

27<sup>th</sup> November 2017

Dear Jean Braun,

Thank you for taking the time to handle our manuscript and provide guidance on how we can improve it.

In response to your major comment, “I did not see [the authors] addressing the important (second) point raised by the second reviewers, that the authors should try to find a better, more general framework to discuss their findings (and not limit their discussion to the PETM and its recording in the Pyrenees as a case example); this would greatly improve the impact of their work”, we have re-written the discussion section, which now includes two introductory paragraphs that try to set our findings in a broader context. We then carefully modified and developed section 4.1 to more broadly discuss the response times and impacts predicted by the models, the asymmetry in model response, and discuss the difficulties in comparing real sedimentary records to these simple numerical models. Subsequently in section 4.2 we have tried to discuss the wider implications of the model as well as providing more details for the three PETM scenarios considered here. And finally in the new section 4.3, we try to summarise the model implications as whole, again highlighting the wider implications and generic findings of this work. We trust these additions, which are tracked in detail, will enable readers to see the generic impact of the work.

In response to your second comment, “Personally, I find also that, in many parts of the paper, the discussion of the results is focused on the comparison between two models and how these can be explained from a mathematical point of view. It would be good if the authors could more often come back to the underlying processes, approximations and hypotheses on which the models are based” we hope in our extensively revised discussion we have managed to focus the text more on the processes. In particular that the stream power model might be more relevant if the sedimentary record is predominantly made up of fine grains that are transported as a suspended load. Conversely the transport limited model is more relevant for bed-load deposits. This means that different parts of the sedimentary record may have different response times and record past climatic change differently. Our new section 4.3 now explicitly considers the limitations of the models in the light of the real physical processes occurring.

The revised discussions include page 24, lines 4 to 19; page 25, lines 1 to 4 and 18 to 30; page 26, lines 13 to 27; page 27, lines 1 to 12; page 29; and page 30 lines 1 to 26.

Finally we have updated the abstract to reflect the wider discussion, and highlight some of the key points that had become lost within our original submission.

Please find below our detailed comments to the two reviews in bold text. We have tried to address all the comments and we hope that our revised manuscript is now considerably improved. We attach a *latexdiff* version of the manuscript to the end of the response to reviews.

Yours sincerely,

John Armitage (on behalf of all the authors).

Interactive comment on “Numerical modelling landscape and sediment flux response to precipitation rate change” by John J. Armitage

L. Guerit (Referee)

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This paper deals with the response of a landscape to a change in precipitation rate in terms of sediment flux and response time. The authors use numerical modelling to explore the behavior of a landscape considering two end-member for the transport law: the stream power law and the sediment transport law. The first one has been widely used and has showed some limitations. It is therefore of high interest to compare this law to another one. The authors propose several simulations that can be compared with each other and show that the two transport laws lead to a similar first-order behavior but with different response time, that can thus be used to discriminate the on-going processes (instantaneous vs diffusive transport). The authors test their models on the Claret Conglomerate, associated to the PETM (a rapid climatic change) and suggest that this formation is best explained by a diffusive model. The purpose of this work is of interest for many fields of research. The methodology is consistent and the results are nicely supported by several numerical experiments. However, this work would benefit from some rewriting and reorganization to make the purpose of the authors more clear and easier to follow: some paragraphs could be reorganized and/or developed in particular to better highlight the state of the art in the domain (Introduction) or to develop some important aspects of the work (Method).

Please find below my specific review.

## INTRODUCTION

p2 l5 responds to tectono-environmental change : The main results of these models should be mentioned here.

**Added information on both tectonic and climatic forcing: p1: “In general terms, numerical studies have found that landscapes typically will recover from a change in tectonic uplift after  $10^5$  to  $10^6$  years (e.g. Romans et al., 2016). These apparently long response timescales to tectonic perturbations have been supported by field observations of landscapes upstream of active faults (e.g. Whittaker et al., 2007; Cowie et al., 2008, Whittaker and Boulton, 2012), although the precise appropriateness of any time-integrated erosion law to specific field sites is not always easy to establish.”**

**Additionally, we note in the introduction:**

**“The response of landscapes and sediment routing systems to a change in the magnitude or timescale of precipitation rates is expected to depend on the long term erosion law implemented (Castelltort and Van den Driessche, 2003; Armitage et al., 2011; 2013). Some numerical modelling studies, based on treating erosion as a length dependent diffusive problem, suggest that landscape responses to a change in rainfall are also on the order of  $10^5$  to  $10^6$  years, similar to tectonic perturbations, although they produce diagnostically different stratigraphic signatures from the latter (e.g. Armitage et al., 2011). However, other modelling contributions with different assumptions suggest that response times to a precipitation change may be more rapid (Simpson and Castelltort, 2012), although field data sets remain equivocal (see Demoulin et al., 2017 for a recent review).”**

p2 l9 to evaluate how the response time varies as a function of the model forcing; a few words about why this is important and how this will support your initial question would be welcome.

**Added: “We will focus on the transient period of adjustment to a perturbation in precipitation rates, and using end-member numerical models attempt to evaluate how the response time varies as a function of the model forcing. To this end we aim to find the model parameters that generate similar landscape morphologies such that we can subsequently explore how the same end-member models respond to change in precipitation rate. We believe that the results of this study have implications for understanding the responses of landscapes to past change in climate, and could potentially be compared with and tested against further laboratory experiments.”**

p2 after l16 in order to highlight the importance of your work, and to better present the general context, the equations of section 2.1 could be presented and discussed here

**We disagree and would rather keep them within the derivations, so that the models can be compared by a non-specialist.**

p3 l2 could you comment briefly the limitations that are suggested by: in principle ?

**No limitations are suggested, rather by “in principle” we were just adding caution because it is never certain that the deposits represent a complete time history of the erosion of the upland catchment. The phrase “in principle” has been deleted to avoid confusion.**

## METHODS

p5 l3-4 Some logical transitions between sentences are necessary here.

**Done.**

p5 l13 the value of the exponent  $m$  is usually related to the value of  $n$  and is therefore not always close to 0.5. You should rather give a range of  $m$  value observed in nature and in experimental landscapes (see for example Kirby and Whipple 200, Lague et al 2003, Wobus et al 2006)

**This is if you relate them to the slope-area analysis, but that is not our point here.**

p5 l17 the last sentences could be supported by more references. In addition, the differences between the linear and non-linear cases should be discussed here.

**We don't quite understand what references are being asked for.**

p5 l24-29 not clear, please consider rewriting

**Rewritten.**

p7 (and Appendix B) please specify which model/soft you used to solve Equation 11

**The 2D equations are solved using a model written in Matlab and developed by Guy Simpson. The 1D version is solved using Matlab and code written by the authors. The text has been adjusted to reflect this. We went through the derivation so that the reader would know what PDE is being solved.**

p7 In Table 1, you mention two model sizes but there is only one in page 7. Please correct.

**Done**

p7 l16-19 you use different grids (square vs triangular) and resolutions (number of nodes) for the two models. Can you please comment on these specific choices and the possible implications (or consider adding a paragraph on this topic in the Discussion) ?

**Grid size matters (see Schoorl JM, Sonneveld MPW, Veldkamp A. 2000. Three-dimensional landscape process modelling: the effect of DEM resolution. Earth Surface Processes and Landforms 25: 1025–1034), therefore we tried to make the two models comparable. The sediment transport model solves the equations using a finite element approach, and a triangular grid is 2<sup>nd</sup> order accurate and appropriate. The stream power model uses a finite difference approach with a rectangular grid, which is also 2<sup>nd</sup> order accurate and appropriate. It is known that resolution can effect the drainage patterns predicted in these sorts of models, where surface water is routed down the steepest slope of descent. However, we focus mainly on the sediment flux signals out, which are not resolution dependent. Furthermore, when comparing model to model, we have attempted to keep the resolutions similar.**

p7 l20-24 (and in Appendix A and B) this paragraph needs to be developed and more precise: why do you consider two different times (5 and 10 Myrs)? by how much is increased or decrease the precipitation rate ? how do you define the values of the different coefficients ? These values are of main importance for your numerical models and should therefore be discussed more extensively (typical values in natural settings, in experiments, implications, etc).

**Two times: because for some models steady state is achieved quite rapidly, while for others it is not. This is now explained in the methods section.**

p8 the last paragraph could be moved to line 20. Also consider a few words about the choice of the output you follow.

**Done.**

## RESULTS

The particular value of  $m/n = 0.5$  has been recently tackled by Kwang and Parker (ESurf 2017). They suggest that  $m/n = 0.5$  leads to unrealistic scale invariance. Can you comment on this ?

**We don't think we can, because we don't think this paper made it through review.**

p9 l1 and l 11 the choice of these values must be explained or supported by some references.

**This was the point of Appendix B, which has now been moved into the Methods section after the comments from the second reviewer.**

p10 Please specify whether you extract only one profile for each model or use several profiles

**We don't understand. On page 10 we discuss the response time, which is calculated from the total volume eroded from the whole model domain. No profiles are used.**

p14 l15-20 these sentences are about the amplitude of the response but it is in a section dedicated to

the response time. Please consider moving this paragraph to a more appropriate section.

**Changed the section heading to “Response to different magnitudes of precipitation rate change”.**

## DISCUSSION

Based on Figure 6, you propose that the response time is shorter for an increase in precipitations than for a decrease and you also show that the response time is related at first order to the precipitation rate (Figure 7). In your examples (Fig. 6), you start from the same initial precipitation rate (1 m/y) and you end with different values. Therefore, the shorter/longer response time be related to this difference in precipitation rates rather than to increase vs decrease. Can you comment on this ? Some simulations with different initial rates but similar final rates would be very interesting. If, even with similar final rates, the response times are still different according to the scenario, it would be very interesting to discuss why.

**We have added a new section exploring how initial precipitation rate impacts the response time. We have found that for the transport model, the response time is sensitive to the initial precipitation rate. However the proportionality does not fit a power law as was found for the relationship between response time and the final precipitation rate. Furthermore, the change in response time is not very large. For the stream power model the response time is not a function of the initial precipitation rate. We believe this difference is in how the transport model responds directly across the whole catchment, so that slopes at the uppermost reaches of the catchment still have a memory of the previous precipitation rate. For the stream power model, the erosion increases bottom-up and so there is no memory of the previous slope and topography.**

In the second part, the authors present the example of the PETM as a rapid change in precipitations and they discuss the timing of the contemporaneous sediment deposits.

1) Rohl et al propose a duration of 170 kyrs for the PETM. Please add some references for the lower value of 90 kyrs.

**We only use the 170 kyr value now, following Rohl.**

2) You assume a constant rate of deposition for these two formations to estimate a duration of deposition. However, average rate of sedimentation are very difficult to estimate and it is a very strong assumption to consider that a conglomerate is deposited at the rate of a paleosol. Therefore, it seems very difficult to consider that the conglomerate account for 1/3 of the total duration or that it is deposited at a rate of  $5 \cdot 10^{-4}$  m/y. In addition, based on your simulations, we expect a change in flux while the system is responding to the change in precipitations. One simple and more robust option is to refer to the value proposed in Schmitz and Pujalte, 2007.

**We have also added the Schmitz and Pujalte reference, giving a range of 10 to 50 kyrs, which we believe to be reasonable and robust. We have deleted the comparison to the rates in the Bighorn basin.**

3) Your work is focused on response time and sediment flux but in this natural example, you document a change in the nature of the deposits. Is there any argument to support a change in  $Q_s$  ?

**Yes. We now write that “These values suggest sedimentation rates of up to 1 mm/yr, and thus elevated sediment fluxes, which if they had been sustained for the duration of the deposition of**

**the Tremp Group (Maastrichtian – end Palaeocene) would have produced >15 km of sediment thickness, an order of magnitude more than actually observed (Cuevas, 1992)."**

## FIGURES

Figure 7 this figure is given for the specific value of 0.5. However, you ran some other simulations with different values of  $m$ . Do you observe the same behavior for different  $m$  values ?

**We have not checked, but as this aspect was covered in Whipple (2001) and Baldwin et al. (2003), we do not believe we need to check this relationship further for other values of  $m$ .**

**We have corrected for the minor comments, where they remained.**

Interactive comment on "Numerical modelling landscape and sediment flux response to precipitation rate change" by John J. Armitage

Anonymous Referee #2

Received and published: 28 August 2017

General Comments: The objective of this paper was explore end-member models of landscape evolution with the goals of finding how changes in model forcing and model assumptions control model response time and whether changes in sediment flux during a perturbation is diagnostic of the end member models. The motivation for the study was whether the end member models can constrain sediment fluxes to depositional basins and allow the interpretation of past climate signals from the sedimentary record. Their main findings were that transport-limited models have a faster response time to a change in precipitation rate and sediment flux is higher in transport limited models. In my opinion, one of the most interesting parts is the asymmetry in the response times of transport limited models, with a longer response time to a drying event than to a wetting event, but this result was not discussed in detail. Overall, the quality of the paper including the motivating questions, the derivation of model equations, and the explanation of model set up (in appendix B) was good.

I strongly suggest that the material in appendix B be moved to the Methods section. This will really improve the reading experience. The results were adequately explained, but at times the text was dense with much discussion about the results without references to appropriate figures. The weakest section of the paper was the discussion and conclusions. The authors conclude that a sedimentary record of the PETM is best explained by using a transport-limited model because this model has a faster response time. But a faster response time for one model doesn't rule out another model. I would like to see a more robust support of the sediment transport model in making this conclusion. Second, the authors favor the transport-limited model to explain a particular sedimentary record of the PETM because the instantaneous transport of bedload (assumed by stream power model) is not justified for specific case cited in the Pyrenees. But I don't think anyone would argue that stream power model is justified, so this weakens potential impact of paper.

**We have incorporated Appendix B material into the main paper. Some people invert continent scale river long profiles using a stream power model, and here we show that SPL models results in large scale topographies which are similar to transport model simulations. Also, all the work of Sean Willett (an his team) is based on the SPL, including the chi value approach. These models are applied rather indiscriminately, therefore we think the**

**comparison is somewhat valid.**

In the discussion, I think the paper could benefit from a more generalized framework of the circumstances under which the findings of this study are relevant. For example, what is needed in the field/sedimentary stratigraphy to distinguish between a landscape that was depositing under a detachment-limited vs. a transport-limited system? I think that exploring end member models to identify diagnostic characteristics of each in transient state is a worthy goal, but the authors could do a better job articulating how the diagnostic characteristics they have identified are useful in a larger context outside of interpretation of sedimentary records.

**We have rewritten the discussion section to focus on this. We identify the circumstances when one could compare erosional end-member models with field data (i.e. where the response time is independently constrained, sedimentation rates are high etc). We are also much more specific in the limitations and comparisons between the model end-members. We stress firstly that we address the erosional response of a catchment system to a perturbation – whether this is “captured” in stratigraphy is an additional problem. We also make the point that we wish to compare the generic behaviour of the end-member models. One can always tune one or other to specific field observables.**

Specific Comments:

P1L19-21: Conclusion that rapid response in sedimentary basins more easily explained by using transport model for two reasons: 1) this model has a faster response time (Is this a new finding?) and 2) instantaneous transport of bedload (assumed by stream power model) is not justified for specific case cited in the Pyrenees. But I don't think anyone would argue that stream power model is justified, so this weakens potential impact of paper.

**We think the faster response time is a new finding. Furthermore, in the work of Mouch  n   et al. (2017), a dominantly stream power based erosion model is applied to the Northern Pyrenees.**

P2L6: “a series of experiments”: give very brief description of experiments. Also examples of real catchments responding to changes in precipitation would be useful to give readers an broader framing for your work.

**We have added the following text:**

**“In laboratory studies, a series of experiments where granular piles of a length scale of order of centimeters are eroded due to surface water, have demonstrated that a change in precipitation rate leads a period of adjustment of the landscape topography until a new steady-state is achieved (Bonnet & Crave, 2003; Rohais et al., 2011). These experiments use a mixture of granular silica of a mean diameter in between 10 and 20  $\mu\text{m}$ , that is eroded by water released from a fine sprinkler system above. Given the complexity of these experiments, unfortunately there have been insufficient different precipitation rates studied to fully understand how the recovery time-scale varies as a function of precipitation or other parameters.”**

P2L15: Can you also point readers to your own modeled examples of landscapes that were created by transport model and stream power model, but are indistinguishable from each other at steady state.

**Not at this point in the paper but we moved Appendix B into the main part of the text, so they will appear later.**

P2L23: Mentioning that river profiles can be inverted to extract climate history at this point seems to bog down the explanation of differences between advective and diffusive dominated systems in transient state.

**OK. Deleted.**

P2L27-31: Transport/diffusive models do not produce knickpoints in transient state. Pointing out that knickpoints occur for reasons other than a transient state in advective-dominated systems doesn't change that, nor does it support the motivation for the study in the following lines (L31-33). That said, I think exploring model end member behavior is a worthy goal.

**Deleted the paragraph.**

P3L7: typo "dirven"

**Fixed.**

P3L7-8: Be more specific here about what experiments show about response time to a perturbation. It's self-evident that there will be a response time for systems to return to steady state.

