We thank the reviewers for their valuable comments. Below we restate the comments in normal font and give our reply in *italics*. We hope that we have clarified everything and satisfactorily answered all the queries.

Reviewer #1

This paper emphasizes the need to think about how cover on a bedrock channel evolves, and I am supportive of that pursuit. I generally like the probabilistic approach of the paper. My review is not as deep as I would like it to be, because I got a bit lost in some of the details of the paper. I also had problems seeing how some of the sections tied together. Because the paper is so heavy in equations, and more importantly symbols, I think more reminders about what different symbols mean could make this a bit easier for the reader to follow. The table of symbols certainly helped. But anything the authors can do to improve the flow would be appreciated.

We agree that we have many equations, but we do not think their number can be reduced without losing necessary mathematical detail (which would make the paper even more difficult to read to those who are only mildly enthusiastic about math…). In revisions, we have tried to clarify were possible.

Line by line comments:

17 alleviated = alluviated?

Corrected.

Equation 3 : When I first read this, I thought "isn't this probability actually a function of many things? Is there a reason that they only show it as A* and M_s*?" It's clear in the text that many variables are important, but I wondered why they were left out of the equation. Eventually I understood that the reason is because this paper focuses on A* and M_s*. Maybe this can be made clear from the start.

We are not sure how to take this comment. We meant to indicate the possible dependence on other variables with the three little dots within the equation. The next sentence in the text makes explicit what we meant by this and gives a list of possible control parameters. We are not sure how to improve clarity here. Maybe the reviewer or the editor have a specific idea? [Text edited to try and clarify further.]

Section 2.1 in general – I know that the authors are not going to change this, but I had a very hard time remembering that A* is the fraction of exposed area, as in my head a cover function goes with fraction of area covered, not exposed. It's not that the authors aren't clear about the meaning, but somehow repeating the definition of A* more would have helped me. For example, I suggest that on all figure axes words accompany symbols, so the meaning of the variable is not mistaken (as I did many times.) I also had a hard time getting used to the meaning of P. In retrospect, after reading line 124 it is clear. However I wonder if this could be emphasized somehow. E.g. State what P is on fig 1A y-axis, or at least restate in the caption. I know repetition is frowned upon in scientific writing, but I need it in this paper.

We try to improve readability in the revisions by repeating the definitions of variables. In particular, we will revise the figures to give both a verbal description of the parameters and their symbol.

Nevertheless, we want to stress here that the exposed fraction, as we have used it, and not the covered fraction is the parameter that is commonly used in equations, because erosion rate is proportional to the exposed fraction.

L 159: I've never seen the word run used like this. Exist instead maybe?
Changed as suggested.

Section 2.2: Again I'm not quite sure that you can do anything about this, but I got confused here because now you are talking about the probability of entrainment, in contrast to above which was the probability of deposition. Maybe just make sure this is clear to readers.
We have added qualifiers to make the relations more clear. To further avoid confusion, we have also changed to small font and added the symbols pi and pc to the notation list. We also added a sentence to the introductory paragraph, explaining the relation between pc and pi and the P-function.

In this model all grains move the same length, right? So where they are deposited is not at all affected by whether or not there are grains in that location, right? This is confusing to me given that in the previous section deposition is probabilistic based on whether there are other grains present. So I had a hard time comparing this model with your framework.
In the model sediment cover is only calculated for grains that are stationary for a time step; grains that are deposited and then immediately re-entrained do not count. Consequently the model implicitly incorporates the effect of local sediment cover on grain deposition. Note also that our P-function quantifies deposition on a reach-scale, while the formulation in the CA model is concerned with the grain scale. When grain scale dynamics are varied, this has a direct effect on the reach-scale, which is expressed in a different P-function. We have added some clarifying sentences.

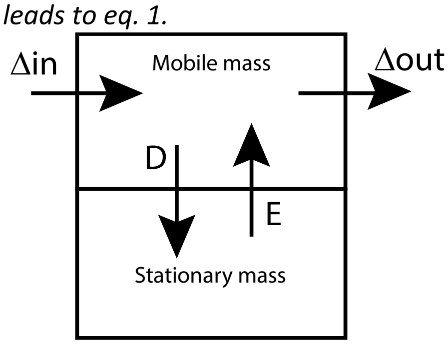
Figure 3: Is this plotting the probability of deposition or entrainment? I think I know the answer, but maybe make this clear.
This is, of course, the probability P defined in eq. (3). We have tried to clarify.

Equation 13 was hard for me. It seems like a new way to write this, but maybe you can walk me through it. Wouldn't E and D be included in d qs / dx? That is, wouldn't material be deposited if qs decreased downstream and entrained if qs increased downstream. I couldn't much evaluate this model because I didn't understand equation 13.
We have explained the meaning of this equation in our brief reply in the discussion of the paper, which is reproduced below. We are at a little loss as how to deal with this comment. On the one hand, we can see that the reviewer was confused by this particular equation and as a result had a hard time understanding the remainder of the paper. On the other, the equation is a standard mass balance (derived from mass conservation), which is routinely used in river dynamics (in a slightly different form as Exner equation), geochemistry and many other disciplines. We think we have made these connections sufficiently clear in the introductory sentences preceding the presentation of the equation. Since a presentation of the derivation, including a cartoon, would take about half a page, we decided to not change the manuscript here. Nevertheless, we are willing to do so if the editor sees the necessity. Some of the confusion may arise because here we have two different reservoirs (Mm and Ms) and so we need to explicitly represent the transitions between them, whereas in a classic Exner the transition of sediment between bedload and bed is assumed and not explicitly represented. That is, if there is a downstream decrease in bedload, in the Exner equation the excess sediment is assumed to be deposited, while we make the exchange between mobile and stationary particle reservoirs explicit. Apart from the obvious advantage when one is interested in the size of the stationary reservoir, which is related to bed cover, the role of particle speed becomes also explicit. We have added a sentence to clarify this relation..

From our previous discussion: Here, we want to briefly comment on eq. 13 that the reviewer found hard to understand. This is a mass balance that is probably more easily understood when viewed in a discrete framework. Consider a mass balance for a control volume in the river (Fig. 1). The rate of change of mobile mass per time, ΔMm/Δt is then the sum of four terms: the mass influx per time from upstream Δin, the mass outflux per time downstream Δout, the entrained mass per time E and the deposited mass per time D. Both outflux Δout and deposition D reduce mass in the control volume and are therefore negative. Thus:

ΔMm/Δt = Δin – Δout +E – D
(eq. 1)
Our equation 13 is essentially a version of this equation where the long-stream variable and the time are treated as continuous corresponding to the limit of infinitesimal length of the control volume and infinitesimal time steps. In this limit the term Δin – Δout becomes dq_s/dx and the term ΔMm/Δt becomes dM/dt.
Note that this is essentially equivalent to the standard Exner equation, which is written in terms of bed height rather than mass, and thus wraps both our reservoirs (mobile and stationary) into a single equation. Since we are explicitly interested in the mass on the bed, and since bed height necessarily varies across a partially covered bed, we opted for using mass as a main variable.

Fig. 1: Cartoon illustration of a control volume in the river. The mass balance for the mobile mass Mm leads to eq. 1.



L 347: I think that this is E*(M*s = 0) = 0, right? If so, maybe state explicitly.
Changed as requested.

L 417 – 419: I was confused about the comparison because you are not plotting the same thing as earlier plots. That is Q* is not M_s*, as was on the x-axes earlier. Can you help the reader relate these two variables?
This is correct. A main point of this chapter is to establish the relation between mass resevoirs Mm and Ms and the relative sediment supply Qs*. This partly has historical reasons – the cover function has so far been expressed in terms of Qs* (see, for example, Sklar and Dietrich, 2004), while experiments give the relation to Ms (see, for example, Sklar and Dietrich, 2001). The transformation between these two variables has been often overseen or done wrong (including in one of the first author's earlier papers, Turowski et al., 2007). So, one of the aims of this chapter is the clarification of this particular relationship and the provision of a sound physical basis to model it. We have added a few sentences at the beginning of the chapter to clarify this aim.

Q* is also not defined in your notation.

Is Q* on the x-axes in figure 4 supposed to be Q*_s with a bar over it? Maybe same for Figure 5 and also in the caption of figure 4?
*Sorry, this was carried over from an early version of the manuscript. Of course we meant Q*_s. We have corrected throughout.*

L 465 Where did 5.7/11.5 come from?
These are dimensionless constants determined from experiment by Fernandez-Luque and van Beek (1976). See equations 35/36. We provided a reference to these equations and the text has been edited for clarification.

Figure 10A – the x-axis is not formatted correctly.
We do not see exactly the problem the reviewer tries to point out. However, we have revised all figures to improve clarity and readability.

Sentence starting on L 611: Does this mean that A* = 0 at all times?
In the natural channel bed of the Erlenbach, yes. But this is irrelevant here, as we were trying to say with the criticized sentence. We merely use the measured time series of bedload transport and discharge to supply realistic input data. We have revised the introduction to the chapter to make this clear.

I guess this gets to a point that I didn't understand in the previous section - when you are talking about the evolution of the mass on the bed through time, this could be under conditions of complete cover the entire time. It might be worth stating this directly. Although I'm not sure that I'm correct about this assumption. In Figure 12, assuming that A on the second from top plot is actually A*, implies that there is not complete cover. I don't think I understood this section very well.
Yes, of course, the cover can be complete all the time. In the simulations we present here for the Erlenbach, the final cover state depends strongly on the initial cover state. This indicates that during the event, there was not enough time to reach a steady cover state. If we had started the simulation with full cover, it would have stayed full for the entire event.
That said, the temporal evolution is also strongly dependent on how one calculates transport capacity, which is a thorny issue for steep streams such as the Erlenbach. We have normalized our transport capacity such that the ratio of supply to capacity is equal to one at the highest discharge. Thus, during most of the event, relative sediment supply is smaller than one (in effect, by choice). The point here was to show the cover evolution for realistic boundary conditions as an illustrative example, rather than trying to predict bed cover throughout an event for the Erlenbach. We have rephrased to improve clarity.

Also in this section, in a single flood event I'm having a hard time understanding what the meaning of p, or the period is. The previous section discussed the period in terms of a sine wave in sediment supply. But is that the case for this flood?
This is meant to be the same as the period discussed earlier. In essence, we assumed that at each time step, a new sinusoidal perturbation with a fixed amplitude commences. From the data we can estimate the local gradient in the variables and use this to calculate an 'effective' period. We agree that we had not done a good job in explaining this and have tried to improve.

Sentence starting on L 669: This is great. Is this shown explicitly and I didn't catch it? If not, can you spell this out more directly?
As is explained in the paragraphs preceding this statement, it is currently not that easy to provide direct comparisons with data or calibrate the P-function directly, mainly because the necessary parameters have not been reported in the experimental studies. However, we have directly demonstrated that the formulation is flexible enough to yield a wide range of cover

functions (see Figs. 4 and 5 for examples). These do encompass (most of) the relationships observed in laboratory and numerical experiments. We do not think that we can meaningfully go any further at the moment, but have provided a reference to the figures.

L 678: This dynamics cover ... typo I think.
Typo; corrected.

Sentence starting on Line 735 has a typo.
Indeed; corrected.

Section 4.3: The comparison with Philips and Jerolmack is a bit confusing to me. You state that in contrast to their findings, your findings suggest that bed cover is adjusted. But you didn't actually have channel morphology as a free variable. So I'm not sure how this study can contrast that one. I don't remember exactly, but I don't think they talked about sediment cover. So is this a fair way to compare the two studies? I like how your conclusions stated this issue - both cover and channel morphology evolve. This makes sense to me, but the discussion in section 4.3 did not.
Here, a main point is that bedrock rivers can adjust cover to achieve grade. And this can be done much more rapidly than adjusting channel morphology. Philips and Jerolmack missed this mechanism – they argue that in bedrock channels, morphological adjustment needs to be quick, because they observe graded channels. But, as stated, the river has other options to achieve grade – by adjusting bed cover. We have rewritten the entire paragraph to clarify.

Reviewer #2

In this manuscript, the authors presented a probabilistic framework for predicting partial cover in mixed bedrock-alluvial channels, which they used to explore how probability of sediment deposition, relative sediment supply, and particle speed interact. It represents a next step in the progress that has been made over the past several years in modeling areal fraction of sediment cover. Overall, I found this to be a good paper, with sound methods and interesting results.
We thank the reviewer for this assessment and the comments. We hope that in our revisions we can satisfactorily answer all queries.

1. My main concern with the manuscript is the equations for entrainment rate and deposition rate. Although the authors explain that eq. 20 approaches Emax* as Ms* goes to infinity, does the same apply to eq. 21? I think Mm* cannot be infinity because it is limited by the capacity value M0*. If so, when Ms* is very large, it is impossible to balance D with E.
Mm is related to particle speed and upstream sediment supply via equation (24). Since both of these other variables are treated as input parameters, Mm is fixed by them (at steady state). Mm can only become infinite in the non-physical cases of zero particle speed but finite transport rate, or infinite supply. The deposition rate is then set by eq. (21). If Mm is large (i.e., enough sediment is available), deposition is limited by upstream sediment supply. In essence, eq. (21) states that when Mm is small, the amount that can be deposited is limited by Mm (if there is only one mobile particle available, then a maximum of one can be deposited), and if Mm is large, deposition is limited by sediment supply. This explanation has been added to the text.

2. The authors assume that increasing sediment deposition decreases local shear stress and increases the critical entrainment shear stress for grains (Line 186-192 and 242-245). I think this assumption is limited to the case of smooth bedrock. Sediment deposition does not necessarily decrease the flow velocity. In rough bedrock, increasing sediment deposition increases local shear stress and decreases the critical shear stress for grains.
This is a good point that we have taken up in the discussion. Indeed, we were thinking of smooth rather than rough bedrock. We have added 'smooth' at the appropriate places to qualify. In addition, feedbacks between cover and roughness have been discussed in some depth in the introduction.

3. In section 4.2, although the authors explain the differences from the model presented by Nelson and Seminara (2011, 2012); the model presented in this paper has more similarities to the model presented by Inoue et al. (2014). In the mentioned paper, they have distinguished between mobile and stationary sediment, and have not assumed a direct correspondence between sediment concentration and degree of cover, which is different from Nelson and Seminara (2011, 2012). I think the main difference is in the sediment continuity equation including entrainment rate and deposition rate. This equation seems very useful. I encourage the authors to explain the differences from Inoue et al. (2014) and the advantages of this sediment continuity equation.
We have included an additional paragraph discussing this model.

Additional comments by line number below:
Line 140: There are situations when sediment does not accumulate even if the exposed part is zero. For example, runaway alluviation in Chatanantavet and Paker (2008).
We are aware of this mechanism and mention it several times in the manuscript. In principle, the mechanism can be modelled in the framework, but entrainment and deposition rate for this case need to be dependent on the state of cover. As a purely descriptive tool, the P-function will still work (there is an example of run-away alluviation in the model data). There could potentially also be hysteresis in cover, which could be modelled by separate P-functions for entrainment and deposition. This is, however, beyond the scope of the paper, in particular as there are currently no data available to constrain the P-function for laboratory experiments or natural channels.
We do not see why the reviewer made this statement in relation to line 140 of the manuscript. All statements there are general and correct.

