# Interactive comment on "Catchment power and the joint distribution of elevation and travel distance to the outlet" by L. S. Sklar et al. 

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We would first like the thank Reviewer 1 for their constructive comments. In this response we provide answers to all the comments and detail the changes that will be applied in the revised manuscript. Please note that line numbers refer to the numbering of the original discussion manuscript.
Comment 1: "In some parts of the paper, it is claimed that the proposed methodology can be used to answer some specific questions. Here are the examples: - Lines 23-23 saying that the empirical algorithm for generating synthetic source-area power distributions can be used to explore the effects of topography on the water and sediment fluxes passing through catchments. - Lines 64-65 saying that do the distributions of elevation and travel distance to the outlet differ in ways that systematically reflect the factors that

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drive landscape evolution, such as weathering, climate, and tectonics? - Lines 71-73 saying that if the synthetic catchments are able to explore how factors such as area, relief, and profile concavity influence catchment power. Unfortunately, none of the above questions are addressed in this paper, except a few qualitative explanations."
Answer: We appreciate the reviewer's interest in seeing these proposed applications of the methodology. The statements referred to in the comment all occur in the abstract and the introduction, and contribute to the motivation for developing the model. It is beyond the scope of this paper to both develop and apply the model. Subsequent applications of the model are best done in subsequent papers. Since discussion of this paper began, already we have submitted and had accepted the first of several planned papers that apply the model to addressing the types of questions referred to in this comment. The paper is Lukens et al., "Grain size bias in cosmogenic nuclide studies of stream sediment in steep terrain", currently in press at the Journal of Geophysical Research - Earth Surface. In that paper, we use the catchment power framework, and the algorithm for generating synthetic distributions of elevation and travel distance developed here, for evaluating risk of bias in estimating catchment average erosion rates when sampling a single sediment size class (e.g. medium sand) given a spatial gradient in the size distribution of sediments produced on hillslopes. It would not have been practical to include the results of that application of the model within this paper.
Changes in the manuscript: Abstract passage beginning on Line 23 has been change to read "We then develop an empirical algorithm for generating synthetic source-area power distributions, which can be parameterized with data from natural catchments. This new way of quantifying the three-dimensional geometry of catchments can be used to explore the effects of topography on the distribution on fluxes of water, sediment, isotopes and other landscape products passing through catchment outlets, and may provide a fresh perspective on problems of both practical and theoretical interest."
Introduction passage beginning on Line 71 has been changed to read "Next, using our analyses of the elevation and travel distance distributions from the study catchments,
we develop an approach for generating synthetic catchments that capture many features of power distributions in natural landscapes. Finally, we discuss how our approach can be used to explore how factors such as area, relief, and profile concavity influence catchment power and more broadly how rivers are influenced by hillslope sources of water, solutes, and sediment (e.g. Lukens et al., 2016)."

Comment 2: "In figure 1, do the given profiles correspond to the longest flow path in those catchments? Also what extra information does this figure provide in comparison to figure 4?"

Answer: As explained in the figure 1 caption, what we refer to as the "channel profile" is the lowest elevation for that travel distance while the "ridge profile" is the highest elevation for that travel distance. The longest flow path in each catchment corresponds to the highest and most distant point shown. This figure has three purposes that set it apart from figure 4. It introduces the study catchments and their geographic locations, and it provides a simple comparison of the relative scale of each in terms of both elevation and travel distance. It also graphically poses the key questions that we seek to answer in this paper, how do we fill in the blank space between these two types of profiles? In other words, what is the joint distribution of elevation and travel distance to the outlet, and how do these distributions differ between catchments? Figure 4 has a very different purpose. It begins to answer those questions by filling in the blank space between the profiles with data and calculated density distributions. In figure 4 it is not practical to plot the three catchments at the same scale, so the relative scales of the catchments must be shown elsewhere; we chose to do that right away in figure 1.

Changes in the manuscript: The first part of the figure 1 caption has been changed to read: "Left: Location map of study catchments in California, USA. Right: Elevation profiles of the lowest point at each travel distance (i.e., the mainstem channel) and the highest point at each travel distance (referred to here as the ridge profile). The longest and shortest travel distances in each catchment are the points where the two profiles meet."