**The point is that the controls on the response time are not fully captured.**

P3L10: Make it clear that this is the new piece this study adds to the existing body of knowledge.

**We have added: "Sediment flux response times for the advective stream power law have been previously characterised by Whipple (2001) and Baldwin et al. (2003), and for the transport model they have been studied by Armitage et al. (2011) and Armitage et al. (2013), but not systematically or using 2-D models. Furthermore, to our knowledge no comparison between the transport model has been previously made." to the end of the first paragraph.**

P3L20-21: Be clear and consistent throughout the paper with terminology when referring to advective/stream power law/detachment limited and diffusive/sediment transport model/transport limited. These are used interchangeably throughout the manuscript. I recommend explaining the meaning of all three descriptions for each endmember model early in the paper, then using one of the terms for the rest of the paper.

**We now only refer to transport model and stream power model.**

Figure 1: include erosion,  $E$ , in the figure.

**Done.**

P4L10: Why ask the question of if mass transport is appropriate at continent scale when this paper doesn't answer that question. It seems to me the paper addresses the question of if advective transport is appropriate for all models with changing boundary conditions.

**Deleted the question. In this study we are not changing boundary conditions, rather the precipitation rate which impacts the transport coefficients.**

P4L13-14: reference here, e.g. Davy and Lague 2009.

**Such a citation is to us a bit odd when we are pointing out the obvious.**

P5L1-8: This discussion of mass transport in suspension seems unnecessary here and unrelated to the point of the paper.

**Well, we think not. If mass transport is not in suspension then it is along the bed, and not rapid. Therefore the idea of instantaneous mass transport, which is implicit in the derivation, is wrong and hence the model is nonsense.**

P5L12: define physical meaning of coefficient  $k_p$ , bedrock erodibility, for readers who are not familiar with erosion models.

**Done.**

P5L20: To this point, the derivation is good, easy to follow for the most part, except authors need to define  $k_p$ , as noted above.

P5L24-26:  $\alpha$  (precipitation rate, I found later) is not defined here and makes this section very confusing.

**Done.**

P5L27: more specific definition of  $k_w$ , width coefficient?

**Done.**

P6L1-5: Before you launch into derivation of transport-limited model, give a few more lines to discussing what this means physically, in a natural system. Also here explicitly say this is the transport-limited erosion equation we use.

**We have addressed this comment in the text.**

P6L23: a fourth name/way of describing stream power as a kinematic wave equation. This is obvious to people who are immersed in the world of erosion models, but many readers are not. So again, be careful and consistent in the terminology used to describe the two end member models.

**We have removed the term “kinematic wave equation”.**

P6L25: can you reference a figure that shows a migrating knickpoint?

**We can't, despite them being mentioned in the article cited. We have therefore removed the sentence and reference to the migrating knickpoint.**

P6L25-27: mentioning shock wave seems unnecessary and distracting to me. General comment about derivations: there is lots of discussion of various exponents, but I would like to see explanations of the link exponent values with natural systems where possible.

**It might be distracting to mention shock waves, yet we think it is not unimportant. The model parameters are almost impossible to relate to real observables in the natural system; you can relate them but not as a unique parameter value set, rather as a broad parameter value space (see e.g. Croissant and Braun 2014). The models are instead trying to capture some gross simplification of the natural system. A natural manifestation of a shock wave would be the**

**upward migration of a waterfall within the catchment. Of course waterfalls could form due to many other natural circumstances. The point however remains that if we chose to use the stream power law, we should be aware that this model can lead to shock waves.**

P7L16-19: why do you use different grid sizes for the two models? Does this affect the outcome of the models?

**See the reply to the same question from Laure Guerit.**

P7L22-24: This is confusing on the first read through. Explain more or reference figure that shows this relationship (maybe Figures 12 or 14)

**Re-written.**

P7L25: you've switched from deriving stream power model first, then transport model to discussing transport model first, then stream power. It would help readers follow if you kept the same order throughout the paper, but I recognize that's difficult and perhaps not always possible. Methods: I strongly recommend moving Appendix B to the methods section so readers have a better idea what you're doing and what these models look like before discussing results. This is very important and will help the readability of the paper.

**We have re-organised the methods so that the sediment transport model is derived first, and the stream power model second. Appendix B is now also in the Methods section.**

Figures 2 and 3: These figures are not high impact by themselves. Suggest combining these figures and then give them clear titles indicating which end member model results are shown.

**Combined.**

P8L16-17: this was the first time it was clear to me that the heart of this paper is model response to changing precipitation rates. Emphasize this more clearly in the methods section.

**We have tried to explain this better in the text.**

P9L2: Sentence that begins, "The response to a reduction in precipitation. . ." is confusing because it's not specific and is a bit of a run on. Needs punctuation or splitting into two sentences to make it clearer.

**Re-written**

P9L3-6: Very long run on sentence. This sentence says "importantly, however, it changes the model elevation. . .". Reference a figure where readers can see this change in elevation with changing c.

**Re-written**

P9L4: Be specific about where in appendix A readers should look

**Re -written**

P9L10-11: Reference one of your figures that shows where these three different sets of parameters result in similar model topography, e.g. figure 14.

**Done**

Figure 4 and 5: These need titles to make it clear that one is the transport model and one is the stream power model.

**Done**

Figure 5: indicate knickpoint on figure 5a

**The text mentioned knickpoints in reference to the paper by Jean Braun (Braun et al., 2015). In Figure 5 we find it hard to pick a knickpoint, therefore we have removed the phrase “The lower reaches of the catchment respond more rapidly than the upper reaches, therefore creating a migrating knickpoint as the landscape responds to the change in model forcing (see Braun et al., 2015).”**

Figure 6: this figure must have titles indicating that one is the transport model and one is the stream power model.

**Done**

P12L3-5: It seems to me that it's at least intuitively known that aggradational (drying) events happen more slowly than an erosional (wetting) event, but I couldn't point to a reference for this. If there aren't many studies that show this, I think this makes a very interesting point. If there are studies, they should be cited.

**We are not aware of any numerical modelling studies which make the explicit point that aggradational drying events are slower than erosional wetting events. If the reviewer has access to studies that show this, we would be very happy to include these in the references.**

Figure 7: include in caption these results apply to catchments where  $L=100\text{km}$  only. Should also make clear where the data to make this figure comes from. Is it just a line through your model data? If made from model data, these should be indicated with points and justification given for how the line was drawn through the model data.

**Done.**

P13L1-2: include reference for knickpoint celerity models, e.g. Berlin and Anderson 2009, JGR

**Why? We were not making any point about knickpoint celerity, unless we are not understanding something here.**

P14L2-3: This seems important that response time takes twice as long when  $L=500\text{ km}$  compared to  $L=100\text{km}$ . I suggest a figure showing this difference in response time.

**A figure has been added.**

P14L12-13: explain what this empirical evidence of  $0.5 < h < 0.7$  means in a physical system.

**Added that this controls the plan view shape of river catchments, but as far as we are aware there is still no explanation for the origin of this scaling (see Dodds & Rothman, “Geometry of river networks. I. Scaling, fluctuations, and deviations”, Physical Review E, 2000).**

Figure 9: show points where you have data from model runs.

**Done**

P16L2-3: reference figure 7 that shows how models respond to precipitation rate.

**Done**

P16L6: reference figure that shows similarities between landscapes created by transport models and advective models.

**Added figure references.**

P16L7-9: I appreciate the goal, but what constraints are needed to make this useful? See comment below on conclusions.

P16L13: reference figure that shows difference in response times.

**Done**

P17L1-2: this asymmetry is interesting. Reference figure that shows it.

**Done**

P17L7-8: Can you say more about catchment response time for the sediment transport model? For example, when is this information useful for evaluating sediment records? I think this point needs to be discussed more thoroughly.

**We have added a section on the “Relevance of model responses to sediment records of climate change” where we now hope these points are discussed more thoroughly.**

P18L2: At Claret: mention this is the site in the Pyrenees.

**Done**

P18L6-10: confusing. What is the justification for comparing paleosols in the Bighorn Basin with paleosols in Claret? Perhaps too much detail in this section to get to the point of rapid deposition.

**We have eliminated this comparison. We now provide two estimates of sedimentation rate for the Claret conglomerate (including the low end-member estimate of 10 kyrs from Schmitz and Pujalte 2007). We adopt a timescale of ca. 200 kyrs based on the work of Brady Foreman for comparison with US PETM sites.**

P19L12-14: I don't see why one would attempt explain evolution of a megafan with a stream power model in the first place. This is a bit of a strawman argument.

**Actually very recently the large megafans of the Northern Pyrenees have been modelled using a model where erosion is calculated using the stream power model (Mouchene et al. 2017). Furthermore, the key point is not the evolution of the mega fan – we are actually talking about how the eroding catchment which is producing the sediment is modelled. One could model catchment erosion using a stream model upstream of a fault and have a sensible sediment flux prediction – yet still want to use this sediment flux model as an input to a**

**depositional stratigraphic model. To make sure the reviewer and readers don't mix these two points, we repeatedly clarify in the discussion that we are talking about the erosional part of the catchment.**

P18L20-24: Run on sentence.

P19L1-3: Again, it seems obvious that bed load transport is more easily described by a diffusive/transport limited model rather than the stream power model. It would be useful to reference studies where authors have used stream power to model bedload dominated systems.

**We reference the Mouchene paper, mentioned in the response above. The point is again, that numerical models can produce sediment flux predictions based on a stream power model. These sediment flux predictions could be used to build stratigraphy in a subsiding basin. We stress the that it is important to separate out erosion and deposition when considering these systems.**

Relevance to sediment record and climate change: This section should include a more generalized framework of the circumstances under which the findings of this study are relevant. What is needed in the field/sedimentary stratigraphy to distinguish between a landscape that was depositing under a detachment-limited vs. a transport-limited system? For example, magnitude and duration of precipitation change, magnitude of sediment flux. It's also important to include an expanded discussion of why the sedimentary record at Claret is difficult to explain with an advective model. How much does precipitation have to change in the time period of deposition for an advective model to work? Is this anywhere close to reality? Generally, I want more to back up the conclusion that the transport model explains deposition in the sedimentary basins simply because it has a faster response time.

**We now write:**

**Erosional source catchment areas were likely < 100 km in length, given the palaeo-geography of the Pyrenees at the time (Manners et al., 2013). The very short duration of the erosional response, which is required for the sediments to be transported and deposited in a timescale of ca.  $10^4$  years is therefore difficult to model within a stream power (advective) end-member model for catchments of this scale (Table 2), although a version of such a model has been recently used to explore the controls on the evolution of later Miocene megafans in the northern Pyrenees (e.g. Mouchen  et al., 2017). To use the stream power model would require a significant increase the bedrock erodibility parameter,  $k$ , within the model (by greater than one order of magnitude), implying slopes and topography in the palaeo-Pyrenees that were highly subdued. In contrast, the sediment transport model more easily reproduces the documented response timescales given an increase in precipitation, is consistent with the volumetrically significant export of bedload transported gravel clasts, and therefore honors the independent field data more effectively. We also note that the transport model displays a response time that has a stronger dependence on precipitation rate change, and has a greater amplitude of perturbation (e.g. Figure 9). We therefore suggest the erosional pulse that led to the deposition of the Claret conglomerate is most appropriately modelled as a diffusive system response to a sharp increase in precipitation over the source catchments of the developing Pyrenean mountain chain at that time."**

**However it is important to stress that our model actually only deals with the erosional part of the catchment, and it does not address whether any erosional signal is "sampled" into stratigraphy. This is beyond the scope of this contribution (it is an important question in its own right!) and we acknowledge this on p24 of the manuscript.**

P19L10-12: I don't think that noting that knickpoints are not a unique indicator of erosional dynamics is helpful for understanding the motivation of this study.

**We would prefer to leave this here as the point about knickpoints not being unique is useful because just because you see one in a river it does mean you should leap for a detachment limited stream power law.**

P19L23-24: This is one of the most interesting outcomes of the paper. I would suggest discussing this and the implications of this effect on interpretation of the sedimentary record.

**Good suggestion. We have added at the end of the discussion that “Nonetheless, a significant finding of this work has been the clear asymmetry in response time of these end-member models in terms of a wetting event (faster) compared to a drying event (slower). This implies that aridification events are harder to preserve in the sedimentary record, not only because they are typically associated with reduced sediment fluxes, but also because the timescale of landscape response may be  $> 10^6$  years”.**

P20L1-4: nicely summarized objective of the paper. I would like to see this in the abstract also  
Figure 10: Figure needs a legend for the lines

**Done.**

Figure 11: It's not immediately clear why both Figure 10 and Figure 11 are necessary. It's explained further down, P22L3-7, but figure 11 is initially confusing.

P22L12-13: show example of how slope-area relationship is sensitive to river network in T-Lim case. Or remove this line as it's not relevant.

**Deleted.**

P22L14-20: The point of this paragraph is unclear even after reading several times. Rewrite and reword.

**Re-written.**

Appendix B: As noted above, I think appendix B should be included before results are discussed. It would make the paper flow much more smoothly.

**Done.**

Figure 12: Labels on the figures: transport limited models.

**Done.**

P24L5: typo, should read steady state.

Figure 14: Figure needs label: stream power models. Also, there is an error in the caption. The caption currently says sediment transport model, should read stream power model.

**Done.**

# NUMERICAL MODELLING LANDSCAPE AND SEDIMENT FLUX RESPONSE TO PRECIPITATION RATE CHANGE

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**Abstract.** Laboratory-scale experiments of erosion have demonstrated that landscapes have a natural (or intrinsic) response time to a change in precipitation rate. In the last few decades there has been a growth in the development of numerical models that attempt to capture landscape evolution over long time-scales. ~~Recently, a sub-set of these numerical models have been~~  
5 ~~used to invert river profiles for past tectonic conditions even during variable climatic conditions.~~ However, there is still an uncertainty over validity of the basic assumption of mass transport that are made in deriving these models. In this contribution we therefore return to a principle assumption of sediment transport within the mass balance for surface processes, and explore the sensitivity of the classic end-member landscape evolution model ~~to~~, and the sediment fluxes they produce, to a change in precipitation rates. One end-member model takes the mathematical form of a kinetic wave equation and is known as the  
10 stream power model, where sediment is assumed to be transported immediately out of the model domain. The second end-member model is the transport model and it takes the form of a diffusion equation, ~~and assumes assuming~~ that the sediment flux is a function of the water flux and slope. We find that both of these end-member models have a response time that has a proportionality to the precipitation rate that follows a negative power law. ~~For~~ However, for the stream power model the exponent on the water flux term must be less than one, and for the ~~sediment~~-transport model the exponent must be greater than  
15 one, in order to match the observed concavity of natural systems. This difference in exponent means that ~~sediment-transport model~~ the transport model generally responds more rapidly to an increase in precipitation rates, on the order of  $10^5$  years for ~~a landscape~~ post-perturbation sediment fluxes to return to within 50 % of their initial values, for theoretical landscapes with a scale of  ~~$10^5$  m.~~ In nature, landscape response times to a rapid environmental change have been estimated for events  
 $100 \times 100$  km. Additionally from the same starting conditions, the amplitude of the sediment flux perturbation in the transport  
20 model is greater, with much larger sensitivity to catchment size. An important finding is that both models respond more quickly to a wetting event than a drying event, and we argue this asymmetry in response time has significant implications for depositional stratigraphies. Finally, we evaluate the extent to which these constraints on response times and sediment fluxes from simple models help us understand the geological record of landscape response to rapid environmental changes in the  
past, such as the Paleocene-Eocene thermal maximum (PETM). In the Spanish Pyrenees, for instance, a relatively rapid, ~~20 to~~  
25 ~~100~~ 10 to 50 kyr, duration of deposition of gravel ~~during the PETM~~ is observed for a climatic shift that is thought to be towards increased precipitation rates. We suggest the rapid response observed is more easily explained through a diffusive ~~sediment~~

transport model, as ~~(1) this the~~ model has a faster response time, consistent with the documented stratigraphic data, ~~and~~; (2) there is a high-amplitude spike in sediment flux; and (3) the assumption of instantaneous transport is difficult to justify for the transport of large grain sizes as an alluvial bed-load. Consequently, while these end-member models do not reproduce all the complexity of processes seen in real landscapes, we argue that variations in long-term erosional dynamics within source  
5 catchments can fundamentally control when, how and where sedimentary archives can record past environmental change.