Line 242-245: It is good to describe a physical reason for smoothing.
Smoothing is applied to prevent the formation of unrealistic piles of grains in one cell when there are far fewer grains in adjacent cells. The text was edited to explain.

Line 345: Can (1-eˆ-Ms)qt* be converted to (1-A)qt*? If so, the entrainment rate is proportional to the areal fraction of sediment cover.
Only for the assumption P=A. We work with this assumption a lot in the later interpretation of the equations and in examples, but at this point, P is left general. But this is a nice way of looking at it.*

Figure 4 and Figure 5: Q* = Qs*?
Apologies, this was a mistake.

Figure 7: What is arbitrary unit? Is the sediment supply rate specified?
One uses arbitrary units when the absolute scale of the relation is irrelevant for the argument.

Figure 8: Please explain the distance from upstream end to downstream end, transport

capacity, bed slope and grain size.
All the variables mentioned by the reviewer are not relevant – what is relevant is not the absolute transport capacity, but the relative sediment supply (supply normalized by transport capacity). We have worked here in the non-dimensional framework specified in 3.1. We have revised the caption for Figure 8, including references to the relevant equation and the one variable that we had missed, particle speed. All relevant information is now contained in the caption.

Figure10: Which equations are used to calculate 99% response time?
We used numerical solutions to obtain these data. The corresponding equations in the paper are (3), (22), (23) and (24). The text has been added to clarify.

A probabilistic framework for the cover effect in bedrock erosion

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Abstract
The cover effect in fluvial bedrock erosion is a major control on bedrock channel morphology and long-term channel dynamics. Here, we suggest a probabilistic framework for the description of the cover effect that can be applied to field, laboratory and modelling data and thus allows the comparison of results from different sources. The framework describes the formation of sediment cover as a function of the probability of sediment being deposited on already alluviated areas of the bed. We define benchmark cases and suggest physical interpretations of deviations from these benchmarks. Furthermore, we develop a reach-scale model for sediment transfer in a bedrock channel and use it to clarify the relations between the sediment mass residing on the bed, the exposed bedrock fraction and the transport stage. We derive system time scales and investigate cover response to cyclic perturbations. The model predicts that bedrock channels achieve grade in steady state by adjusting bed cover. Thus, bedrock channels have at least two characteristic time scales of response. Over short time scales, the degree of bed cover is adjusted such that they can just transport the supplied sediment load, while over long time scales, channel morphology evolves such that the bedrock incision rate matches the tectonic uplift or base level lowering rate.

1. Introduction

Bedrock channels are shaped by erosion caused by countless impacts of the sediment particles they carry along their bed (Beer and Turowski, 2015; Cook et al., 2013; Sklar and Dietrich, 2004). There are feedbacks between the evolving channel morphology, the bedload transport, and the hydraulics (e.g., Finnegan et al., 2007; Johnson and Whipple, 2007; Wohl and Ikeda, 1997). Impacting bedload particles driven forward by the fluid forces erode and therefore shape the bedrock bed. In turn, the morphology of the channel determines the pathways of both sediment and water, and sets the stage for the entrainment and deposition of the sediment (Hodge and Hoey, 2016). Sediment particles play a key role in this erosion process; they provide the tools for erosion and also determine where bedrock is exposed such that it can be worn away by impacting particles (Gilbert, 1877; Sklar and Dietrich, 2004).

The importance of the cover effect - that a stationary layer of gravel can shield the bedrock from bedload impacts – has by now been firmly established in a number of field and laboratory studies (e.g., Chatanantavet and Parker, 2008; Finnegan et al., 2007; Hobley et al., 2011; Johnson and Whipple, 2007; Turowski and Rickenmann, 2009; Turowski et al., 2008; Yanites et al., 2011). Sediment cover is generally modelled with generic relationships that predict the decrease of the fraction of exposed bedrock area A^* with the increase of the relative sediment supply Q_s^*, usually defined as the ratio of sediment supply to transport capacity. Based on laboratory experiments and simple modeling, Turowski and Bloem (2016) argued that the focus on covered area is generally justified on the reach scale and that erosion of bedrock under a thin sediment cover can be neglected. However, the behavior of sediment cover under flood conditions is currently unknown and the assumption that the cover distribution at low flow is representative for that at high flow may not be justified (cf. Beer et al., 2016; Turowski et al., 2008).

The most commonly used function to describe the cover effect is the linear decline (Sklar and Dietrich, 1998), which is the simplest function connecting the steady state end members of an empty bed when relative sediment supply Q_s^* = 0 and full cover when Q_s^* = 1:

$$A^* = \begin{cases} 1 - Q_s^* & \text{for} \quad Q_s^* < 1 \\ 0 & \text{otherwise} \end{cases}$$

(eq. 1)

In contrast, the exponential cover function arises under the assumption that particle deposition is equally likely for each part of the bed, whether it is covered or not (Turowski et al., 2007).

$$A^* = \begin{cases} \exp(-Q_s^*) & \text{for} \quad Q_s^* < 1 \\ 0 & \text{otherwise} \end{cases}$$

(eq. 2)

Here, $\exp$ denotes the natural exponential function.

Hodge and Hoey (2012) obtained both the linear and the exponential functions using a cellular automaton (CA) model that modulated grain entrainment probabilities by the number of neighbouring grains. However, consistent with laboratory flume data, the same model also produced other behaviours under different parameterisations. One alternative behavior is runaway alluviation, which was attributed by Chatanantavet and Parker (2008) to the differing roughness of bedrock and alluvial patches. Due to a decrease in flow velocity, an increase in surface roughness and differing grain geometry, the likelihood of deposition is higher over bed sections covered by alluvium compared to smooth, bare bedrock sections (Hodge et al., 2011). This can lead to rapid alluviation of the entire bed once a minimum fraction has been covered. The relationship between sediment flux and cover is also affected by the bedrock morphology; flume experiments have demonstrated that on a non-planar bed the location of sediment cover is driven by bed topography and hydraulics (e.g., Finnegan et al., 2007; Inoue et al., 2014). Johnson and Whipple (2007) found that stable patches of alluvium tended to form in topographic lows such as pot holes and at the bottom of slot canyons, whereas Hodge and Hoey (2016) found that local flow velocity also controls sediment cover location.

The relationship between roughness, bed cover and incision was explored in a number of recent numerical modeling studies. Nelson and Seminara (2011, 2012) were one of the first to model the impact that the differing roughness of bedrock and alluvial areas has on sediment patch stability. Zhang et al. (2014) formulated a macro-roughness cover model, in which sediment cover is related to the ratio of sediment thickness to bedrock macro-roughness. Aubert et al. (2016) directly simulated the dynamics of particles in a turbulent flow and obtained both linear and exponential cover functions. Johnson (2014) linked erosion and cover to bed roughness in a reach-scale model. Using a model formulation similar to that of Nelson and Seminara (2011), Inoue et al. (2016) reproduced bar formation and sediment dynamics in bedrock channels. All of these studies used slightly different approaches and mathematical formulations to describe alluvial cover, making a direct comparison difficult.

Over time scales including multiple floods, the variability in sediment supply is also important (e.g., Turowski et al., 2013). Lague (2010) used a model formulation in which cover was written as a function of the average sediment depth to upscale daily incision processes to long time scales. He found that over the long term, cover dynamics are largely independent of the precise formulation at the process scale and are rather controlled by the magnitude-frequency distribution of discharge and sediment supply. Using the CA model of Hodge and Hoey (2012), Hodge (in press) found that, when sediment supply was very variable, sediment cover was primarily determined by the recent history of sediment supply, rather than the relationships identified under constant sediment fluxes.

So far, it has been somewhat difficult to compare and discuss the different cover functions obtained from theoretical considerations, numerical models, and experiments, since a unifying framework and clear benchmark cases have been missing. Here, we propose such a framework, and develop type cases linked to physical considerations of the flow hydraulics and sediment erosion and deposition. We show how this framework can be applied to data from a published model (Hodge and Hoey, 2012). Furthermore, we develop a reach-scale erosion-deposition model that allows the dynamic modeling of cover and prediction of steady states. Thus, we clarify the relationship between cover, deposited mass and relative sediment supply. As part of this model framework we investigate the response time of a channel to a change in sediment input, which we illustrate using data from a natural channel.

2. A probabilistic framework

2.1. Development

Here we build on the arguments put forward by Turowski et al. (2007) and Turowski (2009). Consider a bedrock bed on which sediment particles are distributed. We can view the deposition of each particle as a random process, and each area element on the bed surface can be assigned a probability for the deposition of a particle. When assuming that a given number of particles are distributed on the bed, the mean behavior of the exposed area A^* can be calculated from the following equation:

$$dA^* = -P(A^*, M_s^*, \dots)dM_s^*$$

(eq. 3)

~~Here,~~ P is the probability that a given particle is deposited on the exposed part of the bed, which here is ~~may be~~ a function of the fraction of exposed area (A^*) and a dimensionless mass of particles on the bed per area (M_s^*, explained below), but which ~~may in reality~~ can be expected to also be a function of~~,~~ the relative sediment supply, the bed topography and roughness, the particle size, the local hydraulics or other control variables. M_s^* is a dimensionless mass equal to the total mass of the particles residing on the bed per area, which is suitably normalized. A suitable mass for normalization is the minimum mass required to cover a unit area, M_0, as will become clear later. The minus sign is introduced because the fraction of the exposed area reduces as M_s^* increases. Similar to eq. (3), the equation for the fraction of covered area $A_c^* = 1-A^*$ can be written as:

$$dA_c^* = P(A^*, M_s^*, \dots)dM_s^*$$

(eq. 4)

As most previous relationships are expressed in terms of relative sediment supply Q_s^*, the relation of M_s^* to Q_s^* will be discussed later.

We can make some general statements about P. First, P is defined for the range $0 \leq A^* \leq 1$ and undefined elsewhere. Second, P takes values between zero and one for $0 \leq A^* \leq 1$. Third, $P(A^*=0) = 0$ and $P(A^*=1) = 1$. Note that P is not a distribution function and therefore does not need to integrate to one. Neither does it have to be continuous and differentiable everywhere.

For purpose of illustration, we will next discuss two simple forms of the probability function P that lead to the linear and exponential forms of the cover effect, respectively. First, consider the case that all particles are always deposited on exposed bedrock. In this case, formally, to keep with the conditions stated above, we define $P = 1$ for $0 < A^* \leq 1$ and $P = 0$ for $A^* = 0$. Thus, we can write

$$\begin{aligned} dA^* &= -dM_s^* \quad \text{for} \quad 0 < A^* \leq 1 \\ dA^* &= 0 \qquad\quad\ \text{for} \quad A^* = 0 \end{aligned}$$

(eq. 5)

Integrating, we obtain:

$$A^* = -M_s^* + C$$

(eq. 6)

where the constant of integration C is found to equal one by using the condition $A^*(M_s^*=0) = 1$. Thus, we obtain the linear cover function of eq. (1). Note that the linear cover function gives a theoretical lower bound for the amount of cover: it arises when all available sediment always falls on uncovered ground, and thus no additional sediment is available that could facilitate quicker alluviation. In essence, this is a mass conservation argument. Now it is obvious why M_0 is a convenient way to normalize: in plots of A^* against M_s^*, we obtain a triangular region bounded by the points [0,1], [0,0] and [1,0] in which the cover function cannot exist (Fig. 1).

Similarly to above, if we set P to a constant value smaller than one for $0 < A^* \leq 1$, k, we obtain

$$A^* = 1 - kM_s^*$$

(eq. 7)

It is clear that the assumption of $P = k$ is physically unrealistic, because it implies that the probability of deposition on exposed ground is independent of the amount of uncovered bedrock. Especially when A^* is close to zero, it seems unlikely that, say, always 90% of the sediment falls on uncovered ground. A more realistic assumption is that the probability of deposition on uncovered ground is independent of location and other possible controls, but is equal to the fraction of exposed area, i.e., $P = A^*$. In a probabilistic sense, this is also the simplest plausible assumption one can make. Then

$$dA^* = -A^* dM_s^*$$

(eq. 8)

giving upon integration

$$A^* = e^{-M_s^*}$$

(eq. 9)

The argument used here to obtain the exponential cover effect in eq. (9) essentially corresponds to the one given by Turowski et al. (2007). Since this case presents the simplest plausible assumption, we will use it as a benchmark case, to which we will compare other possible functional forms of P.

In principle, the probability function P can be varied to account for various processes that make deposition more likely either on already covered ground by decreasing P for the appropriate range of A^* from the benchmark case $P = A^*$, or on uncovered ground by increasing P from the benchmark case $P = A^*$. As has been identified previously (Chatanantavet and Parker, 2008; Hodge and Hoey 2012), roughness feedbacks to the flow can cause either case depending on whether subsequent deposition is adjacent to or on top of existing sediment patches. In the former case, particles residing on an otherwise bare bedrock bed act as obstacles for moving particles, and create a low-velocity wake zone in the downstream direction. In addition, particles residing on other single particles are unstable and stacks of particles are unlikely. Hence, newly arriving particles tend to deposit either upstream or downstream of stationary particles and the probability is generally higher for deposition on uncovered ground than in the benchmark case. In the latter case, larger patches of stationary particles increase the surface roughness of the bed, thus decreasing the local flow velocity and stresses, making deposition on the patch more likely. In this way, the probability of deposition on already covered bed is increased in comparison to the benchmark case.

A simple functional form that can be used to take into account either one of these two effects is a power law dependence of P on A^*, taking the form $P = A^{*\alpha}$ (Fig. 1A). Then, the cover function becomes (Fig. 1B):

$$A^* = (1-(1-\alpha)M_s^*)^{\frac{1}{1-\alpha}}$$

(eq. 10)

Here, the probability of deposition on uncovered ground is increased in comparison to the benchmark exponential case if $0 < \alpha < 1$, and decreased if $\alpha > 1$.

A convenient and flexible way to parameterize $P(A^*)$ in general is the cumulative version of the Beta distribution, given by:

$$P(A^*) = B(A^*; a, b)$$

(eq. 11)

Here, $B(A^*;a,b)$ is the regularized incomplete Beta function with two shape parameters a and b, which are both real positive numbers, defined by:

$$B(A^*; a, b) = \frac{\int_0^{A^*} y^{a-1}(1-y)^{b-1}dy}{\int_0^1 y^{a-1}(1-y)^{b-1}dy}$$

(eq. 12)

Here, y is a dummy variable. With suitable choices for a and b, cover functions resembling the exponential (a=b=1), the linear form (a=0, b>0), and the power law form (a>>b or a<<b) can be retrieved. Wavy functions are also a possibility (Fig. 2), thus both of the roughness effects described above can be modelled in a single scenario. Unfortunately, the integral necessary to obtain $A^*(M_s^*)$ does not give a closed-form analytical solution and needs to be computed numerically.

In principle, a suitable function P could also be defined to account for the influence of bed topography on sediment deposition. Such a function is likely dependent on the details of the particular bed, hydraulics and sediment flow paths in a complex way and needs to be mapped out experimentally.

