Authors' Response to Reviewer 1

Reviewer 1: *This paper describes a new approach to incorporating lateral channel erosion into landscape evolution models. This is clearly a worthwhile goal and I was excited to read this. That said, there are aspects of the model setup and motivation, as described below, that I think can be improved upon and which I think will lead to a paper with more impact.*

Authors: Thank you, we appreciate your suggestions to help our manuscript have more impact and interest to a broad audience.

Reviewer 1: *1. Specifically, this paper uses a curvature based wall erosion law. While the authors don't expressly say they are modeling meandering, this is the implication of the choice of model. This makes sense as meanders are ubiquitous in bedrock channels and the process is clearly important in many settings.*

Authors: We view the class of "streams with fully developed meandering" as a relatively small subset of "streams able to widen valleys through lateral erosion." In our field experience, there are plenty of examples of streams that most geomorphologists would classify as single-thread, and yet which clearly show evidence of erosion and lateral migration at locations where an outer bend in the channel impinges on a valley wall or terrace. Conceptually, therefore, our approach is not meant to represent exclusively channels with fully developed meandering. To clarify this point for readers, we have added text to the manuscript in the section describing the lateral erosion component of the model.

Reviewer 1: *The first numerical model of river meandering that I am aware of is Howard and Knutson (1984). Their first iteration of the model is one in which erosion scales inversely with the radius of curvature, which is basically the same as the model posed in equation 10. Howard and Knutson point out that such a model results in a channel that breaks down into 3 point bends with alternating positive and negative curvature. When applied to an existing meander bend, the bend can't be maintained. The ultimate conclusion of Howard and Knutson (1984) is that lateral channel motion can't be driven only by local curvature because such a model fails to produce realistic meander kinematics (down stream translation, cutoffs) as well as realistic meander forms. This is what leads to their downstream convolution approach, which in a simply way simulates the advection of the effects of upstream curvature downstream. Given that the setup of the model in the submitted MS is based on a centrifugal acceleration argument, and given that the morphologies of the channels produced in the model are reminiscent of the 3 point bends described by Howard and Knutson, it's not clear to me how this model represents a significant advance in understanding and modeling lateral erosion. Moreover, it's not clear how the river even changes from moving in one lateral direction to the other.*

Authors: These comments arise reflect an understandable confusion about the key differences between a meander model and a landscape evolution model. We have now added a substantial amount of text to the new "Approach and Scope" section to articulate these differences. In brief, the former represents the trace of a single channel whereas the latter represents the topography in which channels are embedded. This is a very important distinction, which we hope is now clear in the revised manuscript.

Example excerpt from text added to new "Approach and Scope" section: "Considerable advances have been made in developing theory and models for the planform dynamics of single-thread meandering channels. As a result, the scientific community has a good understanding of how meander patterns form and evolve, and how meander wavelength and migration rate scale with properties such as water discharge, valley gradient, and sediment grain size (Hooke, 1975; Nanson and Hickin, 1986; Schumm, 1967; Langbein and Leopold, 1966; Lancaster and Bras, 2002, e.g.). This body of work addresses the planform pattern of river channels, but does not deal with the broader drainage-basin topography in which those channels are embedded. [...] There is also a well-developed literature on process models of landscape evolution, and in particular the evolution of ridge-valley topography sculpted around drainage networks. We refer to these models as Landscape Evolution Models, or LEMs. LEMs differ from meander models in treating a self-forming, two-dimensional flow network rather than a single channel reach, and in explicitly modeling the evolution of topography."

Reviewer 1: *What is novel, from my perspective, are the two different formulations of the wall erosion law. Why not, then, simply use the Howard and Knutson meandering model and then explore how the two different wall erosion formulations influence the emergent valley form? Given that field evidence that can discriminate between the two proposed lateral erosion processes should be straightforward to collect, I could see such an exercise leading to numerous field testable hypotheses.*

Authors: Identifying field sites and collecting data to evaluate the model's performance is part of the future plan, but is beyond the scope of this manuscript, which is meant to introduce the model to the community. See also response below.

Reviewer 1: *2. While I like the exploratory aspect of this paper, I think it could benefit from either a sharply formulated research hypothesis or a field example or two that are targeted. As is, it's not clear how we can evaluate the performance of the model other than by simply noting that the river causes the valley walls to move. But I think we could do better.*

Authors: We have added a figure with examples of bedrock valleys and strath terraces that are much wider than their channels in several different environments, including wide valleys created by both meandering and non-sinuous rivers. This figure demonstrates qualitatively the differences between a typical narrow bedrock valley and a valley that has experienced a phase of significant lateral erosion. We have also added a significant amount of text at the end of the discussion section where we discuss different measurements and metrics needed from field or lab experiments in order to test this and future models. We also present a potential test of the model presented in the manuscript in a specific field site.

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Nanson, G. C. and Hickin, E. J. (1986). A statistical analysis of bank erosion and channel migration in western canada. *Geological Society of America Bulletin*, 97(4):497–504.

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Authors' Response to Reviewer 2

Reviewer 2: *This article aims at developing and implementing a model of lateral mobility of rivers in long-term landscape evolution model of mountain ranges. This is timely needed as the lateral mobility of river is now known to play a significant contribution in landscape reshaping, and as most current numerical models of landscape evolution predict valley bottom that are simply 1 pixel wide and fixed in time.*

Authors: Thank you for recognizing the importance of the issue we address in the paper and taking the time to make many relevant and helpful comments to improve our manuscript.

Reviewer 2: *The problem is that I find that the numerical implementation have several flaws which prevent me from trusting the model outcome at this stage. Numerical modellers all know that it is very easy to create landscapes that look ok if you have some large degree of freedom in choosing your model parameters (erodibility, runoff, channel width coefficient etc. . .). Here, the modelling results look ok, as the model is tuned to looks right, but that does not mean that the dynamics and timing are relevant to natural systems, which is what we ultimately expect from a landscape evolution model. And because there's no real attempt to validate model predictions against quantified observables, it is very difficult, given some of the flaw in the implementation, to infer reliable results pertaining to the dynamics of natural mountain valleys.*

Authors: We readily admit that this is a model without a quantitative test... yet. Qualitative reproduction of commonly observed landforms may be a weak test, but it an essential one: if a model does not pass that bar, then it is clearly a failure (albeit possibly an instructive one). Moreover, the history of science is full of examples in which theory precedes empirical confirmation. Nonetheless, it is fair to expect us to provide some ideas on how this model could be tested, and we now do so in the discussion section.

Reviewer 2: *The model presented here is a non-hydrodynamic model aiming at including a "channel mobility" component. This is a great idea, and indeed barely addressed by landscape evolution models. But it is not strictly speaking a "bank erosion model" as it does not resolve 2D flow hydrodynamics. Yet there are many instances in the paper, where the model has some kind of schizophrenic behavior between the two types of models:*

First it uses a relatively small pixel size (10 m), which assumes practically that the channel width must never be larger than 10 m. Unfortunately, this condition is not verified all the time (unless I've missed something in the calculations): the basic model uses a drainage area of 20000 m2 , which coupled with a runoff of 36 mm/hr, and kw=10, gives Wmin = 4.5 m. However, multiplying the drainage area to 160000m2 (section 4.2.1) violates this assumption from the inlet of the model Wmin = 12.65 m. At this point flow should be partitioned over 2 pixels to correctly resolve the equations. I don't know how this bias affects the model predictions, and how such a model could be uspcaled to larger catchments where channel width would be several pixel wide (here we're dealing with small catchments of km2 size).

Authors: It is important to recognize that channel width is not explicitly represented in the model we describe. Rather, it is one element of the lumped parameters K_v and K_l. The channel-width scaling parameter values we discuss, and which the reviewer quotes, are used only in the estimation of reasonable ranges for these parameters. So it is not really correct to say that the model channels are wider in some instances than their grid cells, because channels have no explicitly defined width (though the possibility exists that the "implied width" could potentially be wider than a cell: a problem common to all non-hydrodynamic LEMs). We have added text to section describing vertical and lateral erosion equations to make this point.

The reviewer remarks that grid cells are "relatively small". The word "relatively" is important here. Presumably he means small relative to what one would expect for channel width. If we consider that channel width tends to scale as the square root of drainage area, then all else equal it should also scale with the characteristic length of the drainage basin, or in the case of a model, the side length of the domain. Double this characteristic length scale, and you should also double the expected width of the largest channel. Given this scaling relation, it does not really make sense to speak in terms of the absolute size of model grid cell. Rather, it makes more sense to consider grid size in relation to the scale of the largest drainage basin. In that respect, our model resolution (considered as the ratio of cell size to domain size) is not notably different from that of most other non-hydrodynamic LEMs.

Nonetheless, the reviewer raises an important general critique of LEMs that use single-direction flow-routing schemes: it is possible in principle to have an "implied width" (implied, that is, by the width-discharge relation embedded in K) that is larger than a grid-cell size. This issue is not unique to our particular model; any non-hydrodynamic LEM with sufficient resolution would face the same inconsistency. We agree that it is an issue that should be resolved (interestingly, the same kind of issue arises in other fields, such as the representation of convection cells in atmospheric models or turbulent eddies in 3D flow models). However, our intent here lies not in re-writing the hydrology parameterization for LEMs, but rather with the more narrow goal of investigating how lateral erosion might be implemented within the context of an otherwise fairly generic and common model formulation, without excessive complexity. Therefore, while we acknowledge the "channel in cell" issue as a general problem for non-hydrodynamic LEMs (and indeed related to similar issues we don't think this paper is the right place to roll out a proposed solution. However, we have added text in the supplementary materials section that notes the existence of this issue and the need for an ultimate solution.

Reviewer 2: - *There is no real notion of "bank" in the model given that the channel is defined at sub grid, but rather some kind of "valley side". This makes it difficult to directly relate lateral erosion "end members" (fig.1 section 3.1) to actual physical processes. These are more numerical tricks to resolve vertical feature horizontal migration on fixed horizontal grid, but whose relevance to natural processes is quite debatable. They introduce artificial thresholds in model dynamics whose consequences are not explored thoroughly.*

Authors: Perhaps so, but note that the physical sciences are full of such "tricks". What, for example, is the "true" meaning of viscosity in a liquid? The linear viscosity law is just a parameterization ("trick") too, which happens to work well for certain materials under a certain range of conditions (and fails for others). Maybe our trick will ultimately prove to perform poorly when compared with data, yet by introducing it we draw attention to what we hypothesize is an important process in valley widening: the physical disaggregation of material due to erosional undermining and collapse.

The model end member section was revised (P9L16-30) to emphasize that in one formulation (total block erosion), lateral erosion scales with valley wall height and in the other (undercutting slump), lateral erosion is independent of valley wall height. All discussion of relevance to natural processes is moved to the discussion section.

Reviewer 2: *The model implementation assumes that the channel is always in contact with the neighbour node (there is systematically lateral erosion), which contradicts the underlying assumption that channel width is smaller than the pixel size.*

Authors: See response below under "two components missing in the model", item 2.

Reviewer 2: *The model does not account for lateral deposition which is an important driver of channel migration (but that's not the most critical point)*

Authors: Sustained deposition on surfaces dipping more than several degrees is so rare that we consider it a reasonable thing to neglect.

Reviewer 2: *On top of this, there is an important limitation in the "undercutting- slump" model in assuming that flow depth only depend on discharge (eq. 30) while it must depend on slope (and width, but given that it is fixed by discharge in the model, there's no way to do better).*

Authors: We neglect the influence of slope on water depth because its influence is much less than that of discharge. For example, the Manning equation states that depth scales like $Q^{3/5}$ but scales like $S^{3/10}$.

Reviewer 2: *Hence I see at least two components missing in the model: 1: A proper way to deal with cases in which the channel width becomes larger than the pixel size (as predicted by $kwQ^{0.5}$): either you increase the pixel size (but this also increases the "numerical" threshold for channel migration), or you introduce some kind of flow partitioning/simplified 2D hydrodynamics (but then we're very close to existing models like CAESAR or EROS). I know width is lumped in the model through kw, but either you assume your channel width is never larger than 10 m (that's quite a limiting factor), or you have to partition the flow over several pixels.*

Authors: See prior responses regarding the treatment of width in our model.

Reviewer 2: *2: Adding a way to either explicitly or implicitly account for the sub-pixel position of the channel. For instance a kind of likelihood of bank erosion (which is a function of the ratio of channel width to pixel size) with an asymmetric probability related to along stream curvature.*

Authors: We are aware of course of the excellent work by Hancock and Anderson (2002) that relates valley widening rate to the ratio of channel to valley width. We had originally avoided implementing such a rule because, as noted earlier, the model does not explicitly define channel width. However, even without tracking width explicitly, one could assume $W \sim A^{1/2}$ scaling and therefore allow a similar scaling in lateral erosion rate. One way to address this issue in the model is to multiply the erosion rate by the ratio of channel width/dx so that lateral erosion is decreased in narrow streams and enhanced in larger streams. We have created a version of the model that implements this rule, and run a series test models to evaluate the result. As expected, there is less lateral erosion in the smaller streams in the upper parts of the model, but little change in valley width and channel mobility in the lower parts of the channel. These figures and discussion of the modified model are included in the supplementary materials.

As to the notion of tracking sub-cell channel position: We are delighted that the manuscript is already provoking new ideas about how to address the problem that we've set out to highlight. Indeed, we spent a long time considering various approaches, including one in which channel position within a cell is explicitly tracked. We ultimately settled on the alternative method the paper describes out of considerations for simplicity. Complexity in theory and models comes at a cost. Our philosophy is that the goal of science is to understand things, and if a model becomes too complex to understand, well then all we've succeeded in doing is creating yet another thing we don't understand. In our view, the justification for adding something to a model should be a clear demonstration that the model doesn't "work" (i.e., account for an observed phenomenon) without that thing. So, we've leaned on the side of simplicity. If this paper stimulates others to come up with a demonstrably better approach, then we'll have succeeded in one of our main objectives.

Reviewer 2: *I also note that, even if it is not common practice in the literature of landscape* *evolution models (it should), it is important for any numerical model implementation, to demonstrate that the model results do not systematically depend on grid size (within limits) and time-step, or to acknowledge this dependency and demonstrate how it impact results.*

Authors: A brief overview of model runs with the same domain size and grid size of 15 m and 20 m is included in the supplementary materials.

Reviewer 2: *Also, I would also like to see the model evolve from an initial condition with the lateral erosion "on", and not activated only when the landscape and drainage is already organized: if a model works, it works all the time, and actually exploring drainage development on a plateau could tell us whether you generate realistic patterns or not.*

Authors: The models can be run from an initial condition with lateral erosion. There is no observable difference in model topography. Figures and a brief discussion are included in the supplementary materials.

Reviewer 2: *Other comments Title: it is currently slightly misleading as there is no real evaluation nor comparison of the model prediction with actual results, and the link with the mechanics of bedrock channel bank erosion are extremely tenuous or not really clear. Something like: "Implementing lateral mobility of channels in landscape evolution" models would more represent the actual content of the paper.*

Authors: We have changed the title slightly to more accurately reflect the content of the paper. The new title of the paper is: Developing and exploring a theory for the lateral erosion of bedrock channels for use in landscape evolution models.

Reviewer 2: *Missing literature: The CAESAR numerical model...*

Authors: Missing literature added in background section and throughout the manuscript as noted below.

Reviewer 2: *P2 L23 : I tend to disagree with this statement: some models of channel width adjustment have been proposed, but none can actually fully explain the variety of responses found in nature (see Lague, 2014 ESPL, for a synthesis). As for incision thresholds, which can only been adequately accounted for if discharge variability is explicitly modelled, only two models that I know of properly account for it (CHILD, EROS and LANDLAB ?).*

Authors: Changed text to read: While theories that account for dynamic adjustment to bedrock channel width continue to be refined (Lague, 2014), landscape evolution models that include a relationship between sediment size and cover (e.g. Sklar and Dietrich, 2004), and incision thresholds in bedrock channels (Tucker et al, 2001; Crave and Davy, 2001; Tucker et al., 2013) are available and widely used (Tucker and Hancock, 2013).

Reviewer 2: *P2 L24: rarely: could you specify which models actually includes it?*

Authors: Changed text to read: "existing models do not address the lateral erosion of bedrock channel walls"

Reviewer 2: *Section 2.1 : in this section, the author should emphasize more systematically that the "theory" presented is an assumption of the model. Too often, it is presented almost as a fact or acknowledged theory:*

Authors: Changed text to clarify to readers that vertical incision in our model is represented by the stream power model and added text about we chose this model in the discussion section.

Reviewer 2: *P5L7 : given the emphasis in the introduction of the role of dynamic width, Im surprised that you introduce a fixed width scaling with discharge without more justification. The width scaling should appear as an independent equation number so that it can be discussed much more extensively in the paper.*

Authors: As explained above, the model does not explicitly calculate channel width. Rather, a discussion of width scaling is presented in the paper simply as a consideration of what parameter values might be considered reasonable. We have added text to the section following the lateral erosion equations to clarify this point.

Reviewer 2: *P6L16 : I fail to follow the logic in relating a higher Kl/Kv to the work of Harsthorn et al. 2002 (who studied only one reach with variable discharge, and highlighted the role of bed cover not runoff per se) and to the increase in climate storminess described by Stark et al., which is not accounted for in your description of R (knowing that an increase in climate storminess can very likely affect kw too).*

Authors: Moved these references to Hartshorn et al. and Stark et al. to the discussion section and expanded discussion of the effects of Kl/Kv ratio.

Reviewer 2: *P6L20 : kw : we need more info on the range of possible values. Is this value extracted from alluvial channels (as would suggest the Leopold Maddock, reference) which is inconsistent with your approach of "bedrock channels" as stated in the title, or from bedrock channels (which your model description seems to imply) ? You should also state at some point that kw is assumed fixed, which is a very strong assumption given that width variation with incision rate are very often observed or predicted in models explicitely modelling bed and bank erosion via an hydrodynamic model (e.g., Lague, 2014; Croissant et al., in press).*

Authors: Updated text to discuss use of fixed kw and range of possible values of kw (P8L1-5).

Reviewer 2: *CRITICAL : Is there an internal "safety check" that verifies that the actual channel width in the primary node (kw $Q^{0.5}$) is systematically smaller than the pixel size ? otherwise you violate some of your assumptions.*

Authors: As explained above, there is no explicitly defined width in our model. Text was added to the manuscript in the section describing vertical and lateral erosion equations to make this point.

Reviewer 2: *Figure 1: the legend is quite hard to follow. Similarly there are several black arrows so its hard to clearly understand which one you're referring to in the legend. Please revise this significantly for better clarity. There is also a typo ("after after" L 6). I suggest for instance to give a different color to the area being eroded in the lateral node to make it clearer.*

Authors: Typo fixed and figure revised for clarity.

Reviewer 2: *Figure 1b: it is not clear why you choose to have the neighbouring node set to the downstream elevation node (Zd), not the primary node (Zn). It seems to me that this probably drives artificial mobility in the model without a real justification.*

Authors: Setting the elevation of the lateral node equal to the elevation of the primary node would make the valley slightly wider, but the channel immobile. That is because water flow in that case would continue to prefer going from upstream through primary to downstream, because the "detour" through lateral would have a lower slope (same altitude difference, more distance covered). With flow continuing to prefer the shorter route, that is where the erosive action would be, and the just-eroded lateral node would be left at the original altitude of the primary node.

Setting the elevation of the lateral node equal to the downstream node gives the opportunity for flow to be rerouted through the lateral node, but does not require it.

Reviewer 2: *Lateral erosion : If I understand well, lateral erosion only occurs on a D4 grid, never for diagonal pixels? Would this not generate asymmetric behaviour between orthogonal and diagonal directions favouring one orientation but not the other?*

Authors: A new supplementary materials document has been written and includes a figure detailing how lateral nodes are chosen in the model. To briefly answer the question, a lateral neighbor node can only be the E,S,W,or N neighbor of the primary node, but the lateral node can be to the diagonal direction of a flow line in the case of 45 degree bends or two straight segments that flow across diagonals.

Reviewer 2: *P7L25: I note that if you add a subpixel description of the actual channel position, you would have a much more continuous description of the curvature (albeit with the issue of scale remaining).*

Authors: Yes, but as noted earlier, that would defeat the purpose of having a simple, low-dimensional model formulation.

Reviewer 2: *: P7L25 I fail to really understand this part ? how can you get a curvature with a straight channel ? Again this seems like assuming that you have a sub-pixel variability in the channel position, yet, you do not explicitely account for it and you do not have a model for it.*

Authors: A figure detailing how radius of curvature is calculated is included in the supplementary materials document. Yes, you are correct in that the way radius of curvature for straight channels is calculated is a simple way to account for sub-pixel variability in channel position.

Reviewer 2: *: P7L30 : H only dependent on Q : incorrect assumption to have H independent of slope which can vary alongstream and through time. Why can't you use your local width, slope and friction to back calculate the actual local flow depth?*

Authors: You are that it is possible to calculate flow depth in the model for each cell based on width, slope, and friction, but we choose to use a simpler hydraulic geometry relationship for flow depth as many other non-hydrodynamic landscape evolution models do (CHILD, etc). We emphasize that our goal in developing this model of lateral bedrock erosion is to start with the simplest reasonable erosion model so that we can focus on understanding the dynamics of lateral erosion. Additionally, as noted above, H scales more strongly with Q than with S.

Reviewer 2: *: P7L32 : does all the sediment behaves according to eqs (1) to (6) or is there specific treatment for the collapsed material as mentioned in Fig 1d: 'collapse material" behaves as washload , which would potentially imply that it nevers redeposit in the channel ? More generally, I find that the behaviour of the sediment is not always clear. (note having reread the MS several time, I now understand, but it's really not clear on the first or second read).*

Authors: Text has been added to clarify the treatment of sediment in the model at the end of the numerical implementation section (P9L9-14).