## 1 Introduction

How river networks form and how landscapes erode remains a basic research question despite more than a century of experimentation and study. At a fundamental level, the root of the problem is a lack of an equation of motion for erosion derived from first principles (e.g. Dodds and Rothman, 2000). A range of heuristic erosion equations have however been  
10 proposed, from stochastic models (e.g. Banavar et al., 1997; Pastor-Satorras and Rothman, 1998) to deterministic models based on the St. Venant shallow water equations (e.g. Smith and Bretherton, 1972; Izumi and Parker, 1995; Smith, 2010),  
diffusive transport-limited conditions (e.g. Whipple and Tucker, 2002), or the stream power law (e.g. Howard and Kerby, 1983; Whipple and Tucker, 2002; Willett et al., 2014, amongst many others). These models, in various forms, have been explored to try to understand how landscape evolves and responds to tectono-environmental change. In the laboratory general  
15 terms, numerical studies have found that landscape typically recover from a shift in tectonic uplift after  $10^5$  to  $10^6$  years (reviewed in Romans et al., 2016 ). These apparently long response timescales to tectonic perturbations have been supported by field observations of landscapes upstream of active faults (e.g. Whittaker et al., 2007; Cowie et al., 2008; Whittaker and Boulton, 2012), although the precise appropriateness of any time-integrated erosion law to specific field sites is not always easy to establish. Sediment flux response times for the advective stream power law have been previously characterised by Whipple (2001) and  
20 Baldwin et al. (2003) , and for the transport model they have been studied by Armitage et al. (2011) and Armitage et al. (2013) , but not systematically or using 2-D models. Furthermore, to our knowledge no comparison between the transport model has been previously made.

The response of landscapes and sediment routing systems to a change in the magnitude or timescale of precipitation rates is expected to depend on the long term erosion law implemented (Castelltort and Van Den Dreissche, 2003; Armitage et al., 2011, 2013) .  
25 Some numerical modelling studies, based on treating erosion as a length dependent diffusive problem, suggest that landscape responses to a change in rainfall are also on the order of  $10^5$  to  $10^6$  years, similar to tectonic perturbations, although they produce diagnostically different stratigraphic signatures from the latter (e.g. Armitage et al., 2011) . However, other modelling contributions with different assumptions suggest that response times to a precipitation change may be more rapid (Simpson and Castelltort, 2009), although field data sets remain equivocal (see Demoulin et al., 2017 for a recent review). In laboratory studies, a series of ex-  
30 periments where granular piles of a length scale of order of centimeters are eroded due to surface water, have demonstrated that a change in precipitation rate leads a period of adjustment of the landscape topography until a new steady-state is achieved (e.g. Bonnet and Crave, 2003; Rohais et al., 2011). In this contribution we will focus on this transient period of adjustment to a perturbation in precipitation rates, to evaluate how the response time These experiments use a mixture of granular silica

of a mean diameter in between 10 and 20  $\mu\text{m}$ , that is eroded by water released from a fine sprinkler system above. Given the complexity of these experiments, unfortunately there have been insufficient different precipitation rates studied to fully understand how the recovery time-scale varies as a function of the model forcing precipitation or other parameters.

It has been increasingly recognised over the last two decades that many basic geomorphic measures of catchments, such as the scaling between channel slopes and catchment drainage areas, are typically unable to distinguish the erosional processes behind their formation (e.g. Tinkler and Whol, 1998; Dodds and Rothman, 2000; Tucker and Whipple, 2002; Whipple, 2004). Erosion and transport can be described by equations that encompass both advective and diffusive processes (e.g. Smith and Bretherton, 1972) and at topographic steady state, it is very well-established that fluvial erosion models based on either of these two end-members can produce very similar river longitudinal profiles (e.g. Tucker and Whipple, 2002; van der Beek and Bishop, 2003).

~~The morphology of a landscape that is not at topographic steady state relative to the external forcing depends on the magnitude of the advective relative to diffusive processes (Tucker and Whipple, 2002; Godard et al., 2013). If advective processes dominate there is an expectation, based on both numerical models and field studies, that the landscape will respond to a base level fall through the upstream migration of an erosional ‘knickpoint’ (e.g. Whipple and Tucker, 1999; Snyder et al., 2000; Tucker and Whipple, 2002). Topographic responses to a change in precipitation may also include downstream migrating waves of incision and decreases in channel gradient in these cases (e.g. Wobus et al., 2010). These knickpoints can then be inverted for uplift history assuming erosion is a dominantly advective process (e.g. Pritchard et al., 2009; Roberts and White, 2010), and more recently including the effect of variable water flux (Goren, 2016).~~

~~In contrast, for channels dominated by diffusive processes and perturbed from an initial concave-up steady-state configuration, their longitudinal profiles can maintain a similar shape throughout the adjustment to the new topographic steady state (Whipple and Tucker, 2002). The existence of sharp changes in slope along a river profile might therefore be used as evidence that a diffusive transport model is insufficient to explain landscape evolution. However, knickpoints can also be a product of topographic starting conditions (e.g. Valla et al., 2010), a change in lithology (Grimaud et al., 2014, 2016), or formed during extreme events (Baynes et al., 2015), potentially making them a non-unique signal of system erosion. Therefore, there is arguably a need to further explore how both the end-member models respond to change in the forcing conditions to discover how model assumptions control the tempo of landscape response, and how they approximate or approach the real physical system.~~

Non-uniqueness or equifinality is a common problem when comparing the morphology generated from landscape evolution models (e.g. Hancock et al., 2016). Consequently, we wish-aim to explore if the sediment flux responses of fluvial systems to perturbation may ~~also~~ be diagnostically different for the two end-member deterministic models across a range of parameter space. This issue is pertinent because within sedimentary basins, a change in the erosional dynamics upstream could ~~in principle~~ be recorded by changes in the total sediment volumes stored in sedimentary basins (e.g. Allen et al., 2013; Michael et al., 2014); in sediment delivery or sediment accumulation rates linked to landscape response times (Foreman et al., 2012; Armitage et al., 2015) and/or in the grain-sizes deposited as a function of sediment flux output (Paola et al., 1992; Armitage et al., 2011; Whittaker et al., 2011; D’Arcy et al., 2016). ~~Furthermore, laboratory-scale experiments have demonstrated~~

that within a physical system of erosion driven by the surface flow of water, there is a clear response time for the system to recover to steady state after a perturbation (Bonnet and Crave, 2003, 2006; Singh et al., 2015).

To date sediment flux response times for the advective stream power law have been previously characterised by Whipple (2001) and Baldwin et al. (2003), although to our knowledge no comparison between the transport model has been previously made. In

5 this article we make a comparative study between the transport and stream power model ~~and the transport model~~ to further explore the potential differences between these two end-member hypothetical landscape evolution models. We will focus on the transient period of adjustment to a perturbation in precipitation rates, and using end-member numerical models attempt to evaluate how the response time varies as a function of the model forcing. To this end we aim to find the model parameters that generate similar landscape morphologies such that we can subsequently explore how the same end-member models respond

10 to change in ~~surface run-off. A better understanding of how the landscape evolution models respond to change will allow potentially aid in the interpretation of stratigraphic architecture for past forcing, as it will give an insight as to what processes may be operating within a particular region~~precipitation rate. We believe that the results of this study have implications for understanding the responses of landscapes to past change in climate, and could potentially be compared with and tested against further laboratory experiments.

## 15 2 Methods

### 2.1 Erosion within a single dimension system

We ~~wish~~aim to understand the effects of the most basic assumptions of mass transport in landscape evolution on the sediment flux record. In other words, how do the response times vary for the advective stream power law and the diffusive ~~sediment~~ transport model? To this end we derive the two models from first principles to demonstrate clearly how, from the same starting

20 point, the fundamental assumptions made about mass transport initially give rise to very different model equations. We use this framework as a context for our investigation of an eroding system responding to precipitation change. We first define a one-dimensional system from which the basic equations can be assembled. Following Dietrich et al. (2003) we define a landscape of elevation  $z$  composed of bedrock, thickness  $\eta$  (units of m), and a surface layer of sediment with thickness  $h$  (units of m; see Figure 1). This landscape is forced externally through uplift rate  $U$  (units of  $\text{m yr}^{-1}$ ). The bedrock is transferred into sediment

25 through erosion at a rate  $E$  (units of  $\text{m yr}^{-1}$ ) and the sediment is transported across the system with a flux  $q_s$  (units of  $\text{m}^2 \text{ yr}^{-1}$ ). Assuming that the density of sediment and bedrock are equal, then the change in bedrock thickness is,

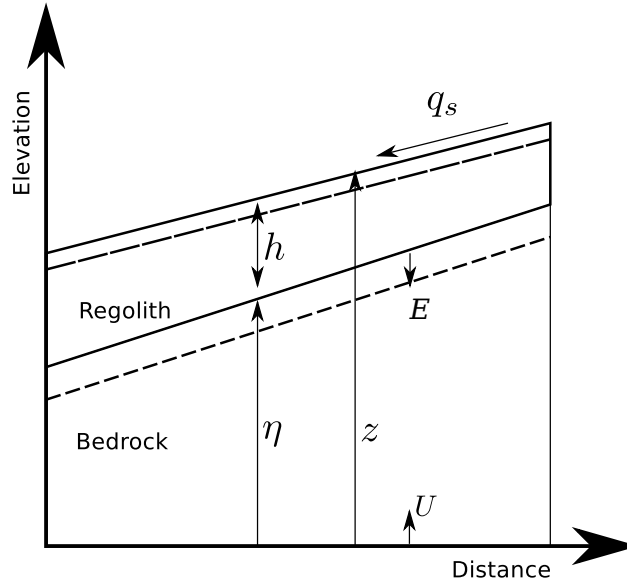
$$\partial_t \eta = U - E, \tag{1}$$

and the rate of change in sediment thickness is,

$$\partial_t h = E - \partial_x q_s. \tag{2}$$

30 It then follows that the rate of change in landscape elevation is,

$$\partial_t z = \partial_t \eta + \partial_t h. \tag{3}$$



**Figure 1.** Diagram showing the conservation of mass within a 2-D domain, where mass enters the system through uplift,  $U$  (units of  $\text{m s}^{-1}$ ), and exists as sediment transported,  $q_s$  ( $\text{m}^2 \text{s}^{-1}$ ) out of the domain.  $P-E$  ( $\text{m s}^{-1}$ ) is the rate of production of regolith,  $h$  (m) is the thickness of regolith,  $\eta$  (m) is the bedrock elevation and  $z$  (m) is the total elevation.

It is important to realise that to solve Equation 3, we are required to make some assumptions that fundamentally affect the erosional dynamics of the modelled system, and we illustrate this below. If-

One basic assumption to make is that there is always a supply of transportable sediment, then we can follow through with the summation in equation 3 giving,

$$\partial_t z = U - \partial_x q_s. \quad (4)$$

This may be appropriate when modelling the transport of sediment along the river bed and when considering the formation of alluvial fans (e.g. Paola et al., 1992; Whipple and Tucker, 2002; Guerit et al., 2014). In the absence of surface water we can assume that sediment flux is simply a function of local slope  $q_s = -\kappa \partial_x z$ . In the presence of flowing water then the sediment flux is a function of the flowing water and local slope  $q_s = -c q_w^\delta (\partial_x z)^\gamma$  where  $c$  is the transport coefficient (units  $(\text{m}^2 \text{yr}^{-1})^{1-n}$ ),  $q_w$  is the water flux per unit width (units  $\text{m}^2 \text{yr}^{-1}$ ) and the exponents  $\delta > 1$  and only-if,  $\gamma > 1$  are dependent on how sediment grains are transported along the bed (Smith and Bretherton, 1972; Paola et al., 1992). Furthermore,  $\delta > 1$  is required to create concentrated flow (Smith and Bretherton, 1972). The change in landscape elevation is then given by,

$$\partial_t z = U + \partial_x (\kappa \partial_x z + c q_w^\delta (\partial_x z)^\gamma). \quad (5)$$

which can be written as,

$$\partial_t z = U + \partial_x \left( \left[ \kappa + c q_w^\delta (\partial_x z)^{\gamma-1} \right] \partial_x z \right). \quad (6)$$

Equation 6 is non-linear in the case that  $\gamma \neq 1$ . In deriving this equation of elevation change due to sediment transport we have simply summed the two terms for sediment flux, the linear and potentially non-linear slope dependent terms. This summation has been done as it is the simplest way to generate landscape profiles that have the desired convex and concave elements observed in natural landscapes (Smith and Bretherton, 1972) .

- 5 To solve this equation in one dimension we assume that the water flux is a function of the precipitation transported down the river network. The water collected is taken from the upstream drainage area,  $a$ , which is related to the main stream length,  $l$ , by  $l \propto a^h$  where  $h$  is the exponent taken from the empirical Hack's law (Hack, 1957) . The main stream length is related to the longitudinal length of the catchment by,  $l \propto x^d$  where  $1 < d \leq 1.1$  (Tarboton et al., 1990; Maritan et al., 1996) . Therefore, we can write that  $x \propto a^{h/d}$ , and the water flux is the precipitation rate,  $\alpha$  units ( $\text{m yr}^{-1}$ ), multiplied by the length of the drainage system,

$$q_w = k_w \alpha x^p \quad (7)$$

where  $k_w$  is the width coefficient (units  $\text{m}^{1-p}$ ), and  $p = d/h$ . Furthermore it is observed that river catchments are typically longer than they are wide, and so  $p < 2$  (Dodds and Rothman, 2000) . Therefore given that  $0.5 < h < 0.7$  (e.g. Rigon et al., 1996) then  $1.4 < p < 2$ , and the transport model (equation 6) becomes,

$$15 \quad \partial_t z = U + \partial_x \left( \left[ \kappa + c k_w \alpha^\delta x^{p\delta} (\partial_x z)^{\gamma-1} \right] \partial_x z \right). \quad (8)$$

However, returning to equation 3, it is clear that the transport model is not the only solution. If we assume that rate of change in sediment thickness is zero over geological time scales, which is to say all sediment created is transported out of the model domain, than Equation 3 becomes,

$$\partial_t z = U - E. \quad (9)$$

- 20 This assumption has been made previously when studying small mountain catchments, where the river may erode directly into the bed-rock. However, recent numerical studies, such as Willett et al. (2014) Rudge et al. (2015) , have expanded this model to cover continent-scale landscapes. ~~Is the assumption of instantaneous mass transport still appropriate at the continent-scale?~~

- It is clearly plausible to suppose that erosion is primarily due to flowing water, so the assumption of geologically instantaneous transport may well be valid for mass that is transported as suspended load within the water column, ~~but such~~. Such an assumption is less clear for bed-load transport. We can assume that the speed at which suspended loads travel down system is a function of the height achieved within each hop, which is a function of the water depth, settling velocity and flow velocity. For small grains,  $< 1 \text{ mm}$ , the settling velocity is given by the force balance between the weight of the grain and the viscous drag given by Stokes law (Dietrich, 1982). For a particle of diameter  $1 \times 10^{-4} \text{ m}$  and density  $2800 \text{ kg m}^{-3}$  the settling velocity is  $\sim 0.01 \text{ m s}^{-1}$ . Therefore the distance traveled assuming a flow velocity of  $1 \text{ km hr}^{-1}$  and an elevation of suspension of  $1 \text{ m}$  is roughly  $3 \text{ km}$ . ~~This distance for the flow velocity and grain size~~ Using a similar argument the travel distance of a sediment grain typical of the Bengal Fan is estimated to be  $\sim 10^4 \text{ m}$  (Ganti et al., 2014). This suggests that rapid transport of sediment across a continent is possible.

The percentage of mass transported in suspension may also be quite significant. For a small Alpine braided river it was found that the majority of mass was transported as suspended load (Meunier et al., 2006), and for the river systems draining the Tian Shan, China, 70 % of mass is transported as suspended and dissolved load (Liu et al., 2011). Therefore significant mass may be transported rapidly, geologically instantaneously, down system suggesting that the assumption that  $\partial_t h \sim 0$  may be valid in some circumstances.

Assuming surface flow is the primary driver of landscape erosion and that positive  $x$  is in the downstream direction then erosion,  $E$ , as a function of the power of the flow to detach particles of rock per unit width can be written as,

$$E = -k_{pb} \rho_w g q_w^m (\partial_x z)^n, \quad (10)$$

where  $k_{pb}$  is a dimensional constant (that parameterises bedrock erodability (Howard and Kerby, 1983 ; units  $(\text{m}^2 \text{yr}^{-1})^{1-m} \text{yr kg}^{-1}$ ),

$\rho_w$  is water density,  $g$  is gravity,  $q_w$  is water flux per unit width (units  $\text{m}^2 \text{yr}^{-1}$ ),  $m$  and  $n$  are constants. The exponent  $m \sim 0.5$ , as it is a function of how the stream flow width is proportional to the water flux (e.g. Lacey, 1930; Leopold and Maddock, 1953; Whittaker et al., 2007). The exponent  $n > 0$  acts upon the slope. Using a version of equation 10 to invert river profiles for uplift histories, it is argued by some authors that  $n$  is close to unity (Rudge et al., 2015) . However, certain river profiles may arguably be indicative of  $n > 1$  (Lague, 2014) . If  $n \neq 1$  equation 10 becomes non-linear.

