex ways, that there is no direct relationship between rock erodibility and erosion rate, and that erosion rates can be greater or less than the uplift rate. They also detect distinct differences in landscape development between cases where either the strong or weak rock is exposed first. In the case of a weak rock on top of a strong rock, a tapered wedge of weak rock forms on top of a steep retreating escarpment in the strong rock. When strong rock is on top of weak rock, the weak rock undercuts the strong rock and forms an extremely steep zone near the contact.

Our simulations and analysis support the conclusions of Forte et al. (2016) on the dynamics of the $n = 1$ case. However, we also show that these dynamics result specifically because the rate of horizontal retreat, or equivalently the knickpoint celerity, is independent of slope when $n = 1$. Consequently, the topography is unable to maintain a state of erosional continuity, and therefore topography is unable to reach continuity steady state. ~~Continuity steady state is also a type of flux steady state , where~~

~~time averaged incision in the channel at any given horizontal position is equal to uplift~~ Landforms developed in layered rocks are driven toward continuity steady state by the same type of negative feedback mechanisms between topography and erosion that generate topographic steady state. In fact, topographic steady state is a special case of continuity steady state. For stream power erosion with $n \neq 1$, landscapes are able to adjust slope to maintain continuity across multiple rock layers. Therefore, a type of equilibrium landscape form does develop sufficiently far from base level when $n \neq 1$. ~~In this state, changes in channel profile shape over time are small and result from changes in basin area. If constant uplift occurs, the channel cannot equilibrate at base level, because~~

If we compare the $n \neq 1$ case with the conclusions above concerning the $n = 1$ cases, several similarities and differences emerge. For both cases, it is true that topographic steady state is only strictly reached if contacts are vertical. Also, for both cases the patterns of steepness in the landscape diverge most strongly from those predicted by topographic steady state when rocks are horizontally layered. However, for $n \neq 1$ erosion rates and steepnesses do exhibit one-to-one relationships with rock erodibility. Since erodibility determines steepness, we do not see any dependence of topography on the order of exposure of the layers, unlike with the $n = 1$ case. Considering two rock types, one strong and one weak, erosion rates bracket the uplift rate, with one rock exhibiting erosion rates higher than uplift and the other lower than uplift. For the subhorizontal case, the weak rock erodes faster when $n < 1$ and the strong rock erodes faster when $n > 1$ (Fig. 6). Contrasts in erosion rates become small for large n (Fig. 6) and very large when $n \approx 1$.

As noted by Forte et al. (2016) , variability in erosion rates across the landscape can produce bias in detrital records, as zones exhibiting faster erosion will contribute a larger proportion of the exported sediment than would be calculated based on areal estimates. Since the framework developed predicts a regular relationship between erosion rates and erodibility for $n \neq 1$, it may help constrain uncertanties in such records. The long term average erosion rate at any location is equal to uplift rate, and therefore continuity steady state ~~results in different vertical incision rates in each rock type . However, the perturbations introduced by thisdisequilibrium at base level rapidly decay over a length scale that is primarily~~ is a type of flux steady state. Because of this, there is also a simple rule that emerges when considering erosion rates as a function of ~~the ratio of rock erodibilities, with larger erodibility contrasts resulting in shorter decay lengths. Practically speaking, for rocks that have erodibilities sufficiently different to have a strong effect on the profile, these perturbations decay after a couple rock contacts are passed~~rock type. For the portion of the landscape that is in flux steady state, the amount of material removed from a given rock layer within a period of time will be proportional to the fraction of the topography that is spanned by that layer, as opposed to its areal extent. For example, in our simulations where each rock type makes up half of the topography, there is an approximately equal volume of material eroded from each rock type within a given timestep.

~~If~~ When contacts between rocks ~~are dipping~~ dip at slopes much greater than the channel slope, then the vertical contact limit from Eq. (4) applies and ~~traditional conceptions for equilibrium channel form hold~~topography approaches the form that would be predicted by topographic steady state. The considerations introduced here become important as rock dips approach values comparable to or less than channel ~~steepness~~slope. This subhorizontal limit, given by Eq. (5), is most likely to apply for rocks that are very near horizontal and/or channels that are very steep. Therefore, these considerations are most applicable in cratonic settings, ~~and~~ in headwater channels, or when considering processes of scarp retreat in subhorizontal rocks (Howard, 1995; Ward et al., 2011). In the subhorizontal limit, slope contrasts are larger than would be predicted by topographic steady state (Fig. 3). In the case of $n < 1$, slope patterns in continuity steady state are also qualitatively different than those predicted by topographic steady state, with steeper channel segments in weaker rocks.

~~The framework presented here provides an intriguing hint concerning the generation of~~ For $n \approx 1$ slope contrasts become extreme, which is particularly important since $n = 1$ is the most common value used in landscape evolution models. In this case, large slope contrasts at contacts may accentuate numerical dispersion. It also must be realized that $n = 1$ is quite a special case in subhorizontal rocks, and the rest of the parameter range for n results in substantially different dynamics and steady state. Field studies have suggested that $n = 1$, where knickpoint retreat rate is independent of slope, can explain the distribution of knickpoints within drainage basins (Crosby and Whipple, 2006; Berlin and Anderson, 2007) . However, it is also clear from our analysis that with $n = 1$ in subhorizontal rocks channels near contacts obtain a steep state, where the stream power model will break down.

During constant uplift, channels cannot attain continuity steady state at base level, because it requires different vertical incision rates in each rock type. However, the perturbations introduced by stream segments in topographic equilibrium at base level rapidly decay over a length scale that is primarily a function of the ratio of rock erodibilities, with larger erodibility contrasts resulting in shorter decay lengths. Practically speaking, for rocks that have erodibilities sufficiently different to have a strong effect on the profile, base level perturbations of continuity steady state decay after a couple rock contacts are passed.