Reviewer 2: *P8L10: I think it would be way more justifiable to present the end-member as exploring lateral erosion laws scaling with bank height (as in Coulthard et al., 2013) or flow depth (as in many hydrodynamic models, Delft3D etc. . . .), and using this terminology all along the paper, and trying to relate these to actual natural processes in the discussion section, rather than the other way around. Because, the link with actual processes is quite tenuous, and there is some* *kind of untold story that the actual erosion model is dependent on the rock resistance chosen in the model. It would be great to beef up the literature here, discussing for comparison how bank erosion is calculated in CAESAR or EROS.*

Authors: Revised this section to emphasize the end member models as representing valley widening as a function of wall height and moved links to natural processes to the discussion section. Discussion of bank erosion in CAESAR and EROS has been added to the background section.

Reviewer 2: *P8L22: Why cannot you use the model with lateral mobility from the beginning ? what kind of hillslope erosion law is used ?*

Authors: It is possible to use the model with lateral erosion from the beginning. Figures comparing model runs with lateral erosion from the beginning and lateral erosion started after topography was initialized are shown in the supplementary materials section and show no difference in model topography.

Reviewer 2: *how were the parameters chosen ? e.g., erodibility, alpha as well as the Kl/Kv ratio and a runoff rate of 14 mm/hr or 36 mm/hr ? I note that 36 mm/hr amounts at 315 m/yr of runoff. . . Given, that nowhere on earth you have this kind of mean annual runoff, I suspect that this is some kind of effective runoff, but it is really not clear. Given that you do not chose the runoff, ending up with such large values should be better discussed. Seems that to get results that look good, you have to end up using boundary conditions that are unrealistic More generally, it is not clear if your choice of parameter is such that the landscape mobility looks "ok", or if at least, some can be independently chosen ? Maybe you should present a reference catchment on which model results could be compared.*

Authors: A range of values for K and alpha were chosen to explore model behavior, specifically channel mobility and valley width. References were added supporting the range of K and alpha values chosen in the model runs. In order to demonstrate the range of possible lateral erosion and valley widths in our new model actually works, we used a high, but justifiable value for runoff on event time scales. Text on how runoff values were chosen was added to the paper. High values of runoff, which are meant to represent peak values, not mean annual values, were chosen to get Kl/Kv ratios of 1 and 1.5 in order to demonstrate the lateral erosion that emerges from the model. The parameter values were chosen from a range of reasonable values found in nature. In some cases, significant channel mobility and lateral erosion occurred (these cases are highlighted in the manuscript), but in some cases, little observable lateral erosion occurred, see Figures 5a,6a; 9a,c;

Reviewer 2: *Given that your parameter choice seems quite ad hoc, I find it quite misleading/dangerous to present "real ages" in the numerical simulations and in the results.*

Authors: The choice, really, is whether to present figures like this in dimensional or non-dimensional form. The latter is of course more elegant, and has the advantage of demonstrating the role of multiple variables and their interactions. However, feedback from colleagues and students indicates that many find dimensional plots more intuitive, so we have stuck with them. As regards danger, all we can say is that no students or colleagues were harmed in the writing of this paper, and we don't think anyone will be harmed by reading it.

Reviewer 2: *P11L14: maybe you could cite Davy and Lague (2009) in which there's the first derivation of the slope-area relationship in the general case of erosion-deposition with a transport distance.*

Authors: Thank you, this paper is now cited here.

Reviewer 2: *P11L15: If you had an independent calibration of your elementary laws, which, when implemented in the numerical model, generates realistic geometries, then you would demonstrate that your new lateral erosion theory and its implementation successfully produce bedrock valleys significantly larger than the channel that created them. But right now, the model is calibrated and constructed to generate these wide valleys, so obviously. . .you get them. . . We are really bordering circular reasoning here.*

Authors: Not at all! The reviewer seems to assume that ANY set of rules or equations could reproduce any desired set of landforms as long as the right parameters are chosen. We disagree. When a model for a particular natural pattern is proposed, it may either succeed or fail at the basic test of qualitative reproduction of the pattern in question. We have shown that our model succeeds at the basic task of qualitative reproduction. This might be a weak test, but does not mean success is inevitable. Of course, it would be wonderful to have independent constraints on parameter values. Nonetheless, we adamantly disagree that demonstrating qualitative consistency between a model and observations, given certain parameter ranges, constitutes circular reasoning.

Reviewer 2: *P12L21: which hillslope processes, you did not describe them and in the discussion you seem to imply that there are no hillslope processes operating.*

Authors: Text changed to reflect that indeed there are no hillslope processes in this model.

Reviewer 2: *P13L13 : careful with the notion of threshold: this is not a true threshold in terms of physical processes (there are no thresholds in the constitutive equations of the problem), but solely an artificial threshold introduced by the numerical implementation and which depends on grid size.*

Authors: The word threshold is removed and text clarified.

Reviewer 2: *Section 4.2.1 : this section needs to be revised in the light that the predicted channel width is very likely larger than the actual pixel width which violates a fundamental assumption of the model (see general comments)*

Authors: We have run the models in section 4.2.1 with with grid size of 15 m and 20 m and compared them to the original models runs with dx=10 m. The new models with larger grid size shows some differences with the original models, but are largely similar in the amount of lateral erosion accomplished and width of bedrock valley created. Figures and discussion of these model runs is included the supplementary materials section.

Reviewer 2: *P14L25 : the increase in lateral erosion rate could be quite dependent on the incorrect assumption that H only varies with discharge (while it varies also with slope), and the flow partitioning errors as at this stage the "channel" theoretically occupies at least 2 pixels which means that discharge should not be as high than predicted given that it is focused in a single pixel.*

Authors: See responses above regarding scaling relationships and treatment of channel width used in this model.

Reviewer 2: *P15 P16 : in this section, assuming that channels only accommodate the increased sediment flux by varying their slope without varying their width (in that case kw), is a pretty strong simplification. Croissant et al., in press at Nature Geosciences have recently demonstrated how important are dynamic width variations (i.e., kw variations) in boosting the transport capacity of mountain rivers, slope variations having secondary effects. This effect, important in driving channel reincision of deposits, terrace generation and channel mobility cannot be captured in your modelling framework if you assume kw is fixed.*

Authors: Added text in this section to remind readers of the fixed width scaling in this model that prevents channels from changing width in response to sediment flux and added a paragraph in the discussion section detailing the implications for the model (P21L8-13).

Reviewer 2: *P17L31: the valley width emerging from any of the lateral erosion model completely depends on the model parametrization which is not properly justified at present. You could obtain narrower valleys with the undercutting-slump model algorithm if the lateral erodibility is much smaller.*

Authors: Model parameters have been more thoroughly explained in model experiments section. We acknowledge that in this initial version of the lateral erosion model, valley width is often strongly related to the imposed Kl/Kv ratio. But we note that the model produced narrow valleys in undercutting slump models with high values of K and low values of alpha for both values of Kl/Kv that were tested in this paper (Figure 3c,d).

Reviewer 2: *P19L20: this is debatable: alpha depends on runoff and settling velocity which can easily be estimated for natural systems. Only d* is more tricky. Setting runoff and settling velocity should set the value of alpha, not the other way around. At least you're sure to evolve in a range of parameters that is realistic.*

Authors: Text here reworded to clarify our intention to note the limitations of the current erosion/deposition model in future work that may address spatial and temporal changes in runoff and changes in and/or multiple grain sizes.

Authors' Response to Short Comment

Commenter 1: *I am excited to see this work come out and I am very much looking forward to seeing the paper published. Below, I am noting a few questions and comments that arose during my read through and I hope that some of these might be helpful in the revision process.*

Authors: Aaron, thanks so much for taking the time to read the manuscript so thoroughly and offer some very helpful comments.

Commenter 1: *In summary, there are three major points that I thought could be clarified (and that I address in more detail in the line comments below).*

First, the treatment of sediment in the model is only mentioned relatively briefly at the very end of the discussion section. In order to better appreciate the modeling result, I suggest that it may be helpful to lay out the role of sediment in the model (and in particular how it "becomes" bedrock when deposited) early in the section about the numerical implementation.

Authors: A section of text that more clearly describes the treatment of sediment in the lateral erosion component was added at the end of the "numerical implementation" section (P9L9-14).

Commenter 1: *Second, as marked below, I suggest to introduce the concept of channel mobility earlier and maybe comment on the difference between mobility within loose sediment (which, in my limited knowledge, much of the literature that uses the term channel mobility; is concerned with), and the mobility linked to movement within a pure bedrock landscape as it is defined in this paper. This distinction may also clarify the discussion about the importance of channel mobility on p 17. I comment more extensively on this below*

Authors: We added text to clarify the distinction between channel mobility in alluvial literature and the use of the term in this paper (P10L31-P11L2) and included this introduction at the beginning of the section on channel mobility. See below for more details on changes made to the paper regarding channel mobility.

Commenter 1: *Third, I would suggest a more extensive discussion of the implications of using the stream power model without a treatment of sediment tools and cover. The lack of the cover effect is mentioned in a sentence on P 19 L11 but in the list of limitation that follows and in the following discussion I did not find a clear mention about the possible limitations that the absence of tools may introduce. An appreciation of the implications of the stream-power model are particularly important before model results are compared with other studies such as those by Hartshorn et al., 2001 and Fuller et al., 2016. The effects of tools and cover seem an integral part of the interpretations of the observations that were made by these cited studies (and in other studies such as Hancock and Anderson 2002 etc.).*

Therefore, without a discussion about the role of tools and cover, the link between the present stream power model and other studies (that is made early in the manuscript) appears problematic. Moreover, tools and cover may affect lateral and vertical erosion in different ways. For example, an increase of lateral erosion rates because of a change in the amount of sediment that is deflected toward a channel wall (Fuller et al., 2016), may or may not be accompanied by changes in vertical erosion rates.

Therefore, the response of the system to a change in water or sediment flux may be more complex than predicted by the model. In short, I can imagine that a more expansive discussion of this limit may be useful. In particular, these complications should probably be mentioned before the model is compared to results from other studies.

Authors: A section of text that acknowledges the limitations of the stream power model and explains why we chose to use the stream power model is added to the end of the "vertical erosion theory" section of the manuscript (P6L15-22). A section of text that discusses the possible effects of using a tools and cover model and how a different incision model would affect model results is added to the discussion (P20L28-P21L4).

Commenter 1: *P.2 L.7 Nitpicky comment: it may or may not be worth noting that cessation of incision and cutting of straths has also been observed in harder lithologies such as quartzites or granites These straths are narrow and they dont contradict the statement that sizes of strath terraces and rates of strath cutting seem strongly linked to bedrock strengths, but the way the sentence is phrased now, it could be understood that straths never form in stronger lithologies:*

Authors: Added sentence to clarify straths also occur in hard lithologies.

Commenter 1: *Somewhere in the setup and introduction (for example somewhere in the paragraph starting P.2 L.11), it might be worth mentioning published models that consider the control of valley wall-height on widening rates e.g.: Malatesta 2016*

Authors: Added text in the introduction to include Malatesta et al. and brief discussion of bank height.

Commenter 1: *P.5 L.7. Setting W=kwQ1/2 may be common knowledge but I wonder if it is worth citing the original works that this scaling is based on.*

Authors: Leopold and Maddock (1953) cited here.

Commenter 1: *P.5 L.16. I like the idea of looking at the centripetal force. I was wondering at this stage what happens to straight segments of rivers. The way straight segments are treated is layed out later: They are treated as having a range of radii of curvature. However, there is also evidence for erosion in perfectly straight channels (e.g. Fuller et al., 2016). Maybe a quick note of the limits and possible alternatives of this model formulation could be made here or in the discussion? At the latest, this should probably be mentioned when the model results are compared to results from Fuller et al., 2016.*

Authors: Text added in discussion on lateral erosion in straight channels: P22L1-5

Commenter 1: *P.6 L. 12 It was a little unclear to me what the word "which" refers to here either equation 12 or the variable Kl.*

Authors: changed to read : "Kl is a dimensional erosion coefficient for lateral erosion composed of known or measurable quantities..."

Commenter 1: *P.6 L. 16 As far as I understand, the result that Kl/Kv scales with Q1/2 or R1/2 is derived within the framework of stream power models. I feel that the link to the field studies is a little misleading in this context. The changes in the ratio of lateral to vertical erosion rates between high and low flows measured in the Liwu river (Hartshorn et al., 2001) have been interpreted to be due to changes in the distribution of sediment in the flow (interpretation by Hartshorn et al., 2001) or to variable shielding of the bed (Turowski et al., 2008, ESPL). Because the importance of tools and cover for lateral and vertical erosion are not considered in the stream power model presented here, whereas the end result may be similar between model and field study (high discharge = high lateral erosion), the processes are likely different. Therefore, a comparison between field study and model without a more extensive discussion might be misunderstood. In turn, the increased sinuosity with storminess found by Stark et al., 2010 was interpreted by the authors as an expression of the importance for hillslope mass wasting in controlling lateral erosion. This interpretation may* *or may not be true, but it again, complicates the link of the model to this field examples. Later (P18 L10), there is a similar issue with the comparison of the model to the study by Fuller et al., 2016. see comment further down.*

Generally, I think it is valuable to discuss whether the model behavior is observed in nature but I think it necessitates a more detailed discussion of the limits of the stream power model before the comparison can be made.

Authors: Removed references to field studies here, and added them to the discussion section. Added text on the motivation for using the stream power model for vertical incision and the potential impact of using a tools and cover model to discussion section.

Commenter 1: *P.7 L.15 I wonder, if, before detailing the way lateral and vertical erosion is calculated, it would be worth detailing one entire timestep and the order in which equations are solved. In particular, at this point of the manuscript, I was unsure how streams migrate. As far as I understand, at the beginning of a timestep, flow is routed across a topography via a D8 algorithm, then the lateral and vertical erosion is calculated, the topography updated and the flow rerouted through the landscape. Could this maybe be briefly laid out step by step? Or as a flow chart figure? Even more importantly, at this point in the manuscript (and up to the very last paragraph of the discussion) it was unclear to me how sediment was treated. This is important to appreciate many of the features of the model (channel migration and channel mobility in particular) Questions that would be good to clarify are: Is deposited material added to the topography of a cell? What happens to a cell that is partly sediment and partly bedrock? Is the difference in erodibility considered or does deposited sediment "become" bedrock? Detailing the treatment of sediment could probably be intertwined with the walkthrough of one model timestep.*

Authors: A section detailing the treatment of sediment was added to the "numerical implementation" section (P9L9-14). We have made a flow chart detailing the steps taken in one model time step and included this in the supplementary material section.

Commenter 1: *Section 3.1: I was a little confused by the (as I understood it) differentiation between resistant lithologies for which the slumped material has to be eroded (therefore bank height is important) and weak lithologies for which all material is swept away after a slump happens (therefore bank erosion is not important). The way it is described in the text is that the material is "transported away". This formulation seems ambiguous to me. Is the material added to the sediment flux Qs or does the material "vanish" in the model. I believe the latter is meant. If the material "vanishes", I was wondering where such a model would be applicable in nature. I had thought that even for loose, nonresistant sediment, there should be a bank height control and that transport capacity is important. As in aside, the importance of wall height, even in loose sediment seems to be implied by the later mentioned study by Bufe et al., 2016. Here, we demonstrated that in loose sediment, the width of valleys across an uplift is a function of the uplift rate (controlling the growth of valley walls) and the channel mobility (controlling the frequency at which a river revisits a given point in the valley). The area of valley that is cut across a fold reaches some equilibrium value that can be maintained and that is flanked by steep, high walls. One interpretation of this finding is that the equilibrium valley area that is actively "maintained" by the river is limited by the bank heights that the stream has to rework as it moves across the valley. For example, when a river moves from point x to point y and back to x, the bank height that has grown at point x during the time the river traveled across the valley depends on the channel mobility. The slower the migration rate of the river, the higher the walls that it encounters at x once the river returns. The observed equilibrium valley area therefore seems to imply that the wall-height and the capacity of the river to transport the material of the walls is important even in loose sediment.*

Authors: Changed wording in this section to clarify that in the undercutting-slump model, slumped material is transported away as wash load and not considered in Qs calculations, but we wouldnt say that it vanishes from the model, just becomes unimportant in this end member model formulation (P9L23-24). We changed text in this section to emphasize the end member models as representing valley widening as a function of wall height and moved links to natural processes to the discussion section. The background section has been expanded to include the work of Malatesta et al. 2016, who found that bank height affects the way alluvial streams erode vertically and laterally. Again, the focus here is to begin to probe how lateral erosion occurs in bedrock channels.

Following the enormous flooding on the Colorado Front Range in September 2013, we observed locations along the creek had stripped the vegetation and sediments from the base of the hillside/terrace, undercut the shale bedrock bank, and the bank slumped into the creek. Because the flood stage was so high and the shale erodes as small, flakey pieces, the slumped material was more or less immediately transported away. So the undercutting-slump model, a model formulation that describes end member behavior, is applicable in a location with an under-capacity stream and lithology that breaks down into a transportable size. We have included an example from Johnson and Finnegan (2015, GSAB) in the discussion section; they document a similar effect of shear stress-driven lateral erosion in weak mudstone.

Commenter 1: *Section 4.1.1: It may be clearer to introduce the concept of channel mobility and the way it is defined in this study either earlier in the paper or at the beginning of this section. At the moment, the channel mobility is defined at the beginning of the second paragraph of the section. As far as I know, the term channel mobility has mostly been used in the framework of alluvial rivers. I am guessing that the processes that limit the mobility of channels in loose sediment and in cohesive bedrock are partly different. Therefore, it would be helpful to clearly make a link here to the treatment of deposition of sediment in the model and to emphasize that in this model, any lateral movement involves bedrock erosion.*

Authors: Added text to clarify the distinction between channel mobility in alluvial literature and the use of the term in this paper. (P10L31-P11L2)

Commenter 1: *P.9 L.15 Because the treatment of deposition was not clear to me, at this point, I found it hard to wrap my head around how alpha affects channel mobility. Is it purely the effect of sediment deposition creating topography and therefore causing channel to switch more frequently? Or does sediment deposition also create an alluvial surface across which channel can migrate rapidly? After reading the end of the manuscript it became clearer that sediment, when deposited, "becomes" bedrock. Therefore, I am guessing the reason that increased sediment flux creates more mobile channels is only because sediment deposition creates "topography" that moves channels? I could imagine, that such questions could be avoided if the treatment of sediment is explained earlier.*

Authors: Yes, you are correct. A clearer explanation of how sediment is handled is now discussed earlier in the paper at the end of the "numerical implementation" section (P9L9-14).

Commenter 1: *P.11 L.18. The expression "The [...] models take [...] 10 ky to respond to lateral erosion" was a little unclear to me. What constitute a "response to lateral erosion"? Is it the time lag between the onset of the lateral erosion after the spin-up and the corresponding appearance of a signature in the topography? In which case, is there some characteristic that was used to define when the topography was thought to show a response? I am sorry if I misunderstood this...*

Authors: This section rewritten to make clearer: "it is not surprising that the total block models take longer to respond to the onset of lateral erosion and valleys are more narrow than in the undercutting-slump models. The total block erosion models take on the order of 10 ky to produce an observable response to lateral erosion and ultimately produce bedrock valleys that are up to 25 meters wide, while the undercutting-slump models take about 5 ky show a response to lateral erosion and ultimately produce valleys that are up to 50 m wide."

Commenter 1: *P. 12 L. 20-21 Typos in this section "runs are easily", "processes due to their low relief", "has recently been shaped"*
Authors: Typos fixed

Commenter 1: *P. 12 L. 20-21: Again, I am sorry if I am misunderstanding but I would be interested in some expansion of the thoughts behind why the widest valleys occur in models with low channel mobility. I am unsure what is meant by "hillslope processes" in this context. I dont think any hillslope processes have been introduced in the model or in the introduction and theory sections of the paper. This word makes me think of landslides, hillslope creep or gullying none of these processes are in the model I believe and I am not sure I understand what is meant here.*
Authors: You are correct, "hillslope processes" is removed and meaning is clarified. Explanation of why the widest valleys occur in models with lower channel mobility is explained lower in the same paragraph.

Commenter 1: *P. 13 L. 20-22: The sequence of incision followed by lateral erosion in the TB models versus simultaneous incision and lateral erosion in the UC models would be nice to see in a figure. It is not clear from Fig. 6. In Figure 8, one panel is missing for the TB and the UC models respectively (the panel for 120 ky, just before lateral erosion starts in the TB model) to appreciate that sequence. Maybe it is possible to add one more panel and to refer to Fig. 8 at this point already? The same added panels would be nice to have at the end of this section (P. 14 L.12-19).*
Authors: Figures revised to show differences in sequence of lateral erosion in two model formulations.

Commenter 1: *P14 L24-26: I did not understand the explanation for why there is no lag time. Is the argument: 1) Lateral erosion rate is increased more than incision rate and 2) the bank height is not important in the UC models -¿ Therefore any increase in lateral erosion rate translates directly into a widening rate?*
Authors: The argument is the first suggestion. The text was rewritten to make this clear.

Commenter 1: *P14 L24-26 typo: "two times" or "two time steps"?*
Authors: Text changed to read "two times".

Commenter 1: *P.16 L. 12. I might be missing something obvious, in which case ignore the comment, but I am unsure of how to distinguish valley width formed via valley infilling or via lateral erosion from the curves of Fig. 11c: : :*
Authors: Interpretation of figure clarified.