In two dimensions the change in elevation is then given by,

$$\partial_t z = U + k q_w^m (\partial_x z)^n, \quad (11)$$

where the constant  $k$  lumps together the other constants (units  $\text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ ). To solve this equation in one dimension we assume that the water flux is a function of the precipitation transported down the river network. The water collected is taken from the upstream drainage area,  $a$ , which is related to the main stream length,  $l$ , by  $l \propto a^h$  where  $h$  is the exponent taken from the empirical Hack's law (Hack, 1957) . The main stream length is related to the longitudinal length of the catchment by,  $l \propto x^d$  where  $1 \leq d \leq 1.1$  (Tarboton et al., 1990; Maritan et al., 1996) . Therefore, we can write that  $x \propto a^{h/d}$ , and the water flux is the precipitation rate multiplied by the length of the drainage system,

$$q_w = k_w \alpha x^p$$

, and if  $n \neq 1$  equation 11 becomes non-linear. Using a version of equation 11 to invert river profiles for uplift histories, it is argued by some authors that  $n$  is close to unity (Rudge et al., 2015) . However, certain river profiles may arguably be indicative of  $n > 1$  (Lague, 2014) . To solve this equation in 1D, as before we will assume that  $q_w = k_w \alpha x^p$  where  $k_w$  is a constant of proportionality (units  $\text{m}^{1-p}$ ), and  $p = d/h$ . Furthermore,  $p < 2$  as it is observed that river catchments are typically elongate (Dodds and Rothman, 2000), and given that  $0.5 < h < 0.7$  (e.g. Rigon et al., 1996) then  $1.4 < p < 2$ . The stream power law for landscape erosion in 1-D is then,

$$\partial_t z = U + k_p \alpha^m x^{mp} (\partial_x z)^n, \quad (12)$$

where  $k_p = k k_w \rho_w g$  (units  $\text{m}^{-p} (\text{m}^2 \text{yr}^{-1})^{1-m}$ ).

However, returning to equation 3, we stress that the stream power-law is not the only solution if we make a different starting assumption. If we assume that sediment transport is not instantaneous so there is always a supply of transportable sediment, then we can follow through with the summation in equation 3 giving,

$$\partial_t z = U - \partial_x q_s.$$

- 5 This may be appropriate when modelling the transport of sediment along the river bed and when considering the formation of alluvial fans (e.g. Whipple and Tucker, 2002; Guerit et al., 2014). In the absence of surface water we can assume that sediment flux is simply a function of local slope  $q_s = -\kappa \partial_x z$ . In the presence of flowing water then the sediment flux is a function of the flowing water and local slope  $q_s = -c q_w^\delta (\partial_x z)^\gamma$  where  $c$  is the transport coefficient (units  $(\text{m}^2 \text{yr}^{-1})^{1-n}$ ),  $q_w$  is the water flux per unit width (units  $\text{m}^2 \text{yr}^{-1}$ ) and the exponents  $\delta > 1$  and  $\gamma \geq 1$  are dependent on how sediment grains are transported along the bed (Smith and Bretherton, 1972; Paola et al., 1992). Furthermore,  $\delta > 1$  is required to create concentrated flow (Smith and Bretherton, 1972). The change in landscape elevation is then given by,

$$\partial_t z = U + \partial_x \left( \kappa \partial_x z + c q_w^\delta (\partial_x z)^\gamma \right).$$

which can be written as,

$$\partial_t z = U + \partial_x \left( \left[ \kappa + c q_w^\delta (\partial_x z)^{\gamma-1} \right] \partial_x z \right).$$

- 15 Equation 6 is non-linear in the case that  $\gamma \neq 1$ . In deriving this equation of elevation change due to sediment transport we have simply summed the two terms for sediment flux, the linear and potentially non-linear slope dependent terms. This summation has been done as it is the simplest way to generate landscape profiles that have the desired convex and concave elements observed in natural landscapes (Smith and Bretherton, 1972).

- The final step is again to estimate how the water flux changes downstream as the drainage area increases. As before we will assume that  $q_w = k_w \alpha x^p$  where  $1.4 < p < 2$ . Therefore equation 6 becomes,

$$\partial_t z = U + \partial_x \left( \left[ \kappa + c k_w^\delta \alpha^{p\delta} x^{p\delta} (\partial_x z)^{\gamma-1} \right] \partial_x z \right).$$

- We have demonstrated two different fundamental equations for change in elevation in 2-D (equations 11 and 6 and 11) and the equivalent 1-D forms (equations 12 and 8 and 12). These two models of elevation change differ in that equation 11 is a kinematic wave an advection equation and equation 6 is a diffusion equation. This means that the time evolution of equation 11 would be a migrating wave of erosion traveling either up or down the catchment (Braun et al., 2015). This wave could also potentially take the form of a shock-wave, where due to the change in gradient, the lower reaches of the migrating wave could travel faster than the upper reaches, creating a breaking wave (Smith et al., 2000; Pritchard et al., 2009). The time evolution of equation 6 is very different because here the evolution is dominated by diffusive processes. The diffusion coefficient is a function of down-system collection of water, which can lead to the concentration of flow and the creation of realistic morphologies (Smith and Bretherton, 1972), however, the model will respond along the length of the system to change. It is not however completely established how the transport model responds differently to changes in tecto-environmental forcing in comparison to the stream power model.

## 2.2 Linear and non-linear solutions

If  $n = 1$  (equations 11 and 12) and  $\gamma = 1$  (equations 6 and 8) then the models are linear, and we can solve the equations both analytically, and in 1-D and 2-D numerical schemes. For the stream power model we use an implicit finite difference scheme (Braun and Willett, 2013) and for the transport model we use an explicit finite element scheme with linear elements (Simpson and Schlunegger, 2003). If  $n \neq 1$  and if  $\gamma \neq 1$  the equations become non-linear. In this case the numerical solutions can become unstable for simple explicit schemes, and may suffer from too much numerical diffusion for implicit schemes, unless the size of the time step is limited by the appropriate Courant-Friedrichs-Lewy (CFL) condition (Campforts and Covers, 2015). Given the short time steps required to obtain an accurate solution, we explore the non-linear solutions for erosion down a river long profile in 1-D. We solve for the stream power model (equation 12) using an explicit total variation diminishing scheme with the appropriate CFL condition (Campforts and Covers, 2015). For the transport model (equation 8) we use an explicit finite element model with quadratic elements and the appropriate CFL condition to find a stable solution.

## 2.3 Generalizing to a two dimensional system

To solve equations ~~11 and 6~~ 6 and 11 over a 2-D domain requires an algorithm to route surface flow down the landscape. In our case, to explore how a model landscape responds to change in uplift and precipitation rate we will make the simplest assumption available; that water flows down the steepest slope. We then solve for equation 11 using the numerical model FastScape (Braun and Willett, 2013), with a resolution of 1000 by 1000 nodes for a 100 by 100 km domain, giving a spatial resolution of 100 m. Erosion by sediment transport in 2D is solved following the ~~methods of Simpson and Schlunegger (2003)~~ MATLAB model of Simpson and Schlunegger (2003), which is available from Simpson (2017). We solve Equation 6 on a triangular grid with a resolution of 316 by 316 nodes for a 100 by 100 km domain, giving a spatial resolution of the order of 300 m. We also explored how the models evolve for a domain that is 500 by 500 km in size. The time step used for both models is 10 kys.

We will explore how an idealized landscape evolves under uniform uplift at a rate of  $0.1 \text{ mm yr}^{-1}$ . The initial condition is of a flat surface with a small amount of noise added to create a roughness. The boundary conditions are of fixed elevation at the left and right sides, and of no flow at the sides. To explore the response of the two models to change in precipitation rate we start the model with ~~a an initial~~ precipitation rate of  ~~$1 \text{ m yr}^{-1}$  for a duration of 5 or 10 Myr. The precipitation rate is then~~ either reduced or increased  $\alpha_0 = 1 \text{ m yr}^{-1}$ . For the linear models we then increase or decrease the precipitation rate to a new value ~~for a duration of 5 or~~  $\alpha_1$ , after 10 Myr of model run time. This is to be sure that the steady state has been reached before applying the perturbation. For the non-linear models (Sections 3.3 and 3.4), the precipitation rate is changed after 5 Myr as in this case steady state was reached earlier. As the coefficients  ~~$k$  and  $c$  and  $k$~~  have units that are related to the exponents  ~~$m$  and  $\delta$~~  in equations 11 and 6 and  $m$  in equations 6 and 11 respectively (e.g. Whipple and Meade, 2006; Armitage et al., 2013), when modelling increasing values of  ~~$m$  and  $\delta$~~  and  $m$  the coefficients are likewise increased.

The response time for the ~~sediment~~ transport model scales by the effective diffusivity, and can be given by,

$$\tau_t = \frac{L^2}{\kappa + cq_w^\delta} \quad (13)$$

where  $L$  is the model length scale (in this case the length of the domain). For the stream power model the response time is a function of the velocity at which the kinematic wave travels up the catchment (e.g. Whipple and Tucker, 1999; Whipple, 2001). The response time is therefore given by the time it takes for this wave of incision to travel up the catchment length,  $l_c$ ,

$$\tau_{sp} = \frac{l_c}{kq_w^m} \quad (14)$$

- 5 Therefore we expect the response time to be a function of the choice of both the constants  $c$  and  $k$ , and the exponents  $\delta$  and  $m$  within both models. The effect of varying the coefficients  $m$  and  $\delta$  independently has been previously explored (e.g. Whipple and Meade, 2006; Armitage et al., 2013), and we therefore will not do so in detail again here. Instead we ~~wish~~ aim to compare the two models, and therefore search for the values of  $c$ ,  $k$ ,  $m$  and  $\delta$  that generate similar topography at steady state. This steady state is then perturbed by a change in precipitation rate.
- 10 ~~We will explore how an idealized landscape evolves under uniform uplift at a rate of  $0.1 \text{ mm yr}^{-1}$ . The initial condition is of a flat surface with a small amount of noise added to create a roughness. The boundary conditions are of fixed elevation at the left and right sides, and of no flow at the sides.~~

### 3 Results

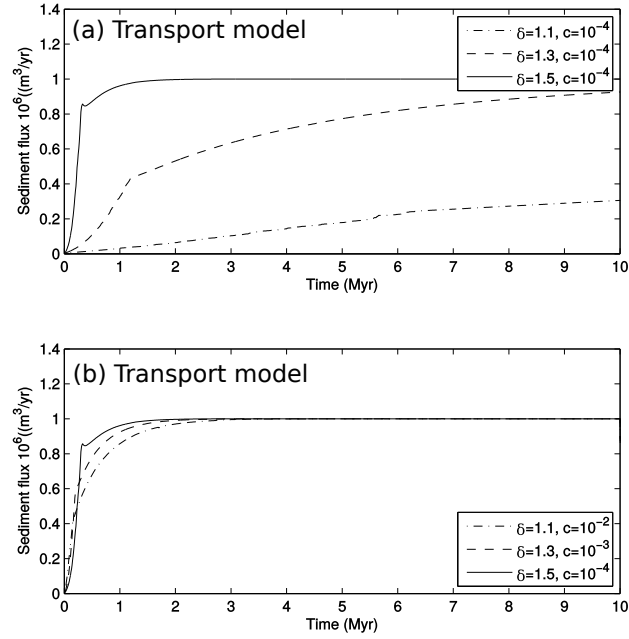
#### 2.1 Generating similar landscapes

- 15 It has been previously demonstrated that both end-member models can generate convex-up long profiles (e.g. Kirkby, 1971; Smith and Bretherton, 1972; Smith et al., 2000; Whipple and Tucker, 2002; Crosby et al., 2007). From solving both equations ~~12 and 8 where  $n=1$  and 8 and 12 where  $\gamma=1$  and  $n=1$~~  we find that in range  ~~$0.3 \leq m \leq 0.7$  and  $1 < \delta \leq 1.5$  and  $0.3 \leq m \leq 0.7$~~  the two end-member models are comparable (see Appendix A). Given the possible additional degree of freedom introduced if we also vary  ~~$n$  and  $\gamma$  and  $n$~~ , it is clear that river-long profiles are not a unique identifier of erosional
- 20 ~~process~~ processes. However, in order to compare how the end-member models respond to change in precipitation rate, it is preferable to perturb catchments of a similar morphology. ~~The stream power~~ We will subsequently explore how the models, in their linear and non-linear forms, respond to a change in precipitation rates within the Results section.

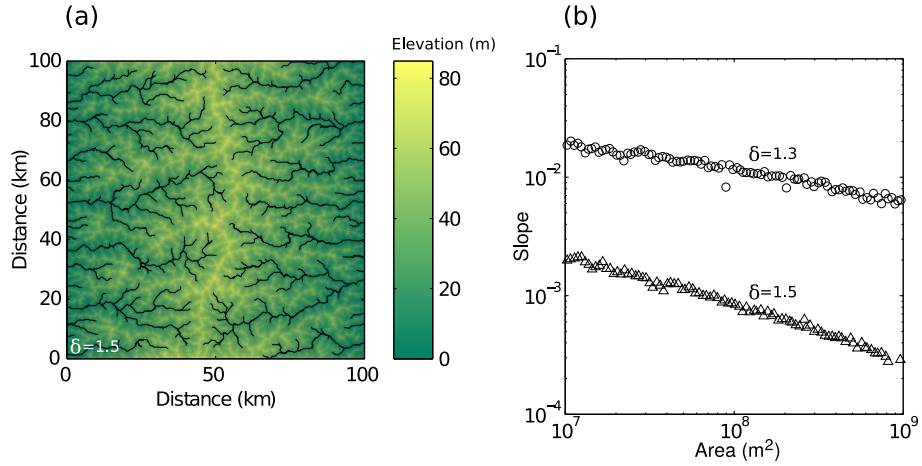
##### 2.1.1 Erosion by Sediment Transport

- Six models have been run without a change in precipitation to find the steady state topography. The models explored are first
- 25 a set of three with varying  $\delta$  and constant  $c$ , i.e.;  $\delta = 1.1, 1.3$  and ~~sediment~~  $1.5$  with  $c = 10^{-4} (\text{m}^2 \text{ yr}^{-1})^{1-\delta}$  (Figure 2a), and a set of three where  $\delta$  and  $c$  co-vary, i.e.;  $\delta = 1.1$  with  $c = 10^{-2} (\text{m}^2 \text{ yr}^{-1})^{1-\delta}$ ,  $\delta = 1.3$  with  $c = 10^{-3} (\text{m}^2 \text{ yr}^{-1})^{1-\delta}$ , and  $\delta = 1.5$  with  $c = 10^{-4} (\text{m}^2 \text{ yr}^{-1})^{1-\delta}$  (Figure 2b).

- When the transport coefficient  $c$  is the same for the three values of the exponent  $\delta$  the model wind-up time increases with decreasing  $\delta$ , and takes several million years where  $\delta < 1.5$  (Figure 2a). Steady state sediment flux is greater for increasing
- 30  $\delta$  when  $c$  is kept constant. The dimensions (units) of  $c$  depend on  $\delta$  which means that the value of the coefficient  $c$  must be adjusted when  $\delta$  is changed to yield the same unit erosion rate per water flux, regardless of  $\delta$  (see Armitage et al., 2013).



**Figure 2.** Sediment flux out of the model domain for the transport model for models where (a)  $\delta = 1.1, 1.3$  and  $1.5$ ,  $\kappa = 10^{-2}$  and  $c = 10^{-4} (m^2 yr^{-1})^{1-\delta}$ , and (b)  $\delta = 1.1, 1.3$  and  $1.5$ ,  $\kappa = 10^{-2}$  and  $c = 10^{-2}, 10^{-3}$ , and  $10^{-4} (m^2 yr^{-1})^{1-\delta}$ .



**Figure 3.** (a) Steady state topography, after 10 Myr, for the transport model where  $\delta = 1.5$  and  $c = 10^{-4} (m^2 yr^{-1})^{1-\delta}$ . (b) Slope area relationship for transport model for  $\delta = 1.3$  and  $\delta = 1.5$ .

Consequently, when  $c$  is suitably adjusted the model can reach a steady state in a similar time for all three values of  $\delta$  (Figure 2b).

**Table 1.** Slope area relationship for trunk streams derived using  $\chi$ -analysis (Perron and Royden, 2012)

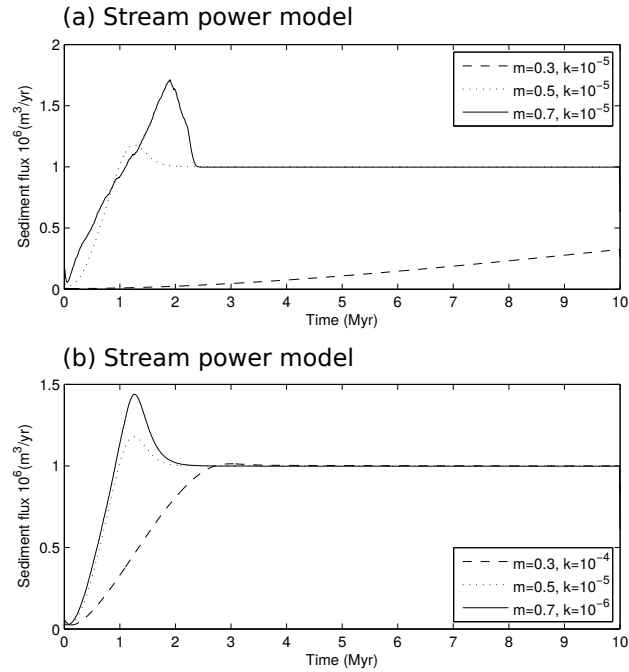
sediment transport	$k_s$	$\theta$
$\delta = 1.3$	0.86	-0.23
$\delta = 1.5$	1.76	-0.42
stream power		
$m = 0.3$	0.95	-0.29
$m = 0.5$	6.52	-0.46
$m = 0.7$	71.42	-0.68

We subsequently analyze the topography for the relationship between trunk river slope and drainage area, Figure 3, using Topotoolbox2 (Schwanghart and Scherler, 2014). For the case where  $\delta = 1.5$  the scaling between channel slopes and catchment drainage areas, the slope area exponent  $\theta$ , is equal to -0.42, and for  $\delta = 1.3$ ,  $\theta$  is equal to -0.23 (Figure 3b). The same value is calculated using the spatial transformation described within (Perron and Royden, 2012), commonly referred to as  $\chi$ -analysis (Table 1). Given the reduction in  $\theta$  from  $\delta = 1.5$  to 1.3, we did not analyze the case for  $\delta = 1.1$  as the slope-area relationship will clearly lie below the observed range ( $0.35 < \theta < 0.7$ ; e.g. Snyder et al., 2000; Wobus et al., 2006). Therefore, for river networks defined by routing water down the steepest slope of descent, the transport model can create catchment morphologies that have a concavity similar to that observed in nature if  $\delta \sim 1.5$ .