Though steepness ratios are a fixed function of rock erodibility in continuity steady state, absolute steepness values depend on rock layer thickness. Since natural systems will not generally have regular patterns of thickness or erodibility, this has implications for the ability of natural systems to approach continuity steady state. As new rock layers with different thicknesses or erodibilities are exposed at base level, the absolute steepness values that would represent continuity steady state change. Therefore, continuity steady state may often represent a moving target, where the landscape is constantly adjusting toward it but never reaching it. The introduction of rock layers with varying thickness and erodibility can produce transience in landscapes that are experiencing otherwise stable tectonic and climate forcing. This only applies, however, for absolute steepness values. Steepness ratios, and their relationship to erodibility, would be expected to be relatively constant in time if sufficiently far from base level.

We speculate that the observed dynamics in subhorizontal rocks provide a potential means to generate caprock waterfalls, a feature that has long fascinated geologists (Gilbert, 1895). Caprock waterfalls, such as Niagara Falls, have a resistant caprock layer that is underlain by a weaker rock. The waterfall has the caprock at its lip, followed by a vertical, or often overhanging, face within the weak rock. This is a case of a very steep channel within a highly erodible rock, which would not be predicted from topographic equilibrium and stream power erosion. Such a state is predicted by the continuity relation developed here for subhorizontal layers with $n < 1$, and somewhat similar features develop in the case of $n = 1$. Values of n might be expected to ~~fall in this range~~ be less than one for erosion processes active in the weak rock layer, such as plucking (Whipple et al., 2000). Furthermore, caprock waterfalls typically form in relatively horizontal strata, and are common within steep headwater channels, which are the settings where differences between topographic and continuity steady state become important. The stream power ~~law~~ model arguably does not apply to waterfalls (Lamb and Dietrich, 2009; Haviv et al., 2010; Lague, 2014), and a variety of erosion mechanisms that are independent of stream power can act in such an oversteepened reach, such as gravity failure, freeze-thaw, shrink-swell, and seepage weathering. However, starting from an initial condition of low relief, topographic equilibrium and stream power erosion would not predict a channel to evolve toward the caprock waterfall state. In contrast, the framework presented here naturally produces features resembling caprock waterfalls from considerations of landscape equilibrium. While further work would be needed to test this hypothesis, it remains plausible that caprock waterfalls are the result of channels steepened within weaker rocks to maintain continuity~~. Once~~ , even if, once the channel becomes sufficiently steep, stream power erosion ~~may no longer provide~~ no longer provides a good approximation to erosion rates~~, though the~~ . The concept of continuity could also be applied to other, more mechanistic, erosion models~~.~~ , as the relation provided by Eq. 2 is independent of erosion model. However, continuity relations are most likely to provide insight for simple erosion models where analytical solutions can be derived, as with stream power erosion. With more complex models, the results of numerical landscape evolution models could be compared against the continuity relation to test whether a similar continuity steady state is attained.

Though the focus of this work is on bedrock channel profiles in layered rocks, the concepts of continuity and flux steady state can be applied in general to any mathematical model for erosion. Much like topographic steady state, both continuity and flux steady state result from negative feedbacks within the uplift-erosion system that drive it toward ~~this~~ steady state as uplift and erosion become balanced. Such feedback mechanisms are likely to be present within most erosional models. Though topographic steady state has been a powerful theoretical tool to understand landscapes, the ~~more general concept of~~ generalized concept of erosional continuity may prove more useful in interpreting steep landscapes in subhorizontal rocks.

6 Conclusions

Topographic steady state has provided a powerful tool for understanding the response of landscapes to climate, tectonics, and lithology. However, within layered rocks, topographic steady state is only attained in the case of vertical contacts. In topographic steady state, vertical erosion rates are equal everywhere, and steepness adjusts with rock erodibility to produce equal erosion. Here we generalize this idea using the concept of erosional continuity, which is a state where retreat rates of the land surface on either side of a rock contact are equal in the direction parallel to the contact rather than in the vertical direction. Using a stream power erosion model with $n = 1$, prior work showed that erosion rates exhibit transient and complex relationships with rock erodibility (Forte et al., 2016) . Our work suggests that these complex and transient effects result because adjustments in steepness cannot produce a state of erosional continuity when $n = 1$. In cases where $n \neq 1$, erosional continuity can be attained, and the landscape sufficiently far from base level exhibits one-to-one relationships between steepness and erodibility that are predicted by continuity. We refer to this as continuity steady state, and show that it is a type of flux steady state. Results from 1D and 2D landscape evolution models confirm the predictions of the erosional continuity equations.

For continuity steady state, the relationships between rock erodibility and landscape steepness differ most from topographic steady state when the rock contacts are subhorizontal, that is, when contact dips are less than channel slope. In the subhorizontal case, contrasts in steepness are larger than predicted by topographic steady state. These contrasts are largest when $n \approx 1$, and in fact may create sufficiently steep channels in one of the rock layers to negate the applicability of the stream power erosion model. For $n \approx 1$, numerical dispersion may also influence the time evolution of the topography because of the large slope contrasts. When $n < 1$, steepness patterns are also qualitatively different than those predicted by topographic steady state, with steeper channel segments in weaker rocks. In continuity steady state, erosion rates bracket the uplift rate and display a regular relationship with erodibility. This may assist in quantifying the uncertainty and bias within detrital records that can result from different erosion rates in different rock types (Forte et al., 2016) . For subhorizontal rocks, continuity steady state is not attained at base level. However, the perturbations to continuity steady state that are introduced at base level decay rapidly when there is a contrast in erodibility of more than a factor of 2-3. We speculate that the framework developed here provides a possible mechanism for the development of caprock waterfalls, since it predicts steep channel reaches within weak rocks. Though we focus on stream power erosion, the concept of erosional continuity is quite general, and may provide insight when applied to other erosion models.

Appendix A: Derivation of the continuity relation

Here we detail how the constraint of channel continuity can be used to derive the relationship given in Eq. (3). Consider a planar contact between lithologies with different erodibilities. We label the downstream and upstream erodibility with K_1 and K_2. Downstream and upstream slopes are S_1 and S_2, the slope of the contact is S_c, and their respective slope angles are θ_1, θ_2, and ϕ, see Fig. A1.