Commenter 1: *P17 L19-26 I am glad to see a discussion about channel mobility and I was thinking about whether this discussion could benefit from a few clarifications and some restructuring. Therefore, I briefly come back to the definition of channel mobility that I mentioned above. The cited studies (Wickert et al., 2013 and Bufe et al., 2016) define channel mobility in the context of the fluvial reworking of an aggrading (or steady) alluvial surface. In my mind, this "alluvial* *channel mobility" is not exactly equivalent to the rate at which actively uplifting valley walls are eroded and therefore to the definition of channel mobility in this study.*

I totally agree that one can define a channel mobility in the context of the cumulative migration metric that was used in this paper. Such a metric can be calculated and defined independently of any regard for whether rivers are migrating across a valley, across an alluvial fan, or whether rivers are eroding valley walls. However, I am unsure that there is an a-priori reason to directly, and without further explanation, use the same terminology for migration rate of alluvial channels within an alluvial valley and the lateral erosion rate of valley walls, and therefore valley width. There are of course reasons to make the link between channel mobility and valley wall erosion. Hancock and Anderson 2002 hypothesized the importance of the frequency of the contacts between the river and valley walls. Malatesta et al, 2016, and this study, demonstrate a potential importance of valley wall height. As mentioned above, if wall height is important, then channel migration rates across the active valley and the uplift rate should control valley width. This was demonstrated by Bufe et al., 2016 at least for loosely consolidated valley walls. In short, there seem to be links between the "classic, alluvial" channel mobility and the lateral erosion of valley walls but I think the link merits expanding upon before the term "channel mobility" is interchangeably in both contexts.

Authors: We have clarified what we mean regarding channel mobility in our model. The commenter defines channel mobility in this model as "rate of erosion for actively uplifting valley walls", which is correct if you define valley walls as nodes immediately to next to the channel. We would say that channel mobility here is the rate of near-channel node erosion (which always occurs in bedrock here). The position of channel does not have to be next to what we define as the valley walls, the high slope nodes to either side of the channel/flat valley bottom. We think using the term channel mobility is justified, as it describes exactly what occurs in the model as well as in nature to describe bedrock valley widening. We do take your suggestion and note the differences between alluvial channel mobility and lateral channel mobility in bedrock valleys.

Commenter 1: *Section 5.2: Maybe here, the comparison with Hartshorn et al., 2001 and Stark et al., 2010 can be made. However, the limits of not considering tools and cover in the models might have to be discussed in more detail before that.*

Authors: Discussion of Hartshorn et al. 2002 and Stark et al. 2010 has been moved here (P20L23-27).

Commenter 1: *P.18 L.11. This paragraph discusses the model setup that links lateral erosion to channel curvature. I think it is worth noting in this context that Fuller et al., 2016 documented lateral erosion in a straight channel. As noted by the authors, the deflection of sediment (tools) toward the walls seemed to control lateral erosion in these experiments, thereby documenting the importance of tools. Because the model (and this paragraph in the paper) discusses the importance of channel curvature and because the significance of the absence of tools in the model has not been discussed, maybe the comparison with the Fuller et al., 2016 study can be moved and/or expanded upon?*

Authors: A significant amount of text was added here discussing Fuller et al. (2016) and the potential effects of implementing a tools and cover model for vertical incision (P20L28-P21L4).

Commenter 1: *P18 L 14 Typo: "has come into equilibrium"?*
Authors: Typo fixed.

Commenter 1: *P18 L23. Maybe worth discussing - Anton, L., A. E. Mather, M. Stokes, A. Munoz- Martin, and G. De Vicente (2015), Exceptional river gorge formation from unexcep-*

tional floods, Nature Communications, 6. This study documents knickpoint retreat and subsequent widening (maybe comparable to the TB models?) in hard bedrock.

Authors: Thank you for the suggestion, this paper is briefly discussed.

Commenter 1: *P18 L 33 Typo: "stream power to carry"?*

Authors: Typo fixed

Commenter 1: *P19 L1-2 As mentioned before, it would be worth to discuss the treatment of sediment in more detail, and earlier in my opinion.*

Authors: Sediment discussed in more detail earlier in manuscript and expanded discussion of tools and cover model in the discussion.

Commenter 1: *Figs. 2-3: It might help to spell out the abbreviations UC, TB, and spin in the figure legend. There should be enough space. If not, it would be useful to have the definitions in the caption. It could also be helpful to add the other variable (K or alpha) to the boxes on top of the figures. For example high K, moderate alpha, high alpha, moderate K etc.*

Authors: Definitions for UC, TB, and spin models now included in figure caption. Text added to figure 2 for improved clarity.

Commenter 1: *Figure 4: The term "spinup model" could be used in panel a to more easily relate these models to the previous figure. Also, I would tend to try not having text and grid overlap. Finally, the axis labels for x and y axes may be useful*

Authors: Text on figure changed and labels for x and y axes added for improved clarity.

Commenter 1: *Fig. 5; the c-axis (slope legend) needs a label and maybe x and y axes could use labels, even though it is fairly obvious what they are*

Authors: Slope legend and labels for x and y axes added for improved clarity.

Commenter 1: *Fig. 6. The last sentence in the caption reads as if there was only waterflux from 100- 150 ky I am guessing "increased drainage area" or "increased waterflux" is meant?*

Authors: Text in caption changed to improve clarity.

Commenter 1: *Fig. 9: Maybe you can add the type of model to the title of panels a and b as well as give the actual number of K instead (or in addition to) low and medium K.*

Authors: Text on figure changed for improved clarity.

Commenter 1: *Fig. 11: Should the y axis label not be "difference in valley width"?*

Authors: Y axis label fixed.

Developing and evaluating exploring a theory for the lateral erosion of bedrock channels for use in landscape evolution models

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Abstract. Understanding how a bedrock river erodes its banks laterally is a frontier in geomorphology. Theory for the vertical incision of bedrock channels is widely implemented in the current generation of landscape evolution models. However, in general existing models do not seek to implement the lateral migration of bedrock channel walls. This is problematic, as modeling geomorphic processes such as terrace formation and hillslope-channel coupling depends on accurate simulation of valley widening. We have developed and implemented a theory for the lateral migration of bedrock channel walls in a catchment-scale landscape evolution model. Two model formulations are presented, one representing the slow process of widening a bedrock canyon, the other representing undercutting, slumping, and rapid downstream sediment transport that occurs in softer bedrock. Model experiments were run with a range of values for bedrock erodibility and tendency towards transport- or detachment-limited behavior and varying magnitudes of sediment flux and water discharge in order to determine the role each plays in the development of wide bedrock valleys. Results show that this simple, physics-based theory for the lateral erosion of bedrock channels produces bedrock valleys that are many times wider than the grid discretization scale. This theory for the lateral erosion of bedrock channel walls and the numerical implementation of the theory in a catchment-scale landscape evolution model is a significant first step towards understanding the factors that control the rates and spatial extent of wide bedrock valleys.

1 Introduction and Background

Understanding the processes that control the lateral migration of bedrock rivers is fundamental for understanding the genesis of landscapes in which valley width is many times the channel width. Strath terraces are a clear indication of a landscape that has experienced an interval where lateral erosion has outpaced vertical incision (Hancock and Anderson, 2002). Broad strath terraces and wide bedrock valleys that are many times wider than the channels that carved them are found in mountainous and hilly landscapes throughout the world (e.g. Chadwick et al., 1997; Lavé and Avouac, 2001; Dühnforth et al., 2012) (e.g., Chadwick et al., 1997; provide clues about the nature of their evolution. Wide bedrock valleys and their evolutionary descendants, strath terraces, are erosional features in bedrock that are several times wider than the channels that carved them and range in spatial scale tens to thousands of meters (Figure 1). Wide bedrock valleys in incising rivers provide the opportunity for sediment storage in the valley bottom, influence hydraulic dynamics by allowing peak flows to spread out across the valley, and decrease the virtual velocity of sediment (Pizzuto et al., 2017) .

Changes in climate that drive changes in sediment flux, changes in discharge magnitude, and/or changes in discharge frequency have been cited as causes of periods of lateral erosion in bedrock rivers. The frequency of intense rain is correlated with higher channel sinuosity and lateral erosion rates on regional scales (Stark et al., 2010). Several studies demonstrate that significant lateral erosion in rapidly incising rivers is accomplished by large flood events (Hartshorn et al., 2002; Barbour et al., 2009), resulting from armoring of the bed during extreme flood events (Turowski et al., 2008) and exposure of the bedrock walls to sediment and flow (Beer et al., 2017). Sediment cover on the bed that suppresses vertical incision and allows lateral erosion to continue unimpeded is a critical element for the development of wide bedrock valleys, as determined from modeling, field, and experimental studies (Hancock and Anderson, 2002; Brocard and Van der Beek, 2006; Johnson and Whipple, 2010). Lateral erosion that outpaces vertical incision and creates wide bedrock valleys and strath terraces is linked to weak underlying lithology, such as shale (Montgomery, 2004; Snyder and Kammer, 2008; Schanz and Montgomery, 2016), although strath terraces certainly exist in stronger lithologies, such as quartzite (Pratt-Sitaula et al., 2004) . The relationships among river sediment flux, discharge, lithology, and rates of lateral bedrock erosion are not well defined. Because we do not sufficiently understand the processes of lateral erosion, landscape evolution models lack a physical mechanism for allowing channels to migrate laterally and widen bedrock valleys, in addition to incising bedrock valleys.

Theory for the vertical incision of bedrock channels has advanced considerably since the first physics-based bedrock incision models were presented in the early 1990's. For example, bedrock incision models now include theories for adjustment of channel width (Wobus et al., 2006; Turowski et al., 2009; Yanites and Tucker, 2010) (Stark and Stark, 2001; Wobus et al., 2006; Turowski et al the role of sediment size and bed cover (Whipple and Tucker, 2002; Sklar and Dietrich, 2004; Yanites et al., 2011), and thresholds for incision (Tucker and Bras, 2000; Snyder et al., 2003b). Rivers respond to changing boundary conditions by adjusting both slope and channel width (Lavé and Avouac, 2001; Duvall et al., 2004; Snyder and Kammer, 2008) and landscape evolution models must be able capture both of these responses if we are to fully describe the behavior and function of landscapes. Research on bedrock channel width gives important insights into the larger scale problem of bedrock valley widening. In particular, the effects of sediment cover on the bed play an important role in the evolution of channel cross-sectional shape because sediment cover on the bed can slow or halt vertical incision (Sklar and Dietrich, 2004; Turowski et al., 2007), while allowing lateral erosion to continue. Models of channel cross-sectional evolution predict that increasing sediment supply to a steady-state stream results in a wider, steeper channel for a given rate of base level fall (Yanites and Tucker, 2010).

Theories While theories that account for adjustment to channel width, dynamic adjustment to bedrock channel width continue to be refined (Lague, 2014) , landscape evolution models that include a relationship between sediment size and cover (Sklar and Dietrich, 2004; Gasparini et al., 2004; Lague, 2010) , and incision thresholds are assimilated in the current generation of landscape evolution models (Tucker and Hancock, 2010) . However, existing models rarely treat lateral erosion of bedrock channel walls and the consequential migration of the channel, in no small part because of the lack of a rigorous understanding of the processes that control bedrock channels (Tucker et al., 2001; Crave and Davy, 2001; Tucker et al., 2013) are available and widely used (Tucker and Hancock, 2010) .

Numerical models for alluvial rivers have made considerable advances in capturing the planform dynamics both meandering and braided rivers, which necessarily include lateral bank erosion. Howard and Knutson (1984) developed the first numerical model that simulates lateral bank movement and produces realistic patterns of river meandering. In this study, bank erosion scales inversely with radius of curvature, such that more rapid erosion occurs in tighter bends with a smaller radius of curvature. A more recent treatment of radius of curvature as a control on lateral erosion rates is developed in CAESAR, a cellular landscape evolution model that calculates a 2-D flow field (Coulthard et al., 2002; Coulthard and van de Wiel, 2006; Coulthard et al., 2013) This model is appropriate for studying alluvial river dynamics in meandering or braided streams at reach and small catchment scales and time scales up to thousands of years (Van De Wiel et al., 2007) , but is not designed to model the evolution of bedrock rivers. The Eros model is a morphdynamic/hydrodynamic model that also allows for lateral erosion of bedrock channel walls. If this theoretical hurdle can be cleared, an algorithm for lateral bank material (Crave and Davy, 2001; Davy and Lague, 2009; Carretier et al., In Eros, lateral erosion of bank material is equal to vertical erosion rate multiplied by the lateral topographic slope and a coefficient of unknown value (Davy and Lague, 2009) . This treatment of lateral erosion must be applied within a framework of models that currently only erode and deposit vertically. To our knowledge, this study is the first attempt at incorporating a generalized physics-based algorithm for lateral bedrock erosion and channel migration on a drainage basin scale to a two-dimensional landscape evolution modelallows for lateral channel mobility and the development of realistic braided rivers, but it lacks a mechanistic process, specifically for the lateral erosion of bedrock channels.

Lateral migration of bedrock channel walls has only been implemented into landscape evolution models in a few specialized limited number of studies (Lancaster, 1998; Hancock and Anderson, 2002; Clevis et al., 2006a; Finnegan and Dietrich, 2011; Limaye and Lamb, 2013). Hancock and Anderson (2002) reproduce model bedrock valley widening using a 1-D stream power model for vertical incision and assume that valley widening rates depend on stream power. They note that the width of the valley floor is related to the duration of steady state in the river, as theorized by Suzuki (1982). This model is based on the key observation that lateral erosion exceeds vertical incision when the channel is carrying the maximum sediment load dictated by the transport capacity. By varying sediment supply to the channel, their model predicts the development of a series of strath terraces. Lateral migration of a meandering channel has been implemented in a few landscape evolution models. Strath terrace sequences have also been produced by coupling a meandering model with a river incision model (Finnegan and Dietrich, 2011). Lateral migration of a meandering channel has been implemented in several landscape evolution models. Clevis et al. (2006a) modeled meandering channels in a valley section using a 2-D landscape evolution model and an adaptive grid approach. A vector-based approach to modeling lateral migration of meandering streams in heterogeneous bed material has been used to reproduce a range of bedrock valley forms (Limaye and Lamb, 2014), but this model is primarily a channel-scale model. While each of these studies model lateral migration of bedrock channel banks, they all operate with a meandering model that is not applicable to lateral migration in low-sinuosity channels or in a generalized landscape evolution model. Existing landscape evolution models do not address the lateral erosion of bedrock channel walls and the consequential migration of the channel, in no small part because of the lack of a rigorous understanding of the processes that control lateral erosion of bedrock channel walls. If this theoretical hurdle can be cleared, an algorithm for lateral erosion must be applied within a framework of models that currently only erode and deposit vertically. To our knowledge, this study is the first attempt at incorporating a generalized physics-based algorithm for lateral bedrock erosion and channel migration on a drainage basin scale to a two-dimensional landscape evolution model.

2 Approach and Scope

Until now, landscape evolution models have lacked a generic mechanism for allowing channels to migrate laterally and widen bedrock valleys, as well as incise bedrock valleys. While advances in controls on bedrock valley width have been made using meandering models, the representation of a sinuous channel doesn't describe all rivers, and often such models are constructed on a channel scale rather than on a drainage basin scale. In this study, we develop a theory for the lateral migration of bedrock channel walls and implement this theory in a 2-D landscape evolution model for the first time. We seek to explore the parameters that exert primary control on the morphology of bedrock valleys and the rate of bedrock valley widening using a series of numerical experiments.

Our As noted above, considerable advances have been made in developing theory and models for the planform dynamics of single-thread meandering channels. As a result, the scientific community has a good understanding of how meander patterns form and evolve, and how meander wavelength and migration rate scale with properties such as water discharge, valley gradient, and sediment grain size (e.g., Hooke, 1975; Schumm, 1967; Nanson and Hickin, 1986; Sun et al., 2001; Lancaster and Bras, 2002; Parker This body of work addresses the planform pattern of river channels, but does not deal with the broader drainage-basin topography in which those channels are embedded. The principal state variable in channel-meander models is the trace of the channel, $\mathbf{x}(\lambda)$, where λ represents streamwise distance $\mathbf{x} = (x, y, t)$ is the channel centerline position. Some more recent models also incorporate a vertical channel coordinate, so that $\mathbf{x} = (x, y, z, t)$ (Limaye and Lamb, 2013, e.g.,) , but the emphasis remains on the channel trace rather than on the topography. For example, the slope of the channel and/or valley is normally treated as a boundary condition rather as an element of topography that evolves dynamically as it steers the flow of water, sediment, and energy.

There is also a well-developed literature on process models of landscape evolution, and in particular the evolution of ridge-valley topography sculpted around drainage networks. We refer to these models as Landscape Evolution Models, or LEMs (e.g., Coulthard, 2001; Willgoose, 2005; Tucker and Hancock, 2010; Valters, 2016; Temme et al., 2017) . With LEMs, the emphasis lies in computing the topographic elevation field, $\eta(x, y, t)$. Water and sediment cascade passively downhill across this surface. In some of these models, channel segments are assumed to exist as sub-grid-scale features that are free to switch direction arbitrarily as the topography around them changes. Other LEMs represent water movement as a two-dimensional flow field, whether through multiple-direction routing algorithms (e.g., Coulthard et al., 2002; Pelletier, 2004; Perron et al., 2008) or with a simplified form of the shallow-water equations (Adams et al., 2017; Simpson and Castelltort, 2006) . Regardless of the approach to flow routing, LEMs differ from meander models in treating a self-forming, two-dimensional flow network rather than a single channel reach, and in explicitly modeling the evolution of topography.

With a few exceptions noted below, most LEMs treat erosion and sedimentation as purely vertical processes. When the flow of water and sediment collects in a "digital valley", the elevation of that location may rise or fall, but lateral erosion by channel impingement against a valley wall is usually neglected. Yet nature seems to be perfectly capable of forming erosional river valleys much wider than the channels they contain (Figure 1). The question arises of how one might honor the process of valley widening by lateral erosion (and narrowing by incision) within the topographically oriented framework of a LEM. In other words, how might the key features of LEMs and channel-planform models be usefully combined?

In addressing this issue, it is useful to consider that the typical LEM treatment of topography as a two-dimensional field $\eta(x,y,t)$ is itself a simplification, albeit a practical one. Consider an alternative framework in which the boundary between solid material (rock, sediment, soil) and fluid (air, water) is treated as a surface in three-dimensional space, $\sigma(x,y,z,t)$ (Braun et al., 2008) . The surface possesses, at each point, a surface-normal velocity, $\dot{\sigma}$, which represents the combined surface-normal rates of erosion, sedimentation, and tectonic motion. Such a framework would lend itself to representing lateral erosion, because any movement of this surface where it is not flat implies a horizontal component of motion. The cost of such an approach lies in computational complexity. For practical reasons, it is desirable to find methods by which a lateral component of erosion by stream channels could be represented within the much simpler framework of a two-dimensional elevation field $\eta(x,y,t)$.

Some models have begun to address this need by treating the erosion of bedrock banks and creation of strath terraces with meandering models (Finnegan and Dietrich, 2011; Limaye and Lamb, 2013) , but these approaches are primarily channel-scale models and assume a meandering channel planform. Hancock and Anderson (2002) also model the widening bedrock valleys on a reach scale of single valley, but they use a 1-D model that does not require a meandering channel.

In this paper, our objective is to define and explore a theory for lateral erosion that has the following characteristics: simple and sufficiently general in nature to be applicable in landscape evolution models; containing as few parameters as possible; requiring relatively few input variables, such as channel gradient and water discharge plus gross channel planform configuration. The aim of this theory is to model valley widening or narrowing over time scales relevant to drainage basin evolution, and across multiple branches within a drainage network. The theory is not designed to predict the movement of a particular channel segment over a period of a few years, but rather is intended to provide a general basis for understanding when and why valleys tend to narrow or widen during the course of their long-term geomorphic evolution. Theoretical predictions about these trends then serve as quantitative, mechanistically based hypotheses that can be tested by experiment and observations. Through a set of numerical experiments, we seek to answer the following set of questions:

- How does this lateral erosion model compare with purely vertical erosion models?
- How do two alternative formulations, which treat bank material differently, compare to each other?
- What combinations of bedrock erodibility, sediment mobility, water flux, sediment flux, and model type result in wide bedrock valleys?
- What are predictions of the model that could be readily tested through experiment and/or observation?

In the following sections we outline our theory for lateral channel wall migration and explain the two algorithms we have developed to apply this theory to an existing model. We then present the results from our set of numerical experiments and discuss how well the model describes the formation of wide bedrock valleys. The approach presented here is intended to be a starting point, but not an ending point. Our main goal is to draw attention to the importance of lateral stream erosion within the context of drainage-basin evolution, and to offer some ideas for how this might be addressed in the framework of a conventional grid-based LEM.

3 Theory

We have deliberately chosen the most simple formulation possible for deposition and erosion, while still capturing the role of sediment. We do this in order to focus on developing the lateral erosion component of our model. Evolution of the height of the landscape, η, through time is described by deposition rate, d, minus erosion rate, e, plus a constant rate of uplift relative to baselevel, U.

$$\frac{\partial \eta}{\partial t} = -e + d + U \quad (1)$$

Deposition rate is assumed to depend on the concentration of sediment (C_s) in active transport and its effective settling velocity, ν_s. Sediment concentration is expressed as the ratio of volumetric sediment flux, Q_s, to water discharge, Q:

$$C_s = \frac{Q_s}{Q} \quad (2)$$

We treat water discharge as the product of runoff rate and drainage area, such that $Q = RA$. Deposition rate is therefore given by:

$$d = \frac{\nu_s d_* Q_s}{RA} \quad (3)$$

where d_* is a dimensionless number describing the vertical distribution of sediment in the water column, which is equal to 1 if sediment is equally distributed through the flow (Davy and Lague, 2009). ν_s, d_*, and R are lumped into a single dimensionless parameter, α, that represents the potential for deposition.

$$\alpha = \frac{\nu_s d_*}{R} \quad (4)$$

A larger α implies more rapid deposition (all else being equal), either because settling velocity, ν_s, is high and sediment is quickly lost from the flow, or because runoff rate, R is low and there is little water in the channels to dilute the sediment. A smaller α represents slower settling velocity, or more intuitively, greater runoff. α can be thought of as a sediment mobility number: when $\alpha < 1$, sediment is easily transported and the model tends towards detachment-limited behavior; when $\alpha > 1$, sediment is less mobile and the model tends towards transport-limited behavior.