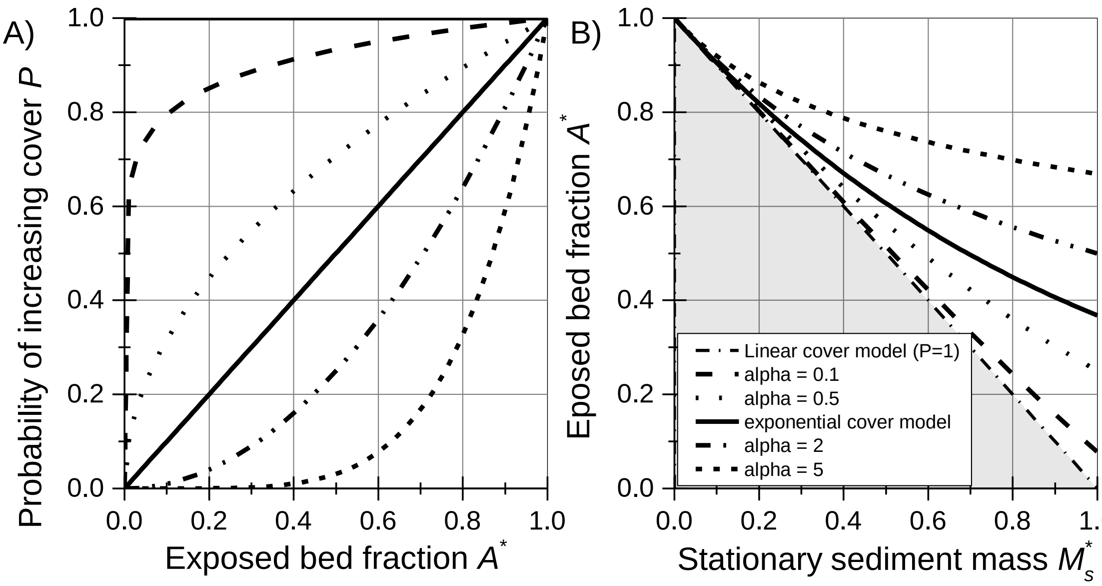

Fig. 1: A) Various examples for the probability function P as a function of bedrock exposure A^*. B) Corresponding analytical solutions for the cover function between A^* and dimensionless sediment mass M_s^* using eq. (7), (9) and (10). Grey shading depicts the area where the cover function cannot run due to conservation of mass.

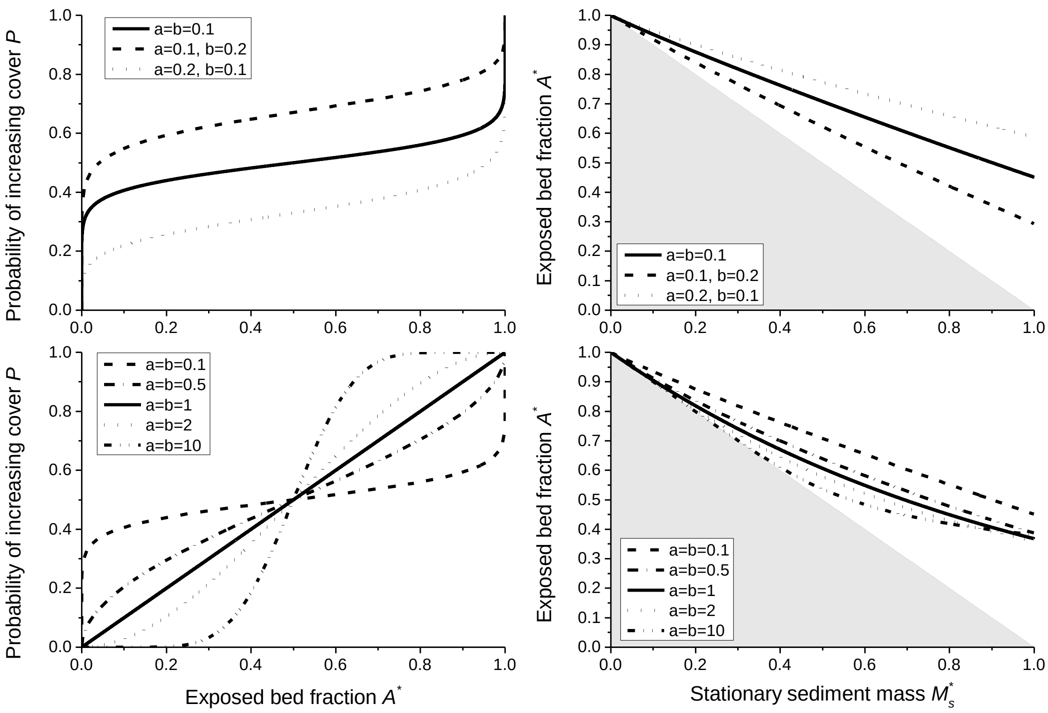

Fig. 2: Examples for the use of the regularized incomplete Beta function (eq. 12) to parameterize P, using various values for the shape parameters a and b. The choice a = b = 1 gives a dependence that is equivalent to the exponential cover function. Grey shading depicts the area where the cover function cannot run due to conservation of mass.

2.2 Example of application using model data

To illustrate how the framework can be used, we apply it to data obtained from the CA model developed by Hodge and Hoey (2012). The CA model reproduces the transport of individual sediment grains over a smooth bedrock surface. In each time step, the probability of a grain being entrained is a function of the number of neighboring grains. If five or more of the eight neighbouring cells contain grains then the grain has probability of entrainment ~~P_c~~p_c, otherwise it has probability ~~P_i~~p_i. In most model runs ~~P_c~~ p_c was set to a value ~~is~~ less than that of ~~P_i~~p_i, thus accounting for the impact of sediment cover in decreasing local shear stress (though increased flow resistance) and increasing the critical entrainment shear stress for grains (via lower grain exposure and increased pivot angles). Thus, in the model, grain scale dynamics of entrainment are varied by adjusting the values of p_i and p_c. This has a direct effect on the reach-scale distribution of cover, which is captured by our P-function (eq. 3).

The model is run with a domain that is 100 cells wide by 1000 cells long, with each cell having the same area as a grain. Up to four grains can potentially be entrained from each cell in a time step, limiting the maximum sediment flux. In each time step random numbers and the probabilities are used to select the grains that are entrained, which are then moved a step length downstream. A fixed number of grains are also supplied to the upstream end of the model domain. A smoothing algorithm is applied to prevent unrealistically ~~local excessively~~ tall piles of grains developing in cells if there are far fewer grains in adjacent cells. After around 500 time steps the model typically reaches a steady state condition in which the number of grains supplied to and leaving the model domain are equal. Sediment cover is measured in a downstream area of the model domain and is defined as grains that are not entrained in a given time step. Consequently grains that are deposited in one time step, and entrained in the following one do not contribute to the sediment cover, and so the model implicitly incorporates the effect of local sediment cover on grain deposition.

Model runs were completed with a six different combinations of ~~P_i~~ p_i and ~~P_c~~p_c: 0.95/0.95, 0.95/0.75, 0.75/0.10, 0.75/0.30, 0.30/0.30 and 0.95/0.05. These combinations were selected to cover the range of relationships between relative sediment supply Q_s^* and the exposed bed fraction A^* observed by Hodge and Hoey (2012). For each pair of P_i and P_c model runs were completed at least 20 different values of Q_s^* in order to quantify the model behaviour.

Cover bed fraction and total mass on the bed given out by the model were converted using eq. (3) into the probabilistic framework (Fig. 3). The derivative was approximated by simple linear finite differences, which, in the case of run-away alluviation, resulted in a non-continuous curve due to large gradients. The exponential benchmark (eq. 9) is also shown for comparison. The different model parameterisations produce results in which the probability of deposition on bedrock is both more and less likely than in the baseline case, with some runs showing both behaviours. Cases where the probability is more than the baseline case (i.e. grains are more likely to fall on uncovered areas) are associated with runs in which grains in clusters are relatively immobile. These runs are likely to be particularly affected by the smoothing algorithm that acts to move sediment from alluviated to bedrock areas. All model parameterisations predict greater bed exposure for a given normalised mass than is predicted by a linear cover relationship (Figure 3b). Runs with relatively more immobile cluster grains have a lower exposed fraction for the same normalised mass. Runs with low values of ~~P_i~~ p_i and ~~P_c~~ p_c seem to lead to behavior in which cover is more likely than in the exponential benchmark, while for high values, it is less likely. However, there are complex interactions and general statements cannot be made straightforwardly.

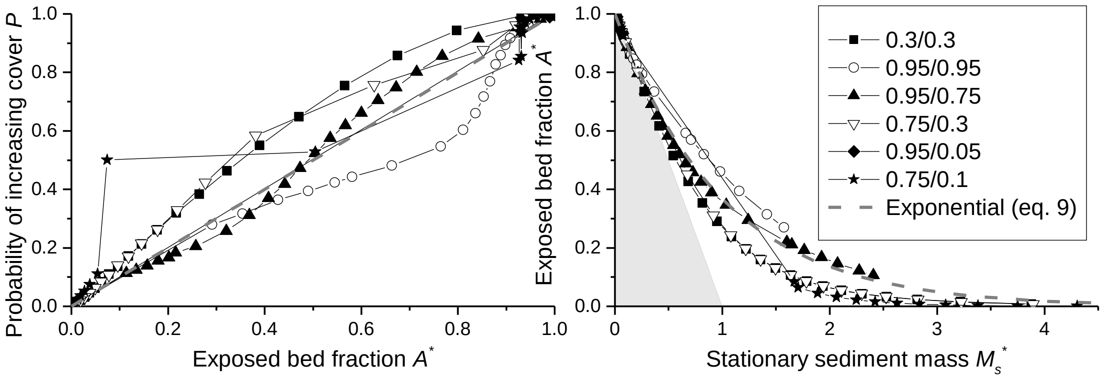

Fig. 3: Probability functions P and cover function derived from data obtained from the model of Hodge and Hoey (2012). The grey dashed line shows the exponential benchmark behavior. Grey shading depicts the area where the cover function cannot run due to conservation of mass. The legend gives values of the probabilities of entrainment ~~P_I~~ p_i and ~~P_ϵ~~ p_c used for the runs (see text).

3. Cover development in time and space

3.1. Model derivation

Previous descriptions of the cover effect relate the exposed fraction of the bed to the relative sediment supply Q_s^* (see eqs. 1 and 2). The relation between Q_s^* and M_s, which we used in eq. (3), has often been muddled and incorrect (see, for example, Turowski et al., 2007). In this chapter, we derive a model to clarify this relationship and put it on a sound physical bases. To this end, t~~T~~he probabilistic formulation introduced above ~~can be~~is extended to allow the calculation of the temporal and spatial evolution of sediment cover in a stream. Here, we will derive the equations for the one dimensional case (linear flume), but extensions to higher dimensions are possible in principle. The derivation is inspired by the erosion-deposition framework (e.g. Charru et al., 2004; Turowski, 2009), with some necessary adaptions to make it suitable for channels with partial sediment cover. In our system, we consider two separate mass reservoirs within a control volume. The first reservoir contains all particles in motion, the total mass per bed area of which is denoted by M_m, while the second reservoir contains all particles that are stationary on the bed, the total mass per bed area of which is denoted by M_s. We need then three further equations, one to connect the rate of change of mobile mass to the sediment flux in the flume, one to govern the exchange of particles between the two reservoirs, and one to describe how sediment transport rate is related to the mobile mass. The first of these is of course the Exner equation of sediment continuity (e.g. Paola and Voller, 2005), which captures mass conservation in the system. Instead of the common approach tracking the height of the sediment over a reference level, we use the total sediment mass on the bed as a variable, giving

$$\frac{\partial M_m}{\partial t} = -\frac{\partial q_s}{\partial x} + E - D$$

(eq. 13)

Here, x is the coordinate in the streamwise direction, t the time, q_s the sediment mass transport rate per unit width, while E is the mass entrainment rate per bed area and D is the mass deposition rate per bed area. The latter two terms ~~give the flux~~describe the exchange of particles between reservoirs; in the single reservoir Exner equation these terms are not needed. It is clear that for the problem at hand the choice of total mass or volume as a variable to track the amount of sediment in the reach of interest is preferable to the height of the alluvial cover, since necessarily, when cover is patchy, the height of the alluvium varies across the bed. It is useful to work with dimensionless variables by defining $t^* = t/T$ and $x^* = x/L$, where T and L are suitable time and length scales, respectively. The dimensionless mobile mass per bed area M_m^* is equal to M_m/M_0, and eq. (13) becomes:

$$\frac{\partial M_m^*}{\partial t^*} = -\frac{\partial q_s^*}{\partial x^*} + E^* - D^*$$

(eq. 14)

Here,

$$q_s^* = \frac{T}{LM_0} q_s$$

(eq. 15)

The dimensionless entrainment and deposition rates, E^* and D^*, are equal to TE/M_0 and TD/M_0, respectively. The rate of change of the stationary sediment mass M_s in time is the difference of the deposition rate D and the entrainment rate E:

$$\frac{\partial M_s}{\partial t} = D - E$$

(eq. 16)

Or, using dimensionless variables

$$\frac{\partial M_s^*}{\partial t^*} = D^* - E^*$$

(eq. 17)

We also need sediment entrainment and deposition functions. The entrainment rate needs to be modulated by the availability of sediment on the bed. If M_s^* is equal to zero, no material can be entrained. A plausible assumption is that the maximal entrainment rate, E^*_{max}, is equal to the transport capacity.

$$E^*_{max} = q_t^*$$

(eq. 18)

Here, q_t^* is the dimensionless mass transport capacity, which is related to the transport capacity per unit width q_t by a relation similar to eq. (15). To first order, the rate of change in entrainment rate, dE, is proportional to the difference of E_{max} and E, and to the rate of change in mass on the bed.

$$dE^* = (E^*_{max} - E^*)dM_s^* = (q_t^* - E^*)dM_s^*$$

(eq. 19)

Integrating, we obtain

$$E^* = E^*_{max}\left(1 - e^{-M_s^*}\right) = \left(1 - e^{-M_s^*}\right)q_t^*$$

(eq. 20)

Here, we used the condition $E^*(M_s^*=0) = 0$ to fix the integration constant to E^*_{max}. As required, eq. (20) approaches E^*_{max} as M_s^* goes to infinity, and is equal to zero when M_s^* is equal to zero. Using a similar line of argument, and by assuming the maximum deposition rate to be equal to q_s^*, we arrive at an equation for the deposition rate D^*.

$$D^* = \left(1 - e^{-M_m^*}\right)q_s^*$$

(eq. 21)

When M_m* is small, then the amount that can be deposited is limited by M_m*. If M_m* is large, then deposition is limited by sediment supply. Substituting eqs. (20) and (21) into eq. (17), we obtain:

$$\frac{\partial M_s^*(x^*,t^*)}{\partial t^*} = D^* - E^* = \left(1 - e^{-M_m^*(x^*,t^*)}\right) q_s^*(x^*,t^*) - \left(1 - e^{-M_s^*(x^*,t^*)}\right) q_t^*(x^*,t^*)$$

(eq. 22)

Note that $q_s^*/q_t^* = Q_s^*$. The equation for the mobile mass (eq. 14) becomes:

$$\frac{\partial M_m^*(x^*,t^*)}{\partial t^*} = -\frac{\partial q_s^*}{\partial x^*} - \left(1 - e^{-M_m^*(x^*,t^*)}\right) q_s^*(x^*,t^*) + \left(1 - e^{-M_s^*(x^*,t^*)}\right) q_t^*(x^*,t^*)$$

(eq. 23)

Finally, the sediment transport rate needs to be proportional to the mobile sediment mass times the downstream sediment speed U, and we can write

$$q_s^*(x^*,t^*) = U^*(x^*,t^*) M_m^*(x^*,t^*)$$

(eq. 24)

Here

$$U^* = \frac{T}{L} U$$

(eq. 25)

After incorporating the original equation between A^* and M_s^* (eq. 3), the system of four differential equations (3), (22), (23) and (24) contains four unknowns: the downstream gradient in the sediment transport rate $\partial q_s^*/\partial x^*$, the exposed fraction of the bed A^*, the non-dimensional stationary mass M_s^*, and the non-dimensional mobile mass M_m^*, while the non-dimensional transport capacity q_t^* and the non-dimensional downstream sediment speed U^* are input variables, and P is a externally specified function. In addition, sediment input q_s^* needs to be specified as an upstream boundary condition and initial values for the mobile mass M_m^* and the stationary mass M_s^* need to be specified everywhere.