In this section we use the subscript i to denote either 1 or 2, as the relationships are valid for the channel within both rock types. Erosion at a rate E_i in the vertical direction, as is calculated by the stream power ~~erosion law~~model, can be transformed to an erosion rate B_i that is perpendicular to the channel bed using the slope of the channel bed, θ_i, with $B_i = E_i \cos\theta_i$, see Fig. A1. The contact and the channel intersect at angle $\theta_i + \phi$, thus the rate of exposure of the contact plane is

$$R_i = \frac{B_i}{\sin(\theta_i + \phi)} = \frac{E_i \cos\theta_i}{\sin(\theta_i + \phi)}. \tag{A1}$$

For the case where $\theta_i + \phi > \pi/2$ the diagram changes, but these same relationships can be recovered using $\sin(\pi - \theta_i - \phi) = \sin(\theta_i + \phi)$. Continuity of the channel bed requires that the contact exposure rates R_1 and R_2 are equal, which gives

$$\frac{E_1 \cos\theta_1}{\sin(\theta_1 + \phi)} = \frac{E_2 \cos\theta_2}{\sin(\theta_2 + \phi)}. \tag{A2}$$

Using a trigonometric identity for angle sums leads to

$$\frac{E_1 \cos\theta_1}{\sin\theta_1 \cos\phi + \cos\theta_1 \sin\phi} = \frac{E_2 \cos\theta_2}{\sin\theta_2 \cos\phi + \cos\theta_2 \sin\phi}. \tag{A3}$$

Simplifying the fractions and multiplying both sides of the equation with $\cos\phi$ we get

$$\frac{E_1}{\tan\theta_1 + \tan\phi} = \frac{E_2}{\tan\theta_2 + \tan\phi}. \tag{A4}$$

Solving for the ratio of erosion rates in the two rock types and converting to slopes rather than angles, ~~the relation becomes~~ using a sign convention where both contact and bed slopes are positive in the downstream direction, the relation becomes

$$\frac{E_1}{E_2} = \frac{S_1 + S_c}{S_2 + S_c}\frac{S_1 - S_c}{S_2 - S_c}. \tag{A5}$$

If erosion rates are given by the stream power ~~law~~model, then it follows that

$$\frac{K_1 S_1^n}{K_2 S_2^n} = \frac{S_1 + S_c}{S_2 + S_c}\frac{S_1 - S_c}{S_2 - S_c}, \tag{A6}$$

which is identical to Eq. (3) with the general subscripts 1 and 2 replaced with s and w for strong and weak.

Appendix B: Derivation of the erosion relation

Using the stream power ~~erosion law~~model, erosion rates in two channel segments above and below a contact are

$$E_1 = K_1 A^m S_1^n \qquad \text{and} \qquad E_2 = K_2 A^m S_2^n, \tag{B1}$$

where A is the recharge area. Taking the ratio of both equations at an arbitrary basin area, we get

$$\frac{E_1}{E_2} = \frac{K_1}{K_2}\left(\frac{S_1}{S_2}\right)^n. \tag{B2}$$

We define H_1 and H_2 to be the thicknesses of the rock layers measured in the vertical direction. If flux steady state is assumed, then the average erosion rate equals the uplift rate U. Therefore, the time needed to uplift a distance equal to the sum of the two layers' thicknesses equals the sum of the times needed to erode through the two layers:

$$\frac{H_1 + H_2}{U} = \frac{H_1}{E_1} + \frac{H_2}{E_2}. \tag{B3}$$

Combining Eq. (B2) and Eq. (B3) gives an expression for the erosion rate in a given rock:

$$E_1 = U\frac{H_1/H_2 + K_1/K_2\,(S_1/S_2)^n}{1 + H_1/H_2}. \tag{B4}$$

While flux steady state seems like a reasonable assumption, simulations also confirm that the erosion rates predicted by Eq. (B4) are approached within a few contacts above base level. Similarly, simulations that alternate uplift rate over time to match the erosion rate of the rock type currently at base level, as given by Eq. (B4), obtain straight line slopes in χ-elevation space all the way to base level. This confirms that the disequilibrium seen in the profiles in Fig. 4B-D is produced by the difference between the constant uplift rate and the equilibrium incision rates experienced in each layer.

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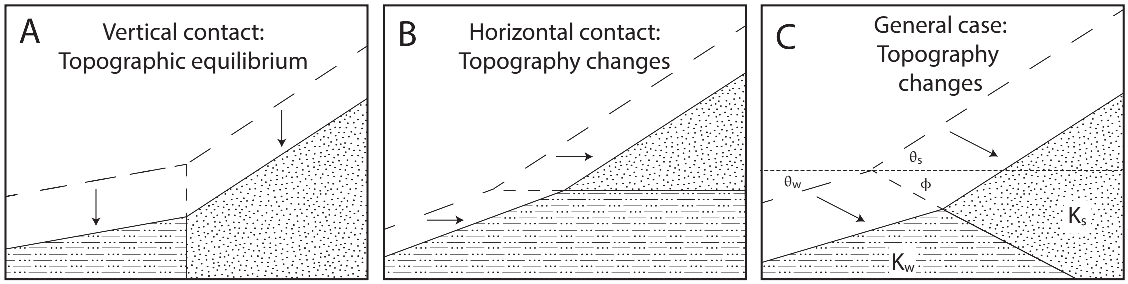

Figure 1. Topographic equilibrium in layered rocks. (A) Response of steepness to rock erodibility is typically derived from a perspective of topographic equilibrium, with equal vertical incision rates in all locations that are balanced by uplift. Topographic equilibrium does not occur in the case of non-vertical contacts. (B) For horizontal strata, horizontal retreat rates, rather than vertical incision, must be equal at the contact. (C) In general, retreat in the direction parallel to the contact must be equal within both rocks to maintain channel continuity. Dashed lines depict former land surface and contact positions, and arrows show the direction of equal erosion at the contact. Uplift is not depicted.