3.1 Vertical erosion theory

In this model, we use the stream power incision model (e.g., Howard, 1994) to represent vertical incision rate because it is the simplest bedrock incision model that represents fluvial erosion for steady state topography. Vertical erosion rate is derived from the rate of energy dissipation on the channel bed, which is given by:

$$\omega_v = \rho g \frac{Q}{W} S \tag{5}$$

where ρ is the density of water, g is gravitational acceleration, Q is water discharge, W is channel width, and S is channel slope. ~~The~~ We assume that the rate of vertical erosion scales as:

$$E_v = K_v^{'} \frac{\omega_v}{C_e} \tag{6}$$

where $K_v^{'}$ is a dimensionless vertical erosion coefficient and C_e is cohesion of bed and bank material. We use bulk cohesion simply as a convenient reference scale for rock resistance to erosion. This choice allows us to express erosion rate as a function of the hydraulic power applied (ω_v), a commonly used measure of material strength (C_e), and a dimensionless efficiency factor ($K_v^{'}$).

We assume that channel width is a function of discharge (Leopold and Maddock, 1953) :

$$W = k_w Q^0.5 \tag{7}$$

where k_w is a width coefficient. It is important to recognize that channel width is not explicitly represented in the model we describe. Rather, it is one element of the lumped parameters K_v and K_l. The channel-width scaling parameter values we discuss (k_w) are used only in the estimation of reasonable ranges for these parameters. The bank width coefficient, k_w, is constant along the channel length based on data sets from both alluvial (Leopold and Maddock, 1953) and bedrock rivers (Montgomery and Gran, 2001) that show a relationship between channel width and discharge. Substituting RA for Q and ~~$k_w Q^{1/2}$~~ equation 7 for W in ~~equation~~ 5, and then combining equations 5 and 6 gives:

$$E_v = \frac{K_v^{'} \rho g R^{1/2}}{k_w C_e} A^{1/2} S \tag{8a}$$

$$E_v = K_v A^{1/2} S \tag{8b}$$

~~where k_w is a width coefficient.~~ Lumping several parameters gives K_v, a dimensional vertical erosion coefficient (with units of years^{-1}), which consists of known or measurable quantities, and one unknown dimensionless parameter, $K_v^{'}$.

Although evidence indicates that sediment in the channel plays an important role in inciting lateral erosion in bedrock channels (Finnegan et al., 2007; Johnson and Whipple, 2010; Fuller et al., 2016) , the model presented here uses the stream power incision model to represent vertical erosion, which does not account for sediment flux-dependent incision(e.g., Beaumont et al., 1992

The standard stream power model (Equation 8) has some limitations, especially in the lack of threshold effects and assumption of constant channel width (Lague, 2014) . Despite these limitations, the stream power model is a good approximation for long term vertical bedrock incision on large spatial scales (Howard, 1994) and is appropriate here given the goal of this work is to explore dynamics of lateral bedrock erosion as a function of channel curvature.

3.2 Lateral erosion theory

Lateral erosion requires hydraulic energy expenditure to damage the bank material and/or dislodge previously weathered particles (Suzuki, 1982; Lancaster, 1998; Hancock and Anderson, 2002). We hypothesize that the lateral erosion rate is proportional to the rate of energy dissipation per unit area of the channel wall created by centripetal acceleration around a bend. Erosion of the channel wall is the result of the force of water acting on the channel wall. We know from basic physics that the force of water acting on the wall is equal to the force of the wall acting on the water, which is equal to centripetal force. Centripetal force is $F_c = m\frac{v^2}{r_c}$, where m is mass, v is velocity, and r_c is radius of curvature. The centripetal force of a unit of water can be found by replacing m with ρLHW, where ρ is the density of water, and L, H, and W are unit length, water depth, and channel width, respectively. Centripetal force of water flowing around a bend can be expressed in terms of centripetal shear stress, which is analogous to bed shear stress, by dividing both sides by HL giving:

$$\sigma_c = \frac{\rho W v^2}{r_c} \tag{9}$$

Centripetal shear stress can be turned into a rate of energy expenditure by multiplying by fluid velocity, giving:

$$\omega_c = \frac{\rho W v^3}{r_c} \tag{10}$$

To express this in terms of discharge, Q, instead of velocity, we employ the Darcy-Weisbach equation, giving $v^3 = gqS/F$, where q is discharge per unit width and F is a friction factor, which yields

$$\omega_c = \frac{\rho g Q S}{r_c F} \tag{11}$$

Equation 11 describes a quantity that might be termed centripetal unit stream power, as it represents the rate of energy dissipation per unit bank area. The centripetal unit stream power is similar to the more familiar quantity unit stream power, except that channel width is replaced by the radius of curvature multiplied by a friction factor.

We hypothesize that lateral erosion rate scales with energy dissipation rate around a bend according to

$$E_l = K_l' \frac{\omega_c}{C_e} \tag{12}$$

where $K_l^{'}$ is a dimensionless lateral erosion coefficient. Combining equations 11 and 12 gives

$$E_l = \frac{K_l^{'} \rho g R}{C_e F} \frac{AS}{r_c} \tag{13a}$$

$$E_l = K_l \frac{AS}{r_c} \tag{13b}$$

where K_l is a dimensional erosion coefficient for lateral erosion , which is composed of known or measurable quantities, and one unknown dimensionless parameter, $K_l^{'}$. If $K_l^{'}$ is equal to $K_v^{'}$, we find a ratio between K_l and K_v, given by

$$\frac{K_l}{K_v} = \frac{R^{1/2} k_w}{F} \tag{14}$$

which consists of runoff rate, R, bank width coefficient, k_w, and friction factor, F. We can measure or make reasonable estimates of each of these parameters in order to determine what the ratio of lateral to vertical erodibility should be. Runoff Mean annual runoff rate can vary widely, but a higher peak runoff intensity will lead to a higher K_l/K_v ratio and more lateral erosion, as suggested by field observations of lateral erosion in bedrock channels (Hartshorn et al., 2002) and correlation of increased sinuosity and storminess of climate (Stark et al., 2010) .

A bank width coefficient fixed k_w is common in landscape evolution models that model long term landscape erosion (e.g., Tucker et al., 2001; Gasparini et al., 2007) , but channel width can vary with incision rate in models and natural systems (Yanites and Tucker, 2010; Duvall et al., 2004) , suggesting there are cases when dynamic width scaling is important (Lague, 2014) . In this model, k_w is given a value of 10 m/(m^3/s)$^{1/2}$, which is reasonable for a range of natural rivers (Leopold and Maddock, 1953). If k_w is lower, then the channel is more narrow and water is deeper, and more vertical incision should occur, but the value can range between 1 and 10 due to differences in runoff variability, substrate properties, and sediment load (Whipple et al., 2013) . The friction factor, F, is the Darcy-Weisbach friction factor, which can range from 0.01–1.0 for natural rivers (Gilley et al., 1992; Hin et al., 2008). With a lower friction factor (representing smooth channel walls), the lateral erosion ratio would be higher due to less energy being dissipated on the channel walls, leaving more energy available for lateral erosion.

4 Numerical implementation

One challenge in modeling both vertical and lateral erosion in a drainage network lies in the representation of topography. NormallyTypically, landscape evolution models use a numerical scheme in which the terrain is represented by a grid of points whose horizontal positions are fixed and whose elevation represents the primary state variable in the model. Such a framework does not lend itself to the motion of near-vertical to vertical interfaces (such as stream banks and cliffs), and for this reason, incorporating lateral stream erosion in a conventional landscape evolution model requires a modification to the basic numerical framework. A vertical rather than horizontal grid (Kirkby, 1999) can be used for near-vertical landforms in isolation, but is inappropriate when one wishes to represent vertical interfaces that are inset within a larger landscape. Grid-node movement combined with adaptive re-gridding (Clevis et al., 2006a, b) provides a possible solution, but is computationally expensive, and particularly difficult to implement when multiple branches of a drainage network may undergo lateral motion. Here, we adopt a simpler approach in which valley walls are viewed as sub-grid-scale features that migrate through the fixed grid. Rather than tracking the position of these vertical interfaces, we instead track the cumulative sediment volume that has been removed from the cell surrounding a given grid node as a result of lateral erosion. When that cumulative loss exceeds a threshold volume, the elevation of the grid node is lowered.

More specifically, at each node in the model, we calculate a vertical incision rate at the primary node and a lateral erosion rate at a neighboring node (Figure 2). The lateral neighbor node for the primary node is chosen on the outside bank of two stream segments that flow into and out of the primary node. The stream segments used to identify the neighboring node over which lateral erosion should occur are the incoming stream segment to the primary node with the greater drainage area and the stream segment that connects the primary node to its downstream neighbor (Figure 2). If the two segments are straight, then a neighboring node of the primary node is chosen at random and lateral erosion occurs at this node until elevation changes at the node.

Calculation of radius of curvature along two stream segments in a raster grid with D8 flow routing presents a challenge, as the angle between segments is discretized; the two segments may form a straight line, in which case the angle is equal to 0°, form a 45° angle, or form a 90° angle. In order to reduce the impact of this discretization, we assume that each of these three cases represents a continuum of possible radii of curvature. Cases of two straight segments are treated as if the actual angle between them ranges anywhere between +22.5° to -22.5°. If one takes the average among these possible angles, the resulting inverse radius of curvature is 0.23/dx, where dx is the cell size in the flow direction. Similarly, we assume that a 45° bend represents a continuum of possible angles between the two segments, ranging from 22.5°~~-63.5~~–63.5°, resulting in ~~a~~ an inverse radius of curvature of 0.67/dx. Following the same principle for a 90° bend gives a mean inverse radius of curvature of 1.37/dx (see Supplementary Materials).

The volumetric rate of material eroded laterally for each lateral node is calculated by $E_l \times dx \times H$, where H is water depth, given in meters. Water depth at each node is calculated by $H = 0.4Q^{0.35}$ (Andrews, 1984), where Q is given in m^3/s. The volume of sediment eroded laterally per time step is sent downstream along with any material eroded from the primary cell. Volumetric erosion rate is multiplied by the time step duration to get the volume eroded at the lateral nodes, and the cumulative volume eroded from each lateral node is tracked throughout the entire model run. The model does not distinguish between sediment and bedrock in the model grid and all material that is eroded has the bedrock erodibility of the K_v or K_l terms. When material is eroded vertically or laterally from bedrock nodes, the volume of the eroded material is sent downstream as part of the Q_s term. If deposition occurs in the model, deposited material is added to the topography of the node as bedrock. Thus, sediment is not "seen" in the model as material that can be re-eroded after deposition, rather sediment works to increase the deposition term (Equation 3).

Lateral erosion rate presented here (Equation 13) relates lateral erosion to radius of curvature, but the application of this model is not limited to meandering streams. Streams with fully developed meandering are part of a relatively small subset of streams that able to widen valleys through lateral erosion; there are examples of streams that classified as single-thread or braided, and yet which clearly show evidence of erosion and lateral migration at locations where an outer bend in the channel impinges on a valley wall or terrace (Cook et al., 2014; Finnegan and Balco, 2013) . Conceptually, therefore, this approach is not meant to represent exclusively channels with fully developed meandering.

4.1 End member model formulations

We have implemented two ways of determining whether enough lateral erosion has occurred to lower the lateral node. The first method, the total block erosion model, dictates that the entire volume of the lateral node above the elevation of the downstream node must be eroded before its elevation is changed (Figure 2a,b). This formulation assumes that the ~~bank material being eroded is resistant and/or blocky. This approach is used to represent, in a simple way, a system in which undermining of a channel bank leads to gravitational collapse of resistant material that must itself then be eroded in place (Lancaster, 1998)~~ height of the valley walls is a controlling factor in the ultimate width a valley can achieve, thus valley width scales with valley wall height. In this method, lateral migration depends on bank height so that taller banks experience slower lateral migration, as all of the volume of the lateral node must be eroded for the valley to widen. The second method, the undercutting-slump model, dictates that only the volume of the water height on the bank times the cell area must be eroded for the elevation to change (Figure 2c,d)~~. This model represents lateral erosion on a bank that has been laterally undercut and~~ , while the remaining material slumps into the channel and is transported away as wash load, ~~and assumes that the bank material slumps easily and rapidly breaks down into small grains that are easily transported.~~ i.e. not redeposited in the model or included in Q_s calculations. This model formulation represents the migration of valley walls independent of valley wall height. With these two end member models, we address whether lateral erosion rate should scale with valley wall height. ~~In the first method, the total block erosion model, lateral migration depends on bank height so that taller banks experience slower lateral migration, as all of the volume of the lateral node must be eroded for the valley to widen (Lancaster, 1998) . On the other hand, if all of the material that has been undercut by the channel is also swept away by the channel, then lateral erosion rate is independent of bank height. However , this undercutting-slump model is not appropriate for landscapes with very hard bedrock (low erodibility), as evidenced by overhanging cliffs along many rivers and persistent blocks of collapsed material following slumping or delivery from adjacent hillslopes (Shobe et al., 2016) .~~ Valley wall or bank height is known to limit lateral channel migration and valley width in transport limited streams where additional sediment from valley walls cannot be transported out of the channel (Nicholas and Quine, 2007; Bufe et al., 2016; Malatesta et al., 2017) . However whether valley wall height should limit valley widening in detachment-limited bedrock channels less clear (Lancaster, 1998) , and likely depends on the bedrock lithology (Finnegan and Dietrich, 2011; Johnson and Finnegan, 2015) . The links between these end member model formulations and the natural processes they represent are explored in the discussion section.

5 Model experiments

In order to ~~explore the factors that control~~ constrain the conditions that result in significant lateral bedrock erosion and valley widening, we ran sets of models using a range of values for bedrock erodibility, α (sediment mobility number), and K_l/K_v ratio using both the total block erosion model and the undercutting-slump model (Table 1). The model domain was 600 m by 600 m with 10 m cell size, three closed boundary edges and uplift rate relative to baselevel of 0.0005 m/yr imposed on the entire model domain. Water flux was introduced in the top of the model by designating a node as an inlet with an area of 20,000 m^2 and sediment flux at carrying capacity so that each model run would have a primary channel on which to measure width and channel mobility. All models were spun up to an initial condition of approximately uniform erosion rate with vertical incision only. The models were then run for 100–200 ky with the lateral erosion component. In order to isolate the effect of bedrock erodibility, a set of model calculations were run where erodibility ranged from 5×10^{-5} to 2.5×10^{-4} (Stock and Montgomery, 1999) while α was held constant at 0.8. In order to isolate the effect of detachment-limited vs. transport-limited behavior, another set of models was run where erodibility was held constant at 1×10^{-4} and α values ranged from 0.1 to 2, which represents a detachment-limited system when $\alpha < 1$ and a transport-limited system when $\alpha > 1$ (Davy and Lague, 2009) (Table 1). K_l/K_v ratios for all model runs were set to 1.0 or 1.5, resulting in a runoff rate of 14 mm/hr or 36 mm/hr from Equ. 14. Water flux was introduced in the top of the model by designating a node as an inlet with an area of 20,000 m^2 and sediment flux at carrying capacity so that each model run would have a primary channel on which to measure width and channel mobilityEquation 14. These runoff rates do not represent a yearly mean annual runoff, rather peak event runoff rates that are likely to result in appreciable lateral erosion due to the scaling with K_l/K_v ratio. Small et al. (2015) found that bedrock erosion rates in abrasion mill experiments are an order of magnitude higher in samples from channel margins compared to the channel thalweg. This suggests that K_l in this model should be at least equal to K_v, and could be much higher (Finnegan and Dietrich, 2011) .

Understanding the model behavior in response to detachment- vs. transport-limited behavior (represented by α) and K_l/K_v ratio is complex and requires understanding how runoff plays into both parameters. The value of α is calculated by v_s, a proxy for grain size, and runoff rate, R, although neither grain size nor runoff is explicitly set in the model runs. Values of α that capture a range of detachment- or transport-limited behavior is set instead (α=0.2–2.0). When K_l/K_v ratio is set for a given model (either 1.0 or 1.5 in all model runs), the runoff rate is calculated inside the model. Once a runoff rate for given K_l/K_v ratio is calculated, by extension, a value of v_s can be calculated from runoff rate and the set α value. Therefore, in model runs with the same K_l/K_v ratio and therefore the same runoff rate, a transport-limited system (α greater than 1) has a larger grain size (approximated by v_s) compared to a detachment-limited system with a low α.

5.1 Measures of lateral erosion in model landscapes

5.1.1 Channel mobility

Channel mobility distinguishes models with lateral erosion from models with only vertical incision. At steady state, channels in models with only vertical bedrock incision do not migrate across the model domain. However, a mobile channel is necessary to carve wide valleys and it is enticing to say that the more mobile the channels, the wider the valley will be. bedrock valley. In our model, channel mobility is not controlled by sediment flux, as found in alluvial channels (Wickert et al., 2013; Bufe et al., 2016) , but by the lateral erosion of bedrock. However the term “channel mobility” is used here in the same sense as in alluvial literature; channel mobility describes lateral channel planform changes along the length of the channel.

Table 1. Model runs and parameters discussed in this paper.

model version	K_l/K_v	K	α	number of runs
total block	1.0–1.5	1×10^{-4}	0.2–2.0	10
total block	1.0–1.5	5×10^{-5}–2.5×10^{-4}	0.8	10
undercutting-slump	1.0–1.5	0.0001	0.2–2.0	10
undercutting-slump	1.0–1.5	0.00005–0.00025	0.8	10
TB water flux	1.0–1.5	0.00005–0.0025	0.8	6
UC water flux	1.0–1.5	0.00005–0.0025	0.8	6
TB sed flux	1.0–1.5	0.0001	0.2–2.0	10
UC sed flux	1.0–1.5	0.0001	0.2–2.0	10

The effect of bedrock erodibility and α on channel migration through time for both model versions is shown in Figure 3. Channel migration over 200 ky is shown for six selected runs that span the range of bedrock erodibility and α values for the two different model formulations: the undercutting-slump model where K_l/K_v=1.5 and the total block erosion model where K_l/K_v=1.5. In all runs, the total block erosion model produced more confined channels compared to the undercutting-slump model. The undercutting-slump model produces more dynamic channel migration over the model domain, especially in the high K model. In both model formulations, the high K and high α runs have the widest extent of channel migration (recall that high α represents lower sediment mobility) and the low K and low α runs have the most restricted channel migration.

In order to describe channel mobility in our model runs in a single term, we calculate a cumulative migration metric, λ. λ is calculated by first determining the migration distance of the channel between time steps at all model cells the main channel occupies. Most often the migration distance between time steps at a single cell will be 0 or 10 m, indicating no migration or migration to a neighboring cell. The mean of migration distances between time steps is taken and summed over the duration of the model run to give the cumulative migration metric. λ, indicates how often the channel has migrated during the model run; a model run can have the same λ value if the channel marches across the entire model domain or if the channel repeatedly switches between two nearby channel courses. λ can also be used as an indicator for the maximum lateral extent occupied by the channel during the model run. That is, the maximum possible extent of x positions occupied by the channel is equal to λ~~at a maximum~~, but the actual x distance occupied by the main channel could be lower as the channel migrates over the same area repeatedly.

Bedrock erodibility and K_l/K_v ratio have the strongest control on channel migration distance. Channel mobility increases as bedrock erodibility increases in both the total block erosion model and the undercutting-slump model (Figure 4a,b). When K is low, representing strong bedrock lithology, there is limited channel movement in the total block erosion models with λ values between 15–35 m. This means that on average during the model run the channel occupied 1–3 cells (Figure 3c). With low values of K, the undercutting-slump model had λ values around 200 m, but a lateral extent of only 5 model cells (Figure 3c). This indicates that in the undercutting-slump model, the channel was actively migrating within a small area of the model domain. In model runs with high K values representing weak bedrock, total channel migration, λ increases, as well as the spatial extent of the channel migration (Figure 3a). With the total block model, λ appears to be a good proxy for total spatial extent of channels, but for the undercutting-slump model, λ tends to over estimate lateral extent of channel occupation (Figure 3).

Increasing the K_l/K_v ratio from 1.0 to 1.5, results in 1.5–2 times more channel mobility, with the largest relative increases in total block erosion model runs with high erodibility and higher α values (Figure 4a,b). This is because the undercutting-slump models already have high channel mobility with K_l/K_v equal to 1. Increasing K_l/K_v ratio to 1.5 increases channel mobility in UC models, but the total block erosion models have a larger threshold for lateral erosion so the increased K_l/K_v ratio results in relatively more channel mobility in the total block models.

For model runs with the same bedrock erodibility, but different α values (which represents sediment mobility), channel mobility is lower in models with lower values of α (representing high sediment mobility) and higher when $\alpha > 1$ (representing less mobile sediment) (Figure 4b). This effect is most pronounced in the total block erosion models, where channel mobility increases by a factor of four as α increases. In the undercutting-slump models, channel mobility also increases with α, especially when $K_l/K_v = 1.5$. When $K_l/K_v = 1$ in the undercutting-slump models, the trend in channel mobility vs. α is less well defined.

5.1.2 Valley width

Valley width is the primary indicator of lateral erosion; a wide bedrock valley implies that significant lateral erosion has occurred. Valleys can be defined in a few different ways; valley width needs to be quantified in our model. Many studies use low gradient areas of a DEM to determine valley width (e.g., Brocard and Van der Beek, 2006; May et al., 2013). This gives the width for the valley bottom that has been shaped by channel processes, but excludes areas that have been recently shaped by channel processes and then reworked by hillslope processes. Another way to measure valley width is by determining the width of the valley at a certain height above the channel. This simple metric is often used for finding valley width in the field, for example using eye height above the channel (e.g., Snyder et al., 2003a; Whittaker et al., 2007). Using a certain height above the channel to determine valley width in the models cannot distinguish between a fluvially carved bedrock valley and low relief in a landscape with weak bedrock. Instead we define valley width as the width of the area next to the main channel, where slope is characteristic of the fluvial channel rather than hillslopes for a given bedrock erodibility and α value. The reference slope for a fluvial channel is given by the slope-area relationship, assuming that the height of the landscape and Q_s are steady in time. When the height of the landscape is in equilibrium, Equations 1 and 3 are combined and rewritten as:

$$U = e - \frac{\nu_s d_* Q_s}{RA} \quad (15)$$

At steady state, Q_s is the total upstream eroded material, given by $Q_s = AU$. Substituting the steady state equation for Q_s and Equation 8 into Equation 15 gives

$$U = K_v A^{1/2} S - \alpha U \tag{16}$$

Solving the above equation for S gives the equation for reference slope that determines whether a model cell is shaped by fluvial or hillslope processes (Davy and Lague, 2009) .

$$S = \frac{U}{K_v A^{1/2}}(\alpha + 1) \tag{17}$$

Our models successfully produce bedrock valleys that are several model cells wider than the channels that created them (Figure 5). Models with only vertical incision have v-shaped valleys that are only 1 model cell wide (10 meters in our experiments) and the channels do not shift laterally (Figure 5a). Given the specifications of the total block and undercutting-slump models, it is not surprising that valleys in the total block models take longer to respond to the onset of lateral erosion and valleys are more narrow than in the undercutting-slump models. The total block erosion models take on the order of 10 ky to respond produce an observable response to lateral erosion and ultimately produce bedrock valleys that are up to 25 meters wide, while the undercutting-slump models take about 5 ky to respond to show a response to lateral erosion and ultimately produce valleys that are up to 50 m wide.