Comment 3: "Lines 118-120 are not clear at all from figure 2. Authors might want to clarify it directly in the figure 2 ."

Answer: The text in question reads "Conversely, for a given travel distance, elevations are highest at the ridges and lowest in the valley axis. These patterns are especially clear at Inyo Creek (Fig. 2a) and Providence Creek (Fig. 2b), which drain small, relatively undissected catchments." The best way to see this is to follow the boundary between two color bands, which represents a contour of fixed travel distance, and consider how the distance to the nearest elevation contour (black) line changes. Fig. 2a (Inyo Creek) is provides the clearest example. Consider the travel distance contour of 0.6 times the maximum travel distance, which is the boundary between the dark green and yellow color bands. An elevation contour line crosses this travel distance contour right at the valley axis. As one follows either the color band boundary or the elevation contour, the contours diverge. The elevation contour crosses into lower travel distance bands while the travel distance contour crosses higher elevation contour lines. With that example in mind, the same pattern can be discerned in the other two catchments, although the increase in scale introduces substantial variability.
Changes in the manuscript: Starting with the sentence beginning at line 117, the passage has been changed to read: "This pattern is especially clear at Inyo Creek (Fig. 2a) and Providence Creek (Fig. 2b), which drain small, relatively undissected catchments. In particular, as can be seen in Fig. 2a by following a given elevation contour (black lines), travel distances (color bands) are longest in the valley axis and shortest at the ridges. Conversely, for a given travel distance (i.e. following a boundary between color bands), elevations are highest at the ridges and lowest in the valley axis."
Comment 4: "In lines 166-167, it is said that "the joint distribution plots generally show dense concentrations of data points at low elevations for any given travel distance". This is not definitely true based on the color bar given in figure 4d. For instance in the Providence Creek, very small concentrations of data points exist at low elevations over a wide range of travel distances less than 4000 m . Similar observation can be made for
the Inyo Creek for travel distances less than 3000 m , except some high concentration data points spanning around travel distances of 1500 m ."
Answer: The original text was unclear, thank you for pointing this out. The observation is about relative elevation for a given travel distance. In other words, looking only at a vertical column of data points, the thinnest concentrations occur at relatively high elevations (for that travel distance), and the thickest concentrations appear to occur near the bottom of that vertical stack of points. However, the question of where the point densities are greatest is addressed in the next paragraph, so the best solution is to simply cut this sentence entirely, and better explain the point in the subsequent paragraph.
Changes in the manuscript: The final sentence of the paragraph (which began at line 157) now reads "Meanwhile, many paired values are so common that they overlap, particularly along flowpaths that converge near the mainstem channel."
We have also modified the subsequent paragraph to better address this point, the relevant passage now reads: "These binned representations of the raw data show that, for a given travel distance, the lowest point densities (point area $=100 \mathrm{~m} 2$ ) generally occur at the highest relative elevations. As relative elevation decreases within a vertical stack of data, point density typically increases to a peak and then approaches zero at the channel elevation. In general, peak densities for a given travel distance occur closer to the channel than the ridge elevation, although there are notable exceptions."
Comment 5: "In lines 169-171, it is said that "for a given travel distance, as elevation decreases, data point density generally increases to a peak and then quickly tapers to zero." Should not it be as elevation increases? At least all the plots (4c, 4e, 4f) show that for any given travel distance, the data point density goes to zero at the highest elevations (depicted by black colors)."

Answer: This sentence was unclear. The changes made in response to Comment 4 also address the problems noted in this comment.

Comment 6: "In lines 171-172, it is said that "they also show that the density of paired values is highest at 60 and $80 \%$ of the maximum travel distance". This is not true at all except for the Noyo River, while the statement is given in a general sense.

Answer: Thank you again for highlighting the lack of clarity in this paragraph overall. In this sentence we wish to draw the reader's attention to where in each catchment the high point-densities are most common. We refer here to the bins colored red and brown, i.e. normalized density $>0.6$. For Inyo and Providence Creeks, that occurs in the upstream-most third of the catchment; for Noyo it is the entire upper half of the catchment.