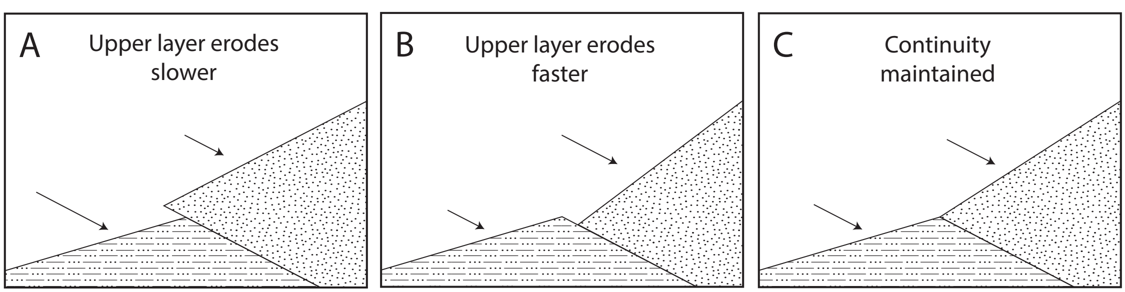

Figure 2. Erosional continuity. (A) If the upper layer at a contact erodes slower, this produces a discontinuity at the contact and the resulting steepening ~~in~~ or undercutting of the upper layer will drive the system toward erosional continuity. (B) If the upper layer erodes faster, this produces a low or reversed slope zone near the contact, which will ~~reduce erosion rates in~~ also drive the ~~upper layer~~ system toward continuity (C) ~~In~~ We hypothesize that, in general, ~~the system~~ topography will tend to approach a state where continuity is maintained.

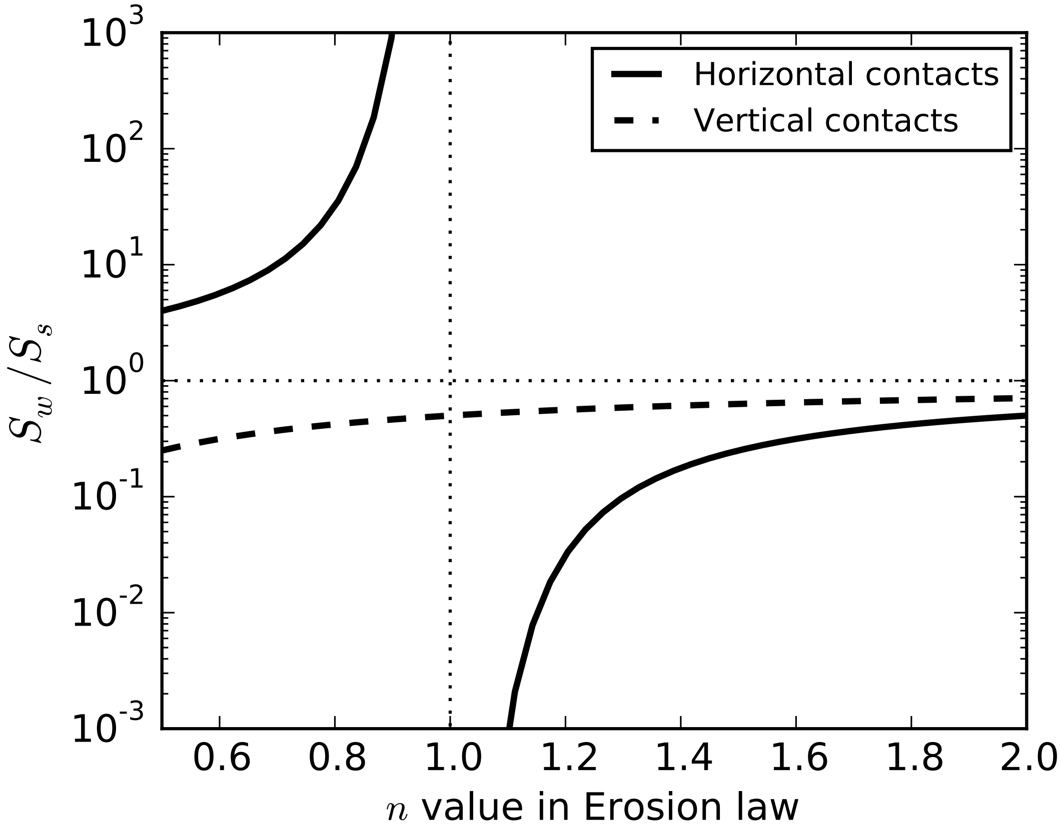

Figure 3. Channel slope response at a subhorizontal contact from an assumption of continuity. The ratio of slope within the weaker rock (S_w) and the slope within the stronger rock (S_s) near a horizontal contact (solid line) with differing values of the ~~power~~ exponent n in the stream power ~~erosion law~~model. Erodibility in the weaker rocks (K_w) is twice that of the stronger rocks (K_s). This subhorizontal case applies when the dip of the contact is small compared to channel slope. The dashed line displays the ~~traditional~~ standard topographic equilibrium relationship, which applies for cases where the contact slope is much larger than the channel slope.