3.2. Time-independent solution

Setting the time derivatives to zero, we obtain a time-independent solution, which links the exposed area directly to the ratio of sediment transport rate to transport capacity. From eq. (23) it follows that in this case, the entrainment rate is equal to the deposition rate and we obtain

$$\left(1 - e^{-\overline{M_m^*}}\right) \overline{q_s^*} = \left(1 - e^{-\overline{M_s^*}}\right) q_t^*$$

(eq. 26)

Here, the bar over the variables denotes their steady state value. Substituting eq. (24) to eliminate $\overline{M_m^*}$ and solving for $\overline{M_s^*}$ gives

$$\overline{M_s^*} = -\ln\left\{1 - \left(1 - e^{-\overline{q_s^*}/U^*}\right)\frac{\overline{q_s^*}}{q_t^*}\right\} = -\ln\left\{1 - \left(1 - e^{-\frac{q_t^*}{U^*}\overline{Q_s^*}}\right)\overline{Q_s^*}\right\}$$

(eq. 27)

Note that we assume here that sediment cover is only dependent on the stationary sediment mass on the bed and we thus neglect grain-grain interactions known as the dynamic cover (Turowski et al., 2007). In analogy to eq. (24), we can write

$$q_t^* = U^* M_0^*$$

(eq. 28)

Here, M_0^* is a characteristic dimensionless mass that depends on hydraulics and therefore implicitly on transport capacity (which is independent of and should not be confused with the minimum mass necessary to fully cover the bed M_0). When sediment transport rate equals transport capacity, then M_0^* is equal to the mobile mass of sediment normalized by the reference mass M_0. It can be viewed as a proxy for the transport capacity and is a convenient parameter to simplify the equations. The mobile mass can then, in general, be written as follows (cf. Turowski et al., 2007), remembering that the relative sediment supply $Q_s^* = 1$ when supply is equal to capacity:

$$M_m^* = M_0^* Q_s^*$$

(eq. 29)

If we use the exponential cover function (eq. 9) with eqs. (27), (28) and (29) we obtain

$$\overline{A^*} = 1 - \left(1 - e^{-\overline{q_s^*}/U^*}\right)\frac{\overline{q_s^*}}{q_t^*} = 1 - \left(1 - e^{-\frac{q_t^*}{U^*}\overline{Q_s^*}}\right)\overline{Q_s^*} = 1 - \left(1 - e^{-M_0^*\overline{Q_s^*}}\right)\overline{Q_s^*}$$

(eq. 30)

Similarly, equations can be found for the other analytical solutions of the cover function. For the linear case (eq. 7), we obtain:

$$\overline{A^*} = 1 + \ln\left\{1 - \left(1 - e^{-M_0^*\overline{Q_s^*}}\right)\overline{Q_s^*}\right\}$$

(eq. 31)

For the power law case (eq. 10), we obtain:

$$\overline{A^*} = \left[1 + (1-\alpha)\ln\left\{1 - \left(1 - e^{-M_0^*\overline{Q_s^*}}\right)\overline{Q_s^*}\right\}\right]^{\frac{1}{1-\alpha}}$$

(eq. 32)

It is interesting that the assumption of an exponential cover function essentially leads to a combined linear and exponential relation between $\overline{A^*}$ and $\overline{Q_s^*}$. Instead of a linear decline as the original linear cover model, or a concave-up relationship as the original exponential model, the function is convex-up for all solutions (Fig. 4). Adjusting M_0^* shifts the lines: decreasing M_0^* leads to a delayed onset of cover and vice versa. The former result arises because a lower M_0^* means that the sediment flux is conveyed through a smaller mass moving at a higher velocity. The original linear cover function (eq. 1) can be recovered from the exponential model with a high value of M_0^*, since the exponential term quickly becomes negligible with increasing $\overline{Q_s^*}$ and the linear term dominates (Fig. 4C). Note that for the linear (eq. 6) and the power law cases (eq. 10), high values of M_0^* may give $\overline{A^*} = 0$ for $\overline{Q_s^*} < 1$ (Fig. 4B,D), which is consistent with the concept of runaway alluviation. Using the beta distribution to describe P, a numerical solution is necessary, but a wide range of steady-state cover functions can be obtained (Fig. 5). By varying the value of M_0^*, an even wider range of behavior can be obtained.

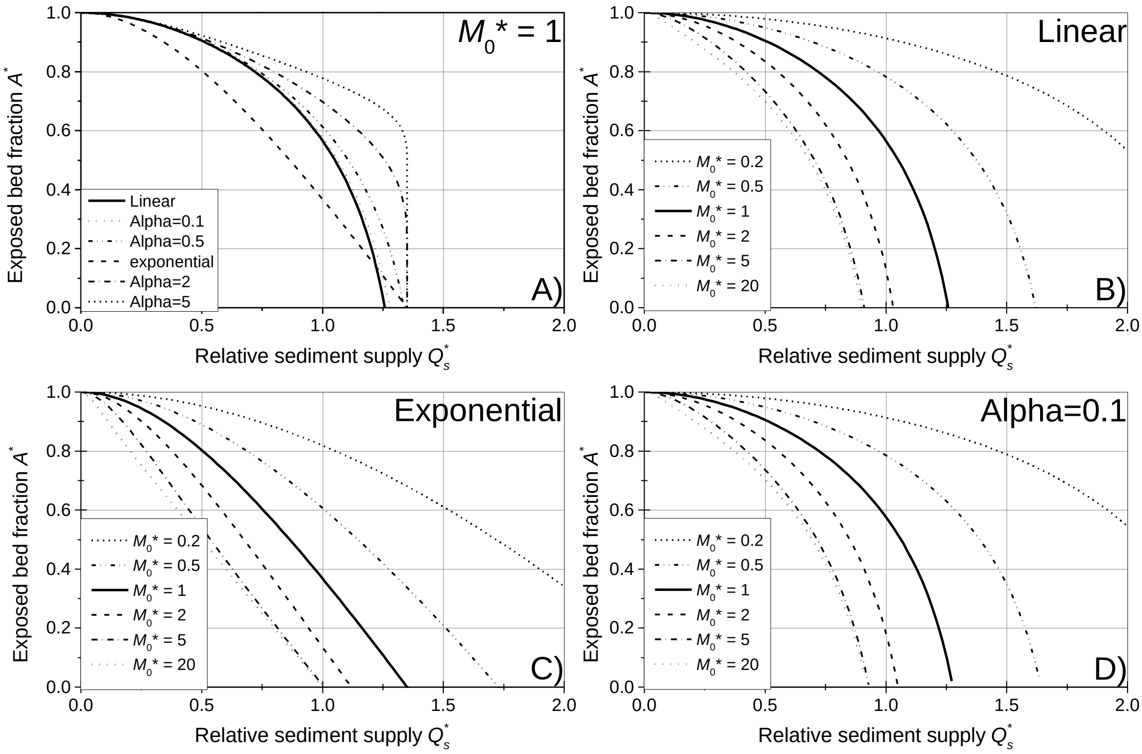

Fig. 4: Analytical solutions at steady state for the exposed fraction of the bed (A^*) as a function of relative sediment supply (Q^*, cf. Fig. 1). A) Comparison of the different solutions, keeping M_0^* constant at 1. B) Varying M_0^* for the linear case (eq. 31). C) Varying M_0^* for the exponential case (eq. 30). D) Varying M_0^* for the power law case with α = 0.1 (eq. 32).

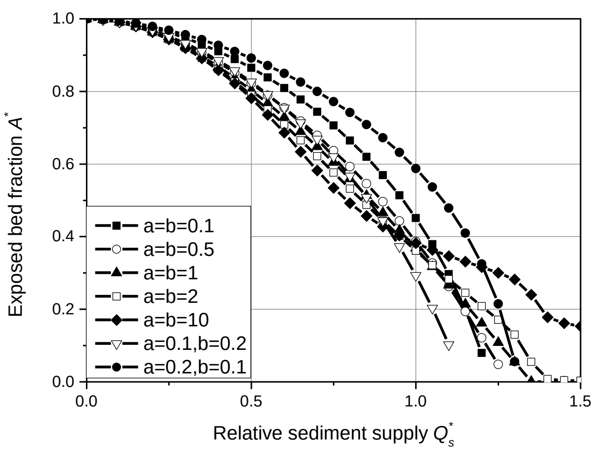

Fig. 5: Steady state solutions using the beta distribution to parameterize P (eq. 11) for a range of parameters a and b, and using M_0^* = 1 (cf. Fig. 2). The solutions were obtained by iterating the equations to a steady state, using initial conditions of A^* = 1 and M_m^* = M_s^* = 0.

The previous analysis shows that steady state cover is controlled by the characteristic dimensionless mass M_0^*, which is equal to the ratio of dimensionless transport capacity and particle speed (eq. 28). Converting to dimensional variables, we can write

$$M_0^* = \frac{q_t^*}{U^*} = \frac{q_t}{M_0 U}$$

(eq. 33)

The minimum mass necessary to completely cover the bed per unit area, M_0, can be estimated assuming a single layer of close-packed spherical grains residing on the bed (cf. Turowski, 2009), giving:

$$M_0 = \frac{\pi \rho_s D_{50}}{3\sqrt{3}}$$

(eq. 34)

Here, ρ_s is the sediment density and D_{50} is the median grain size. We use equations derived by Fernandez-Luque and van Beek (1976) ~~derived equations both~~ from flume experiments that describe ~~for the~~ transport capacity and ~~the~~ particle speed ~~from flume experiments, using~~as a function of bed shear stress ~~as a control parameter~~ (see also Lajeunesse et al., 2010, and Meyer-Peter and Mueller, 1948, for similar equations):~~.~~

$$q_t = 5.7 \frac{\rho_s \rho}{(\rho_s - \rho) g} \left(\frac{\tau}{\rho} - \frac{\tau_c}{\rho}\right)^{3/2}$$

(eq. 35)

$$U = 11.5 \left(\left(\frac{\tau}{\rho}\right)^{1/2} - 0.7 \left(\frac{\tau_c}{\rho}\right)^{1/2} \right)$$

(eq. 36)

Here, τ_c is the critical bed shear stress for the onset of bedload motion, g is the acceleration due to gravity and ρ is the water density. Combining eqs. (34), (35) and (36) to get an equation for M_0^* gives:

$$M_0^* = \frac{3\sqrt{3}}{2\pi} \frac{(\theta - \theta_c)^{3/2}}{\theta^{1/2} - 0.7\theta_c^{1/2}} = \frac{3\sqrt{3}\theta_c}{2\pi} \frac{(\theta/\theta_c - 1)^{3/2}}{(\theta/\theta_c)^{1/2} - 0.7}$$

(eq. 37)

Here, the Shields stress $\theta = \tau/(\rho_s - \rho)gD_{50}$, and θ_c is the corresponding critical Shields stress, and we approximated 5.7/11.5 = 0.496 with 1/2 (compare to eqs. 35/36). At high θ, when the threshold can be neglected, eq. (37) reduces to a linear relationship between M_0^* and θ. Near the threshold, M_0^* is shifted to lower values as θ_c increases (Fig. 6). The systematic variation of U^* with the hydraulic driving conditions (eq. 36) implies that the cover function evolves differently in response to changes in sediment supply and transport capacity. For a first impression, by comparing equations (35) and (36), we assume that particle speed scales with transport capacity raised to the power of one third (Fig. 7).

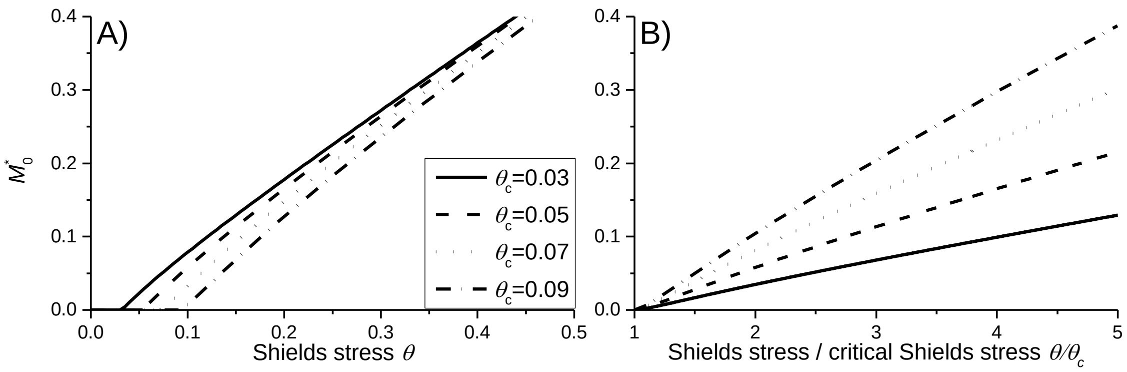

Fig. 6: The characteristic dimensionless mass M_0^* depicted as a function of A) the Shields stress and B) the ratio of Shields stress to critical Shields stress (eq. 37).

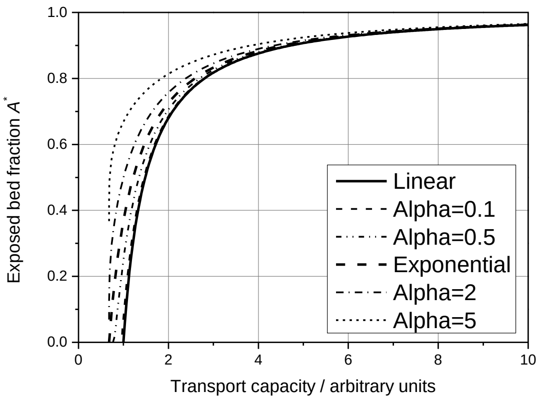

Fig. 7: Variation of the exposed bed fraction as a function of transport capacity, assuming that particle speed scales with transport capacity to the power of one third.