Figure 6 shows slope maps of total block and undercutting-slump models that show the width of the valley shaped by fluvial processes. The blue areas have slopes that are characteristic of fluvial channels and red areas have slopes that are characteristic of hillslopes. The total block erosion model with a low α value shows very little bedrock valley widening as evidenced by the thin band of blue along the main channel 1–2 model cells wide (Figure 6a). Increasing transport-limited behavior (higher α) results in wider valleys that have been shaped by the channel that are 2-3 model cells wide in the total block erosion model (Figure 6b). The landscape in the undercutting-slump model has wider valleys that result from more extensive carving by channels. The fluvially carved valleys in the detachment-limited model are about 2-3 model cells wide and the valleys in the transport-limited model are over 50 meters wide in some places (Figure 6c,d).

Figure 4c,d shows valley width for the lower two-thirds of the model channels averaged over the duration of the model runs in 54 model runs. To ensure that using characteristic fluvial slope as the criterion for a valley in all model runs gives valley width resulting from lateral erosion, and not valley width inherent in the model, we first use this criterion to measure valley width for the spin up models that include no lateral erosion component. Valley width for the spin up models is consistently 10 m, the width of one model cell. Valley width does not change significantly for any of the total block model runs in which K is varied and α is held constant (Figure 4c). When the K_l/K_v ratio is increased from 1 to 1.5, valley width increase slightly for all model runs, but wide valleys are not possible in the total block erosion model with this value of α. Valley width in the undercutting-slump model for changing bedrock erodibility shows a somewhat counter-intuitive signal (Figure 4c); the undercutting-slump model results in wider valleys for lower values of bedrock erodibility. The reasons for this signal are discussed in the section below.

When α is varied and K is constant, valley width increases with the tendency towards transport-limited conditions ($\alpha > 1$) in all undercutting-slump models, but only in total block erosion models when the K_l/K_v ratio is equal to 1.5 (Figure 4b). The widest valleys for a given bedrock erodibility occur with high α values as a result of higher slope. The models predict more channel mobility and wider valleys under transport-limited streams (set by α) compared to detachment-limited streams (Figure 4b,d). As α increases, the deposition term increases, and a steeper slope is needed to maintain the landscape in steady state relative to uplift. Higher channel slopes in transport-limited model runs also cause increased lateral erosion according to equation 13.

5.1.3 Linking channel mobility and valley width

We have shown that the greatest channel mobility occurs in the undercutting-slump models and increases significantly with increasingly soft bedrock (Figure 4a). However, maximum channel mobility does not translate into maximum valley width. In the undercutting-slump models, the widest valleys occur in the low erodibility model runs that have relatively low channel mobility. This reflects that the areas visited by the migrating channel in the low-relief, high K model runs ~~is~~ are easily overprinted by ~~hillslope processes due to its low relief~~small scale fluvial processes and lose the slope signature of the larger channel. This prevents our algorithm from finding where an area of the model ~~has recently~~ that has recently been shaped by the channel. The mismatch between channel mobility and valley width also reflects that hard bedrock valleys are allowed to erode very easily in the undercutting-slump model and the ~~channel smoothed surface~~ surface smoothed by the channel is persistent through time. The relationship between hard bedrock and wide valleys reflects the use of the undercutting-slump model, which is inappropriate for hard bedrock wall erosion in natural systems. With the undercutting-slump model, only a small volume threshold must be overcome for lateral erosion to occur, and the rest of the node material is transported downstream as wash load. However, it is these models that have resistant bedrock (low K) that are least suitable for the undercutting-slump model. In order for this to be a good description of how nature works, the bed material would need to be able to break up into small pieces that are easily transported away. The total block erosion model is more appropriate for representing the erosion of hard bedrock channels.

5.2 Adding complexity: water flux, sediment flux

5.2.1 Effects of increased discharge on lateral channel migration

In order to investigate how transience in landscapes affects lateral erosion, we introduce increased discharge at the inlet point in the upstream end of the model. Using drainage area as a proxy for discharge, increasing water flux in the model represents how a larger stream on the same landscape will influence valley width. Increasing drainage area also allows us to observe the extent of landscape change and how rapidly the different model runs respond to an event such as stream capture. The drainage area at this input point is increased from 20,000 m^2 to 160,000 m^2 and sediment load is set to the carrying capacity of the new drainage area. For a typical model run, the additional drainage area approximately doubles the drainage area at the outlet of the main channel in the model domain. Models with increased water flux were run using both model formulations, $K_l/K_v = 1.0$ and 1.5, and erodibility values that ranged from 5×10^{-5}–2.5×10^{-4}, with alpha held constant at 0.8 (Table 1).

Recall that lateral erosion scales with drainage area (Equation 13), while vertical incision scales with the square root of drainage area (Equation 8), and therefore we expect that increasing drainage area will increase lateral erosion and valley width in every case for the undercutting-slumping model, where the ~~threshold~~ numerically imposed condition for lateral erosion to occur is much smaller than in the total block erosion model. In the total block erosion model, lateral erosion will temporarily stall because of the volume threshold that must be exceeded before lateral erosion occurs. There is no threshold for vertical incision, which will speed up when additional water flux is added to the model.

5.2.2 Total ~~Block~~ block erosion models

In all of the model runs, increased water flux resulted in increased lateral erosion and wider valleys. Figure 7 shows valley width averaged over the ~~lower half of the~~ model domain vs. model time for all of the water flux models. The total block erosion model and undercutting-slump model respond differently to a step change in water flux. The total block erosion models first incise vertically to a new steady state stream profile, then erode laterally as a result of the increased water flux, while the undercutting-slump model incise vertically and erode laterally simultaneously (Figure 10a,b).

Total block erosion models where the K_l/K_v ratio is equal to 1.5 (TB1.5) show an interesting pattern in valley widening after increased water flux (Figure 7~~b~~c). All of the TB1.5 model runs show a significant increase in valley width during the 50 ky period of increased water flux. After 6 ky of increased water flux (model time = 106 ky), the high and medium erodibility model runs have greater valley widths, but the low erodibility model shows a gradual increase in valley width over 14 ky of increased water flux (model time ~~100-114~~ 100–114 ky). For the first 14 ky of the increased water flux, the channel of the low K model run incises rapidly, increasing the gradient between the channel and the adjacent cells and preventing lateral erosion. After the channel profile comes into new equilibrium, the increased water flux accelerates lateral erosion on the valley walls and valleys widen by 10 m compared to before increased water flux in the total block erosion models.

After the increased water flux stops at 150 ky, the wider valleys persist in the low and medium erodibility models (Figure 7~~b~~c) for two reasons. First, after the cessation of increased water flux, the channel returns to equilibrium through aggradation and uplift. While aggradation is occurring, lateral erosion can occur more easily in the total block erosion models. In this case, the total volume that must be eroded from any lateral node cell is reduced as the channel floor moves up in vertical space. The second reason for persistent wide valleys is that in the medium and low K model runs, the increase in water flux eroded wide valleys into relatively resistant bedrock. These flat surfaces near the channel persist in harder bedrock, even after water flux has decreased to original levels. Following end of the period of increased water flux, valley width in the the TB1.5 medium K model run remains elevated for 10 ky (model time 160 ky), before channel narrowing that propagates upstream (Figure 8). After cessation of the increased water flux at 150 ky, the channel profile returns to equilibrium through uplift and aggradation (Figure 8a). Channel aggradation begins at the bottom of the channel profile and results in a convexity that propagates upstream (Figure 8a). At model position y=400, from 150–158 ky, the channel increases in elevation due to uplift (Figure 8b). Wide valleys created during increased water flux are maintained, and new lateral erosion of valley walls is seen

(Figure 8b). At 159 ky, 9 ky after the cessation of increased water flux, the aggradational knickpoint reaches y=400 and incision and valley narrowing is observed (Figure 8d,e).

Figure 9a,b shows surface topography and cross sections across the model domain for two times in the low erodibility model run using the total block erosion model. This figure demonstrates the effect of valley deepening, then widening in response to increased water flux. Before water flux is increased, the channel is narrow and has steep valley walls (Figure 9a, Figure 10a~~,b~~). After 20 ky of increased water flux and increased vertical incision, channel erosion and baselevel fall reach a new equilibrium and channel elevation is stationary. Only after this period of re-equilibration can lateral erosion begin to widen the valleys. After 30 ky of increased water flux, the entire channel has incised, especially in the upper valley. At y=420, the position of the cross section, the channel has been incised by 3 m, and the valley has widened to about 20 m (Figure 9b). This response of primarily vertical incision is expected when using the total block erosion model, which sets a high threshold for lateral erosion.

5.2.3 Undercutting-slump models

In the undercutting-slump models, all of the model runs show a significant increase in channel mobility with additional water flux (Figure 7~~e~~b,d). The largest valley widths occur in the models with low bedrock erodibility for reasons discussed above. Unlike the total block erosion models, there is no discernible lag between onset of water flux and valley widening in the undercutting-slump models (Figure 10b). This is because ~~the volume that must be eroded from neighboring nodes is set by the water surface height in this~~ erosion of the valley wall is independent of the height of the valley wall for the undercutting slump model formulation and the increase in drainage area ~~increases~~ results in larger increases in lateral erosion rates ~~more than~~ faster compared to vertical incision rates ~~.~~ (Equation 8, 13).

Figure 9c,d shows topography and cross sections for two ~~time~~ times in the low erodibility model run using the undercutting-slump model. Before water flux is increased, the channel is significantly wider than in the total block erosion model. The cross section shows a wide valley spanning three model cells, and low gradient areas on the neighboring interfluves, indicating that these areas were shaped by the lateral erosion from the channel. After 40 ky of increased water flux, the valley is much wider across the entire model domain, especially at the upstream segments of the channel. At y=420, the channel migrated 50 m across the model domain in 40 ky. The undercutting-slump model runs with medium and low erodibility maintain increased valley width after water flux has decreased, particularly in $K_l/K_v = 1.5$ models (UC1.5) (Figure 7d). This indicates that wide valley floors can persist for long periods of time after the conditions that created them have stopped.

5.2.4 Effects of increased sediment flux on lateral erosion

In order to explore how the addition of sediment to a stream affects lateral erosion and valley widening, we added sediment to the inlet point at the top of the model. The sediment flux models were run for 100 ky with 50 ky of standard lateral erosion followed by 50 ky of increased sediment flux. Before additional sediment flux was added, the sediment flux at the inlet was equal to the carrying capacity of the stream, which is equal to UA. Models with increased sediment flux were run using both model formulations, $K_l/K_v = 1.0$ and 1.5, and α values that ranged from 0.2–2.0, with bedrock erodibility held constant at 1×10^{-4} (Table 1). During the 50 ky periods of increased sediment flux, five times more sediment flux was added, forcing all of the streams to aggrade initially. Adding sediment increases the deposition term (Equation 3), which will result in aggradation if the model is initially in steady state, that is $e-d=U$. Aggradation in the channels continues until the channel slopes become steep enough to increase the vertical erosion term so that $e-d=U$ again, and the landscape is in a new equilibrium state. The model only responds to changes in sediment flux by adjusting channel slope, rather than both slope and channel width as observed in natural systems (Yanites et al., 2011) because of the fixed width scaling in this model.

Figure 11 shows valley width averaged over the upper half of the model domain (closest to the sediment source) plotted against model time. After sediment is added to the models, all of the model runs show a significant increase in valley width, except the low α model runs, which show little change in width. Valley width increases more and valleys stay wide for longer with higher values of α. Valleys are narrowest and least persistent through time in the TB1 model group (Figure 11a), and valleys are widest and most persistent through time in the UC1.5 model group (Figure 11d). Valley widths and duration of wide valleys after the addition of sediment are similar between the TB1.5 group and the UC1 group (Figure 11b,c).

The addition of sediment to these models results in channel aggradation and valley filling that accounts for a substantial fraction of measured increases in valley width for all of these model runs. It is not possible to distinguish between widening due to valley filling and widening due to bedrock wall retreat from this spatially averaged value of valley width. However, we know that the TB1 models have little lateral bedrock erosion during the runs with no additional sediment flux, as seen in valley widths from 0–50 ky of the model runs (Figure 11a). Therefore, the valley widening that occurs from the TB1 model group is from valley filling only (Figure 10c)~~(further discussion in section below)~~. We then subtract the values of valley width through time for the TB1 group from the valley width through time for the other models runs to determine a metric we assume serves as a proxy for valley widening from lateral erosion alone (Figure 12).

Figure 10c,d shows model cross sections through time for the TB1 model and the TB1.5 model with α=1.5. The TB1 model shows valley widening exclusively through valley filling after the addition of sediment. Other channels shown in the cross section (at 80 m and 250 m) are immobile and show little evidence of lateral erosion (Figure 10c). Figure 10d shows an example of simultaneous valley filling and significant bedrock erosion in the TB1.5 model group. Before the addition of sediment flux (t=50ky), the channel is 10 meters wide. After the addition of sediment to the model, the channel aggrades by 2.5 meters while also shifting 50 meters to the right, eroding a significant amount of bedrock valley wall over 12 ky.

Figure 12 shows the difference in width through time between the model groups with significant widening and the total block model $K_l/K_v=1$ model group, which has valley widening only in response to valley filling. This reveals interesting behaviors of the model groups through time, both before and after the addition of sediment flux. In Figure 12, the first 50 ky of the model runs show the differences in width between the control model group (TB1) and the other model groups under normal lateral erosion conditions. Differences are greatest in the undercutting-slump $K_l/K_v=1.5$ (UC1.5) group and smallest in the total block $K_l/K_v=1.5$ (TB1.5) group. After the addition of sediment flux, not all runs in the model groups showed an increase in valley width compared to the control run. Lower values of α showed little or no increase in bedrock valley width after the addition of sediment flux. This is because channels in the low α runs (high sediment mobility) easily adapt to the increased sediment flux without significant or far-reaching changes to the channel slope. In the TB1.5 and UC1 model groups, α values of 0.8–1.0 tend towards increased variability in valley width following the addition of sediment flux, but no convincing signal of increased valley width, with the exception of model run $\alpha = 0.8$ in model group TB1.5 (Figure 12a). Model runs with $\alpha > 1.0$ tend to have valley widths that are 10–30 meters wider than would be expected from valley filling alone. This effect is small, but detectable in the TB1.5 model group (Figure 10d).

Figure 10c,d shows model cross sections through time for the TB1 model and the TB1.5 model with α=1.5. The TB1 model shows valley widening exclusively through valley filling after the addition of sediment. Other channels shown in the cross section (at 80 m and 250 m) are immobile and show little evidence of lateral erosion (Figure 10c). Figure 10d shows an example of simultaneous valley filling and significant bedrock erosion in the TB1.5 model group. Before the addition of sediment flux (t=50ky), the channel is 10 meters wide. After the addition of sediment to the model, the channel aggrades by 2.5 meters while also shifting 50 meters to the right, eroding a significant amount of bedrock valley wall over 12 ky.

The response to increased sediment flux in the UC1.5 model group is different from the responses in the UC1 and TB1.5 groups. In the UC1.5 group, increased valley width following increased sediment flux is more clearly defined for the low-medium α values and the highest α value shows increased valley width due to sediment filling rather than from lateral erosion (Figure 12c). It is interesting to note that mean valley width increases at 50 ky for all model runs, then declines to close to pre-sediment values by about 80 ky. Mean valley width begins to decline as the models come into steady state with the increased sediment flux, indicating that lateral erosion can most readily occur when the channel is in a transient, aggradational state.

Figure 13 shows the $\alpha = 1.5$ run from model group UC1.5, before and after added sediment flux that results in true bedrock valley widening. At 50 ky in the model run before the additional sediment is added, the valley in the upper half of the model domain (y=240) is flat and about 30 m wide (Figure 13a). Over 50 ky, sediment is added to the model and the channel aggrades for $\sim$20 ky before it comes into steady state, i.e., its slope is steep enough to carry the additional sediment load and aggradation stops. During the 20 ky of aggradation, this model run shows both retreat of the valley walls and channel aggradation. By 70 ky in the model run, the channel has aggraded by 5 meters and the valley is 50 m wide (Figure 13b). During this 20 ky period, the channel has migrated 50 m to the right, eroding the hillslope and forming steep valley walls.

Before the increase in sediment flux, all channels are in equilibrium by definition. Adding sediment to the inlet point in the models causes the channels to aggrade in all model runs, increasing the channel slope. This increase in channel slope increases the lateral erosion term and the vertical erosion term (Eqs. 8, 13); but while the channel is aggrading, vertical incision is effectively zero. Therefore, for the total block erosion models, most new lateral erosion should occur while the channel is aggrading, because the threshold volume that must be eroded becomes smaller when relief between the channel node and neighboring nodes decreases (Figure 2). Figure 11 shows that after sediment flux is added, there is a persistent increase in valley width for many model runs even after the channel profile has come into steady state with respect to the added sediment flux. The permanent increase in slope should result in higher lateral erosion rates, resulting in permanently wider valleys because the increased vertical incision rates that result from the higher slope is offset by increased deposition. This suggests the possibility that if a channel experiences increased slope through aggradation, then more lateral erosion occurs.

6 Discussion

6.1 Comparison among purely vertical incision models and end member lateral erosion models

~~This~~ The simple theory for lateral bedrock channel erosion presented here, combined with a landscape evolution model produces valleys that are several times wider than the channels they hold. The development of wide valleys is sensitive to the end member model formulation selected, which is discussed below. The widest valleys in this set of models occur in transport-limited model runs (high α values) when using the undercutting-slump model formulation~~. The undercutting-slump algorithm represents easily erodible bedrock allowing the development of wider bedrock valleys, as observed in many natural systems (e.g., Montgomery, 2004; Snyder and Kammer, 2008; Schanz and Montgomery, 2016)~~ , which represents lateral erosion that is independent of valley wall height. Wider bedrock valleys under conditions of relatively immobile sediment (high α value) (Figure 6) reflect conditions observed in natural systems, where wide bedrock valleys are considered a diagnostic feature of transport-limited streams (Brocard and Van der Beek, 2006).

The results presented here show that the lateral erosion component allows for mobile channels in all model runs (Figure 4a,b), even when the model has reached steady state, unlike models with vertical incision only which have stationary channels at steady state. The modeling experiments show that landscapes with highly erodible bedrock have the most mobile channels. In ~~both model formulations presented, easily erodible~~ the total block erosion model formulation, weak bedrock allows greater channel mobility because the amount of lateral erosion that must occur to erode valley walls is lower in low-relief landscapes with easily eroded bedrock. The model also predicts more channel mobility and wider valleys in models with high values of α (low sediment mobility), especially in the total block erosion models.

Channel mobility is a critical factor in the development of wide bedrock valleys, because all of the erosion of the valley must be accomplished through erosion by the channel (e.g., Tomkin et al., 2003). The width of surfaces beveled by lateral erosion has been framed as a competition between channel mobility and relative rock uplift rate (Bufe et al., 2016)~~. Channel mobility is also important because measures of channel mobility during periods of lateral planation can be used to validate lateral erosionin models~~, with greater channel mobility resulting in more area shaped by lateral erosion. The mobility of river channels increases with increasing sediment flux (Wickert et al., 2013), which emphasizes the potential importance of high sediment load as a requirement for the development of wide bedrock valleys. Landscapes in weaker bedrock are more likely to have more channel mobility and wider ~~valleys because in natural systems, rivers~~ valley (e.g., Montgomery, 2004; Snyder and Kammer, 2008; Schanz and Mon Rivers flowing through soft bedrock are also more likely to behave as transport-limited rivers, as a result of the increased sediment flux in the stream from the surrounding hillslopes and lower channel slopes in easily eroded bedrock. Channel mobility as a parameter extracted from the model is also important because measures of channel mobility during periods of lateral planation (e.g., Reimann et al., 2015) can be used to validate future lateral erosion models.

The two model formulations presented here describe end member behavior for ~~different widening responses in hard and soft bedrock~~how lateral erosion of valley walls scales with wall height, and can also be considered in terms of the physical processes of valley widening found in natural systems. The total block erosion model, in which the entire volume of a neighboring node must be eroded before lateral erosion can occur, best describes ~~the behavior of resistant bedrock. In the~~ lateral erosion in resistant and/or blocky material. This approach is used to represent, in a simple way, a system in which the undermining of a channel bank leads to gravitational collapse of resistant material that must itself then be eroded in place (Lancaster, 1998) . The dependence of rates of valley widening on wall height has been demonstrated in alluvial systems where sediment transport rates in the channel are low relative to the sediment eroded from valley walls (Bufe et al., 2016; Malatesta et al., 2017) . One can imagine a similar limitation in bedrock gorges where lateral valley wall movement is accomplished through rockfall into the river (Shobe et al., 2016) . Valley widening may also be limited when valley wall height exceeds the height of the flood stage, as vertical erosion of terrace surfaces can result in orders of magnitude greater valley erosion rates (Collins et al., 2016) .