Changes in the manuscript: The sentence beginning on line 171 has been rewritten to read: "Figure 4 (d-f) also shows that the greatest frequency of the high point-density (normalized density $>0.6$ ) primarily occurs in the upper third of Inyo and Providence Creeks, and in the upper half of Noyo Creek."

Comment 7: "First of all, direct comparison of figures 3 and 4 is not easy as the horizontal axes show the same quantity, but different ranges (authors might want to make it consistent throughout the paper). Second, figure 3 shows that at the Noyo River, the majority of the area pertains to long travel distances and low elevations. Can the authors explain why this is not reflected in figure $4 f$ where highly dense data points correspond to mid travel distances and relatively high elevations?

Answer: This is a very helpful comment in that it illuminates the challenges in thinking beyond the conventional use of hypsometry and width function, which represent elevation and travel distributions separately. In response to the first part of this comment, the horizontal axes in the six figure 4 panels are travel distance, scaled to extend over the same length on the page for each catchment. So in that respect they are directly comparable to the normalized travel distance shown on the horizontal axis in figure 3b. Figure 3 a has normalized elevation on the horizontal axis, so comparison is a bit more difficult because in figure 4 elevation is shown on the vertical axes. But the scales of
vertical exaggeration are adjusted to the ranges span the same length on the page, so if the reader can mentally rotate the figures the elevation axes are directly comparable.

What truly makes these two sets of figures difficult to compare is how area is represented. In both panels in figure 3, area is on the vertical axis, while in figure 4 area has no axis, rather it is distributed throughout the interior of the figure (and plotted as color contours in panels $4 d-f)$. To see how the same data can produce these quite different graphical representations, one must mentally integrate across figure 4 to compare with figure 3. In response to the second part of this comment, in Figure 4f, when one integrates horizontally across a band of elevation, one sums area across nearly all travel distances for the relatively low elevations. This sum includes many if not most of the highest point density regions, and thus highest area regions. When one sums across the higher elevation bands, data are only encountered (and summed) at the highest travel distances and thus sum to lower totals. This explains why the (hypsometric) distribution of area with elevation shown in figure 3a shows the majority of the area at relatively low elevations. There is no contradiction between the two figures, the underlying data are the same. The same exercise can be done in comparing figures 4 f and $3 b$ by integrating figure $4 f$ vertically, thus summing area for a given travel distance. The vertical extent of the data cloud (the local relief for a given travel distance) gradually increases toward the right, as does the point density (and thus area), which is much greater on average for the longer travel distances. This pattern of vertical integration corresponds exactly to the distribution of area with travel distance shown in Figure 3b.
A key point is that plotting the joint distribution of elevation and travel distance reveals why the width function and hypsometric curves covary the way they do. It shows where area is concentrated in the vertical and horizontal structure of the catchment. Another way to put it is Figure 3 can be derived from Figure 4, but not the other way around.
Changes to the manuscript: We have added a new paragraph that will be inserted at line 173, which reads: "These patterns in the density of paired values of elevation and travel distance help explain the shapes of the corresponding hypsometry and width
functions. For example, Figure 3 shows that in the Noyo Creek catchment the majority of area occurs at relatively long travel distances and relatively low elevations. Yet Figure $4 f$ shows that this does not mean that the highest densities of catchment area occur at points that have both long travel distance and low elevation. Rather, low elevations dominate across all travel distances, and summing area horizontally across figure 4 f leads to higher total area in the lower elevation bins of Figure 3a. Similarly, the Noyo catchment has greater relief at longer travel distances, and summing area vertically across fig. $4 f$ leads to higher total area in the longer travel distance bins of Figure 3b. This comparison demonstrates that the joint distribution of elevation and travel distance reveals where area is distributed in the vertical and horizontal structure of the catchment in ways that the hypsometry and width function cannot."

We have also changed the first sentence of the next paragraph, to read: "Comparisons of the joint distributions between catchments also reveal significant differences that cannot be inferred from the conventional representations of vertical and horizontal catchment structure in Fig. 3."
Comment 8: "In lines 231-233, it is said that "in landscapes where rates of precipitation and erosion are spatially variable and sometimes correlated, we expect the distributions of power and mean slopes to differ". Then the Inyo Creek catchment is mentioned as an example of this case. But comparing figures 5 and 6 a does not support this at all, i.e., the spatial patterns of "water" power and mean slopes are very identical to each other in this catchment. How do the authors explain this?"