3.3 Temporal evolution of cover within a reach

3.3.1 System timescales

To calculate the temporal evolution of cover on the bed within a single reach, we solved the equations numerically for a section of the bed with homogenous conditions using a simple linear finite difference scheme. Then, the sediment input is a boundary condition, while sediment output, mobile and stationary sediment mass and the fraction of the exposed bed are output variables. In general, a change in sediment supply leads to a gradual adjustment of the output variables towards a new steady state (Fig. 8). Unfortunately, a general analytical solution is not possible, but a ~~results~~ result can be obtained for the special case of q_s^* = 0. Such a situation is rare in nature, but could be easily created in flume experiments as a model test. Then, the time derivative of stationary mass is given by:

$$\frac{\partial M_s^*}{\partial t^*} = -\left(1 - e^{-M_s^*}\right) q_t^*$$

(eq. 38)

Using the exponential cover model (eq. 9), we obtain:

$$\frac{1}{A^*(1 - A^*)} \frac{\partial A^*}{\partial t^*} = q_t^*$$

(eq. 39)

Equation (39) is separable and can be integrated to obtain

$$\ln(A^*) - \ln(1 - A^*) = t^* q_t^* + C$$

(eq. 40)

Letting $A^*(t^*=0) = A^*_0$, where A^*_0 is the initial cover, the final equation is

$$\frac{1 - A^*}{1 - A_0^*} \frac{A_0^*}{A^*} = e^{-t^* q_t^*}$$

(eq. 41)

To clarify the characteristic time scale of the process, equation (41) can also be written in the form of a sigmoidal-type function:

$$A^* = \frac{1}{1 + \left(\frac{1 - A_0^*}{A_0^*}\right) e^{-t^* q_t^*}}$$

(eq. 42)

By making the parameters in the exponent on the right hand side of eq. (42) dimensional, we get:

$$t^* q_t^* = \frac{t}{T}\frac{T}{LM_0} q_t = \frac{t q_t}{LM_0}$$

(eq. 43)

which allows a characteristic system time scale T_E to be defined as

$$T_E = \frac{LM_0}{q_t}$$

(eq. 44)

Since this time scale is dependent on the transport capacity q_t, we can view it as a time scale associated with the entrainment of sediment from the bed (cf. eq. 20) – hence the subscript E on T_E. From eq. (42), the exposed bed fraction evolves in an asymptotic fashion towards equilibrium (Fig. 9). We can expect that there are other characteristic time scales for the system, for example associated with sediment deposition or downstream sediment evacuation.

We can make some further progress and define a more general system time scale by performing a perturbation analysis (Appendix A). For small perturbations in either q_s^* or q_t^*, we obtain an exponential term describing the transient evolution, which allows the definition of a system timescale T_S

$$\exp\left\{-\left(\overline{q_t^*} - \left(1 - e^{-\overline{q_s^*}/\overline{U^*}}\right)\overline{q_s^*}\right)t^*\right\} = \exp\left\{-\frac{t}{T_S}\right\}$$

(eq. 45)

The characteristic system time scale can then be written as

$$T_S = \frac{LM_0}{\bar{q}_t\left(1 - \left(1 - e^{-\overline{q_s^*}/\overline{U^*}}\right)\frac{\bar{q}_s}{\bar{q}_t}\right)} = \frac{LM_0}{\bar{q}_t} e^{\overline{M_S^*}}$$

(eq. 46)

Note that for $q_s^* = 0$, eq. (46) reduces to eq. (44), as would be expected. Since $\overline{M_S^*}$ is directly related to steady state bed exposure $\overline{A^*}$, we can rewrite the equation, for example by assuming the exponential cover function (eq. 3), as

$$T_S = \frac{LM_0}{\bar{q}_t \overline{A^*}}$$

(eq. 47)

Since bed cover is more easily measurable than the mass on the bed, eq. (47) can help to estimate system time scales in the field. Further, $\overline{A^*}$ varies between 0 and 1, which allows estimating a minimum system time using eq. (44). As $\overline{A^*}$ approaches zero, the system time scale diverges.

To illustrate these additional dependencies, we have used numerical solutions of eqs. (3), (22), (23) and (24) to calculate~~d~~ the time needed to reach 99.9% ~~(chosen due to the asymptotic behavior of the~~

~~system)~~ of total adjustment after a step change in transport stage (chosen due to the asymptotic behavior of the system), produced by varying particle speed U over a range of plausible values (Fig. 10). Response time decreases as particle speed increases. This reflects elevated downstream evacuation for higher particles speeds, resulting in a smaller mobile particle mass and thus higher entrainment and lower deposition rates. Response time also increases with increasing relative sediment supply Q_S^* ~~q_s/q_t~~. As the runs start with zero sediment cover, and the extent of cover increases with Q_S^* ~~q_s/q_t~~, at higher Q_S^* ~~q_s/q_t~~ the adjusted cover takes longer to develop.

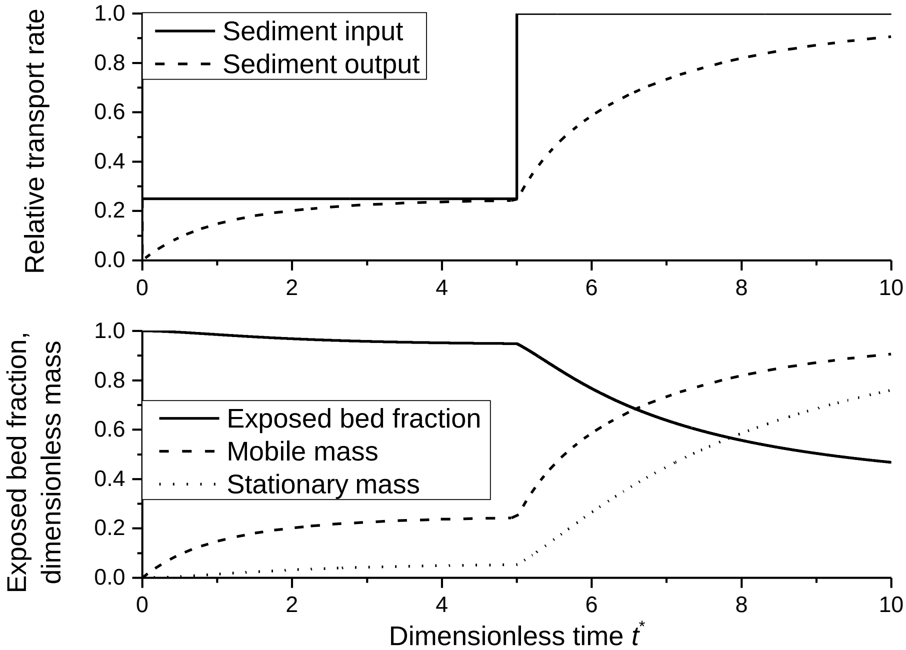

Fig. 8: Temporal evolution of cover for the simple case of a control box with sediment through-flux, based on eqs. (3), (22), (23) and (24). Relative sediment supply (supply normalized by transport capacity) was specified to 0.25 and increased to 1 at t^* = 5. The response of sediment output, mobile and stationary sediment mass and the exposed bed fraction was calculated. Here, we used the exponential function for P (eq. 9) and M_0^* = U^* = 1. The initial values were A^* = 1 and M_m^* = M_s^* = 0.

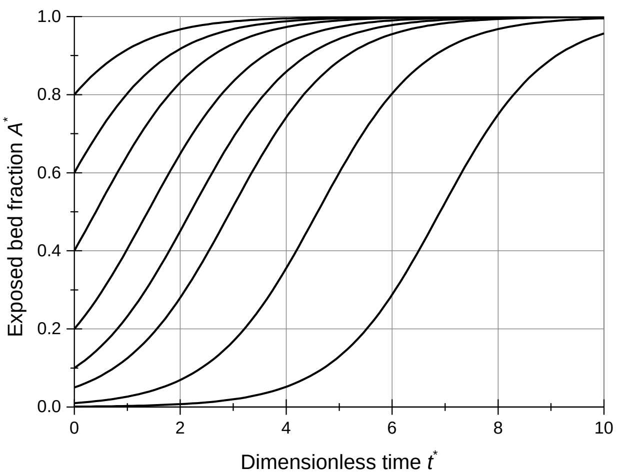

Fig. 9: Evolution of the exposed bed fraction (removal of sediment cover) over time starting with different initial values of bed exposure, for the special case of no sediment supply, i.e., $q_s^* = 0$ (eq. 41) and $q_t^* = 1$.

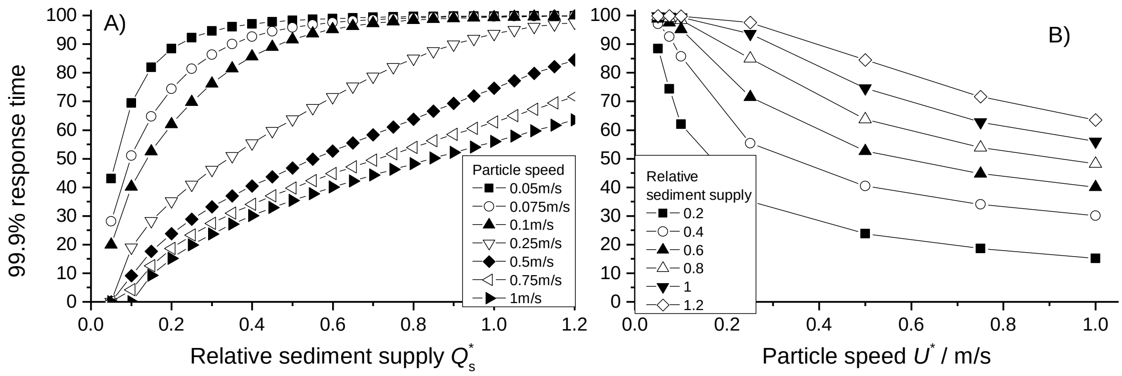

Fig. 10: Dimensionless time to reach 99.9% of the total adjustment in exposed area as a function of A) transport stage and B) particle speed. All simulation were started with $A^* = 1$ and $M_m^* = M_s^* = 0$.

3.3.2 Phase shift and gain in response to a cyclic perturbation

The perturbation analysis (Appendix A) gives some insight into the response of cover to cyclic sinusoidal perturbations. Let sediment supply be perturbed in a cyclic way described by an equation of the form

$$q_s^* = \overline{q_s^*} + \delta q_s^* = \overline{q_s^*} + d \sin\left(\frac{2\pi t}{p}\right)$$

(eq. 48)

Here, the overbar denotes the temporal average, δq_s^* is the time-dependent perturbation, d is the amplitude of the perturbation and p its period. A similar perturbation can be applied to the transport capacity (see Appendix A). The reaction of the stationary mass and therefore cover can then also be described by sinusoidal function of the form (Appendix A)

$$\delta M_s^* = G \sin\left(\frac{2\pi t}{p} + \varphi\right)$$

(eq. 49)

Here, δM_s^* is the perturbation of the stationary sediment mass around the temporal average, G is known as the gain, describing the amplitude response, and φ is the phase shift. If the gain is large, stationary mass reacts strongly to the perturbation; if it is small, the forcing does not leave a signal. The phase shift is negative if the response lags behind the forcing and positive if it leads. The phase shift can be written as

$$\varphi = \tan^{-1}\left(-2\pi \frac{T_S}{p}\right)$$

(eq. 50)

The gain can be written as

$$G = \frac{p}{T_S} \frac{Kd}{\sqrt{\left(\frac{p}{T_S}\right)^2 + 4\pi^2}}$$

(eq. 51)

Here, d is the amplitude of the perturbation, and K is a function of the time-averaged values of q_s, q_t and U and differs for perturbations in transport capacity and sediment supply (see Appendix A).

Thus, the system behavior can be interpreted as a function of the ratio of the period of perturbation p and the system time scale T_s. The period p is large if the forcing parameter, i.e., discharge or sediment supply, varies slowly and small when it varies quickly. According to eq. (50), the phase shift is equal to $-\pi/2$ for low values of p/T_s (quickly-varying forcing parameter), implying a substantial lag in the adjustment of cover. The phase shift tends to zero as p/T_s tends to infinity (Fig. 11). The gain varies approximately linearly with p/T_s for small p/T_s (quickly-varying forcing parameter), while it is approximately constant at a value of Kd for large p/T_s (slowly-varying forcing parameter) (eq. 51). Thus, if the forcing parameter varies slowly, cover adjustment keeps up at all times.

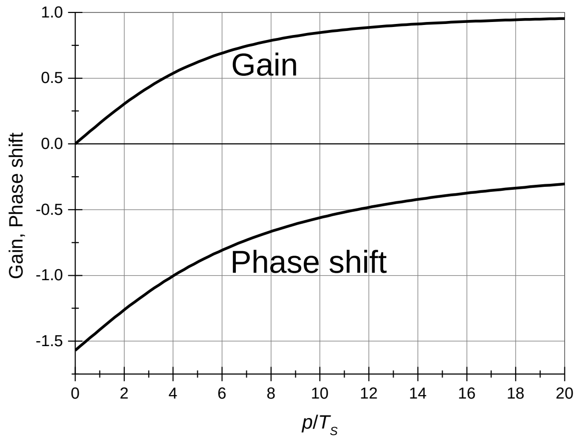

Fig. 11: Phase shift (eq. 50) and gain (eq. 51) as a function of the ratio of the period of perturbation ~~period~~ p and the system time scale T_s. For the calculation, the constant factor in the gain (Kd) was set equal to one.

3.3.3 A flood at the Erlenbach

To illustrate the magnitude of the timescales using real data, we use a flood dataset from the Erlenbach, a sediment transport observatory in the Swiss Prealps (e.g., Beer et al., 2015). There, near a discharge gauge, bedload transport rates are measured at 1-minute resolution using the Swiss Plate Geophone System, a highly developed and fully calibrated surrogate bedload measuring system (e.g., Rickenmann et al., 2012; Wyss et al. 2016). We use data from a flood on 20th June 2007 (Turowski et al., 2009) with highest peak discharge that has so far been observed at the Erlenbach. The meteorological conditions that triggered this flood and its geomorphic effects have been described in detail elsewhere (Molnar et al., 2010; Turowski et al., 2009, 2013). ~~Although t~~The Erlenbach does not have a bedrock bed in the sense that bedrock is exposed in the channel bed, however, the data provide a realistic natural time series of discharge and bedload transport over the course of a single event. Rather than predicting bed cover evolution for a natural system, for which we do not currently have data for validation, we use the Erlenbach data to ~~and are ideal for illustrating~~ illustrate possible cover behavior during a fictitious event ~~and~~ with different initial sediment cover extents, using natural data to provide realistic boundary conditions.

Using a median grain size of 80 mm, a sediment density of 2650 kg/m^3 and a reach length of 50 m, we obtained M_0 = 128 kg/m^2. We calculated transport capacity using the equation of Fernandez Luque and van Beek (1976). However, it is known that this and similar equations strongly overestimate measured transport rates in streams such as the Erlenbach (e.g., Nitsche et al., 2011). Consequently, we rescaled by setting the ratio of bedload supply to capacity to one at the highest discharge. The exposed fraction was then calculated iteratively assuming $P = A^*$ (i.e., the exponential cover formulation, eq. 9). In a real flood event, water discharge and sediment supply obviously do not follow a small cyclic perturbation (Fig. 11). But we can tentatively relate the observations to the theory by assuming that at each time step, the change in sediment supply can be represented by the commencement of a sinusoidal perturbation with varying period ~~commences~~. To estimate the effective period p, one needs to take the derivatives of eq. (48).

$$\frac{dq_s^*}{dt} = \frac{d\delta q_s^*}{dt} = \frac{2\pi d}{p}\cos\left(\frac{2\pi t}{p}\right)$$

(eq. 52)

Setting t = 0 for the time of interest, we can relate p to the local gradient in bedload supply, which can be measured from the data.

$$\frac{2\pi d}{p} = \frac{\Delta q_s^*}{\Delta t}$$

(eq. 52)

Assuming that all change in the response time is due to changes in the period (i.e., assuming a constant amplitude, $d = 1$), we can obtain a conservative estimate of the range over which p varies over the course of an event.

$$p = 2\pi\frac{\Delta t}{\Delta q_s^*}$$

(eq. 52)

In the exemplary event, the evolution and final value of bed cover depends strongly on its initial value (Fig. 12), indicating that the adjustment is incomplete. The system timescale is generally larger than 1000s and is inversely related to discharge via the dependence on transport capacity. The p/T_s ratio varies around one, with low values at the beginning of the flood and large values in the waning hydrograph. Both the high values of the system time scale and the smooth evolution of bed cover over the course of the flood imply that cover development cannot keep up with the variation in the forcing characteristics. This dynamic adjustment of cover, which can lag forcing processes, may thus play an important role in the dynamics of bedrock channels and probably needs to be taken into account in modelling.