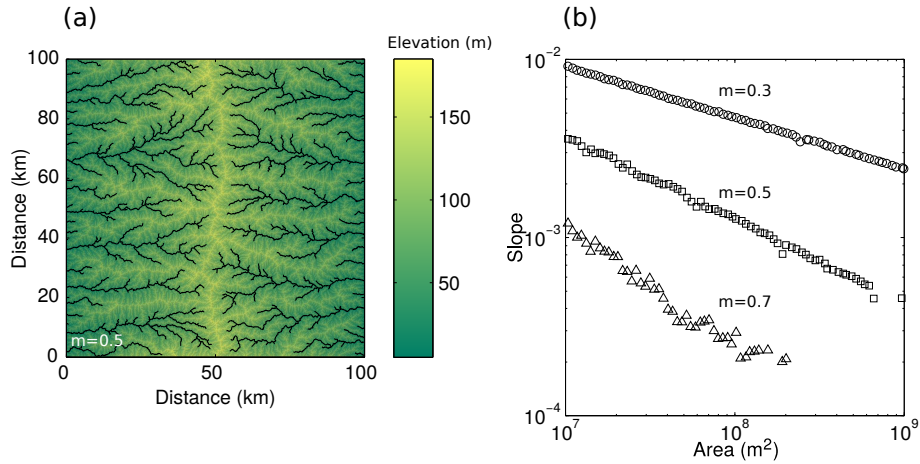
### 2.1.2 Comparison to Erosion by Stream Power

In order to provide a comparison for the morphology of the transport model we explore how the stream power model evolves to a steady state. The landscape derived from the stream power model, equation 11, evolves towards a steady state with a slightly different behaviour in comparison to the transport model (Figure 4). As before we run six models where in this case the first set of three are  $m = 0.3, 0.5$  and  $0.7$  with  $k = 10^{-5} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$  (Figure 4a). The second set of three are of  $m = 0.3$  with  $k = 10^{-4} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$ ,  $m = 0.5$  with  $k = 10^{-5} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$ , and  $m = 0.7$  with  $k = 10^{-6} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$  (Figure 4b). This range of  $m$  is chosen as it spans the range of observed concavities within catchments. As with the transport model the coefficient  $k$  can be adjusted along with  $m$  as they are related, where increasing  $k$  reduces the model wind-up time (Figure 4b). Decreasing the exponent  $m$  increases the timescale taken to reach a steady state (Figure 4a), however by varying  $k$  by a factor of 100, the steady state sediment flux is reached within 3 Myrs for the three values of  $m$  (Figure 4b).

Following the previous examples, we analyze the topography for the relationship between trunk river slope and drainage area (Figure 5). Both the transport model and the stream power model can create landscapes with similar slope-area relationships calculated using the  $\chi$ -analysis approach (Table 1). For both models, the value of the intercept,  $k_s$ , and the gradient,  $\theta$ , are of similar magnitudes for  $\delta = 1.5$  and  $m = 0.5$ . Absolute elevation for the model shown in Figure 5a is higher than the transport

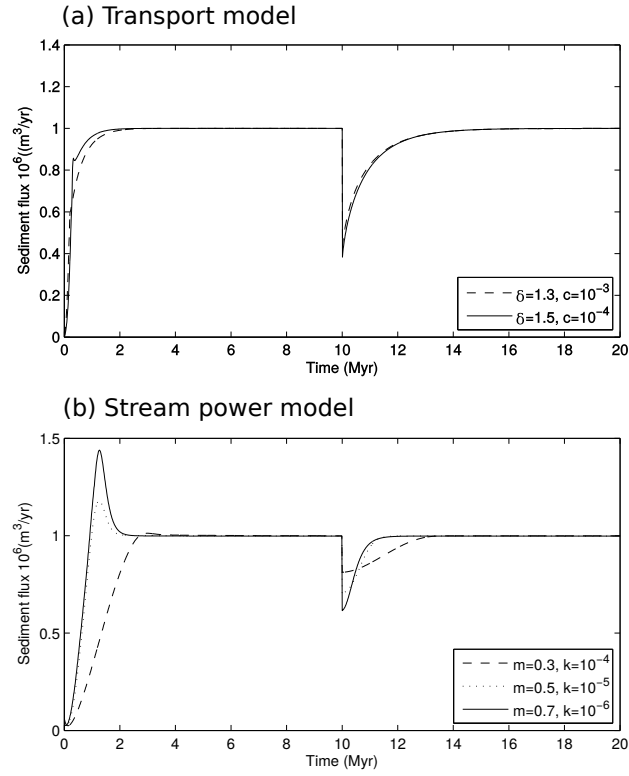


**Figure 4.** Sediment flux out of the model domain for the stream power model for models where (a)  $m = 0.3, 0.5$  and  $0.7$ , and  $k = 10^{-5} (\text{m}^2 \text{yr}^{-1})^{1-m}$ , and (b)  $m = 0.3, 0.5$  and  $0.7$ , and  $k = 10^{-4}, 10^{-5}$ , and  $10^{-6} \text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ .



**Figure 5.** (a) Steady state topography, after 10 Myr, for the transport model where  $m = 0.5$  and  $k = 10^{-5} \text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ . (b) Slope area relationship for transport model for  $m = 0.3, 0.5$  and  $0.7$ .

model example due to the larger value of  $k$  relative to  $c$ . However, importantly, both models can create similar landscape morphologies at steady state.



**Figure 6.** Response of the transport and stream power model to a reduction in precipitation rate (a) Sediment flux ~~out-of-for~~ the ~~sediment~~ transport model for a step reduction in precipitation from 1 to  $0.5 \text{ myr}^{-1}$  after 10 Myr. Two models are plotted, where  $\delta = 1.3$  and  $1.5$ ,  $\kappa = 10^{-2}$  and  $c = 10^{-3}$ , and  $10^{-4} (\text{m}^2 \text{ yr}^{-1})^{1-\delta}$ . (b) Sediment flux for the stream power model for a step reduction in precipitation from  $\alpha_0 = 1 \text{ m yr}^{-1}$  to  $\alpha_1 = 0.5 \text{ m yr}^{-1}$  after 10 Myr. Three models are plotted, where  $m = 0.3, 0.5$  and  $0.7$ , and  $k = 10^{-4}, 10^{-5}$ , and  $10^{-6} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$

### 3 Results

The stream power and transport model can both fit observed slope-area relationships of the present day landscape morphology ~~within a certain parameter range~~ (e.g.  $\theta$  ranging from 0.35 to 0.7 (Snyder et al., 2000; Wobus et al., 2006) , when the water flux exponent is  $m \sim 0.5$  or  $\delta \sim 1.5$  ; see Appendix B) for the stream power and transport model respectively. Therefore, both

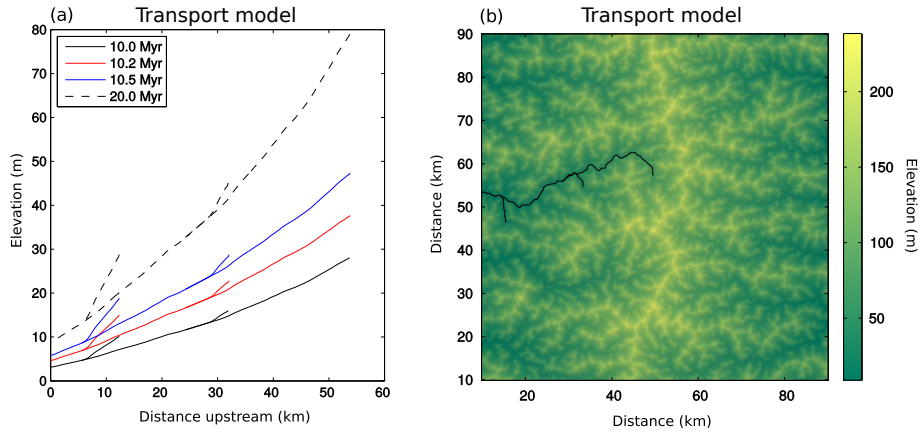
5 models may be a reasonable representation of how, on a gross scale, a landscape erodes. We therefore ~~fix the slope exponents at these values~~ keep the exponents in the range  $0.3 \leq m \leq 0.7$  and  $1.3 \leq \delta \leq 1.5$ , and explore how the models, in their linear and non-linear forms, respond to a change in precipitation rates.

**Table 2.** Response to change in precipitation rate where  $\alpha_1$  ~~is represents the~~ value the precipitation rate changes to from  $\alpha_0 = 1 \text{ mm yr}^{-1}$ . Response time is given for two model sizes, 100 and 500 km, and as the time for the model to recover by half and a tenth towards the steady state sediment flux.

$L = 100 \text{ km}$	transport		detachment	
	$\tau_{1/2}$	$\tau_{1/10}$	$\tau_{1/2}$	$\tau_{1/10}$
	Myr	Myr	Myr	Myr
$\alpha_1$				
$\text{mm yr}^{-1}$				
0.25	1.42	6.07	0.98	1.66
0.50	0.53	2.19	0.70	1.18
0.75	0.30	1.21	0.57	0.98
2.00	0.09	0.31	0.34	0.60
$L = 500 \text{ km}$	transport		detachment	
	$\tau_{1/2}$	$\tau_{1/10}$	$\tau_{1/2}$	$\tau_{1/10}$
	Myr	Myr	Myr	Myr
$\alpha_1$				
$\text{mm yr}^{-1}$				
2.00	0.17	0.64	0.34	0.60

### 3.1 Response to precipitation rate reduction

- In Figure 6a we display the response of erosion for the ~~sediment~~-transport model, in terms of sediment flux out of the model domain, for a reduction in precipitation rate from 1 to  $0.5 \text{ mm yr}^{-1}$  at 10 Myr of model evolution. We explore how the transport model responds for  $\delta = 1.5$ ,  $c = 10^{-4}$ ;  $\delta = 1.3$ ,  $c = 10^{-3}$  as these two values of  $\delta$  generate resonable slope area relationships (Figure 3b, Table 1). The response to a reduction in precipitation is ~~very similar~~ similar for the two model parameter sets, with the flux initially reducing by a half and then recovering to within 10 % of steady state values within  $\sim 2$  Myr (Figure 6a; see Table 2). Changing the transport coefficient,  $c$ , does not affect predicted ~~slope-area-relationship (Appendix A); importantly, however, it~~ gradient of cathcment slope versus catchment area (see Appendix A, Figure 16). However, changing  $c$  changes the model elevation, ~~larger  $c$  creates more transport and it alters the model response time;~~ (Figure 16). Furthermore, the larger the value of  $c$  the faster the response (Equation 13; see Armitage et al., 2013). ~~Response is slower for smaller values of  $A$  small increase in the exponent  $\delta$  , therefore the increase will strongly reduce reponse times, as it will increase the water flux term (Equation 13). Therefore an order of magnitude decrease in  $c$  counters this effect, and for the the change in  $\delta$  for the two model sets (Figure 6a). For the~~ values chosen both models respond at a similar rate to the change in precipitation (Figure 6a; see Table 2).
- The response of the stream power model to an identical reduction in precipitation at a model time of 10 Myr takes a similar form, with an initial decease in sediment flux out followed by a gradual recovery (Figure ??6b). In a similar manner as the transport model, response is a function of the exponent  $m$  and the coefficient  $k$  (Equation 14). We have modeled three parameter sets:  $m = 0.3$  and  $k = 10^{-4}$ ,  $m = 0.5$  and  $k = 10^{-5}$ , and  $m = 0.7$  and  $k = 10^{-6}$ . ~~Response~~ (Figure 6b). The response time



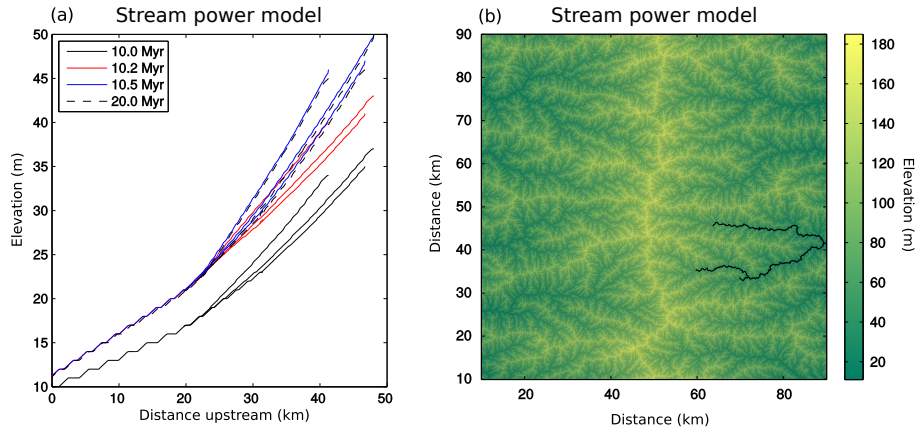
**Figure 7.** Transport model evolution due to a reduction in precipitation. (a) Selected river long profile response to change in precipitation. Black line is the profile just before a factor of two reduction in precipitation. The red and blue lines are 200 kyr and 500 kyr after the reduction in precipitation. The dashed black line is the steady state profile. (b) Trunk stream used for the analysis with the steady state elevation.

to achieve return to 10 % of the steady state sediment flux varies from 3 Myr in the case of  $m = 0.3$  to less than 1 Myr when  $m = 0.7$ . As well as response time being longer for smaller values of  $m$ , the peak magnitude of the flux response is reduced for smaller values of  $m$  (Figure ??6b).

~~Sediment flux-out-of-the-stream-power-model-for-a-step-reduction-in-precipitation-from  $\alpha_0 = 1 \text{ m yr}^{-1}$  to  $\alpha_1 = 0.5 \text{ m yr}^{-1}$  after 10 Myr. Three models are plotted, where  $m = 0.3, 0.5$  and  $0.7$ , and  $k = 10^{-4}, 10^{-5}$ , and  $10^{-6} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$ .~~

The magnitude of the response for all the runs is greater for the ~~sediment~~ transport model when compared to the stream power model (Figures 6and ??Figure 6). Consequently, response time, while being a function of the transport coefficients  $c$  and  $k$  respectively, may still systematically differ between the two models: The ~~sediment~~ transport model with  $\delta = 1.5$  and  $c = 10^{-4}$  generates a maximum model elevation of  $\sim 240 \text{ m}$ , and the stream power model with  $m = 0.5$  and  $k = 10^{-5}$  generates a maximum elevation of  $\sim 180 \text{ m}$ . These two models have a similar slope area relationship at steady state (Table 1) and are therefore comparable suggesting a faster response to a reduction in precipitation rates for the stream power model (Figures 6and ??Figure 6).