The undercutting-slump ~~model, the neighboring node need only be undercut over the area of the model cell before the remainder of the node is transported out of the model~~ algorithm represents lateral erosion of valley walls that is independent of bank height. This model represents lateral erosion on a bank that has been laterally undercut and the remaining material slumps into the channel and is transported away as wash load, ~~and more accurately reflects behavior from weakly cohesive bedrock that tends to weather into small pieces, such as shale~~i.e. not redeposited in the model. The undercutting-slump model is applicable in locations with an under-capacity stream and lithology that slumps easily and rapidly breaks down into small grains that are easily transported (Finnegan and Dietrich, 2011; Johnson and Finnegan, 2015) . Lateral erosion that is independent of valley wall height may be more likely occur in weak bedrock, allowing the development of wider bedrock valleys, as observed in many natural systems (e.g., Montgomery, 2004; Snyder and Kammer, 2008; Schanz and Montgomery, 2016) . The undercutting-slump model consistently produces wider bedrock valleys and more mobile channels than the total block erosion model because less lateral erosion is required to erode valley walls in the undercutting-slump model algorithm.

However, this undercutting-slump model is not appropriate for landscapes with very hard bedrock (low erodibility), as evidenced by overhanging cliffs along many rivers and persistent blocks of collapsed material following slumping or delivery from adjacent hillslopes (Shobe et al., 2016) . The behavior of the models varies significantly based on which model is selected, although the same general trends are seen in both models. In nature, lateral erosion of valley walls will not follow either one of these end members perfectly, but will operate on a continuum between the two (Lancaster, 1998). Tomkin et al. (2003) presented two end member relationships between channel erosion and valley erosion that are similar to the models presented ~~here and they~~ in this study, and found similar behavior between their two models.

6.2 Model limitations and future directions

While the model captures several important markers of lateral bedrock erosion, such as mobile channels and bedrock valleys that are several times the channel width, the model did not develop broad, smooth, valleys that are many times the width of their channel and that are sustained over many years, as observed in flights of strath terraces in the Front Range of Colorado, for example (Foster et al., 2017) . The model also did not show a strong relationship between increased sediment flux and protection of the channel bed from vertical incision and increased lateral erosion of valley walls as we expected to see (Figure 12). Some important elements of reality have been simplified or omitted in this model, and future versions should address 1) setting runoff variability and magnitude separately from grain size, 2) including tools and cover effects and thresholds in the vertical incision model, 3) treating sediment and bedrock erodibility separately, 4) hillslope processes, 5) differences in grain sizes, 6) changes in bank material through time from weathering or water content. The first three items are the most important in our opinion.

In order to focus on implementing the equations for lateral erosion into the model, the simplest possible erosion-deposition model was used. This erosion-deposition model (Equation 1) has the advantage of not requiring the calculation of transport capacity and prevents potential problems with abrupt transitions from erosion to deposition, but does so at the expense of losing some details of runoff rate and grain size, which are lumped into the parameter α. In this model, detachment- or transport-limited behavior is set through α, which works well for general model exploration, but becomes problematic when exploring specific model responses to spatial and temporal changes in runoff rate and multiple grain sizes. Setting runoff and grain size explicitly is a important next step for determining how these factors independently impact bedrock valley width and channel mobility. Including a dynamic K_l/K_v that is calculated with runoff from discrete events and channel widths is a target for future models. Runoff rate can vary widely, but a higher runoff intensity will lead to a higher K_l/K_v ratio and more lateral erosion, as suggested by field observations of lateral erosion in bedrock channels during large flood events (Hartshorn et al., 2002) and correlation of increased sinuosity and storminess of climate (Stark et al., 2010) . The model presented here does not have the capability to represent changes in K_l/K_v based on processes that cause increased lateral erodibility, such as changes in the distribution of sediment during high flow (Hartshorn et al., 2002) or increased mass wasting of hillslopes (Stark et al., 2010) . More process specific representation of K_l/K_v ratio is a target for future model development.

The model presented in this paper uses the stream power incision model, the simplest reasonable vertical incision model, in order to focus on our goal of exploring the novel application of lateral bedrock erosion in a landscape evolution model. Using a tools and cover incision model (e.g., Beaumont et al., 1992; Sklar and Dietrich, 2004; Gasparini et al., 2004; Turowski et al., 2007) in a future lateral erosion bedrock model would be closer to the way we conceptualize lateral erosion in natural systems. The main impact of using a tools and cover incision model in a lateral erosion model would be less efficient vertical incision as relative sediment flux increases (Hobley et al., 2011) . Slowing vertical erosion so that lateral erosion can catch up is an important part of the mechanism cited by many studies for allowing lateral erosion in incising streams (Hancock and Anderson, 2002; Turowski et al., 200 Slowing vertical incision may be a necessary condition for significant lateral erosion and bedrock valley widening, but it is not by itself a sufficient condition. A model that describes how sediment tools carry out lateral erosion needs to be addressed (Fuller et al., 2016) , but tools and cover incision models do not offer any mechanism for changing the rate of lateral erosion, just decreasing the efficiency of vertical incision.

Another limitation of the current model is that sediment is not treated explicitly, but rather is tracked in the model through the Q_s term. No distinction in erodibility is made between sediment and bedrock. In the current model, when the landscape is in steady state, vertical erosion plus deposition is equal to the uplift rate. Increasing sediment flux, Q_s, in the deposition term immediately results in channel aggradation and increasing channel slope. In natural systems, channels respond to increased sediment flux by increasing both slope and width. Changes in channel width are not captured in this model due to the fixed value of k_w, which is appropriate for landscapes in quasi-equilibrium (Whipple et al., 2013) . How bedrock channel width responds to changes in boundary conditions, such as uplift rate and sediment, is the subject of ongoing research (e.g., Lague, 2014; Whittaker et al., 2007; Turowski et al., 2009) , with important implications for driving channel incision of slump deposits and terrace generation (Croissant et al., 2017) .

In model formulations that use the concept of transport capacity of a stream, adding sediment to a river that is far below transport capacity will not cause aggradation, but will easily carry the sediment load downstream. If sediment is continually added to a such a stream, the ratio of sediment flux, Q_s, to transport capacity, Q_t, will increase until Q_s/Q_t=1 and the stream becomes transport-limited (Willgoose et al., 1991) . As Q_s/Q_t for a stream increases, the bed of the stream is progressively covered by more sediment, protecting the underlying bedrock from further incision (Sklar and Dietrich, 2004) . Under these kinds of scenarios, adding sediment to a detachment-limited stream eventually reduces vertical incision, and allows lateral erosion to widen the bedrock channel walls while the bed remains stationary (Hancock and Anderson, 2002) .

In not differentiating between sediment and bedrock explicitly in this model, the different erodibilities of sediment and bedrock are not accounted for. In most cases, sediment in a channel should be much easier to erode than the bedrock in a channel, allowing more rapid lateral migration through cells that have previously been occupied and are contain some amount of sediment (Limaye and Lamb, 2013) . But in some cases, sediment in a soft bedrock channel can be composed of coarse grained, resistant lithology sourced from upstream. For example, the streams that drain the Colorado Front Range flow from hard, crystalline bedrock onto soft, friable shale bedrock (Langston et al., 2015) . The granitic cobbles that armor the channel bed in stream segments underlain by shale bedrock, take much more energy to move than it does to transport the friable flakes of shale that line the walls of the channel. Different erodibilities should also result in more active channel migration once a wide valley is established because the channel erodes laterally through sediment that is more easily eroded than bedrock (Limaye and Lamb, 2014) .

6.3 Comparison between models and field studies

Lateral erosion rates depend on the magnitude of shear stress and tools applied to channel walls, and the resistance of the bedrock to erosion. Our model of lateral bedrock erosion proposes that channel curvature controls lateral erosion rate. Cook et al. (2014) showed that extremely efficient bedrock wall erosion of up to ∼80 m over 5 years occurred where the river encountered sharp bends. They attribute this rapid lateral bedrock erosion in river bends to abrasion from sediment particles that detach from flow lines in the curve and impact the wall. Fuller et al. (2016) also suggest that lateral erosion rate by bedrock abrasion depends on how often sediment particles are deflected towards the channel walls~~.~~ , specifically by channel roughness elements. There is an important distinction between this study and the work of Fuller et al. (2016) in that their conclusions are based on observations of lateral erosion in a straight flume. Lateral erosion that occurs in the absence of channel curvature highlights the point that channel curvature is not the only control on lateral erosion, but it is an important one.

The total block erosion model demonstrates how landscapes with hard bedrock and detachment-limited conditions respond to increased discharge by first incising the channel bed, and then widening after the channel has come into equilibrium (Figure 10a,b). This behavior is similar to narrowing and incision of bedrock channels in response to increased uplift ~~or increased discharge (Duvall et al., 2004)~~ (Duvall et al., 2004) or vertical incision followed by channel widening in response to increased discharge (Anton et al., 2015) . The model predicts that not only will channels in easily eroded bedrock reach equilibrium more quickly than channels in resistant bedrock, but channels in easily eroded bedrock will begin to widen valleys faster than in more resistant bedrock (Lavé and Avouac, 2001).

One of the few studies that has been able to report bedrock valley widening through time is from a unique case in Death Valley (Snyder and Kammer, 2008). Stream capture increased the drainage area of a small basin by 75 fold in the 1940's and channel response over the following 60 years was mapped by aerial photos. Snyder and Kammer (2008) found that mean valley width in a channel segment with weak bedrock increased by 9 meters in 60 years. In contrast, in channel segments in hard bedrock, they found vertical channel incision and the development of knickpoints. They attribute the difference in response to lithological differences and suggest that the presence of sediment on the bed in the weak bedrock channel segments protects the bed from incision, allowing the valley walls to migrate laterally. This difference in response is similar to the behavior of the end-member models presented here: the total block erosion model shows rapid incision and narrowing in response to increased water flux, whereas the undercutting-slump models show incision and valley widening.

In nature, we often assume that lateral erosion is achieved by adding sediment, suppressing vertical erosion and giving lateral erosion a chance to outpace vertical incision. If this is the case, then we expect increased sediment flux to have the largest effect on the low α/detachment-limited model runs. The same amount of new sediment was added to each model run, but the sediment resulted in more aggradation the high α runs. In the high α/transport-limited runs, the channels already behave as if they are loaded with sediment. In low α runs, the model tends towards detachment-limited behavior, so there is abundant stream power to carry away the sediment. The slope needed to transport the additional sediment is lower in the detachment-limited runs, resulting in less aggradation in response to the increased sediment flux.

The addition of sediment in this model does not lead to increased sediment cover on the bed, as bedrock and sediment are not differentiated in the model, but rather results in immediate channel aggradation. This channel aggradation in the model certainly indicates that vertical incision has stopped, allowing lateral erosion to become the primary erosive agent, even in models where K_l/K_v ratio is low or in the total block erosion models. This predicted increase in lateral erosion during periods of aggradation occurs in ~~many~~ some of the model runs, especially those with high α values. When the channel has reached a new equilibrium following increased sediment flux, many model runs maintain wider valleys due to the higher slope and increased lateral erosion rates.

6.4 ~~Model limitations~~Potential tests of model with field data

~~While the model captures several important markers~~

Researchers have only recently started to study the mechanistic processes of lateral bedrock erosion ~~, the model did not develop broad, smooth, valleys that are many times the width of their channel and that are sustained over many years, as observed in flights of strath terraces in the Front Range of Colorado, for example. The model also did not show a strong relationship between increased sediment flux and protection of the channel bed from vertical incision and increased lateral erosion of valley walls. Some important elements of reality have been omitted or set with a lumped parameter in this model~~(Fuller et al., 201 The model presented here does not include all of the processes the community has identified as relevant to lateral erosion;

rather, we formulated the simplest reasonable model to test the hypothesis that stream power exerted on channel walls is a primary control on lateral bedrock erosion. Therefore, we do not consider the presumed significant role of changes in climate when developing criteria to evaluate if this model is successful. We also do not consider small scale processes, such as 1) treating sediment and bedrock erodibility separately, 2) setting variability and magnitude of runoff separately from grain size, 3) threshold effects of sedimenton bed, 4) hillslope processes, 5) differences in grain sizes, 6) changes in bank material through time from weathering or water content. The first three items are the most important in our opinion.

In order to focus on implementing the equations for lateral erosion into the model , the simplest possible erosion-deposition model was used. This erosion-deposition model (Equation 1) has the advantage of not requiring the calculation of transport capacity and prevents potential problems with abrupt transitions from erosion to deposition, but does so at the expense of losing the details of runoff rate and grain size, which are lumped into the parameter α. In this model abrasion of channel walls by sediment, detachment- or transport-limited behavior is set through α, which works well for general model exploration, but becomes problematic when exploring specific model responses to changes in runoff rate and sediment size. Setting runoff and grain size explicitly is a important next step for determining how these factors independently impact bedrock valley width and channel mobility. rather landscape-scale drivers of valley wall erosion.

Another limitation of the current model is that sediment is not treated explicitly, but rather is tracked in the model through the Q_s term. No distinction in erodibility is made between sediment and bedrock. In the current model, when the landscape is in steady state, vertical erosion plus deposition is equal to the uplift rate. Increasing sediment flux, Q_s, in the deposition term immediately results in channel aggradation. In model formulations that use the concept of transport capacity of a stream, adding sediment to a river that is far below transport capacity will not cause aggradation, but will easily carry the sediment load downstream. If sediment is continually added to a such a stream, the ratio of sediment flux, Q_s, to transport capacity, Q_t, will increase until Q_s/Q_t=1 and the stream becomes transport-limited (Willgoose et al., 1991) . As Q_s/Q_t for a stream increases, the bed of the stream is progressively covered by more sediment, protecting the underlying bedrock from further incision (Sklar and Dietrich, 2004) . Under these kinds of scenarios, adding sediment to a detachment-limited stream eventually reduces vertical incision, and allows lateral erosion to widen the bedrock channel walls while the bed remains stationary (Hancock and Anderson, 2002) . One of the goals of developing landscape evolution models is to develop and test hypotheses about how dynamics in natural systems work over spatial and temporal scales that are not readily observable. A challenge remains of how to test a newly developed numerical model with field data. The robust data set that could be used to test the model presented here are the following: knowledge of the duration of widening; measurements of steady state valley geometry, including valley width, channel width, and vertical offset; measurements of valley geometry during the active widening phase; and perhaps most importantly, the processes of lateral erosion must be well characterized. This would dictate that channel curvature must be identified as the primary mechanism for lateral erosion in order to test the model presented here, for example, rivers in mudstone bedrock where detachment from the bank is from fluid stresses alone (Finnegan and Dietrich, 2011; Johnson and Finnegan, 2015) . A field data set to test this lateral erosion model could conceivably be derived from experimental data, a well constrained "natural experiment" of wide bedrock valley that developed over geologic time scales (Tucker, 2009) , or from rapid valley widening associated with an extraordinary event. To our knowledge, experimental data sets that describe the effect of channel curvature on lateral bedrock erosion do not exist, nor have we identified an appropriate natural experiment to evaluate bedrock valley widening over geologic timescales.

In not differentiating between sediment and bedrock explicitly in this model, the different erodibilities of sediment and bedrock are not accounted for. In most cases, sediment in a channel should be much easier to erode than the bedrock in a channel. But in some cases, sediment in a soft bedrock channel can be composed of coarse grained, resistant lithology sourced from upstream. For example, the streams that drain the Colorado Front Range flow from hard, crystalline bedrock onto soft, friable shale bedrock (Langston et al., 2015) . The granitic cobbles that armor the channel bed in stream segments underlain by shale bedrock, take much more energy to move than it does to transport the friable flakes of shale that line the walls of the channelFrom 2004–2010, Cook et al. (2014) documented the rapid lateral erosion of a bedrock gorge that was created as a result of coseismic uplift in 1999. They propose a mechanism of rapid gorge eradication, termed “downstream sweep erosion”, by which a bedrock valley is rapidly created when the channel must make a sharp bend to enter the gorge downstream. The upstream boundary of the uplifted bedrock block is eroded by rapid erosion focused on the outside bend when the channel makes a sharp bend to enter the gorge. This particular set of field data meets all of the requirements to test the lateral erosion model presented here: control on duration of widening, measurements of valley geometry through time, and an apparently strong dependence of lateral erosion rate on channel curvature. While it is beyond the scope of this paper to fully validate the lateral erosion model, we note that many observations from the field data set of Cook et al. (2014) are outcomes that are also seen in the lateral erosion model presented here (Langston and Tucker, 2017) .

7 Conclusions

We have shown that a simple, physics-based theory for lateral bedrock channel migration, when combined with a landscape evolution model, produces several interesting behaviors observed in natural systems. During transient channel incision, lateral erosion in the model temporarily stalls until channel equilibrium is re-established. Following a transient disturbance, wide bedrock valleys develop more quickly in weaker bedrock. The model predicts wider bedrock valleys with easily erodible bedrock, as many have observed in natural landscapes (Montgomery, 2004; Brocard and Van der Beek, 2006). Weaker bedrock also results in more channel mobility, which is a fundamental factor for developing and maintaining a bedrock valley that is several times wider than the channel it holds (Tomkin et al., 2003). Increased channel mobility and wider flat-bottomed valleys under transport-limited conditions in the model, suggests that sediment cover on the bed that is present under transport-limited conditions is an effective way to slow vertical incision and amplify the effect of lateral erosion (Hancock and Anderson, 2002). However, the model lacks some important elements of reality, especially variations in runoff and separate handling of bedrock and sediment in the channels. Our theory for the lateral erosion of bedrock channel walls and the numerical implementation of the theory in a catchment-scale landscape evolution model is a significant first step towards understanding the factors that control the rates and spatial extent of wide bedrock valleys.

Code availability. The lateral erosion models described in this text will be made available as a Landlab component in the summer of 2017.

Competing interests. The authors declare that there are no competing interests present.

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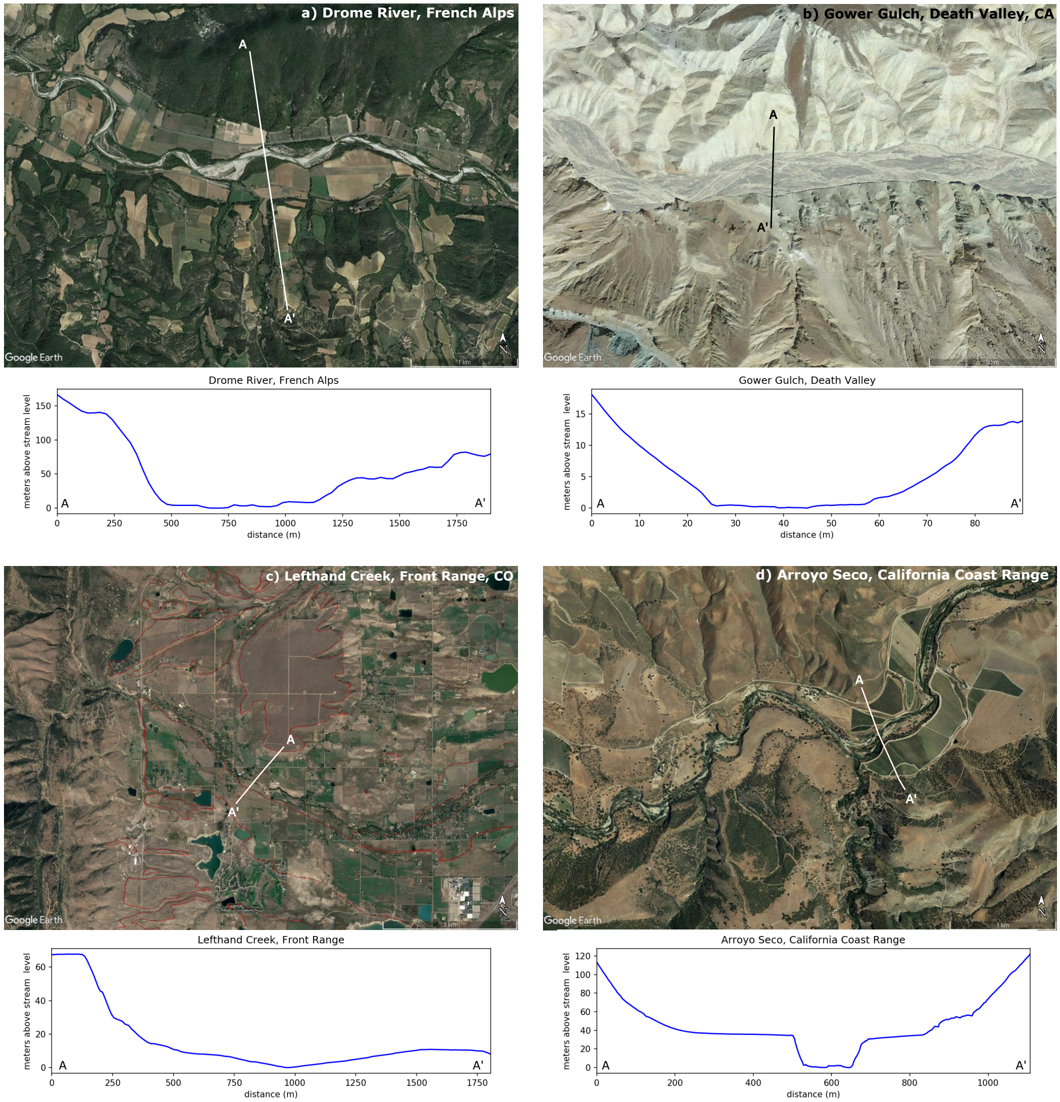

Figure 1. Field examples of lateral bedrock erosion and wide bedrock valleys. All cross sections are from north to south. a) The Drôme River in the French Alps is transport-limited and meandering in reaches that carve wide bedrock valleys. The bedrock valley at this location (44.69°N, 5.14°E) is 500 m wide and the channel is ∼45 m wide. b) Gower Gulch (36.41°N, 116.83°W) in Death Valley, USA widened significantly in response to increased discharge from a stream diversion in the 1940's (Snyder and Kammer, 2008) . The bedrock valley is 30 m wide and the channel braids are ∼2 m wide. c) Lefthand Creek drains the Colorado Front Range (40.11°N, 105.25°W) and has undergone multiple cycles of lateral erosion that produced flights of strath terraces, outlined in red on the image. The cross section shows Table Mountain at ∼70 m above the current stream height on the north side of cross section and a lower terrace level at 10 m above current stream level on the south side of the cross section. d) Arroyo Seco in the California Coast Range (36.27°N, 121.33°W) carved a 600 m wide strath terrace during a period of lateral erosion that is 30 m above the current stream level. The current bedrock valley is 125 m wide and the channel is ∼15 m wide.