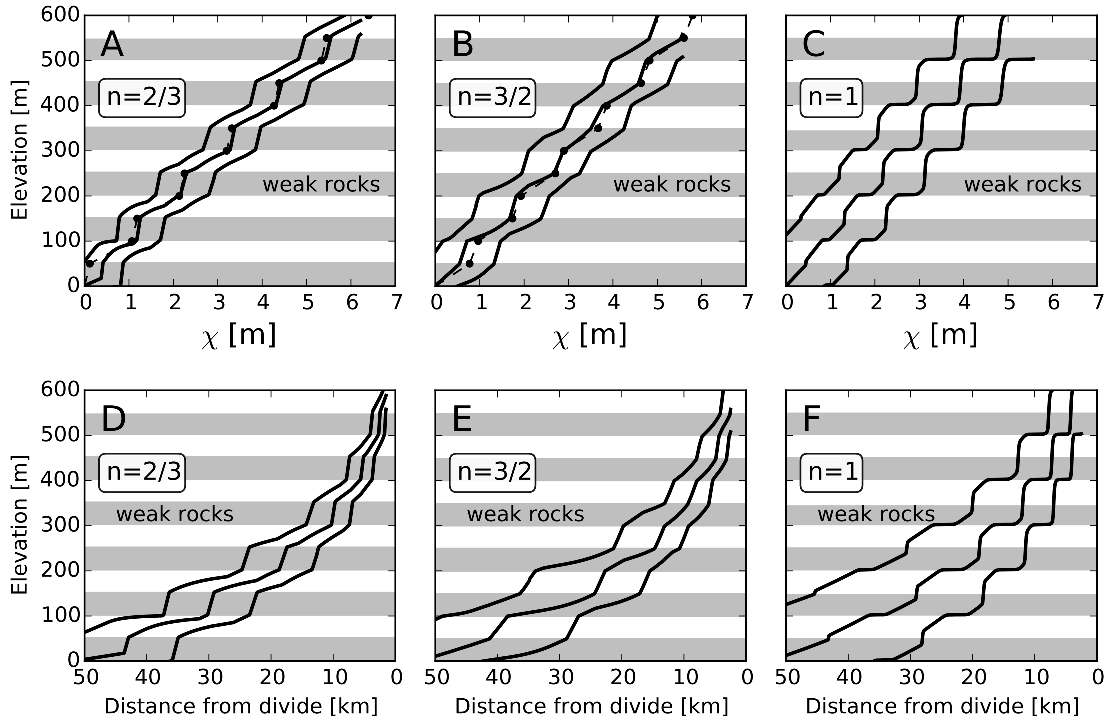

Figure 4. Channel profiles in subhorizontally layered rocks with high uplift ($2.5\ \mathrm{mm\,yr^{-1}}$). (A-C) Channel profiles in χ-elevation space for cases where $n = 2/3$ (A), $n = 3/2$ (B), and $n = 1$ (C). (D-F) Channel profiles as a function of distance from divide. Each panel contains three time snapshots of the profile with uplift subtracted from elevation so that the profiles evolve from left to right. Grey bands represent the weak rock layers. The dashed lines (A,B) show the ~~continuity steady state profile~~ profiles predicted by the continuity steady state theory (Eqs. 5 and 8), with filled circles depicting predicted crossing points of the contacts. Channel profiles obtain ~~an equilibrium~~ a steady state shape except near base level, where a constant rate of base level fall is imposed. For $n \neq 1$ the equilibrium profile steepness (slope in χ space) has a one-to-one relationship with rock erodibility, with steeper channels in weaker rock if $n < 1$. For $n = 1$ there is no unique relationship between erodibility and ~~χ slope~~steepness, as continuity cannot be maintained along the entire profile.

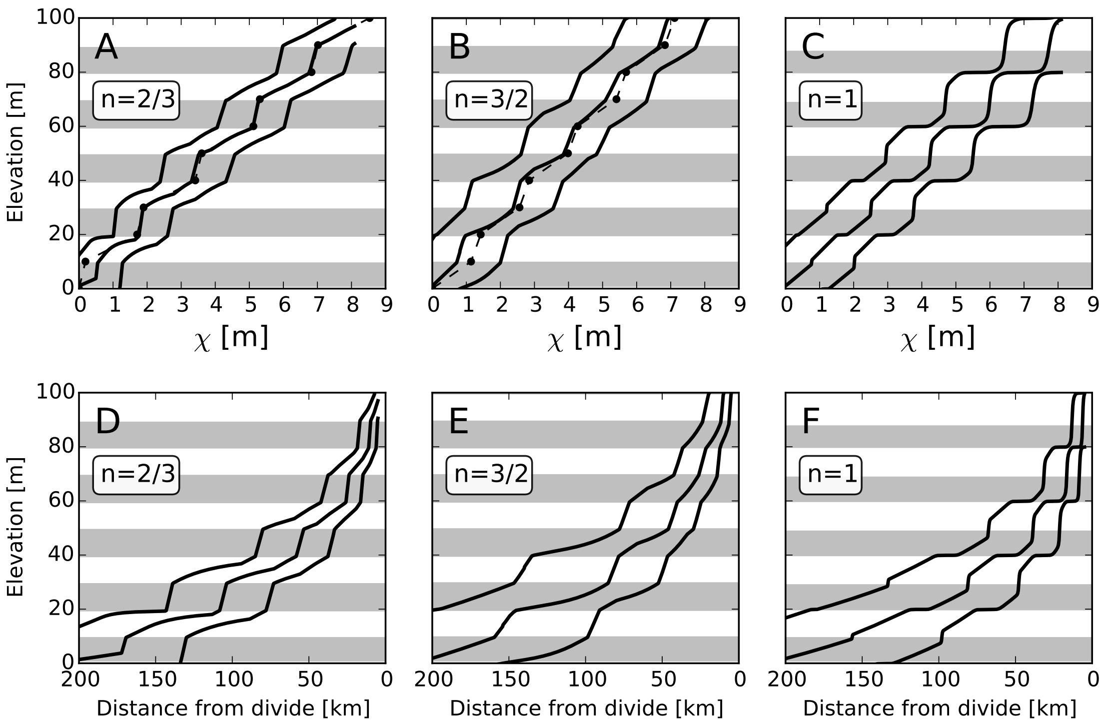

Figure 5. Channel profiles in subhorizontally layered rocks with low uplift ($0.25\ \mathrm{mm\,yr^{-1}}$). (A-C) Channel profiles in χ-elevation space for cases where $n = 2/3$ (A), $n = 3/2$ (B), and $n = 1$ (C). (D-F) Channel profiles as a function of distance from divide. Grey bands indicate weaker rocks. The low uplift simulations utilize longer distances and thinner rock layers in order to obtain a similar number of rock layer cycles. ~~Profile~~ These profile shapes are qualitatively similar ~~for~~ to the ~~low and~~ high uplift cases (Fig. 4).