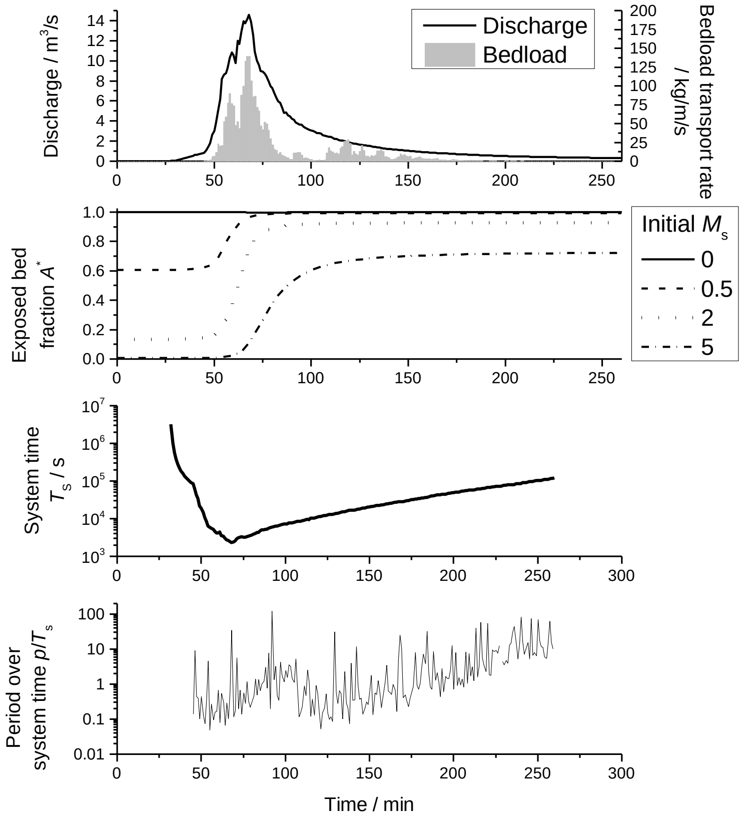

Fig. 12: Calculated evolution of cover during the largest event observed at the Erlenbach on 20th June 2007 (Turowski et al., 2009). Bedload transport rates were measured with the Swiss Plate geophone sensors calibrated with direct bedload samples (Rickenmann et al., 2012). The final fraction of exposed bedrock is strongly dependent on its initial value.

4. Discussion

4.1 Model formulation

In principle, the framework for the cover effect presented here allows the formulation of a general model for bedrock channel morphodynamics without the restrictions of previous models (e.g. Nelson and Seminara, 2011; Zhang et al., 2015). To achieve this, the dependency of P on various control parameters needs to be specified. In general, P should be controlled by local topography, grain size and shape, hydraulic forcing, and the amount of sediment already residing on the bed. Furthermore, the shape of the P function should also be affected by feedbacks between these properties, such as the development of sediment cover altering the local roughness and hence altering hydraulics and local transport capacity (Inoue et al., 2014; Johnson, 2014). Within the treatment presented here, we have explicitly accounted only for the impact of the amount of sediment already residing on the bed. However, all of the mentioned effects can be included implicitly by an appropriate choice of P. The exact relationships between, say, bed topography and P need to be mapped out experimentally (e.g., Inoue et al., 2014), with theoretical approaches also providing some direction (cf. Johnson, 2014; Zhang et al., 2015). Currently available experimental results (Chatanantavet and Parker, 2008; Finnegan et al., 2007; Hodge and Hoey, 2016; Inoue et al., 2014; Johnson and Whipple, 2007) cover only a small range of the possible parameter space and, in general, ~~do~~ not ~~generally report~~ all necessary parameters to constrain P were reported. Specifically the stationary mass of sediment residing on the bed is ~~generally~~ usually not reported and can be difficult to determine experimentally, but is necessary to determine P. Nevertheless, depending on the choice of P, our model can yield a wide range of cover functions that encompasses reported functions both from numerical modelling (e.g., Aubert et al., 2016; Hodge and Hoey, 2012; Johnson, 2014) and experiments (Chatanantavet and Parker, 2008; Inoue et al., 2014; Sklar and Dietrich, 2001) (see Figs. 4 and 5).

The dynamic model put forward here is a minimum first order formulation, and there are some obvious future alterations. We only take account of the static cover effect caused by immobile sediment on the bed. The dynamic cover effect, which arises when moving grains interact at high sediment concentration and thus reduce the number of impacts on the bed (Turowski et al., 2007), could in principle be included into the formulation, but would necessitate a second probability function specifically to describe this dynamic cover. It would also be possible to use different P-functions for entrainment and deposition, thus introducing hysteresis into cover development. Such hysteresis has been observed in experiments in which the equilibrium sediment cover was a function of the initial extent of sediment cover (Chatanantavet and Parker, 2008; Hodge and Hoey, 2012). Whether such alterations are necessary is best established with targeted laboratory experiments.

4.2 Comparison to previous modelling frameworks

We will briefly outline in this section the main differences to previous formulations of cover dynamics in bedrock channels. Thus, the novel aspects of our formulation and the respective advantages and disadvantages will become clear.

Aubert et al. (2015) coupled the movement of spherical particles to the simulation of a turbulent fluid and investigated how cover depended on transport capacity and supply. Similar to what is predicted by our analytical formulation, they found a range of cover function for various model set-ups, including linear and convex-up relationships (compare the results in Fig. 4 to their Fig. 15). Despite short-comings, Aubert et al. (2015) presented the so far most detailed physical simulations of bed cover formation and the correspondence between the predictions is encouraging.

Nelson and Seminara (2011, 2011) formulated a morphodynamic model for bedrock channels. They based their formulation on sediment concentration, which is in principle similar to our formulation based on mass. However, Nelson and Seminara (2011, 2012) did not distinguish between mobile and stationary sediment and linked local transport directly to sediment concentration. Further, a given mass can be distributed in multiple ways to achieve various degrees of cover, a fact that is quantified in our formulation by the probability parameter P. Nelson and Seminara (2011, 2012) assumed a direct correspondence between sediment concentration and degree of cover, which is equivalent to the linear cover assumption (eq. 7), with the associated problems outlined earlier. Practically, this implies that the grid size needs to be of the order of the grain size. Although different in various details, Inoue et al. (2016) have used essentially the same approach as Nelson and Seminar (2011, 2012) to link bedload concentration, transport and bed cover. Both of these models allow the 2D modelling of bedrock channel morphology. Although we have not fully developed such a model in the present paper, our model framework could easily be extended to 2D problems.

Inoue et al. (2014) formulated a 1D model for cover dynamics and bedrock erosion. There, they distinguish between stationary and mobile sediment using an Exner equation to capture sediment mass conservation. The degree of bed cover is related to transport rates and sediment mass via a saturation volume, which is related to our characteristic mass M_0^* (see section 3.2). A key difference between Inoue et al.'s (2014) model and the one presented here lies in the sediment continuity equation (eq. 26), in which we explicitly take ~~explicitly~~ account of both entrainment and deposition. In addition, with the function P, describing the relationship between deposited mass and degree of cover, we provide a more flexible framework for complex simulations where the bed needs to be discretized (e.g., 2D models or reach-scale formulations).

Zhang et al. (2015) formulated a bed cover model specifically for beds with macro-roughness. There, deposited sediment always fills topographic lows from their deepest positions, such that there is a reach-uniform sediment level. While the model is interesting and provides a fundamentally different approach to what is suggested here, its applicability is limited to very rough beds and the assumption of a sediment elevation that is independent of the position on the bed seems physically unrealistic. In principle, the probabilistic framework presented here should be able to deal with macro-rough beds as well and thus allows a more general treatment of the problem of bed cover.

Within this paper, we focused on the dynamics of bed cover, rather than modelling the dynamics of entire channels. The probabilistic formulation using the parameter P provides a flexible framework to connect the sediment mass residing on the bed with the exposed bedrock fraction. This particular element has not been treated in any of the previous models and could be easily implemented in other approaches dealing with sediment fluxes along and across the stream and the interaction with erosion and, over long time scales, channel morphology. However, it is as yet unclear how flow hydraulics, sediment properties and other conditions affect P and this should be investigated in targeted laboratory experiments. Nevertheless, the proposed formulation provides a framework in which data from various sources can be easily compared and discussed.

4.3 Further implications

Based on field data interpretation, Phillips and Jerolmack (2016) argued that bedrock rivers adjust such that, similar to alluvial channels, medium sized floods are most effective in transporting sediment, and that channel geometry therefore can quickly adjust their transport capacity to the applied load and therefore achieve grade (cf. Mackin, 1948). They conclude that bedrock channels can adjust their morphologic parameters (channel width and shape) quickly in response to changing boundary conditions, a somewhat counter-intuitive notion for slowly-eroding channels. C~~ontrary to the suggestion of Phillips and Jerolmack (2016) that this is achieved by changing channel morphologic parameters such as width~~In contrast, our model suggests that bed cover ~~is~~ can be adjusted to achieve grade. ~~Furthermore changes in sediment cover can occur far more rapidly than morphological changes.~~ In steady state, time derivatives need to be equal to zero. Thus, entrainment equals deposition (eq. 16), implying that the downstream gradient in sediment transport rate is equal to zero (eq. 14). When sediment supply or transport capacity change, the exposed bedrock fraction can adjust to achieve a new steady state and a change of the channel geometry is unnecessary. These changes in sediment cover can occur far more rapidly than changes in width and cross-sectional shape (compare to eq. 47). Whether a steady state is achieved depends on the relative magnitude of the timescales of perturbation and cover adjustment (see section 3.2). Our results imply that bedrock channels have two distinct time scales to adjust to changing boundary conditions to achieve grade. Over short times, bed cover is adjusted. This can occur rapidly. Over long time scales, channel width, cross-sectional shape and slope are adjusted.

5. Conclusions

The probabilistic view put forward in this paper offers a framework into which diverse data on bed cover, whether obtained from field studies, laboratory experiments or numerical modeling, can be easily converted to be meaningfully compared. The conversion requires knowledge of the mass of sediment on the bed and the evolution of exposed fraction of the bed. Within the framework, individual data sets can be compared to the exponential benchmark and linear limit cases, enabling physical interpretation. Furthermore, the formulation allows the general dynamic sub-grid modelling of bed cover. Depending on the choice of P, the model yields a wide range of possible cover functions. Which of these functions are appropriate for natural rivers and how they vary with factors including topography needs to mapped out experimentally.

It needs to be noted here that the precise formulation of the entrainment and deposition functions also affects steady state cover relations. When calibrating P on data, it cannot always be decided whether a specific deviation from the benchmark case results from varying entrainment and deposition processes or from changes in the probability function driven for example by variations in roughness. For the prediction of the steady state cover relations and for the comparison of data sets, this should not matter, but the dynamic evolution of cover could be strongly affected.

The system timescale for cover adjustment is inversely related to transport capacity. This time scale can be long and in many realistic situations, cover cannot instantaneously adjust to changes in the forcing conditions. Thus, dynamic cover adjustment needs to be taken into account when modelling the long-term evolution of bedrock channels.

Our model formulation implies that bedrock channels adjust bed cover to achieve grade. Therefore, bedrock channel evolution is driven by two optimization principles. On short time scales, bed cover adjusts to match the sediment output of a reach to its input. Over long time scales, width and slope of the channel evolve to match long-term incision rate to tectonic uplift or base level lowering rates.

Appendix A: Perturbation analysis

Here, we derive the effect of a small sinusoidal perturbation of the driving variables, namely sediment supply q_s^* and transport capacity q_t^*, on cover development. The perturbation of the driving variables can be written as

$$q_s^* = \overline{q_s^*} + \delta q_s^*$$

(eq. A1)

$$q_t^* = \overline{q_t^*} + \delta q_t^*$$

(eq. A2)

Here, the bar denotes the average of the quantity at steady state, while δq_s^* and δq_t^* denote the small perturbation. The exposed area can be similarly written as

$$A^* = \overline{A^*} + \delta A^*$$

(eq. A3)

Steady state cover is directly related to the mass on the bed M_s^* by eq. (3), which we can rewrite as

$$\frac{dA^*}{dt} = -P\frac{dM_s^*}{dt}$$

(eq. A4)

Substituting eq. (A3) and a similar equation for M_s^*,

$$M_s^* = \overline{M_s^*} + \delta M_s^*$$

(eq. A5)

we obtain

$$\frac{d\delta A^*}{dt} = -P\frac{d\delta M_s^*}{dt}$$

(eq. A6)

Here, the averaged terms drop out as they are independent of time. If P and the steady state solution for A^* are known, a direct relationship between A^* and M_s^* can be derived. For example, for the exponential cover model (eq. 2), substituting eqs. (A3) and (A5), we find

$$\overline{A^*} + \delta A^* = e^{-\overline{M_s^*}-\delta M_s^*} = e^{-\overline{M_s^*}}e^{-\delta M_s^*} = \overline{A^*}e^{-\delta M_s^*} \approx \overline{A^*}(1-\delta M_s^*)$$

(eq. A7)

Here, since the δ variables are small, we approximated the exponential term using a Taylor expansion to first order. We obtain

$$\delta A^* = -\overline{A^*}\delta M_s^*$$

(eq. A8)

It is therefore sufficient to derive the perturbation solution for M_s^*, the time evolution of which is given by eq. (22). Eliminating M_m^* using eq. (24), we obtain

$$\frac{\partial M_s^*}{\partial t^*} = \left(1 - e^{-q_s^*/U^*}\right)q_s^* - \left(1 - e^{-M_s^*}\right)q_t^*$$

(eq. A9)

Perturbation of sediment supply

First, let's look at a perturbation of sediment supply q_s^*, while other parameters are held constant. Substituting eq. (A1) and (A5) into (A9), we obtain

$$\frac{\partial \delta M_s^*}{\partial t^*} = \left(1 - e^{-\left(\overline{q_s^*}+\delta q_s^*\right)/U^*}\right)\left(\overline{q_s^*} + \delta q_s^*\right) - \left(1 - e^{-\overline{M_s^*}-\delta M_s^*}\right)q_t^*$$

(eq. A10)

Again, since the δ variables are small, we can replace the relevant exponentials with Taylor expansion to first order:

$$e^{-\delta q_s^*/U^*} \approx 1 - \frac{\delta q_s^*}{U^*}$$

(eq. A11)