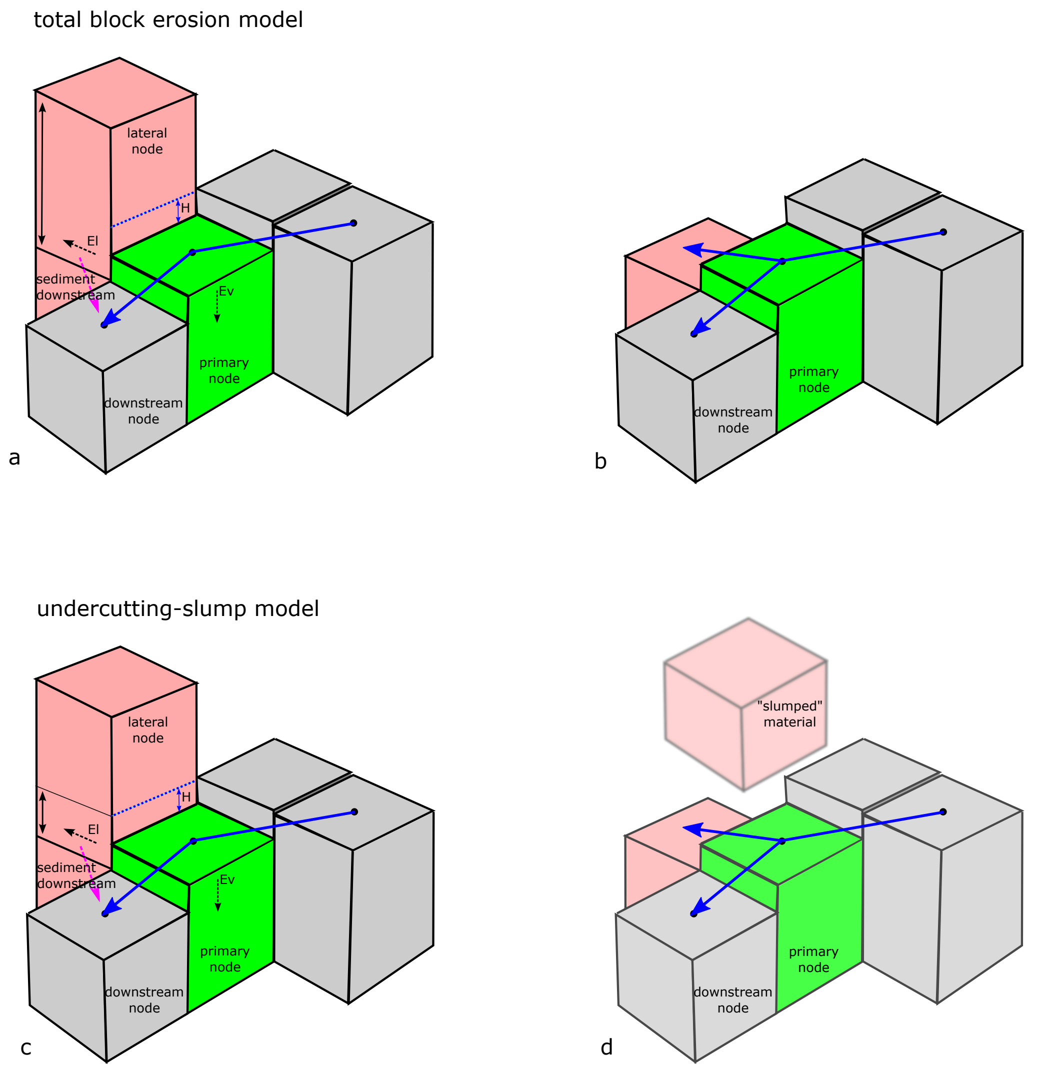

Figure 2. Conceptual illustration of model nodes showing the stream segments (in light blue) from the upstream node to the primary node (in green), to the downstream node. Vertical erosion (E_v) occurs at the primary node. The neighbor node (in pink) where lateral erosion (E_l) occurs is located on the outside bend of the stream segments. The height over which lateral erosion occurs, H, is shown in the dashed blue line. a) For the total block erosion model, the volume that must be laterally eroded before elevation is changed is $(Z_n - Z_d)dx^2$, the difference in elevation between the neighbor node and the downstream node (indicated with black arrow) times the surface area of the neighbor node. b) Elevation of the lateral node is changed after ~~after~~ the entire block is eroded and flow can ~~(~~potentially ~~)~~ be rerouted. c) In the undercutting-slump model, the volume that must be laterally eroded (representing bank undercutting) before elevation is changed is $(H - Z_d)dx^2$. $H - Z_d$ is the difference in elevation between the water surface height and the elevation of the downstream node, indicated with black arrow. d) When the neighbor node has been undercut, elevation is changed, allowing water to be re-routed, while the slumped material is transported downstream as washload.

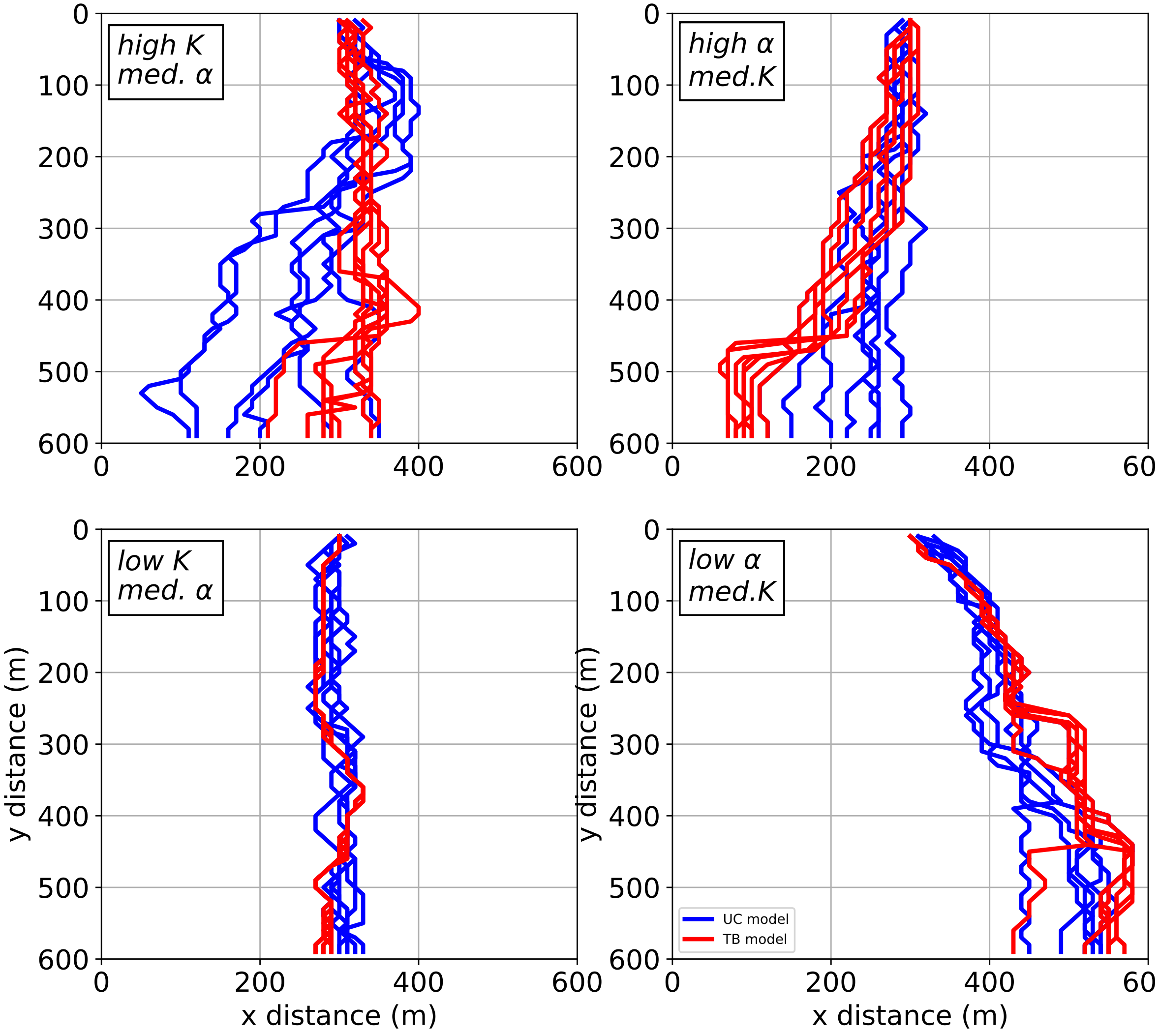

Figure 3. Channel positions over 200 ky with different values for bedrock erodibility and α in the undercutting-slump model (UC model, blue lines) and total block erosion model (TB model, red lines). a) high bedrock erodibility ($K = 2.5 \times 10^{-4}$), medium α value (α=0.8). b) high α (value, indicating low sediment transport (α=2.0), medium bedrock erodibility ($K = 10^{-4}$). c) low bedrock erodibility ($K = 5 \times 10^{-5}$), medium α value (α=0.8). d) low α(, indicating high sediment transport (α=0.2), medium bedrock erodibility ($K = 10^{-4}$).

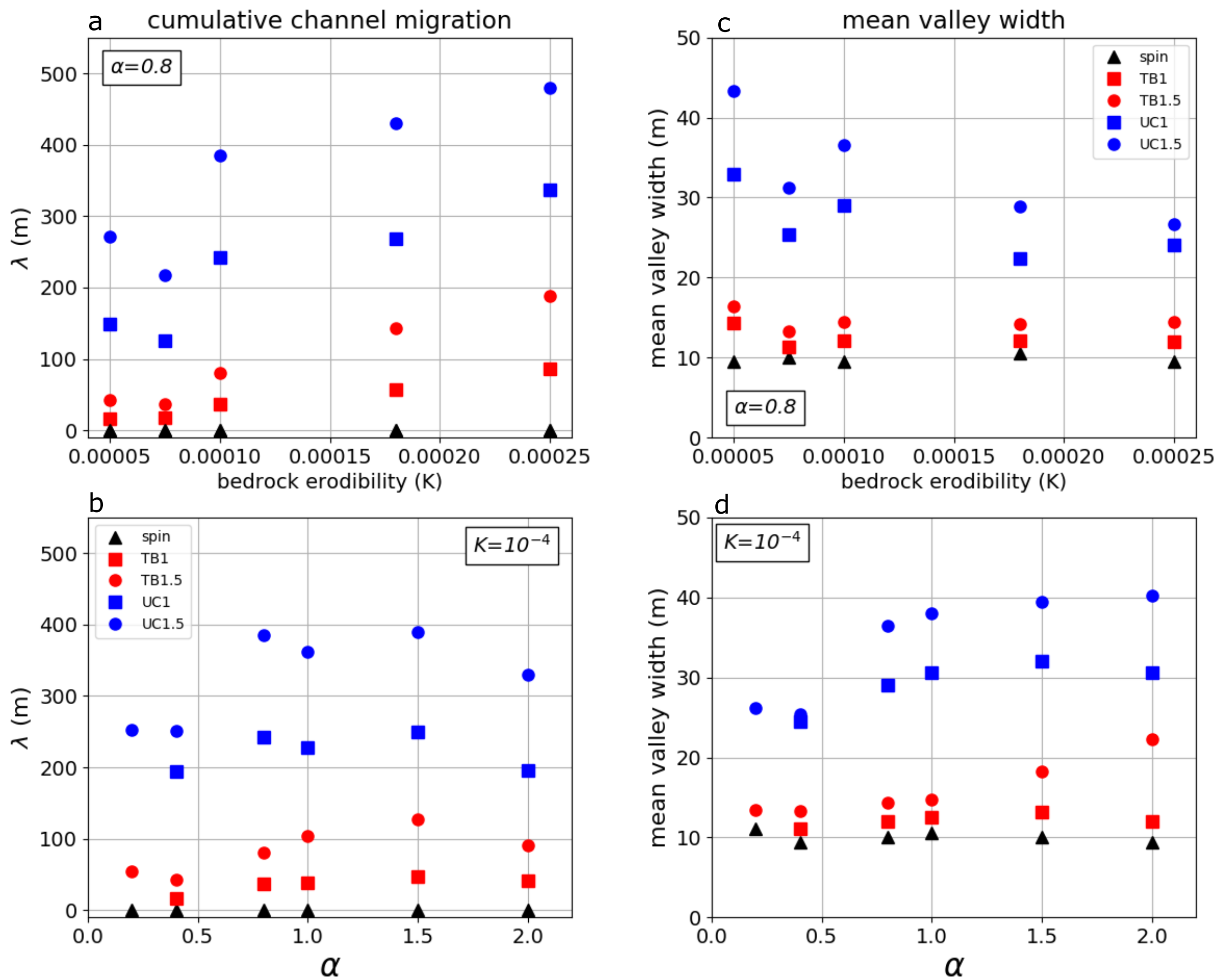

Figure 4. Cumulative channel-averaged migration (a,b) and mean valley width (c,d) over 100 ky for spin up models with no lateral erosion (spin, black triangles), total block erosion models (TB, red markers) and undercutting-slump models (UC, blue markers) with $K_l/K_v = 1$ (square markers) and 1.5 (circle markers). a) Cumulative channel-averaged migration (λ) for model runs with $\alpha = 0.8$ plotted against bedrock erodibility, K. b) λ for model runs with $K = 10^{-4}$ plotted against α. Mean valley width averaged over 100 ky of the model runs. c) Mean valley width for model runs with $\alpha = 0.8$ plotted against bedrock erodibility, K. d) Mean valley width for model runs with $K = 10^{-4}$ plotted against α.

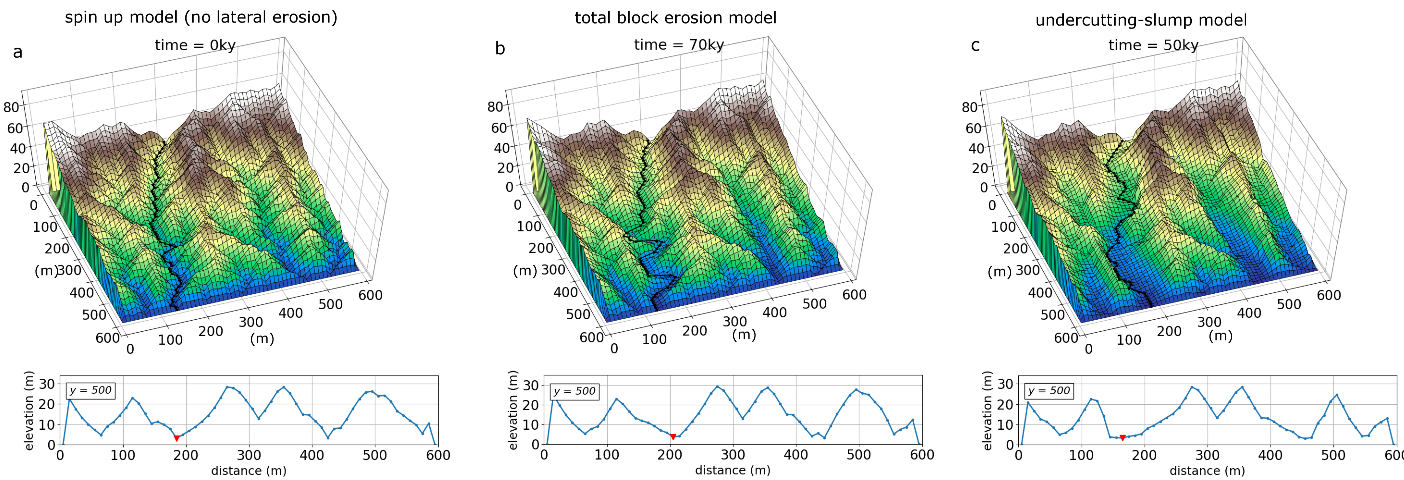

Figure 5. Model topography and cross sections at y=500 showing examples of valley widening. Black line indicates position of the main channel on the landscape. Red triangle shows position of the main channel in the cross section. a) Model with vertical incision only. b) Total block erosion model after 70 ky of lateral erosion. c)Undercutting-slump model after 50 ky of lateral erosion.

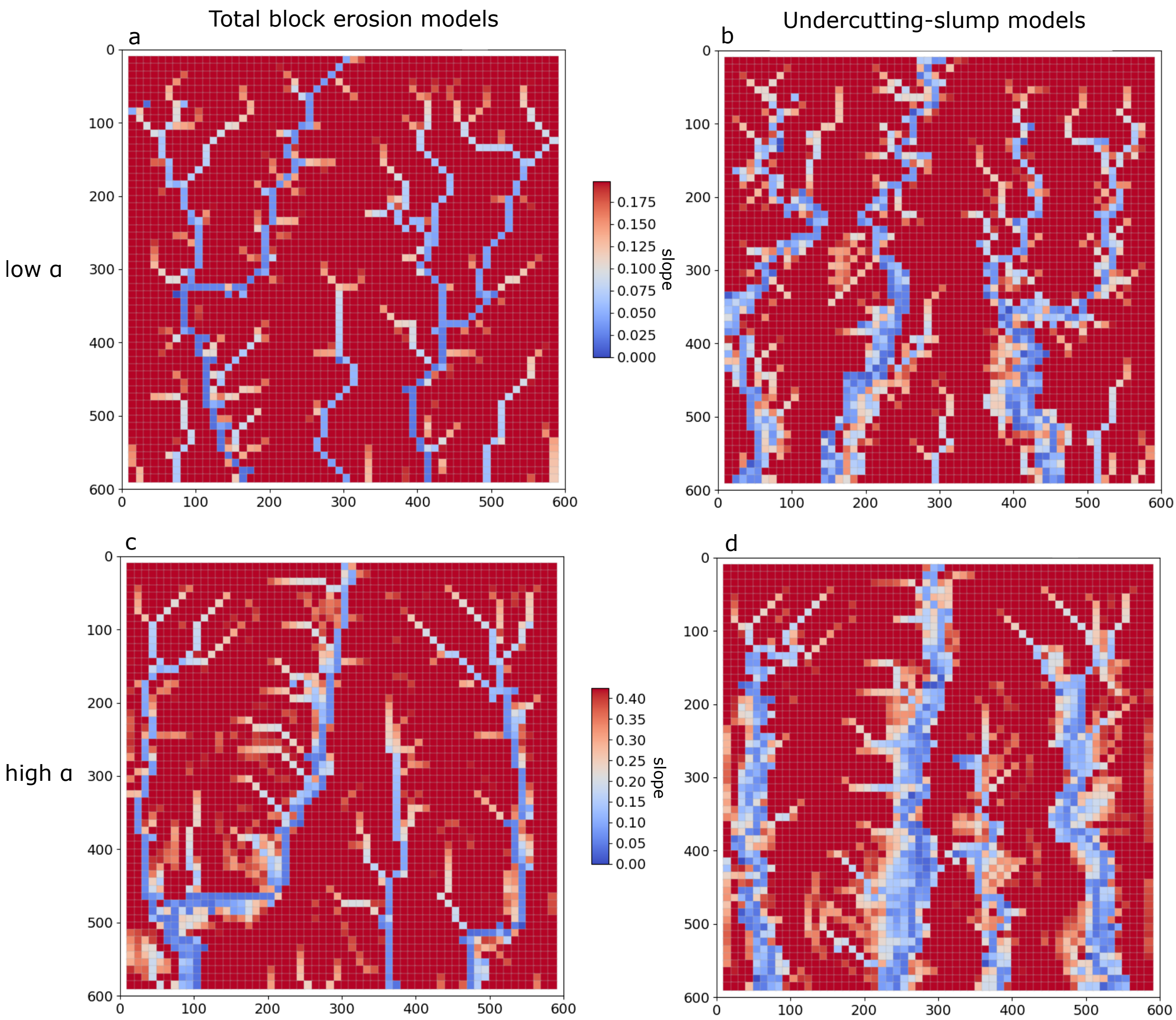

Figure 6. Slope maps showing fluvially carved valleys in total block erosion and undercutting-slump models with high and low values of α. The white and blue areas in the maps that indicate slopes that are characteristic of fluvial channels, i.e. lower than the reference slope value (Equation 17). a. Total block erosion model, low α (detachment-limited) b. Undercutting-slump model, low α (detachment-limited) c. Total block erosion model, high α (transport-limited) d. Undercutting-slump model, high α (transport-limited)

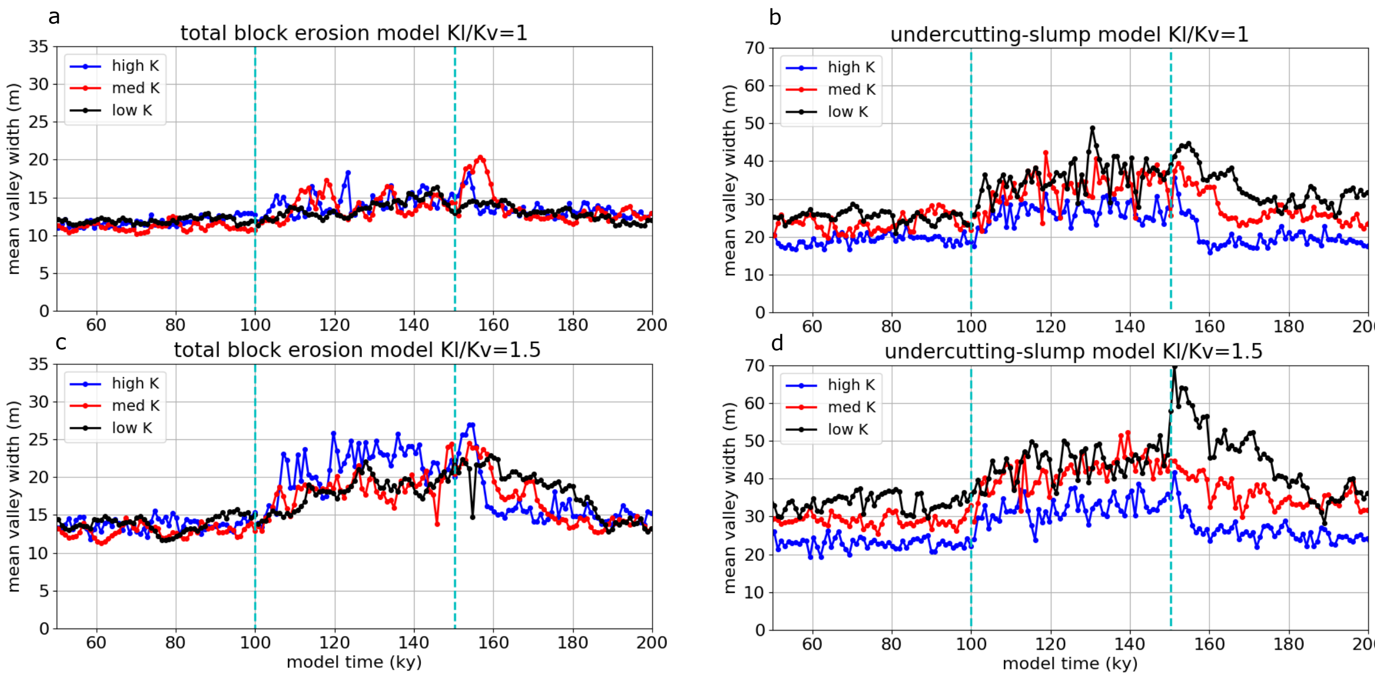

Figure 7. Valley width averaged over the ~~upper half of the~~ model domain vs. model time for total block erosion and undercutting-slump models with $K_l/K_v =$ to 1 and 1.5. ~~Water~~ Increased water flux occurs from 100 ky to 150 ky, indicated by vertical dashed blue lines.