To explore how the difference in response time and magnitude is expressed in the landscape, we extract the river profiles of the main trunk systems for models where  $\delta = 1.5$  and  $m = 0.5$  during the response to the reduction in precipitation rate while uplift rate is constant (Figures 7 and 8). For the ~~sediment~~ transport model in which  $\delta = 1.5$  and  $c = 10^{-4}$ , the catchment elevation increases to a new steady state that has an elevation that is roughly 2.6 times higher than the steady state elevation after 10 Myr (Figure 7). Just under half of this new topographic elevation is achieved within the first 500 kyr, ~~and the increase in elevation occurs without any knickpoint migration~~. In contrast, for the stream power model where  $m = 0.5$  and  $k = 10^{-5}$ , the steady state topography is achieved within a fraction of the time when compared to the transport model ( ~~$\sim 500 \text{ kyr}$~~ ). This is in line with the more rapid response of this model to a relative drying of the climate using these parameters (compare Figures 6and ??Figure 6a and b). Furthermore the increase in elevation due to the reduced surface water flux is only a factor of  $\sim 1.2$ ,



**Figure 8.** Stream power model model evolution due to a reduction in precipitation. (a) Selected river long profile response to change in precipitation. Black line is the profile just before a factor of two reduction in precipitation. The red and blue lines are 200 kyr and 500 kyr after the reduction in precipitation. The dashed black line is the steady state profile. (b) Trunk stream used for the analysis with the steady state elevation.

which is less than half of the increase for the **sediment**-transport model. ~~The lower reaches of the catchment respond more rapidly than the upper reaches, therefore creating a migrating knickpoint as the landscape responds to the change in model forcing (see Braun et al., 2015).~~

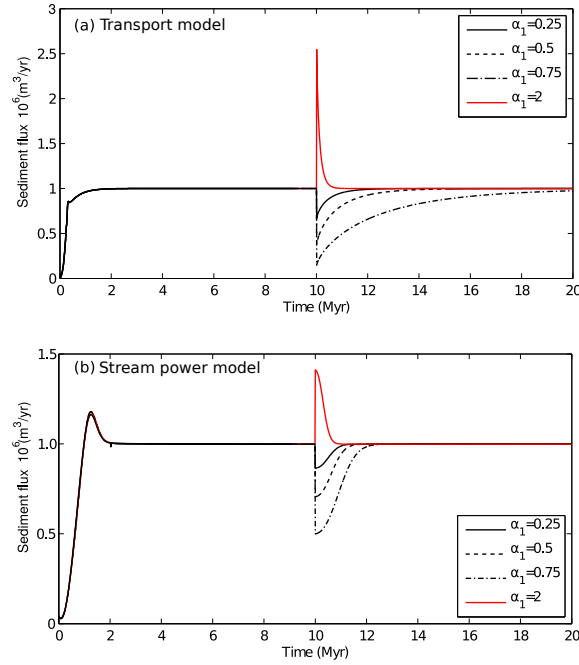
### 3.2 Response timescales to different magnitudes of precipitation rate change

- 5 (a) Response of the sediment transport model to change in precipitation rate. Equation 6 is solved for  $\delta = 1.5$  and  $c = 1 \times 10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-m}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 0.25, 0.5, 0.75$  or  $2 \text{ m yr}^{-1}$  after 10 Myr. (b) Response of the stream power model to change in precipitation rate. Equation 6 is solved for  $m = 0.5$  and  $k = 1 \times 10^{-5} \text{ m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 0.25, 0.5, 0.75$  or  $2 \text{ m yr}^{-1}$  after 10 Myr.

Our results confirm that two different end-member erosion models, encompassing advective and diffusive **mathematics** phenomena, can produce landscapes with similar morphologies, if particular parameter sets are selected accordingly. However the key question here is whether these landscapes produce different sediment flux responses if perturbed from their steady state configuration, and if so, how do they differ in magnitude and timescale?

### 3.2 Response to different magnitudes of precipitation rate change

The response time of the **sediment**-transport model is known to be a function of the transport coefficient and the magnitude of the precipitation rate (c.f. Armitage et al., 2013). This behavior is displayed in Figure 9a, where the response of the transport model with  $\delta = 1.5$  and  $c = 10^{-4}$  for a change in precipitation from 1 to 0.25, 0.5, 0.75 and  $2 \text{ m yr}^{-1}$  is plotted. The response time, measured as the time for the sediment flux to recover by half and by 90 % to the steady state value, is shown additionally in Figure 10 as black solid and dashed lines respectively, and in Table 2. For a reduction to  $0.25 \text{ m yr}^{-1}$  the prediction is for

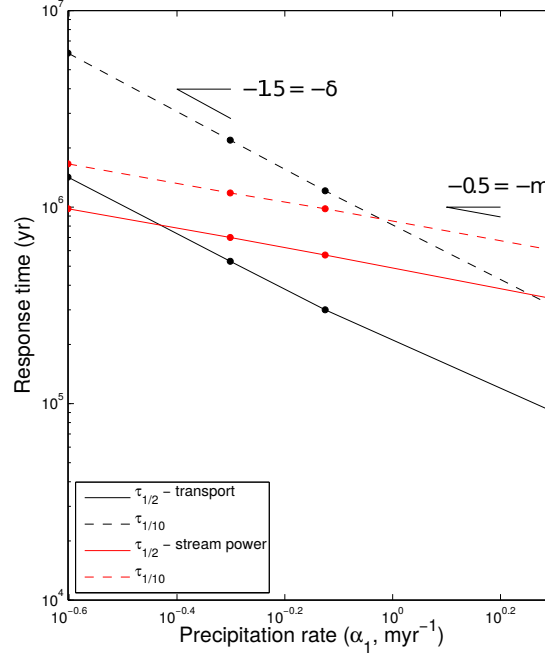


**Figure 9.** (a) Response of the transport model to change in precipitation rate. Equation 6 is solved for  $\delta = 1.5$  and  $c = 1 \times 10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 0.25, 0.5, 0.75$  or  $2 \text{ m yr}^{-1}$  after 10 Myr. (b) Response of the stream power model to change in precipitation rate. Equation 6 is solved for  $m = 0.5$  and  $k = 1 \times 10^{-5} \text{ m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 0.25, 0.5, 0.75$  or  $2 \text{ m yr}^{-1}$  after 10 Myr.

a long response time of 6.07 Myr, while for an increase to  $2 \text{ m yr}^{-1}$  the prediction is a rapid response time of 310 kyr for 90 % recovery towards previous sediment flux values. The equivalent half life, recovery by 50 % towards previous sediment flux values, is 1.42 Myr and 90 kyr.

The stream power model likewise has a response time that is a function of precipitation rate (Figure 9b). For a reduction to  $0.25 \text{ m yr}^{-1}$  the prediction is for a response time of 1.66 Myr, while for an increase to  $2 \text{ m yr}^{-1}$  the prediction is for recovery time of 600 kyr for 90 % recovery (Table 2). The equivalent half life is 0.98 Myr and 340 kyr (Table 2). The stream power model is therefore faster to recover for a reduction in precipitation rate yet slower ~~for to respond to~~ an increase in precipitation rate. This is because the response time of the stream power model is more weakly a function of precipitation rate. Importantly, these results therefore suggest there is a fundamental asymmetry in the response timescale to a climate perturbation, ~~in which a drying event takes much longer. The models suggest that it takes longer for surface processes to recover from ,compared to wetting events~~ drying event compared to a wetting event.

Both models display a response time that is a function of the precipitation rate (Figures 9 and 10). ~~Given that for the models The water flux term is given by,~~  $q_w = \alpha l_d$ , where  $l_d$  is the drainage network length and  $\alpha$  is the precipitation rate. Therefore,



**Figure 10.** Log-log plot of response time to change to a precipitation rate  $\alpha_1$  from an initial value of  $\alpha_0 = 1 \text{ m yr}^{-1}$  when the model domain is 100 by 100 km (see Table 2).  $\tau_{1/2}$  is the time for the sediment flux to recover by a half of the magnitude change in sediment flux and  $\tau_{1/10}$  is the time for the sediment flux to recover by 90 %.

the relationship between precipitation rate and the **sediment**-transport model response can be expressed as,

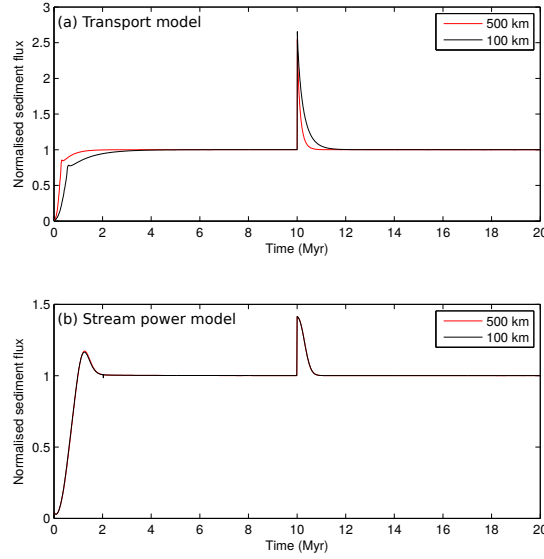
$$\tau_t \propto \alpha^{-\delta} \quad (15)$$

where in this case  $\delta = 1.5$ . This proportionality is in agreement with our numerical model results, where the slope of trend for the **sediment**-transport model in the log-log plot is -1.5 (Figure 10).

- 5 In contrast, the response time of the stream power model is not as strongly inversely dependent on the precipitation rate (Figure 10). For this model, the response time is a function of the velocity at which the wave of incision travels up-stream, **and the portion of the channel downstream of the knock-point provides the locus of enhanced erosion**. This velocity is directly related to the inverse of the water flux,  $q_w^m$ , which is in turn again a function of the drainage length and precipitation rate,  $\alpha$ . Therefore for the stream power model we can write that response time is,

$$10 \quad \tau_{sp} \propto \alpha^{-m}. \quad (16)$$

This proportionality, which is in agreement with the approximate analytical solutions of Whipple (2001), is likewise in agreement with our numerical model results, where the slope of trend for the stream power model in the log-log plot is -0.5 (Figure



**Figure 11.** (a) Response of the transport model to change in precipitation rate for two different model dimensions, 100 by 100 km and 500 by 500 km. Equation 6 is solved for  $\delta = 1.5$  and  $c = 1 \times 10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 2 \text{ m yr}^{-1}$  after 10 Myr. (b) Response of the stream power model to change in precipitation rate for two different model dimensions, 100 by 100 km and 500 by 500 km. Equation 6 is solved for  $m = 0.5$  and  $k = 1 \times 10^{-5} \text{ m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 2 \text{ m yr}^{-1}$  after 10 Myr.

10). Consequently, for these two models, which were derived from the same starting point (Figure 1), and applied to catchments of similar topography and morphology, we find that above a certain magnitude of precipitation rate change, the **sediment** transport model responds more rapidly than the stream power model and vice versa.

The position of the critical point where the stream power model responds more rapidly than the **sediment**-transport model is a function of the water flux and the collection of coefficients. In the model comparison developed here, we have compared two model catchments that have similar concavity,  ~~$\beta$  between 0.4 and 0.5~~  $\theta$  between -0.4 and -0.5 ( $\delta = 1.5$  and  $m = 0.5$ ) and model domain length of  $L = 100 \text{ km}$  giving catchments of roughly 50 km length. In this case the 90 % recovery of the sediment flux signal is predicted to be more rapid for the **sediment**-transport model when compared to the **detachment limited stream power** model for an increase in precipitation rate (Figure 10). If however the model domain is increased to  $L = 500 \text{ km}$  then ~~a 90% recovery it takes twice as long for the transport model to recover from an increase in precipitation rate from 1 to  $2 \text{ m yr}^{-1}$ : 0.63 Myr compared to 0.31 Myr for  $L = 100 \text{ km}$  (Figure 11a and Table 2). This recovery time is now similar to the equivalent stream power model where  $m = 0.5$ , which remains 0.60 Myr.~~

The stream power model is insensitive to the size of the model domain because of the particular choice of  ~~$m$~~   $m = 0.5$  and the shape of drainage network that forms under the assumptions of routing water down the steepest slope of descent (Figure

11b). Taking the drainage length to be directly proportional to the catchment area,  $l_d \propto a$ , and given that catchment length is proportional to drainage area raised to the Hack exponent,  $h$ , we can re-write equation 14 as,

$$\tau_{sp} \propto \frac{a^h}{(\alpha a)^m}. \quad (17)$$

Therefore, in the case that  $h = 0.5$  and  $m = 0.5$ , as in the numerical model here, the response time becomes independent of system length (c.f. Whittaker and Boulton, 2012). If  $h < m$  then response times would reduce with increasing drainage basin size, and if  $h > m$  then response times would increase with drainage basin size. There is good empirical evidence for  $0.5 < h < 0.7$  (e.g. Rigon et al., 1996), which fundamentally controls the plan view shape of catchments, yet there is not a complete consensus on the value of  $m$  (see Lague, 2014; Temme et al., 2017).

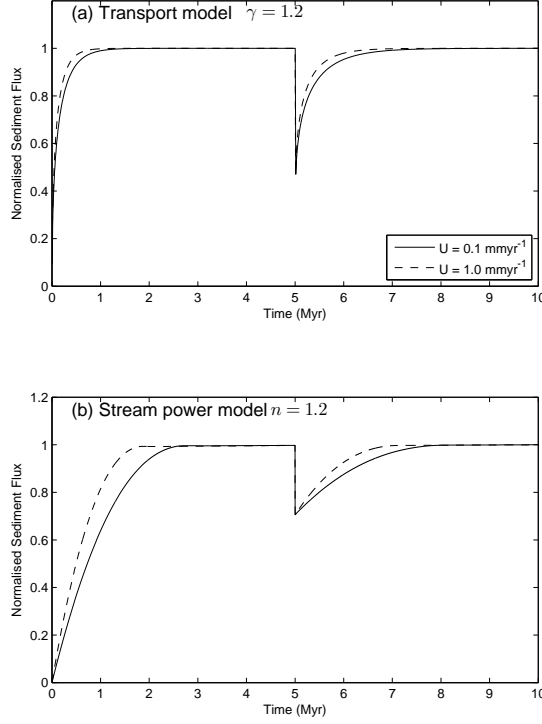
A final key difference between the transient sediment flux responses of the two models is that the peak magnitude of system response to a change in precipitation rate is systematically larger for the ~~sediment~~-transport model (Figure 9). For an increase in precipitation rates from 1 to 2  $\text{m yr}^{-1}$ , the sediment flux increases from  $1 \times 10^6 \text{ m}^3$  to  $2.5 \times 10^6 \text{ m}^3$  for erosion by sediment transport. This is three times greater than the equivalent increase for the stream power model. The reduction in sediment flux is likewise larger for the ~~sediment~~-transport model (Figure 9). Therefore, although response time is a function of precipitation rate, the magnitude of change is consistently larger for the ~~sediment~~-transport model.

### 15 3.3 Non-linear response timescales

Up to this point we have compared how the models respond to a precipitation rate change when the solutions are linear. However, there is reasonable debate as to the value of the slope exponent  $n$  in the stream power model (e.g. Lague, 2014; Croissant and Braun, 2014; Rudge et al., 2015) and likewise within the transport model it is plausible that the slope exponent  $\gamma > 1$ . The response time for the stream power model for various values of  $n$  has been explored within Baldwin et al. (2003). Here we expand on this by exploring the equivalent response times for the transport model. To explore the implications of the non-linearity introduced by relaxing the constraint that  $n = 1$  and  $\gamma = 1$  for both models, we solve equations ~~12 and 8~~ 8 and 12 for  $p = 1.1$ ,  $\delta = 1.5$ ,  $c = 5 \times 10^{-5}$ , and  $m = 0.5$ ,  $k = 10^{-4}$  ~~and  $\delta = 1.5$  and  $c = 5 \times 10^{-5}$~~ , respectively with different uplift rates. We have modelled the response due to an uplift rate of between 0.1 and 1.0  $\text{mm yr}^{-1}$  for the case where  ~~$n = 1.2$  and  $\gamma = 1.2$~~  in equations 12 and 8 and  $n = 1.2$  in equations 8 and 12 (Figure 12).

We find that for both the transport and stream power model, when the slope exponent is greater than one, the model response time is a function of uplift rate. The faster the rate of uplift, the faster the system responds to a change in precipitation rate. If the response time for a system recovery to steady state by 50 % or 10 % is plotted on a log-log plot against uplift rate we find that the response time is proportional to the uplift rate raised to a negative power (Figure 13). In the case of  $n = 1.2$  or  $\gamma = 1.2$  the slope of trend is -0.1667, and for  $n = 2$  or  $\gamma = 2$  the slope of trend is -0.5 (Figure 13). These slopes are in agreement with the approximate analytical solutions of Whipple (2001) and numerical models of Baldwin et al. (2003), i.e. the stream power response time  $\tau_{sp}$  has a proportionality,

$$\tau_{sp} \propto U^{\frac{1}{n}-1} \quad (18)$$

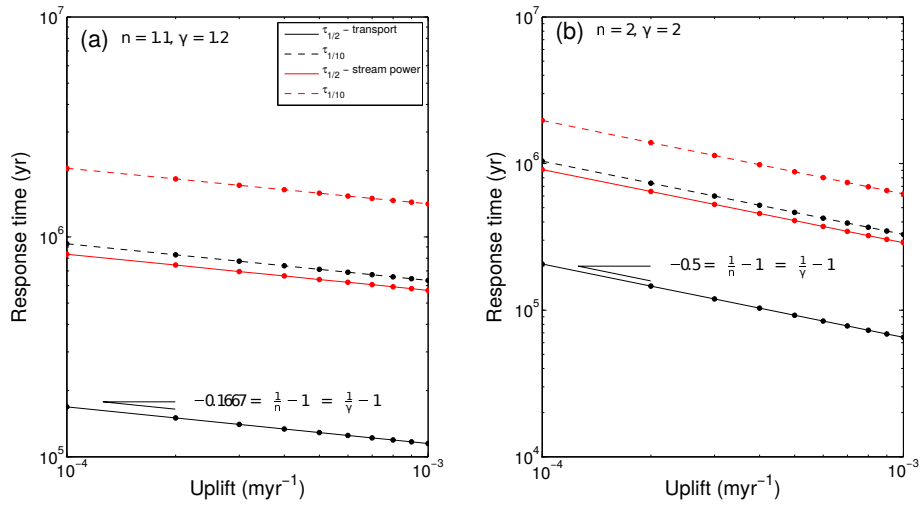


**Figure 12.** (a) Response of the ~~sediment~~-transport model to change in precipitation rate for two values of uplift,  $0.1$  and  $1.0 \text{ mm yr}^{-1}$ . Equation 8 is solved for  $\gamma = 1.2$ ,  $\delta = 1.5$ ,  $p = 1.1$  and  $c = 5 \times 10^{-5} (\text{m}^2 \text{ yr}^{-1})^{1-\delta}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 0.5$ . (b) Response of the stream power model to change in precipitation rate for two values of uplift,  $0.1$  and  $1.0 \text{ mm yr}^{-1}$ . Equation 12 is solved for  $n = 1.2$ ,  $m = 0.5$ ,  $p = 1.1$  and  $k = 1 \times 10^{-4} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$ . The precipitation rate is initially  $\alpha_0 = 1 \text{ m yr}^{-1}$  and changes to  $\alpha_1 = 0.5$  after 5 Myr.

and equivalently we infer from our numerical model (Figure 13) that the transport ~~limited~~-model response time as,

$$\tau_t \propto U^{\frac{1}{\gamma}-1}. \quad (19)$$

This implies that both models have the same form of response dependency on uplift rates, ~~and~~. Therefore, regardless of the rate of uplift we should expect the transport model to respond more rapidly to a large increase in precipitation rate and the stream power model to respond more rapidly to a reduction in precipitation rate (Figure 10). Our results are also consistent with the field-based findings of Whittaker and Boulton (2012) who showed that landscape response times for rivers close to the detachment-limited end-member were shorter, even accounting for drainage area, where they eroded terrain uplifted by faster slipping active faults.