A similar approximation applies for the exponential in M_s^*. Substituting eq. (A11) into eq. (A10), expanding the multiplicative terms, dropping terms of second order in the δ variables and rearranging, we get

$$\frac{\partial \delta M_s^*}{\partial t^*} = \delta q_s^* \left(1 - e^{-\overline{q_s^*}/U^*} + \frac{\overline{q_s^*}}{U^*} e^{-\overline{q_s^*}/U^*} \right) - \delta M_s^* \left(q_t^* - \left(1 - e^{-\overline{q_s^*}/U^*} \right) \overline{q_s^*} \right)$$

(eq. A12)

The perturbation is assumed to be sinusoidal

$$\delta q_s^* = d \sin\left(\frac{2\pi t}{p}\right)$$

(eq. A13)

Here, p is the period of the perturbation and d is its amplitude. Note that, to be consistent with the assumptions previously made, d needs to be small in comparison with the average sediment supply. Substituting, eq. (A12) can be integrated to obtain the solution

$$\delta M_s^* = G_{q_s^*} \sin\left(\frac{2\pi t}{P} + \varphi_{q_s^*}\right) + C\exp\left\{ -\left(q_t^* - \left(1 - e^{-\overline{q_s^*}/U^*} \right) \overline{q_s^*} \right) \frac{t}{T} \right\}$$

where C is a constant of integration. The gain is given by

$$G_{q_s^*} = \frac{p}{T} \frac{\left(1 - e^{-\overline{q_s^*}/U^*} + \frac{\overline{q_s^*}}{U^*} e^{-\overline{q_s^*}/U^*} \right) d}{\sqrt{\left(q_t^* - \left(1 - e^{-\overline{q_s^*}/U^*} \right) \overline{q_s^*} \right)^2 \left(\frac{p}{T}\right)^2 + 4\pi^2}}$$

(eq. A14)

And the phase shift by

$$\varphi_{q_s^*} = \tan^{-1}\left[-\frac{2\pi}{\frac{p}{T}\left(q_t^* - \left(1 - e^{-\overline{q_s^*}/U^*} \right) \overline{q_s^*} \right)} \right]$$

(eq. A15)

Perturbation of transport capacity

The perturbation of the transport capacity q_t^* is a little more complicated, since both q_t^* and U^* are explicitly dependent on hydraulics (e.g., shear stress; see eqs. 43 and 44), and thus U^* is implicitly dependent on q_t^* and δq_t^*. To circumvent this problem, we expand the exponential term featuring $U^*(\delta q_t^*)$ in eq. (A9) using a Taylor series expansion around $\delta q_t^* = 0$.

$$\exp\left\{-\frac{q_s^*}{U^*(\delta q_t^*)}\right\} \approx \exp\left\{-\frac{q_s^*}{U^*(\delta q_t^* = 0)}\right\}\left[1 - \frac{q_s^*}{U^{*2}(\delta q_t^* = 0)}\frac{\partial U^*}{\partial \delta q_t^*}(\delta q_t^* = 0)\delta q_t^*\right]$$

(eq. A16)

Both U^* and its derivative are constants when evaluated at $\delta q_t^* = 0$. We can thus write

$$\exp\left\{-\frac{q_s^*}{U^*}\right\} = \exp\left\{-\frac{q_s^*}{\overline{U^*}}\right\}\left[1 - \frac{q_s^*}{\overline{U^*}^2}\overline{\left(\frac{\partial U^*}{\partial \delta q_t^*}\right)}\delta q_t^*\right] = [1 - C_0\delta q_t^*]e^{-q_s^*/\overline{U^*}}$$

(eq. A17)

Here, C_0 is a constant. Proceeding as before by substituting eq. (A2), (A8) and (A17) into (A9), expanding exponential terms containing δ variables, dropping terms of second order in the δ variables and rearranging, we obtain:

$$\frac{\partial \delta M_s^*}{\partial t^*} = \left(Bq_s^* e^{-q_s^*/\overline{U^*}} + e^{-\overline{M_s^*}} - 1\right)\delta q_t^* - \delta M_s^* \overline{q_t^*} e^{-\overline{M_s^*}}$$

(eq. A18)

A sinusoidal perturbation of the form

$$\delta q_t^* = d \sin\left(\frac{2\pi t}{p}\right)$$

(eq. A19)

yields the solution

$$\delta M_s^* = G_{q_t^*}\sin\left(\frac{2\pi t}{P} + \varphi_{q_t^*}\right) + C\exp\left\{-\left(\overline{q_t^*} - \left(1 - e^{-q_s^*/\overline{U^*}}\right)q_s^*\right)\frac{t}{p}\right\}\left\{-\left(\overline{q_t^*} - \left(1 - e^{-q_s^*/\overline{U^*}}\right)q_s^*\right)\frac{t}{T}\right\}$$

with

$$G_{q_t^*} = \frac{p}{T}\frac{\left(\frac{q_s^{*2}}{\overline{U^*}^2}\overline{\left(\frac{\partial U^*}{\partial \delta q_t^*}\right)}e^{-q_s^*/\overline{U^*}} - \left(1 - e^{-q_s^*/\overline{U^*}}\right)\frac{q_s^*}{\overline{q_t^*}}\right)d}{\sqrt{\overline{q_t^*}^2\left(\frac{p}{T}\right)^2\left(1 - \left(1 - e^{-q_s^*/U^*}\right)\frac{q_s^*}{\overline{q_t^*}}\right)^2 + 4\pi^2}}$$

(eq. A20)

and

$$\varphi = \tan^{-1}\left(-\frac{2\pi}{\frac{p}{T}\left(\overline{q_t^*} - \left(1 - e^{-q_s^*/\overline{U^*}}\right)q_s^*\right)}\right)$$

(eq. A21)

Summary

Using the system timescale T_S, the phase shift and gain can be generally rewritten as

$$\varphi = \tan^{-1}\left(-2\pi\frac{T_S}{p}\right)$$

(eq. A22)

$$G = \frac{p}{T_S} \frac{Kd}{\sqrt{\left(\frac{p}{T_S}\right)^2 + 4\pi^2}}$$

(eq. A23)

Here, K differs for perturbations in sediment supply and transport capacity, given by the equations

$$K_{q_s^*} = 1 - e^{-\overline{q_s^*}/U^*} + \frac{\overline{q_s^*}}{U^*} e^{-\overline{q_s^*}/U^*}$$

(eq. A24)

$$K_{q_t^*} = \frac{\overline{q_s^{*2}}}{\overline{U^{*2}}} \overline{\left(\frac{\partial U^*}{\partial \delta q_t^*}\right)} e^{-\overline{q_s^*}/\overline{U^*}} - \left(1 - e^{-\overline{q_s^*}/\overline{U^*}}\right) \frac{\overline{q_s^*}}{\overline{\overline{q_t^*}}}$$

(eq. A25)

Notation

Overbars denote time-averaged quantities.

a	Shape parameter in the regularized incomplete Beta function.
A^*	Fraction of exposed (uncovered) bed area.
A_c^*	Fraction of covered bed area.
b	Shape parameter in the regularized incomplete Beta function.
B	Regularized incomplete Beta function.
C	Constant of integration.
C_0	Constant [m^2s/kg].
d	Amplitude of perturbation [kg/m^2s].
D	Sediment deposition rate per bed area [kg/m^2s].
D^*	Dimensionless sediment deposition rate.
D_{50}	Median grain size [m].
e	Base of the natural logarithm.
E	Sediment entrainment rate per bed area [kg/m^2s].
E^*	Dimensionless sediment entrainment rate.
E_{max}	Maximal possible dimensionless sediment entrainment rate.
g	Acceleration due to gravity [m/s^2].
G	Gain [kg/m^2s].
I	Non-dimensional incision rate.
k	Probability of sediment deposition on uncovered parts of the bed, linear implementation.
k_I	Non-dimensional erodibility.
K	Parameter in the gain equation.
L	Characteristic length scale [m].
M_0	Minimum mass per area necessary to cover the bed [kg/m^2].
M_0^*	Dimensionless characteristic sediment mass.
M_m	Mobile sediment mass [kg/m^2].
M_m^*	Dimensionless mobile sediment mass.
M_s	Stationary sediment mass [kg/m^2].
M_s^*	Dimensionless stationary sediment mass.
p	Period of perturbation [s].
p_c	Probability of entrainment, CA model, blocked grains.
p_i	Probability of entrainment, CA model, free grains.
P	Probability of sediment deposition on uncovered parts of the bed.
q_s	Mass sediment transport rate per unit width [kg/ms].
q_s^*	Dimensionless sediment transport rate.
q_t	Mass sediment transport capacity per unit width [kg/ms].
q_t^*	Dimensionless transport capacity.
Q_s^*	Relative sediment supply; sediment transport rate over transport capacity.
Q_t	Mass sediment transport capacity [kg/s].
t	Time variable [s].
t^*	Dimensionless time.
T	Characteristic time scale [s].
T_E	Characteristic time scale for sediment entrainment [s].

T_S	Characteristic system time scale [s].
U	Sediment speed [m/s].
U^*	Dimensionless sediment speed.
x	Dimensional streamwise spatial coordinate [m].
x^*	Dimensionless streamwise spatial coordinate.
y	Dummy variable.
α	Exponent.
γ	Fraction of pore space in the sediment.
δ	denotes time-varying component.
θ	Shields stress.
θ_c	Critical Shields stress.
ρ	Density of water [kg/m^3].
ρ_s	Density of sediment [kg/m^3].
τ	Bed shear stress [N/m^2].
τ_c	Critical bed shear stress at the onset of bedload motion [N/m^2].

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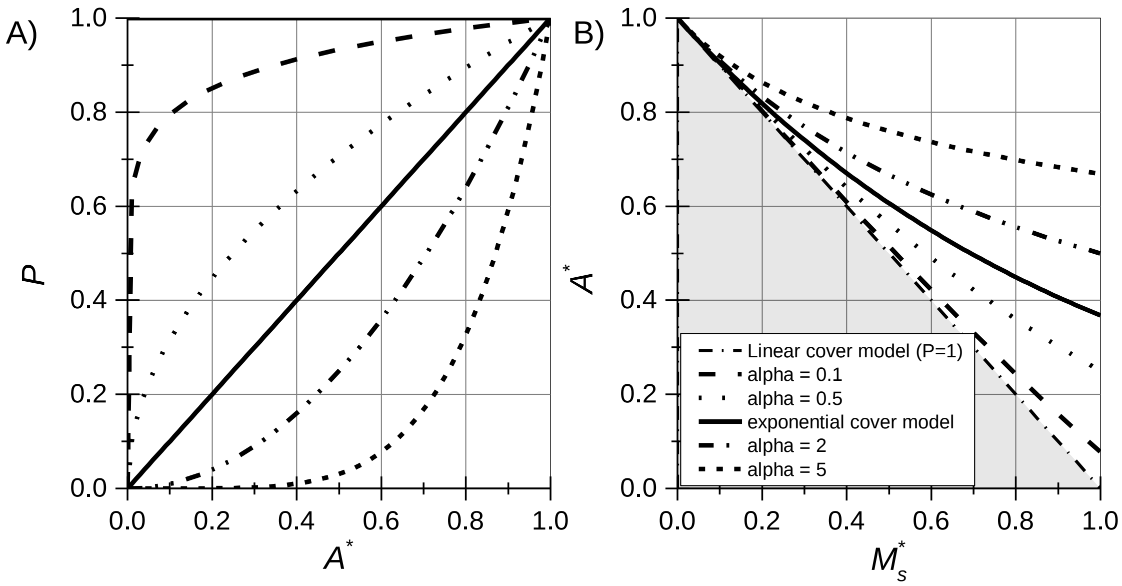

Fig. 1: A) Various examples for the probability function P as a function of bedrock exposure A^*. B) Corresponding analytical solutions for the cover function between A^* and dimensionless sediment mass M_s^* using eq. (7), (9) and (10). Grey shading depicts the area where the cover function cannot run due to conservation of mass.

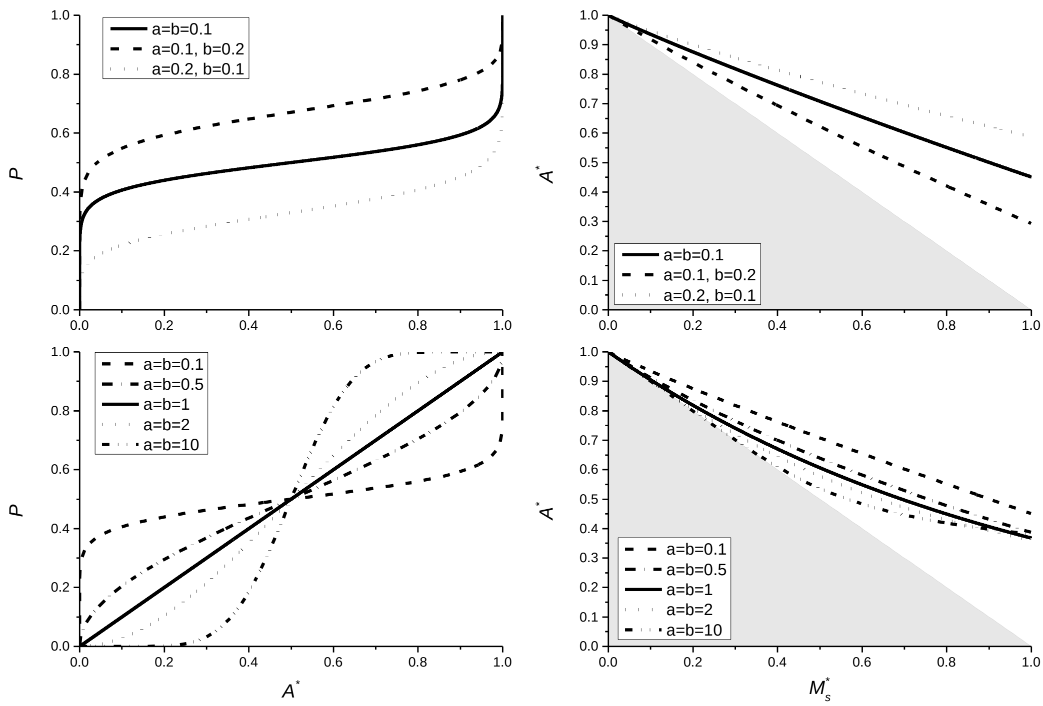

Fig. 2: Examples for the use of the regularized incomplete Beta function (eq. 12) to parameterize P, using various values for the shape parameters a and b. The choice $a = b = 1$ gives a dependence that is equivalent to the exponential cover function. Grey shading depicts the area where the cover function cannot run due to conservation of mass.