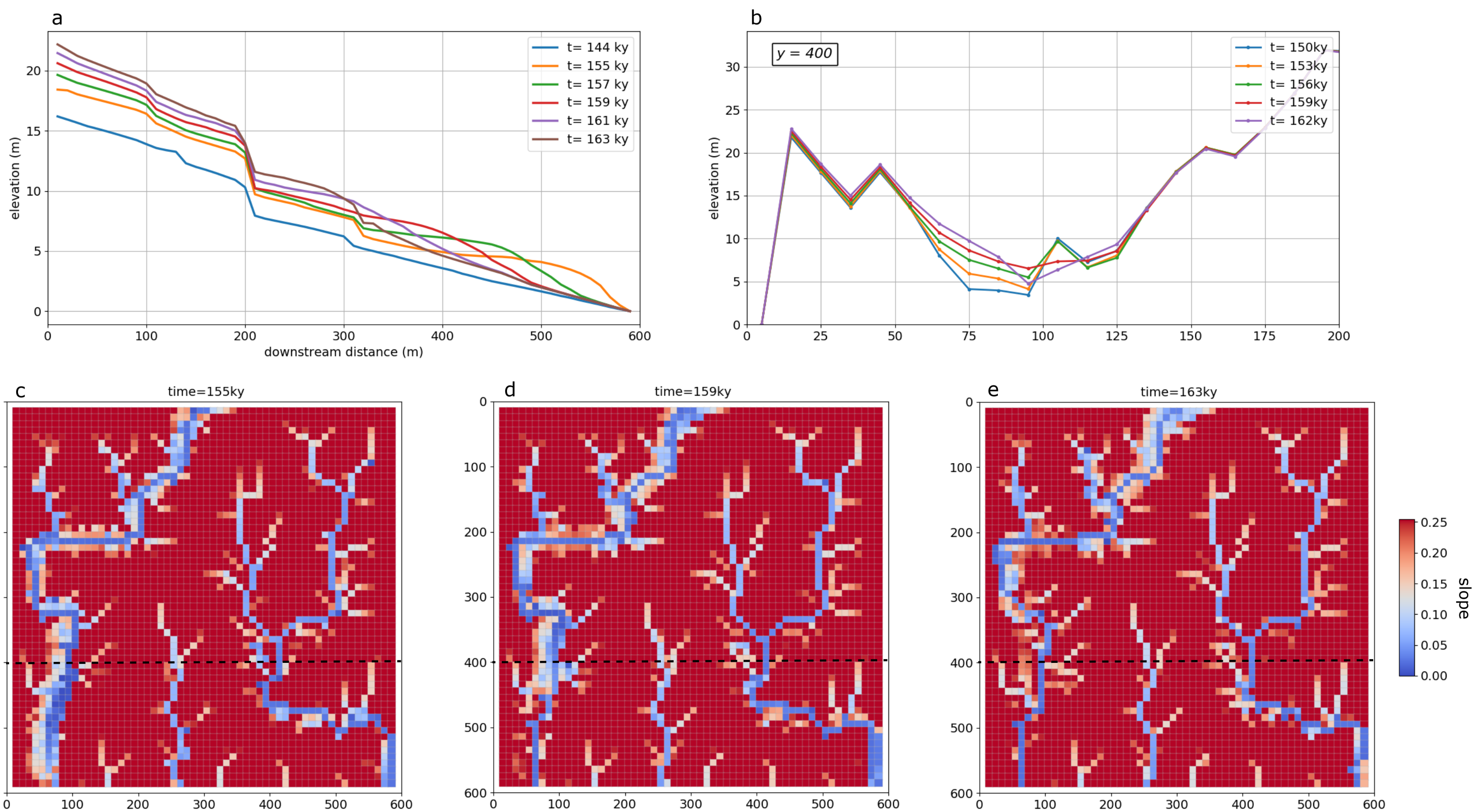

Figure 8. Longitudinal profile, cross sections, and slope maps from model run TB1.5, medium K after cession of increased water flux. a)Longitudinal channel profiles show uplift and aggradation, which produces a convexity that propagates upstream. b) Cross sections across the model domain at y=400 show channel aggradation and new lateral erosion of valley walls. c,d,e) Slope maps show valley narrowing following the passage of the knickpoint where y=400 (dashed line) at 155 ky, 159 ky, and 163 ky.

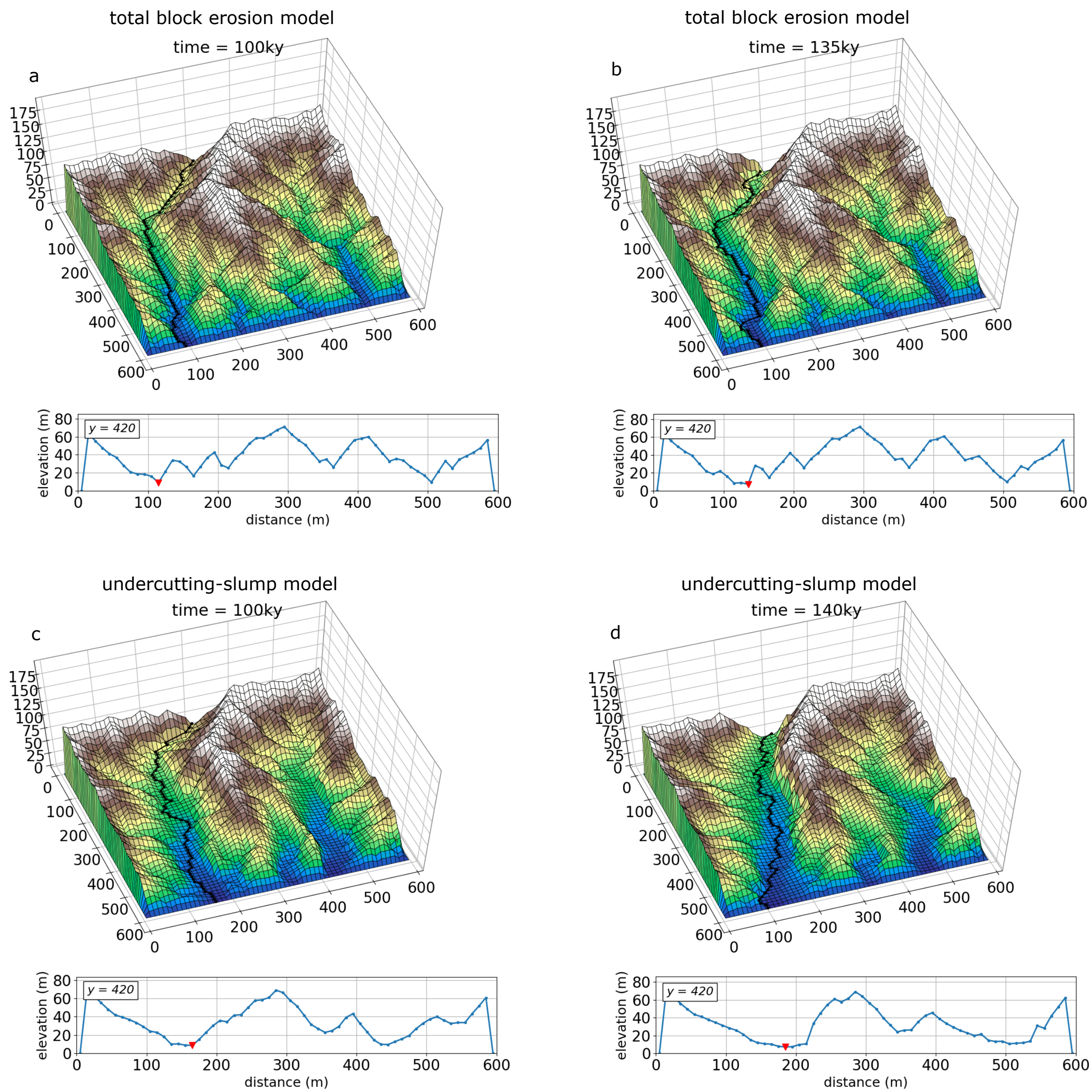

Figure 9. Surface topography and cross section at y=420 during period of increased water flux for the total block erosion models (a,b) and undercutting-slump models (c,d). Red triangle on cross sections indicates the channel position. a) Total block erosion model with low K and $K_l/K_v = 1.0$ at 100 ky, before the increase in water flux. Note that this model looks similar to the spin up model runs with no lateral erosion. b) After 35 ky of increased water flux. Cross section shows incision in the channel and increased relief between the channel and the hillslopes with some valley widening. c) Undercutting-slump model with low K and $K_l/K_v = 1.5$ at 100 ky, before the increase in water flux. Valley is 20 m wide. d) After 40 ky of increased water flux, the channel is slightly lower elevation than before the addition of water flux and the right wall of the valley has eroded by 50 m.

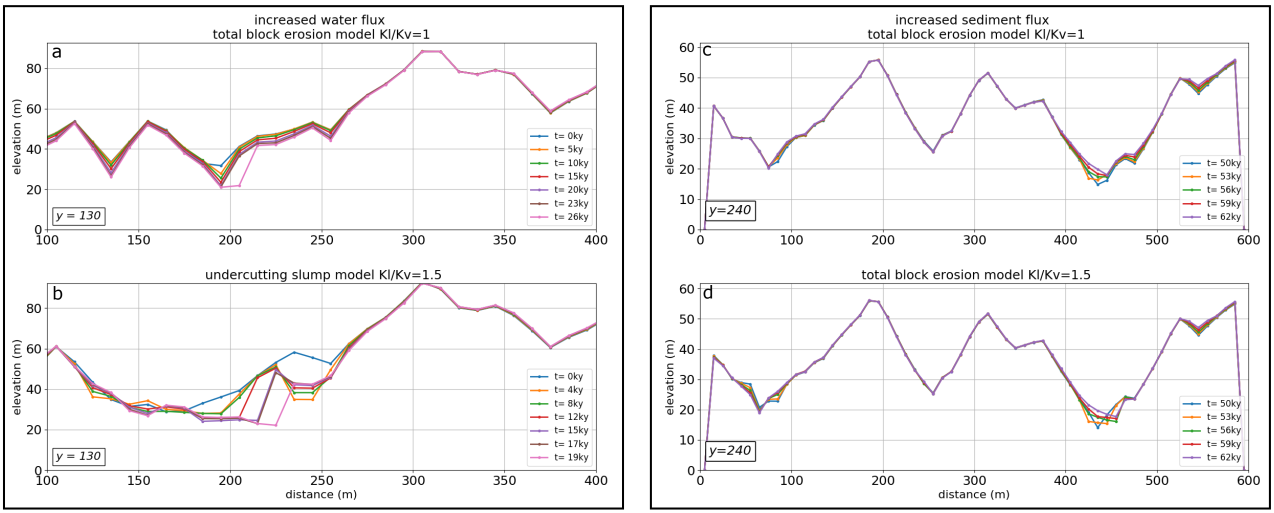

Figure 10. Cross sections across model domain for increased water flux and increased sediment flux models. a~~,b~~) Cross ~~section~~ sections at y=~~120~~ 130 for total block erosion ~~models~~ model with ~~low erodibility (~~K=5×10^{-5} ~~)~~ and ~~medium erodibility (~~~~K~~ K_l/K_v=~~10^{-4})~~ 1 during period of increased water flux. Cross sections over 26 ky show vertical incision of channel and increasing relief between the channel and hillslopes initially. After equilibrium is reached, lateral erosion can begin at an increased rate compared to before the additional water flux. b) Cross sections at y=130 for undercutting-slump model with K=5×10^{-5} and K_l/K_v=1.5 during period of increased water flux show simultaneous channel incision and widening over 19 ky of the model run. c,d) Cross sections at y=240 for total block erosion models with K_l/K_v=1 and K_l/K_v = 1.5 during period of increased sediment flux. c) In the TB1 model, the channel aggrades in response to increased sediment flux without eroding bedrock walls d)In the TB1.5 model, the channel aggrades and simultaneously erodes bedrock walls.

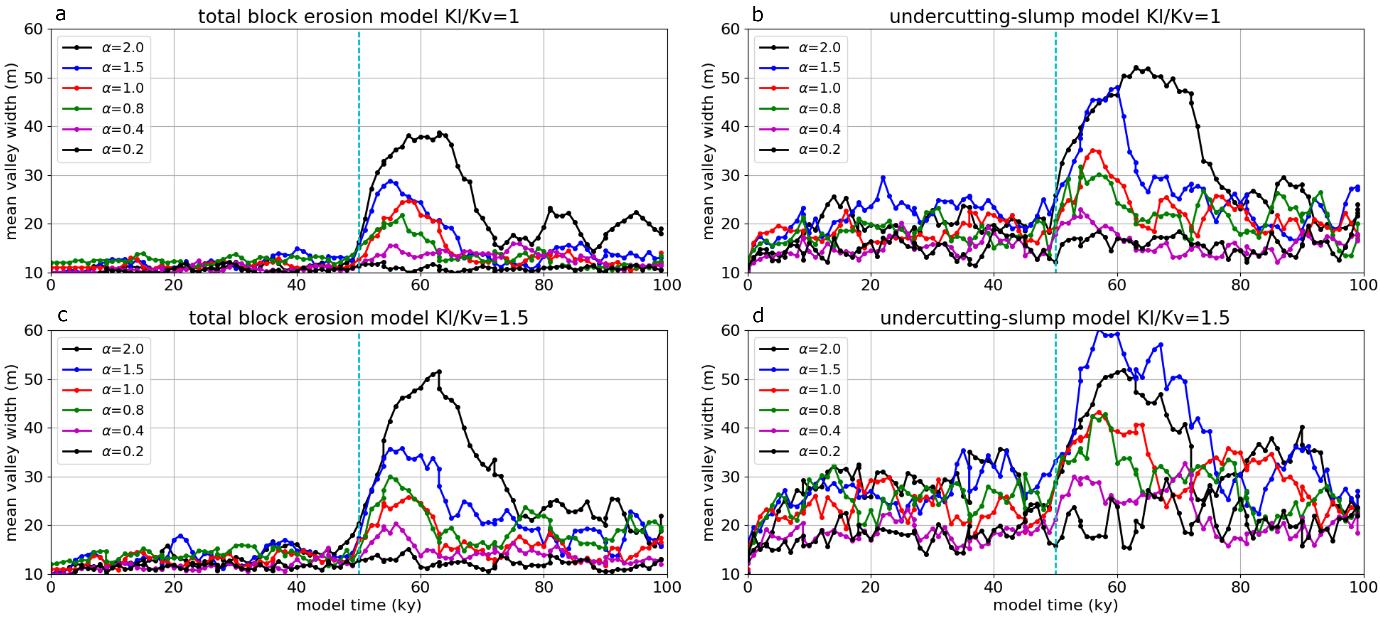

Figure 11. Mean valley width for the upper half of model domain over duration of additional sediment flux model run for total block erosion and undercutting-slump models with K_l/K_v ratio of 1 and 1.5. Dashed light blue line shows when addition of sediment flux began

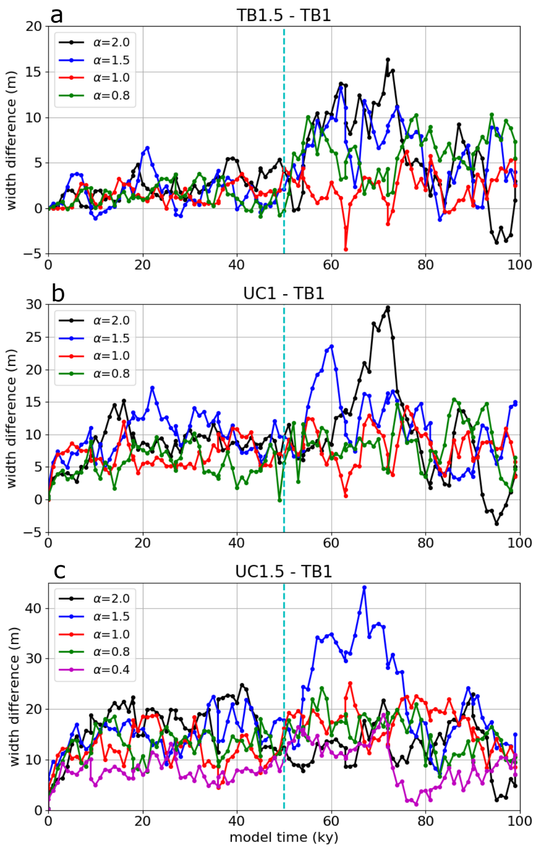

Figure 12. Difference from total block erosion model with $K_l/K_v = 1$ for a) total block erosion model with $K_l/K_v = 1.5$ b) undercutting-slump model with $K_l/K_v = 1$ c) undercutting-slump model with $K_l/K_v = 1.5$. Dashed light blue line shows when addition of sediment flux began.

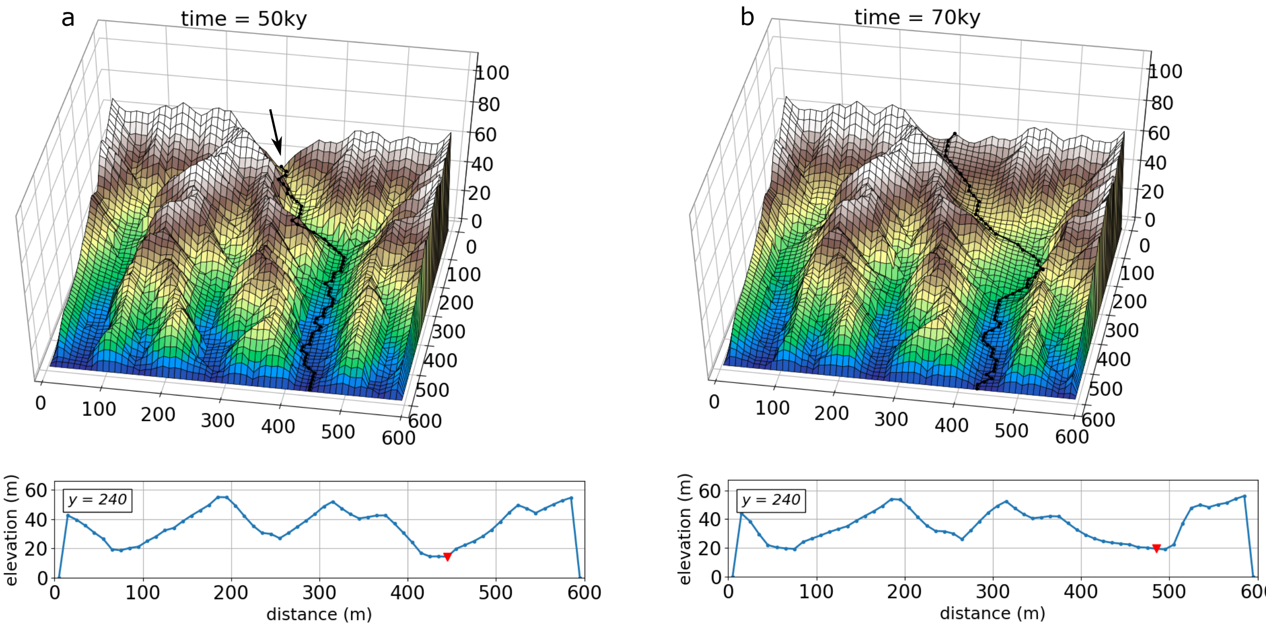

Figure 13. Model topography and cross sections at y=420 during period of increased sediment flux for the undercutting-slump model with α=1.5 and K_l/K_v=1.5. Black line indicates position of the main channel on the landscape. Red triangle shows position of the main channel in the cross section. a) Before increased sediment flux is introduced at input point, indicated with the arrow. b) After 20 ky of increased sediment flux, the channel has aggraded by 5 m and has eroded the valley wall by 50 m.