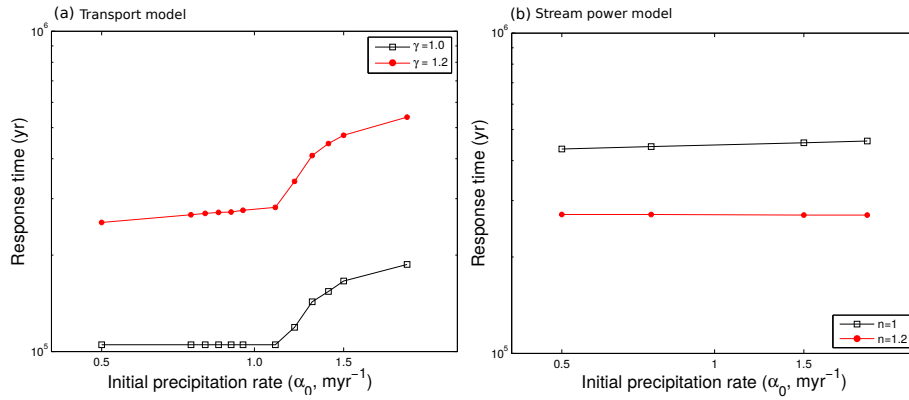
**Figure 13.** Log-log plot of response time for different slope exponents and uplift rates to change to a precipitation rate from an initial value of  $\alpha_0 = 1 \text{ m yr}^{-1}$  to  $\alpha_1 = 0.5 \text{ m yr}^{-1}$ .  $\tau_{1/2}$  is the time for the sediment flux to recover by a half of the magnitude change in sediment flux and  $\tau_{1/10}$  is the time for the sediment flux to recover by 90 %. (a) Response time for the transport model (equation 8) and stream power model (equation 12) when the slope exponent  $\gamma = 1.2$  and  $n = 1.2$  respectively. A linear trend is found with a gradient of  $-1.667$ . (b) Response time for the transport model and stream power model when the slope exponent  $\gamma = 2$  and  $n = 2$  respectively. A linear trend is found with a gradient of  $-0.5$ .

### 3.4 Response time as a function of the initial precipitation rate

Up until this point we have only explored how the numerical models respond to a increase or decrease in precipitation rate by keeping the initial precipitation rate fixed at  $\alpha_0 = 1 \text{ m yr}^{-1}$  and varying the final precipitation rate  $\alpha_1$ . In this final section we will now instead keep the final precipitation rate fixed at  $\alpha_1 = 1 \text{ m yr}^{-1}$ , and vary the initial precipitation rate  $\alpha_0$  from values of  $0.5$  to  $1.5 \text{ m yr}^{-1}$ . We will focus again on the 1-D models, and look at the linear and non-linear cases with  $n = 1.2$  and  $\gamma = 1.2$ .

For the linear and non-linear transport model we find that if the initial precipitation is less than the final precipitation ( $\alpha_0 < \alpha_1$ ) then the response time is not very sensitive to the initial precipitation rate (Figure 14a). If  $\alpha_0 > \alpha_1$  then the response time is a function of the initial precipitation rate, but the relationship cannot be explained by a simple power law (Figure 14a). The change in response time as a function of the initial precipitation rate is however small compared to the change in response time as a function of the final precipitation rate.

In the case of the linear and non-linear stream power model, the response time has a no dependence on the initial precipitation rate, and is only a function of the final precipitation rate (Figure 14b). With all other parameters being held constant, the initial precipitation rate will set up the topography and hence slope of the pre-perturbation landscape. Elevations will be lower for higher precipitation rates, and the topographic gradient will be smaller. For the case of the stream power model, the change in erosion rates migrates up the catchment and so the old topography does not impact the response time. For the transport model,



**Figure 14.** Log-log plots for the transport model and the stream power model in 1-D for a step change in precipitation rate, where the initial precipitation rate,  $\alpha_0$  varies from 0.5 to 1.5  $\text{m y}^{-1}$  and the final precipitation rate is fixed at  $\alpha_1 = 1 \text{ m y}^{-1}$ . (a) Results for the transport model. (b) Results for the stream power model.

however, the remnant topography does have a small effect on the response time, but only if the previous precipitation rate was higher than the new post perturbation precipitation rate.

## 4 Discussion

In deriving the two end member models to describe landscape evolution, we showed that if the rate of transport of sediment were assumed to be instantaneous (i.e. all sediment is transported out of the model domain) then the stream power model would be appropriate to describe catchment erosion. However, if it is instead assumed that the rate of sediment transport is not instantaneous, then we arrive at a model in which erosion scales with the rate of change of sediment flux, which itself is dependent on both linear and potentially non-linear slope-dependent terms. These two end-members can produce similar steady-state landscapes, as noted by a number of previous workers (e.g. Whipple and Tucker, 2002; Tucker and Whipple, 2002). However, as we demonstrate above, when perturbed by a change in conditions such as rainfall rate, they can produce very different landscape responses, which vary in terms of their style, magnitude and tempo. We explore the nature and implications of these responses below.

It is also important to stress that the catchment responses, and the predicted sediment fluxes out of these two model domains might be relevant variously to different erosional and depositional domains (c.f. Lague, 2014; Temme et al., 2017). A model of instantaneous sediment transport might be more relevant for suspended sedimentary loads, where transport times can be very small, while the transport model might be more appropriate for bedload dominated systems, even in cases where bedrock is clearly incised (e.g. Paola et al., 1992; Valla et al., 2010). Furthermore, given that these two models have different response times, it is possible that fine grained deposits might record signals of climate change differently from e.g. the gravel deposits within alluvial valleys. Below, we will therefore first discuss how the two model responses compare in terms of their response

time, and place our results in the wider context of sediment routing system response to environmental change. Second, we will compare the model results with three records of change in sediment deposition during the Paleocene Eocene Thermal Maximum (PETM), a known and well-constrained period of rapid climate change. Finally, we will summarise the key implications from our results.

#### 5 4.1 Response times as a function of model choice

Under certain parameter sets it is relatively straightforward to generate two landscapes, eroded by ~~diffusive or advective mathematics~~ the transport or stream power model, that have similar elevation, slope, and area metrics (~~Appendix B~~ Figures 3 and 5). To find a path to break the apparent ~~non-unique non-uniqueness of these~~ solutions we have explored the ~~response in terms of sediment flux~~ transient sediment flux response out of the model domain for two end-member solutions to erosion.

10 The first observation is that both models respond, at a first order, in a broadly similar way to a precipitation rate (climate) driver (~~Figure~~ Figures 9 and 10). Both models have a response that is an inverse function of the magnitude of precipitation rate change. Both models have a response that is related to uplift in an identical manner (Figure 13). However, the responses for catchments that are comparable in slope-area relationship and maximum elevation, but which are governed by different erosional dynamics defined by  $c$ ,  $k$ ,  $m$  and  $\delta$ , actually display different response times by almost one order of magnitude (Figures  
15 2 and 4).

We have demonstrated that models limited by their ability to transport sediment tend to have shorter response times to an increase in rainfall rate, and thus re-achieve pre-perturbation sediment flux values more rapidly compared to stream power dominated systems, particularly when catchment length-scales are small (e.g.  $< 100$  km, Figure 10). ~~The~~ These model observations suggest that the sediment fluxes from small alluvial catchments, even when captured in downstream depocentres,  
20 may be difficult to tie to changing climate parameters, unless depositional chronologies are exceptionally-well constrained (e.g. D'Arcy et al., 2017) . Conversely catchments whose erosional dynamics lie close to the stream power end-member model, may be well-placed to record longer-term climate shifts, but may be buffered to very high frequency variations in the climate driver (c.f. Simpson and Castelltort, 2012; Armitage et al., 2013) . It is important to stress that the trend in response is asymmetric, by which we mean that both models show a faster response for a precipitation increase relative to a precipitation  
25 decrease ~~-(Figure 10)~~. This is an important outcome, which has to-date not been widely recognised or investigate in field scenarios. In particular, it raises the prospect that for glacial-interglacial cycles, where characterised by wetter, cooler stadial periods and dryer, warmer interstadials, the rapid climate recovery from peak glacial conditions typically seen in  $\delta^{18}\text{O}$  records might be mediated by a longer landscape response time to this change. Conversely, physically slower boundary condition changes towards wetter conditions may give rise to faster landscape response times. We suggest that an exploration of these  
30 differences may be a promising avenue of future research.

Given that the ~~the~~ response time is a function of the water flux exponent ( $m$  or  $\delta$ ), and that the water flux exponent for the ~~sediment~~ transport model is greater than that for the stream power model, there will be a cross over point where the stream power model responds faster than the transport model. This cross-over point is a function of the erodability coefficient  $k$  and the transport coefficient  $c$ . In the scenario where we have tried to initiate the perturbation in precipitation rates from similar

catchments, we find that this cross-over point is towards large reductions in precipitation rates (Figure 10). This implies that the ~~sediment~~-transport model generally responds faster than the stream power model ~~, when the parameters produce similar~~ (10<sup>5</sup> to 10<sup>6</sup> years), for examples in which the parameter combinations used here produce similar steady-state landscapes.

~~The~~ For such conditions, the stream power model predicts a landscape response time to a change in precipitation of the order of 10<sup>6</sup> yr, and this time is related to the precipitation rate to the inverse power of  $m$  (Figure 10). ~~Furthermore this time scale is not sensitive to catchment length (Table 2). The sediment~~ The transport model predicts a wider range of response times of order 10<sup>6</sup> to 10<sup>5</sup> yr that is related to the precipitation rate to the inverse power of  $\delta$ , also in this case the response time is length dependent (Figure 10 and Table 2). It has been suggested that a transition from a landscape controlled by ~~detachment-limited erosion~~ detachment limited erosion (stream power model) to sediment transport at longer system lengths may explain the longevity of mountain ranges (Baldwin et al., 2003). This hypothesis is somewhat backed up by the analysis of response times for the ~~sediment~~-transport model, as the response time increases with system length (Table 2) unlike the stream power model, which has a response that is only slightly modified by system length (Whipple, 2001; Baldwin et al., 2003). To-date physical constraints on landscape and sediment flux response times to climate changes in the geologic past are relatively scarce (Ganti et al., 2014; Romans et al., 2016; Temme et al., 2017), because real systems are complex, they include internal dynamics such as vegetation and autogenic behaviour which are often omitted from model studies, and because of the need of stratigraphic archives to be complete with well-established chronologies (Allen et al., 2013; Forman and Straub, 2017). In principle, however, the dominant long-term incision process governing catchment behaviour fundamentally determines the sediment flux response and may itself help identify catchment erosional dynamics; we explore this question in section 4.2.

Finally, it is worth noting that the model response time has implications for the inverse modeling of river profiles. When river long profiles are inverted for uplift, erosion is typically assumed to be captured by the stream power model (e.g. Pritchard et al., 2009). Studies of continent scale inversion have found that the best fit value of  $k$ , for the stream power model, increases by two orders of magnitude to fit river profiles in Africa relative to Australia (Rudge et al., 2015). Such a large change in  $k$  would result in a highly significant difference in response time from continent to continent, which in itself would imply that tectonic and climatic signals are preserved in landscapes and sediment archives over vastly different time periods (c.f. Demoulin et al., 2017). Such an outcome may reflect fundamentally differences in bedrock erodibility (Roy et al., 2015), but alternatively could be satisfactorily explained by differing long-term erosional dynamics and sediment transport. These differences are enhanced in the case where  $n > 1$  in the stream power erosion model.

## 4.2 Relevance of model responses to sediment records of climate change

To what extent do these model results, which start from similar steady-state topographies, help us to understand ~~how whether~~ stratigraphic records of sediment accumulation through time do, or do not, reflect the effects of climatic change on sediment routing systems governed by differing long-term erosional dynamics?

One motivation for this study ~~came from a~~ has come from the increasing number of field and stratigraphic investigations of terrestrial sedimentary deposits ~~contemporaneous with~~, apparently contemporaneous with (and taken to record) known past climate perturbations, such as the Palaeocene Eocene thermal maximum (PETM), a hyperthermal event that occurred around

56 Ma. Stratigraphers often correlate changing stratigraphic characteristics to changing environmental boundary conditions in a qualitative way (c.f. Romans et al., 2016; Allen, 2017) . However, to evaluate quantitatively how sediment routing systems respond to climate with reference to real examples, it is imperative to consider systems in which the timescales of erosion (or as a proxy, deposition) are known, stratigraphic sections are complete, and the driving mechanisms well-documented (c.f. Allen et al., 2013; D'Arcy et al., 2017) .

The PETM is arguably the most rapid global warming event of the Cenozoic, with a rise in global surface temperatures by 5 to 9°C (Dunkley Jones et al., 2010). forming a clear step-change in climate for which depositional records are well-constrained in a number of basins world-wide (Foreman et al., 2012) . It is therefore a good example for high-level comparison with our model outputs. While the large magnitude glacial-interglacial cycles of the past 1 Myrs are also plausible candidates to investigate these links in principle (c.f. Armitage et al., 2013) , we note that many terrestrial records of sedimentation over ca. 100 kyrs, such as fluvial terraces and alluvial fans, have depositional chronologies that are often incomplete or reworked (D'Arcy et al., 2017; Demoulin et al., 2017) .

The initial warming associated with ~~this event~~ the PETM event occurred at ca. 55.5 Ma, and may have been as abrupt as 20 kyr, with a duration of 100 to 200 kyr based on synthesis of  $\delta^{13}\text{C}$  and  ~~$\delta^{18}\text{O}$  records (e.g. Foreman et al., 2012)~~  $\delta^{18}\text{O}$  records (e.g. Schmitz and Pujalte, 2007; Foreman et al., 2012) . The event has been associated with clear changes to global weather patterns, for instance hydrogen isotope ~~records~~ records suggest increased moisture delivery towards the poles at the onset of the PETM, consistent with predictions of storm track migrations during global warming (Sluijs et al., 411). This event has also been argued by ~~a~~ an increasing number of authors to have produced a significant geomorphic and erosional impact based on sedimentary evidence, and its apparent effect on the global hydrological cycle and catchment run-off (e.g. Foreman et al., 2012; Foreman, 2014).

A clear response to the PETM is recorded within both the Spanish Pyrenees and the western U.S. in North America; however the responses are arguably not the same. At the onset of the PETM there is strong evidence for the contemporaneous increase in precipitation rates and the deposition of coarse gravels ~~, ranging from known as~~ the Claret Conglomerate (Schmitz and Pujalte, 2007), in the Tresp basin of the Spanish Pyrenees ~~to the~~. In the western U.S. the PETM is marked by the deposition of well-documented channel sandstone bodies in the Piceance Creek and Bighorn Basins , western USA (Foreman et al., 2012; Foreman, 2014, e.g.). In the US cases, the deposits include coarse channelized sands, marked by upper flow regime bed forms, some of which are consistent with a synchronous increase in both water and sediment discharge. At Claret, where the style of sedimentation abruptly changes from a semi-arid alluvial plain to an extensive braid plain or megafan deposit, the conglomerate has a thickness of ~ 10m while the total carbon isotope excursion (CIE) in the same section measures ~ 35 m (Manners et al., 2013).