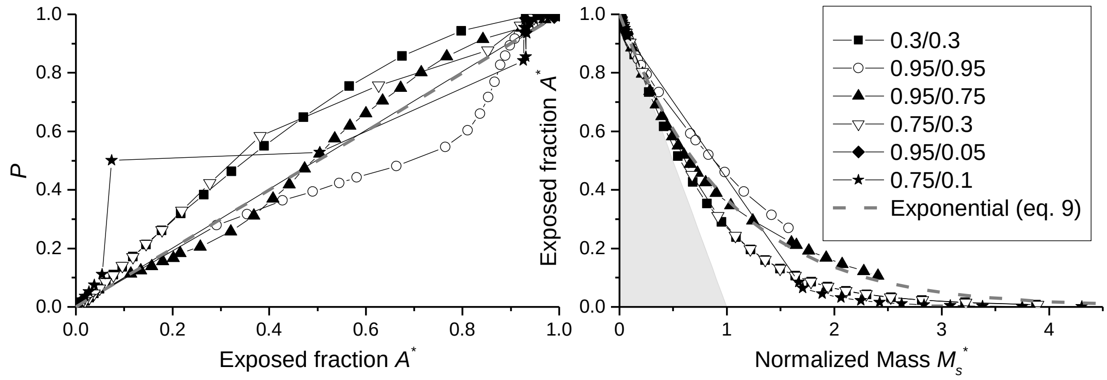

Fig. 3: Probability functions P and cover function derived from data obtained from the model of Hodge and Hoey (2012). The grey dashed line shows the exponential benchmark behavior. Grey shading depicts the area where the cover function cannot run due to conservation of mass. The legend gives values of ~~P_i~~ p_j and ~~P_ϵ~~ p_c used for the runs (see text).

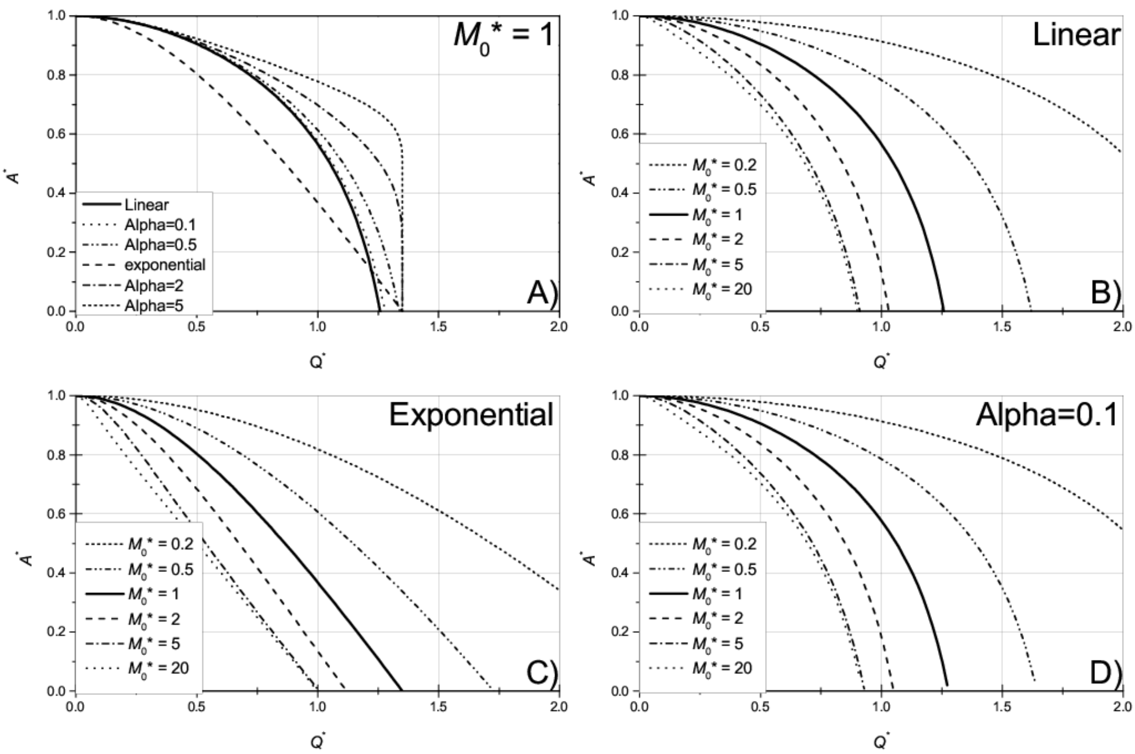

Fig. 4: Analytical solutions at steady state for the exposed fraction of the bed (A*) as a function of relative sediment supply (Q*, cf. Fig. 1). A) Comparison of the different solutions, keeping M_0^* constant at 1. B) Varying M_0^* for the linear case (eq. 31). C) Varying M_0^* for the exponential case (eq. 30). D) Varying M_0^* for the power law case with α = 0.1 (eq. 32).

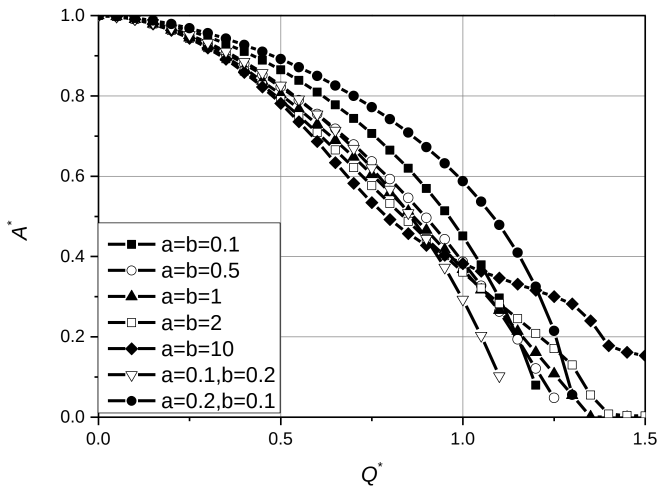

Fig. 5: Steady state solutions using the beta distribution to parameterize P (eq. 11) for a range of parameters a and b, and using M_0^* = 1 (cf. Fig. 2). The solutions were obtained by iterating the equations to a steady state, using initial conditions of A^* = 1 and M_m^* = M_s^* = 0.

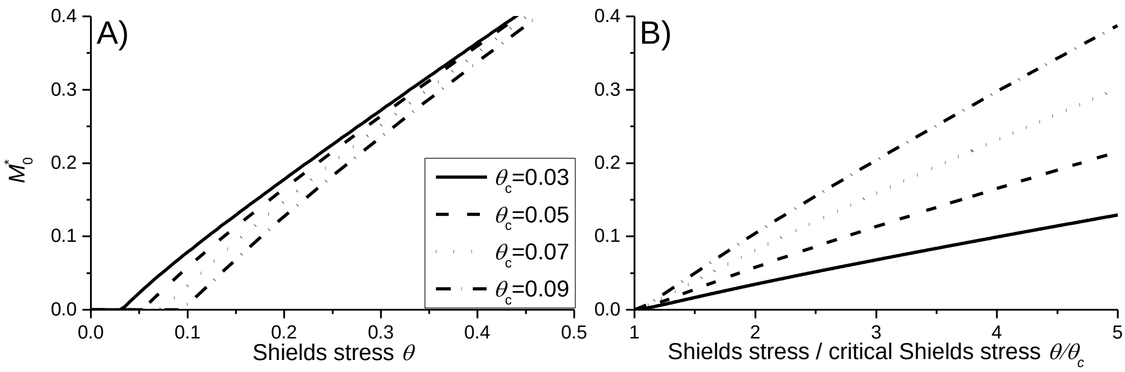

Fig. 6: The characteristic dimensionless mass M_0^* depicted as a function of A) the Shields stress and B) the ratio of Shields stress to critical Shields stress (eq. 37).

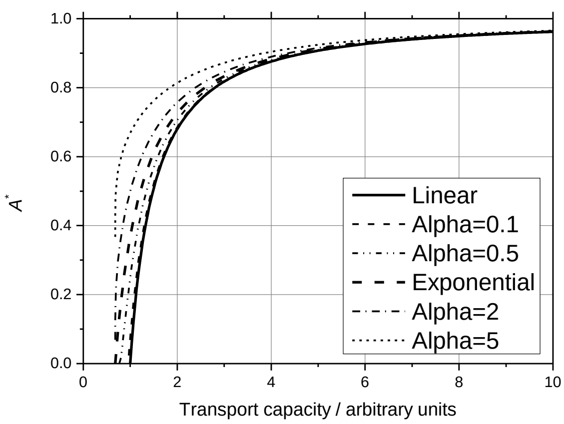

Fig. 7: Variation of the exposed bed fraction as a function of transport capacity, assuming that particle speed scales with transport capacity to the power of one third.

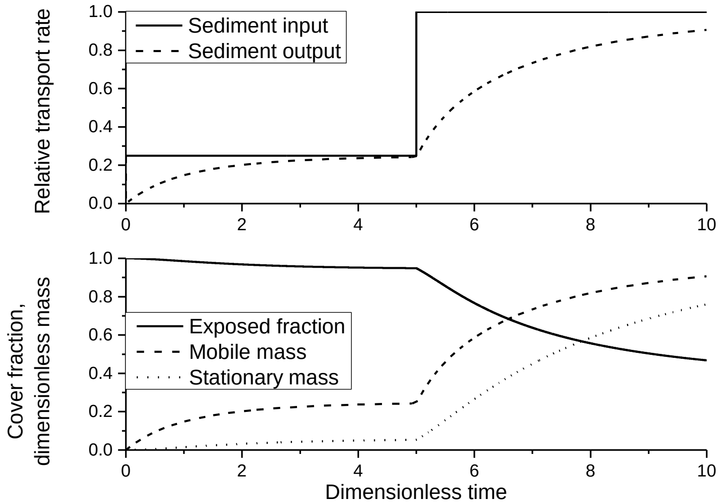

Fig. 8: Temporal evolution of cover for the simple case of a control box with sediment through-flux, based on eqs. (3), (22), (23) and (24). Relative sediment supply (supply normalized by transport capacity) was specified to 0.25 and increased to 1 at t^* = 5. The response of sediment output, mobile and stationary sediment mass and the exposed bed fraction was calculated. Here, we used the exponential function for P (eq. 9) and $M_0^* = U^* = 1$. The initial values were $A^* = 1$ and $M_m^* = M_s^* = 0$.~~Fig. 8: Temporal evolution of cover for a simple case. Here, we used the exponential function for P (eq. 9) and $M_0^* = 1$. The initial values were $A^* = 1$, $M_m^* = M_s^* = 0$ and $q_s^* = 0.25$. Sediment supply was increased to $q_s^* = 1$ at $t^* = 5$.~~

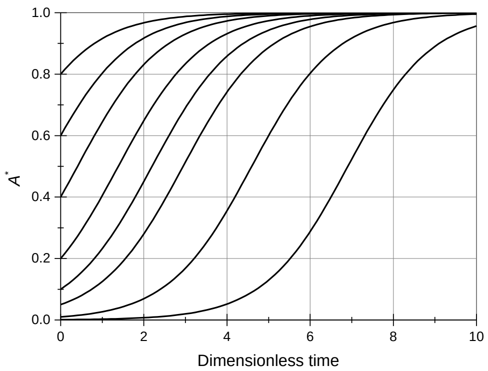

Fig. 9: Evolution of the exposed bed fraction (removal of sediment cover) over time starting with different initial values of bed exposure, for the special case $q_s^* = 0$ (eq. 41) and $q_t^* = 1$.

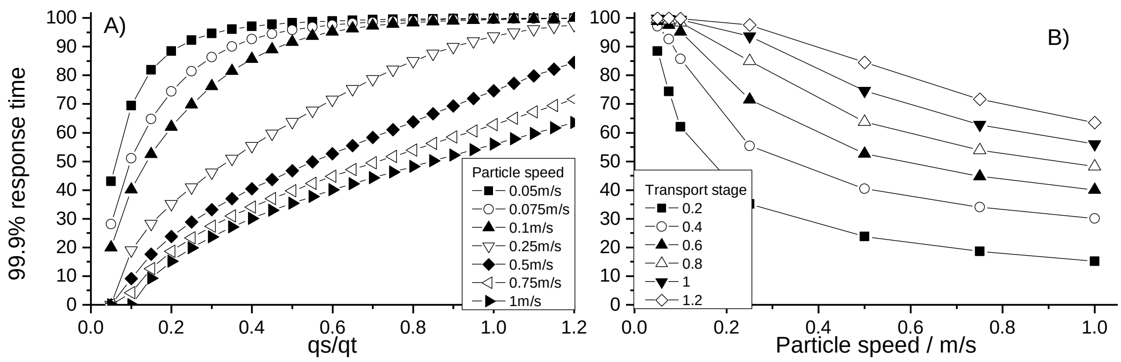

Fig. 10: Dimensionless time to reach 99.9% of the total adjustment in exposed area as a function of A) transport stage and B) particle speed. All simulation were started with $A^* = 1$ and $M_m^* = M_s^* = 0$.

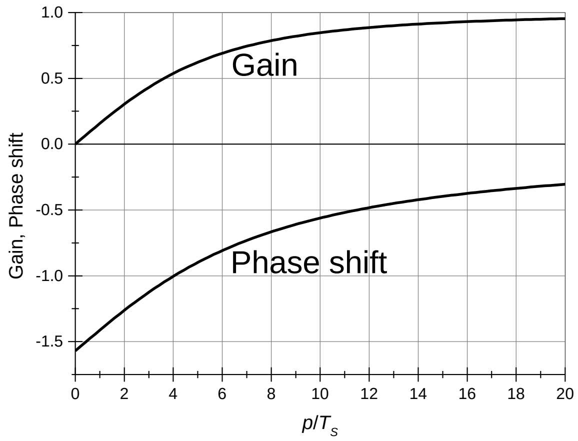

Fig. 11: Phase shift (eq. 50) and gain (eq. 51) as a function of the ratio of the period of perturbation ~~period~~ p and the system time scale T_s. For the calculation, the constant factor in the gain (Kd) was set equal to one.

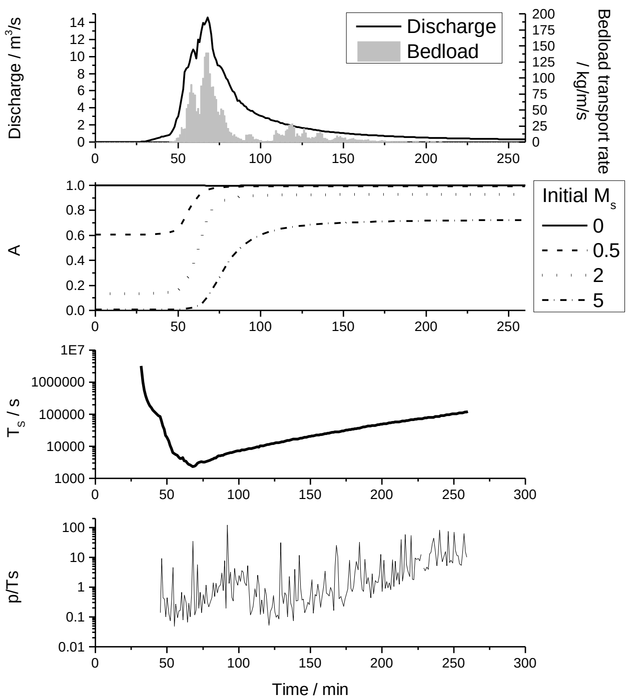

Fig. 12: Calculated evolution of cover during the largest event observed at the Erlenbach on 20th June 2007 (Turowski et al., 2009). Bedload transport rates were measured with the Swiss Plate geophone sensors calibrated with direct bedload samples (Rickenmann et al., 2012). The final fraction of exposed bedrock is strongly dependent on its initial value.