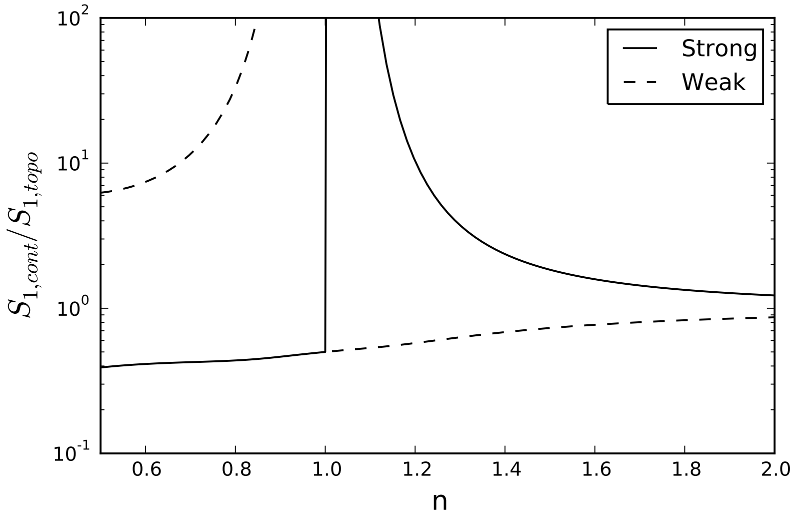

Figure 6. An example case of the ratio of slopes predicted by continuity and topographic steady states. This example assumes a choice of equal rock thicknesses in both rock types and a weak rock erodibility that is twice that of the strong rock. Contrasts are in general strongest for $n < 1$ and gradually disappear for large n.

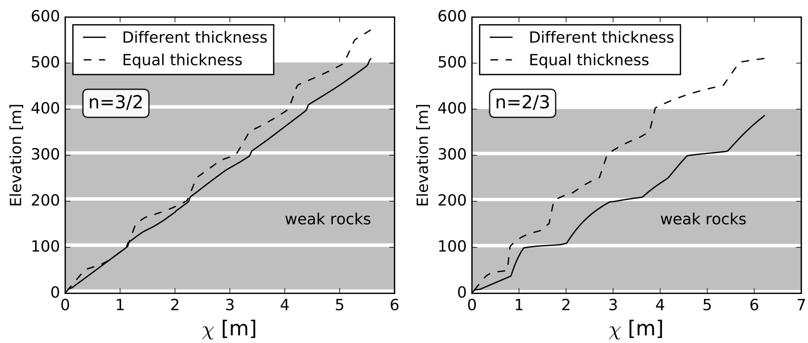

Figure 7. The influence of relative layer thickness on slopes in continuity steady state. If the relative thickness of the strong and weak layers is changed, the far from base level slopes in both rocks adjust correspondingly (solid lines), as predicted by continuity steady state. Grey bands depict the locations of weak rocks in the differing thickness model. The dashed lines depict channel profiles for simulations with equal layer thickness but the same erosional parameters. Increasing the weak layer percentage reduces topography overall.

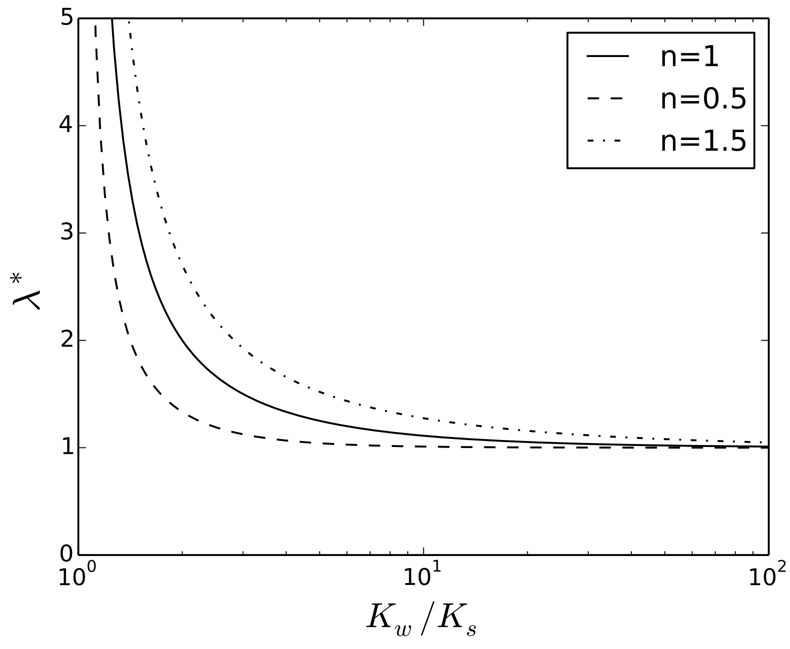

Figure 8. The dimensionless damping length scale, λ^*, as a function of erodibility ratio. Damping of base level perturbations is strong when the erodibility ratio is greater than three. λ^* can be interpreted as roughly the number of pairs of strong and weak rock layers that base level perturbations must pass through before substantial damping toward continuity steady state.

Simulation	K_s $[\mathrm{m}^{1-2m}\,\mathrm{a}^{-1}]$	K_w $[\mathrm{m}^{1-2m}\,\mathrm{a}^{-1}]$	m	U $[\mathrm{m\,a}^{-1}]$
High uplift cases				
$n = 2/3$	$1 \cdot 10^{-4}$	$2 \cdot 10^{-4}$	$1/3$	$2.5 \cdot 10^{-3}$
$n = 1$	$2 \cdot 10^{-5}$	$2.4 \cdot 10^{-4}$	$1/2$	$2.5 \cdot 10^{-3}$
$n = 3/2$	$1.5 \cdot 10^{-6}$	$3 \cdot 10^{-6}$	$3/4$	$2.5 \cdot 10^{-3}$
Low uplift cases				
$n = 2/3$	$4 \cdot 10^{-5}$	$8 \cdot 10^{-5}$	$1/3$	$2.5 \cdot 10^{-4}$
$n = 1$	$2 \cdot 10^{-5}$	$2.4 \cdot 10^{-4}$	$1/2$	$2.5 \cdot 10^{-4}$
$n = 3/2$	$3 \cdot 10^{-6}$	$6 \cdot 10^{-6}$	$3/4$	$2.5 \cdot 10^{-4}$

Table 1. Parameters used in the 1D model runs.