Answer: Thank you for pointing out the need for greater clarity in describing the differences in between the "distributions of power and mean slopes" at Inyo Creek (note that we did not use the phrase "spatial patterns"). The text beginning on line 237 is intended to describe the differences in question. Careful examination of the histograms in figures 5 a and 6 a (as well as 6 b ) shows that the shapes of the distributions are significantly different: for mean slope the distribution is negatively skewed, with a long tail of relatively low values, whereas the power distributions have a positive skew, with a
long tail of relatively high values. As noted in the original text, the shape of the power contours are also shifted toward the ridges. These differences in contour pattern within one catchment are not as great as the differences between catchments shown in figure 5 , however some of those differences can be attributed to the relative size of the catchments.

Changes to the manuscript: We have changed the passage that began on line 237 to read: "When we combine these relationships for water and sediment production with the distribution of mean slopes using Equation 3, we can create histograms and maps showing the distributions of source-area power for the two materials, water and sediment (Fig. 6a-b). For both materials, the shape of the distributions shift from negative skew to positive skew, and the power contours are stretched towards the catchment divide, relative to the case of uniform precipitation and erosion (equivalent to Fig. 5a). The difference is greatest for the case of spatially varying erosion (Fig. $6 b)$, due to the nonlinear relationship between erosion rate and elevation. Thus for catchments with spatial variation in the rate of production of water or sediment, mean slope distributions cannot reliably predict distributions of source-area power."

Comment 9: "In figure 12, it seems that the hypsometric curves and width functions generated with the partially-synthetic formulation using actual profiles fits better to the real data than the fully-synthetic formulation using modeled profiles in the Noyo River. But the reverse is observed for the Providence Creek. Can the authors explain why the partially synthetic formulation using actual profiles should not always result in better fitting?"

Answer: Thank you for this comment because it reveals that we mislabeled the model curves in Figure 12e, the hyposometry for the Noyo River catchment. The two model curves should be reversed. The reviewer is correct in assuming that the partially synthetic formulation using actual channel and ridge profiles should always result in a closer fit to the curves calculated from the DEM. This is the case for all six of the comparisons shown in Figure 12. To quantify the relative goodness of fit we have calculated
the RMSE of the deviations between the model curves and the DEM. Comparing the partially synthetic with fully synthetic, for the Inyo Creek hypsometry and width function the values are 4.2 versus 5.7 , and 4.6 versus 5.0 respectively. Making the same comparison for Providence Creek, the values are 12.5 versus 15.7 , and 12.3 versus 14.2 respectively. And for Noyo River, the values are 2.1 versus 9.2 , and 7.8 versus 10.1 respectively.

Change to the manuscript: We have revised Figure 12 to show the correct labeling of the curves for the Noyo River Hypsometry, and to list the RMSE values in the legends on each panel. The revised figure will be uploaded with this reply.

The caption to Figure 12 has been revised to include an additional sentence at the end: "Values in parenthesis indicate RMSE calculated by comparing model curves with DEM."

Technical corrections: Line 438: figure 11 instead of 10 Line 444: figure 11A instead of 10A Line 456: figure 12 instead of 11 Line 477: figure 11 instead of 10 Line 478: figure 12 instead of 11 Line 482: figure 12a-b instead of 11a-b Line 484: figure 12c-d instead of 11 c -d Line 486: figure 12 c -d instead of $11 \mathrm{c}-\mathrm{d}$

Response: Thank you for noting the incorrect numbering of the figures. One source of the errors is that figures 10 and 11 should be reversed. The fully synthetic model result should be figure 10 and the best fit beta distributions for Providence Creek and Noyo River should be figure 11.
Changes to the manuscript: Figures 10 and 11 have been reversed in the sequence of figures. The figure numbering in the text has been double checked throughout the manuscript and all needed corrections have been made.

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Fig. 1. Figure 12. Comparison of actual with modeled hypsometric curves and width functions for three study catchments.


[^0]:    Interactive comment on Earth Surf. Dynam. Discuss., doi:10.5194/esurf-2016-9, 2016.