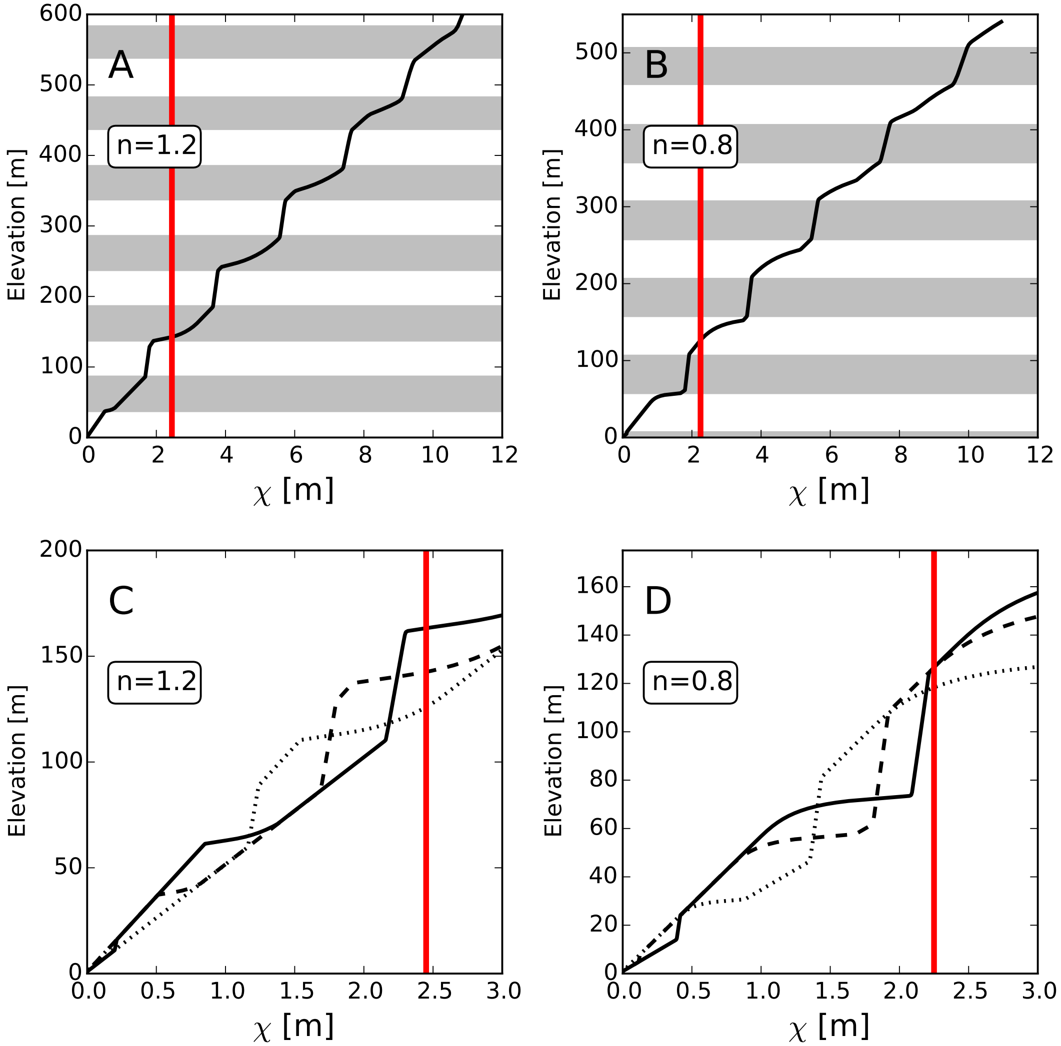

Figure 9. Simulations of knickpoint propagation and damping from base level. Entire equilibrium profiles are depicted for cases where $n = 1.2$ (A) and $n = 0.8$ (B). Panels C and D show zoomed figures that depict three separate timesteps (dotted, dashed, and then solid) as fast knickpoints catch up with slow knickpoints at the ~~calculating~~ calculated damping length scale (λ, thick red line). The interaction of the two knickpoints can be visualized as the reduction in size of a slope patches that are at the topographic equilibrium slope as these patches approach λ.

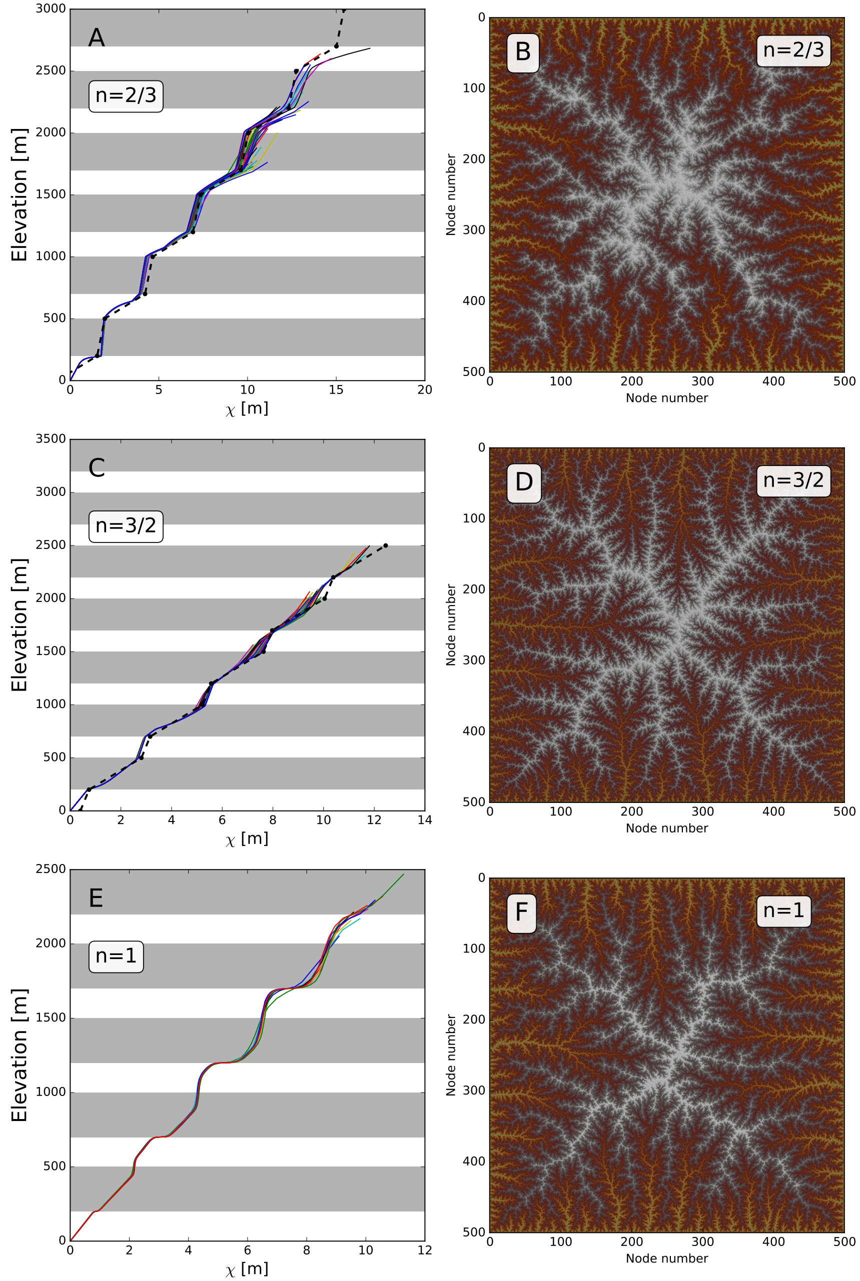

Figure 10. Results of the *FastScape* simulations. Lines in the left-hand panels are profiles extracted from the DEMs. Simulations were run at constant uplift with alternating bands of weak and strong rocks. Grey bands indicate the weaker rocks. The individual panels show simulations where $n = 2/3$ (A), $n = 3/2$ (C), and $n = 1$ (E). The dashed lines (A,C) show the equilibrium profile predicted by the theory, with circles depicting predicted crossing points of the contacts. Profiles obtain similar shapes as in the 1D simulations (Figure 2B-D). Panels B, D, and F show DEMs of the landscapes formed in each simulation. Color represents elevation with white being high.

Simulation	K_{fw} $[\mathrm{m}^{1-3m}\,\mathrm{a}^{-1+m}]$	K_{fs} $[\mathrm{m}^{1-3m}\,\mathrm{a}^{-1+m}]$	m	$P\,[\mathrm{m\,a}^{-1}]$	$U\,[\mathrm{m\,a}^{-1}]$
$n = 2/3$	$1.2\cdot10^{-4}$	$0.5\cdot K_{fw}$	$1/3$	1	$2.5\cdot10^{-3}$
$n = 1$	$\cancel{2.5\cdot10^{-5}}\,1.5\cdot10^{-5}$	$0.83333\cdot K_{fw}$	$1/2$	1	$2.5\cdot10^{-3}$
$n = 3/2$	$1\cdot10^{-6}$	$0.5\cdot K_{fw}$	$3/4$	1	$2.5\cdot10^{-3}$

Table 2. Parameters used in the *FastScape* model runs.

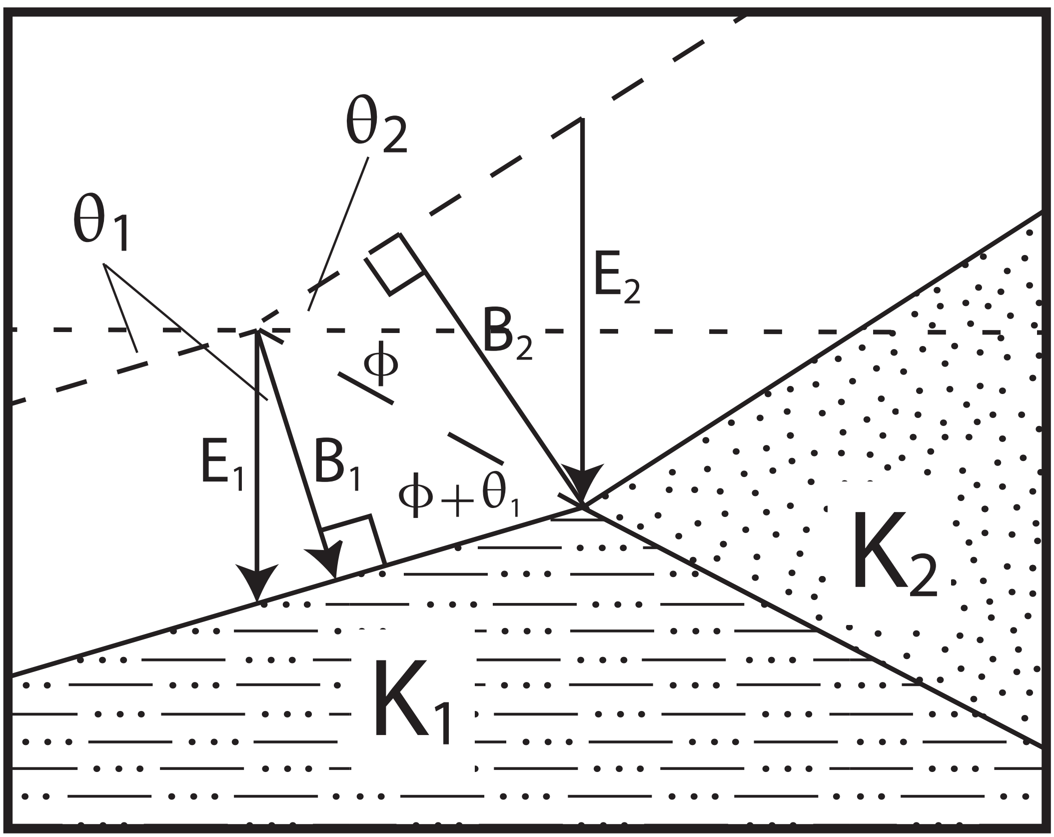

Figure A1. Geometric relationships used to derive the equation for continuity of the channel at a contact between two rock types. Note that the slope of the contact plane ($\cancel{\phi}$$S_c = -\tan\phi$) is defined as positive when the contact dips in ~~a~~ the downstream direction~~opposite that of channel bed slope~~.