#### 4.2.1 Claret Conglomerate, Spanish Pyrenees

The Claret Conglomerate was likely deposited rapidly, representing a fast response to climate change. If we assume a constant rate of deposition, then the ~~conglomerate~~ Claret Conglomerate accounts for roughly 30 % of the total duration of deposition for the CIE (~~90 to~~ 170 kyr; Röhl et al., 2007), suggesting deposition occurred over a duration of ~~25 up~~ to 50 kyrs.

If we however drop the simple assumption of constant accumulation, and assume that the sediment accumulated at a rate of e.a.  $5 \times 10^{-4} \text{ m yr}^{-1}$ , in line with lower accumulation rates from paleosols within the Bighorn Basin during the PETM (e.g. Bowen et al., 2001), then the duration for the deposition of paleosols at Claret is  $\sim 70 \text{ kyr}$ . This leaves a duration of conglomeratic deposition at between 25 and 70 kyr. Indeed, Schmitz and Pujalte (2007) argue that the deposition of this unit may have been markedly quicker than the conservative estimate above, perhaps taking less than 10 kyrs, based on their detailed comparison of  $\delta^{13}\text{C}$  and  $\delta^{18}\text{O}$  records at the field site, compared to marine records of the excursion. Therefore unless there is a major unconformity within the CIE, the implication is that the erosional system responded rapidly, in less than 100 at this particular field site, in 10 to 50 kyr to a significant shift in climatic conditions. These values suggest sedimentation rates of up to  $1 \text{ mm yr}^{-1}$ , and thus elevated sediment fluxes, which if they had been sustained for the duration of the deposition of the Tremp Group (Maastrichtian – end Palaeocene) would have produced  $> 15 \text{ km}$  of sediment thickness, an order of magnitude more than actually observed (Cuevas, 1992).

Erosional source catchment areas were likely  $< 100 \text{ km}$  in length at the time, given the palaeo-geography of the Pyrenees at the time (Manners et al., 2013). The very short duration of the depositional response, coupled with the extensive deposition of gravel clasts within a mega-fan setting in which sediment was clearly abundant is erosional response, which is required for the sediments to be transported and deposited in a timescale of ca.  $10^4$  years is therefore difficult to model or explain within an advective within a stream power (advective) end-member model (e.g. for catchments of this scale (Table 2)). In contrast, the, although a version of such a model has been recently used to explore the controls on the evolution of later Miocene megafans in the northern Pyrenees (e.g. Mouchen  et al., 2017). To use the stream power model would require a significant increase the bedrock erodibility parameter,  $k$ , within the model (by greater than one order of magnitude), implying slopes and topography in the palaeo-Pyrenees that were highly subdued. In contrast, the sediment transport model of sediment transport more easily reproduces the documented response timescales given an increase in precipitation, is consistent with the volumetrically significant export of bedload transported gravel clasts, and therefore honors the independent field data more effectively. We also note that the transport model displays a response time that has a stronger dependence on precipitation rate change, and has a greater amplitude of perturbation (e.g. Figure 9). We therefore suggest the erosional pulse that led to the deposition of the Claret conglomerate is most appropriately modelled as a diffusive system response to a sharp increase in precipitation over the source catchments of the developing Pyrenean mountain chain at that time.

#### 4.2.2 Sandstone Bodies in the Piceance Creek and Bighorn Basins, Western U.S.

The time-equivalent sections in the Bighorn and Piceance Creek basins of the western US (Foreman et al., 2012; Foreman, 2014) U.S., also provide clear evidence of anomalous sedimentation at the PETM, however in this case with a somewhat longer duration of  $> 200 \text{ kyrs}$ . However, the basin responses, including the deposition of coarse sand bodies, are clearly longer than the CIE associated with the PETM event, suggesting a more complex relationship between increased precipitation and the export of volumetrically significant coarse sediments. Our approach evidently does not reproduce complex dynamics such as vegetation turn-over and sediment reworking that may influence large-scale stratigraphic responses and we note the relative difficulty of reliably reconstructing climatic and hydrological variables from geologic data in the past makes it challenging to compare field

outputs directly and unambiguously with models that illustrate end-member responses. Furthermore, it is possible that the system response is sensitive to the initial condition assumed for the landscape (e.g. Perron and Royden, 2012). the duration of deposition is somewhat longer, > 100kyrs (Foreman et al., 2012; Foreman, 2014). Here the deposits are of smaller grain sizes, with the Boundary Sandstone sequence in the Bighorn basin being made up of fine to coarse sand with little gravel (Foreman, 2014). In the Piceance Creek basin, the PETM section documents the rapid progradation of coarse-grained sands, consistent with greater discharges, over silty underlying strata and in that sense these observations also match sediment transport model predictions for rapid increases in sediment flux, driven by enhanced precipitation (Foreman et al., 2012). However, it is notable the documented changes in fluvial style persisted beyond the PETM and we therefore suggest that fast response of system to the increase in precipitation, but the persistence of coarser grain sedimentation as the climate presumably dried and cooled may indeed reflect the marked asymmetry in sediment flux responses to wetting and drying noted in Figure 10.

For In contrast, the Bighorn basin boundary sandstone sediments are contained within the PETM time period, and indicate uniform flow depths and widths during this time, while also being coarser than the underlying horizons. Moreover, proxy data suggest a net decrease rather than increase in precipitation (Foreman et al., 2012; Foreman, 2014). As the authors note, while progradation of such coarse-grained facies could be represented as a diffusive process driven by increasing rainfall-driven discharge (Paola et al., 1992; Armitage et al., 2011), this is apparently inconsistent with the sedimentological characteristics of the deposit. Although sediment fluxes are not explicitly reconstructed in this work, this response apparently requires greater volumes and grain sizes of sediment delivered despite lowered rainfall conditions, and is thus difficult to capture in either of the end-member models used here. The authors argue for the preferential removal of fine-grained flood plain deposits, speculatively linked to changing vegetation and the reduced cohesion of overbank sediments. Consequently, while two of the PETM sections considered here are broadly consistent with landscape responses governed by a sediment transport model, some depositional stratigraphies are not immediately consistent with either end-member model, and may reflect important complexity such as the effects of vegetation lacking from simple model solutions.

### 4.3 Summary and model limitations

In this discussion section we have considered the implications of our model outputs, both generally for interpreting sediment routing system response to boundary condition change and specifically in the context of the well-studied PETM event. While the sediment flux response of the models to a change in precipitation are at a first order level broadly similar, there are four key differences to highlight. First, starting from the same initial conditions, the sediment transport model ~~it has been previously demonstrated that an increase in the transport coefficient~~ appears to be more sensitive to precipitation change than the equivalent stream power model. It therefore is a good candidate where rapid catchment-wide responses are recorded to e.g. a climate change event, as we argued for the PETM Claret conglomerate in the Spanish Pyrenees. Second we note that in both model cases there is a quicker response to wetting than drying event, something which has not been well-established or demonstrated from field observations. Nevertheless we argue that field data sets, including PETM studies may already have recorded this asymmetry, although it may not have been recognised as such. Third, the sediment transport model has greater magnitude

of peak sediment flux and is particularly sensitive to catchment size. Finally we note that response time in both models is a function of uplift rate for  $n > 1$ , which means in such cases, perhaps counter-intuitively, that the more perturbed the system the faster it responds (c.f. Whittaker and Boulton, 2012).

However, it is important to recognise that in deriving these two classic end-member models we have simplified landscape evolution considerably. We acknowledge that change in the model parameters,  $c$  will reduce the model response time predictably (Armitage et al., 2013). The erodability parameter,  $k$  has a similar effect for the stream power model. In this paper we have attempted to find the,  $m$  and  $\delta$ , will alter the response times depicted here (c.f. Armitage et al., 2013). However, in order to compare the two models we have specifically used values of  $c$ ,  $k$ ,  $m$  and  $\delta$  that generate comparable model landscapes, and we then changed the precipitation rate to understand the form of the model response. Above we discussed the geological relevance of the models with such a choice of  $c$ ,  $k$ ,  $m$  and  $\delta$ , yet of course both models could have the values of their transport coefficient or erodability adjusted further to tune them to the observations. Furthermore, while inverse models suggest that the slope exponent  $n \sim 1$  (e.g. Rudge et al., 2015), there is room for argument over the non-linear nature of the fundamental equations (see Harel et al., 2016). Therein lies the rub of these heuristic models, and the potential difficulties. No model incorporates all the complexities that characterize sediment routing systems from source to sink (c.f. Allen, 2017) and the act of simplification inherent in considering erosional end-member models necessitates that in arguing for the applicability of one over the other. However, we note that the transport model does display a response time that has a stronger dependence on precipitation rate change. Furthermore, in the Spanish Pyrenees the PETM shift in climatic conditions is recorded as a short duration deposition of gravels down-system. The transport of such large clasts is most likely in the form of bed-load movement, which conceptually is more easily described by transport equations rather than the instantaneous stream power model, we simply consider the broad styles of behavior suggested by model outputs. We do not, for example, consider autogenic behaviours (e.g. Forman and Straub, 2017), nor do we consider coupled issues of vegetation turn-over in response to climate change, which may play an important role in examples such as the Bighorn basin considered here (c.f. Foreman, 2014). Nonetheless, a significant finding of this work has been the clear asymmetry in response time of these end-member models in terms of a wetting event (faster) compared to a drying event (slower). This implies that aridification events are harder to preserve in the sedimentary record, not only because they are typically associated with reduced sediment fluxes, but also because the timescale of landscape response may be  $> 10^6$  years.

## 5 Conclusions

Deterministic numerical models of landscape evolution rest on fundamental assumptions on how sediment is transported down system. The kinematic wave equation that is the stream power law is based on the assumption that all sediment generated is transported instantaneously out of the landscape. Sediment transport models assume that there is an endless supply of sediment to be transported. The existence of knickpoints within river long profiles, assumed to be produced by a system perturbation such as a base level, has been used to provide evidence in support of the stream power law in upland areas (e.g. Whipple and Tucker, 1999; Snyder et al., 2000; Whittaker et al., 2008). Knickpoints however can likewise be a result of

changes in lithology (Grimaud et al., 2014; Roy et al., 2015) and are certainly not a unique indicator of erosion dynamics (e.g. Tucker and Whipple, 2002; Valla et al., 2010; Grimaud et al., 2016). In this contribution we therefore attempted to understand of the sediment flux signal out of the eroding catchment may generate a distinguishable difference between the end-member models in term of a response to a change in run-off. This idea is motivated from field observations of past landscape responses to climate excursions, such as the PETM, which are manifested in the rapid deposition of coarse sedimentary packages in terrestrial depocentres (Armitage et al., 2011; Foreman et al., 2012).

Both models suggest that the response time of landscape to change in precipitation rate has a proportionality of the form of a negative power law (equations 15 and 16). The key difference is in the value of the exponent. For the stream power model, the exponent must be less than one in order to match the observed concavity of river profiles. In contrast, for the transport model the exponent on the precipitation rate must be greater than one in order to generate a river network (Smith and Bretherton, 1972), and to generate the observed concavity of river profiles. This results in the ~~sediment~~-transport model responding more rapidly to an increase in precipitation rate in comparison to the stream power law model (Figure 10). ~~Additionally~~In contrast, the stream power model is faster to respond to a reduction in rainfall rate. This is fundamentally because the response time of this model is more weakly a function of precipitation than the sediment transport model. Significantly, therefore, our results show that there is a fundamental asymmetry in the response of ~~the transport limited both~~ models to a climatic perturbation, with the response time to a drying event longer than that to an increase in rainfall. In general terms, the magnitude of the response to a change in precipitation rate appears greater across the range of model space investigated here for the sediment transport (diffusive) model solutions, while for the stream power (advective) model, the magnitude of the sediment flux perturbation is smaller, but is more localised within the catchment with respect to knickpoint retreat.

While ~~our models obviously do~~ this study does not address whether or not these sediment flux signals will be preserved in the stratigraphic record, a problem that fundamentally rests on the availability of accommodation to capture the eroded sediment (~~e.f. Allen et al., 2013~~) (c.f. Allen et al., 2013; Whittaker et al., 2011), it does suggest that landscapes governed by these simple erosional end-members should be sensitive to climate change; and moreover that there are some important diagnostic differences between their sediment flux responses to an identical perturbation, including the amplitude, timescale and locus of the erosional response. Using published stratigraphic examples, we suggest that the timescales and magnitude of coarse sediment deposition in the Spanish Pyrenees at the time of the PETM are best described using the ~~transport limited diffusive transport model~~ end-member, ~~while other examples are more equivocal. Consequently. Moreover,~~ we argue that these model end-members allow us to constrain the range of likely sediment flux scenarios that precipitation changes may generate, and that numerical models, in conjunction with a range of field and independently-constrained proxy data sets are best placed to tease apart when and in what circumstances climate signals are likely to have been generated ~~and preserved in sedimentary systems~~ in erosional catchment systems, which fundamentally determines whether they can be subsequently captured in sedimentary depocentres downstream.

## 6 Code availability

The ~~codes developed here are~~ 1-D solution to the transport model is available from John Armitage (armitage@ipgp.fr). The 1-D solution to the stream power model is available from Benjamin Campforts (benjamin.campforts@kuleuven.be). Fastscape is available from Jean Braun (GFZ Potsdam) by request. The 2-D solution to the transport model was developed by Guy Simpson  
 5 (University of Geneva) and is available as part of Simpson (2017) .

## Appendix A: Steady state 1-D profiles

The solution to the one dimensional stream power law (Equation 12) assuming that at the end of the catchment at  $x = L$  elevation  $z = 0$  and  $mp \neq 1$  is,

$$z_{sss} = \frac{U}{mk\alpha^m(mp-1)} \left( x^{(1-mp)} - Lx^{(1-mp)} \right) \quad (A1)$$

10 and for the case where  $mp = 1$  this simplifies to,

$$z_{sss} = \frac{U}{k\alpha^m} \log_e(L/x) \quad (A2)$$

For the ~~sediment~~-transport model (Equation 8) there is an exact solution for the case that  $\delta p = 2$ , which assuming at  $x = 0$ ,  $\partial_x z = 0$  and at  $x = L$ ,  $z = 0$  is,

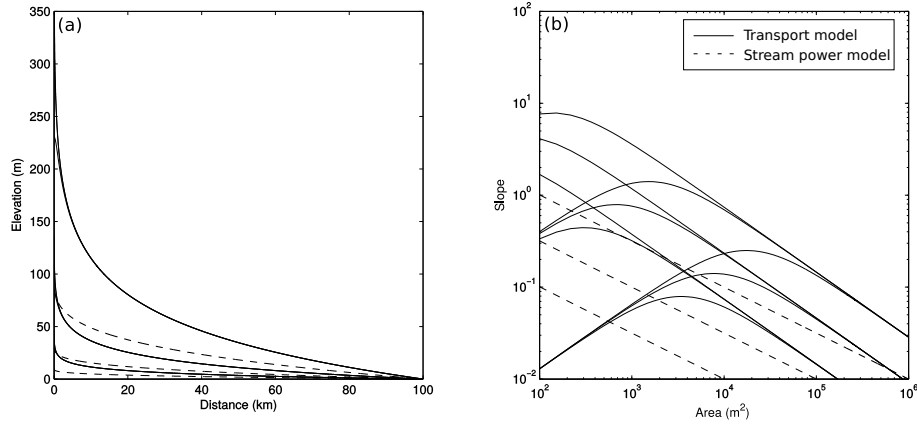
$$z_{sst} = -\frac{UL}{2\kappa D_e} \left( \log(D_e x^2 + 1) + \log(D_e + 1) \right) \quad (A3)$$

15 where,

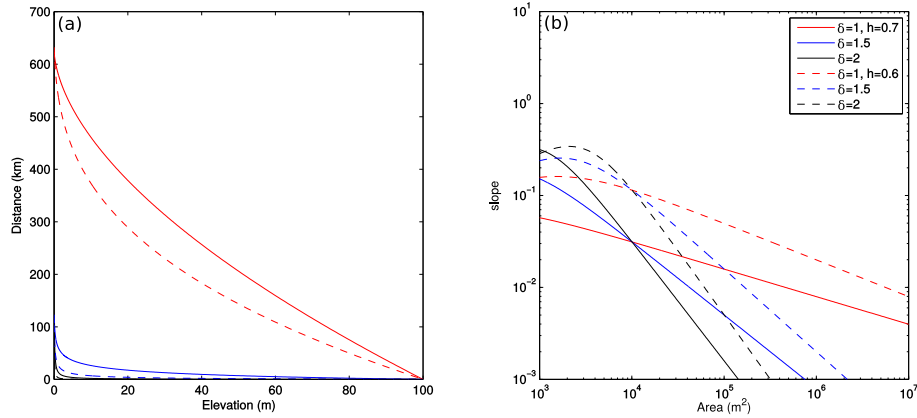
$$D_e = \frac{ck_w\alpha^{2/p}L^2}{\kappa}. \quad (A4)$$

For other values of  $\delta p$  the steady state solution is solved for numerically, where Equation 8 is solved using the finite element method with linear weight functions. We use a non-uniform 1-D nodal spacing, where the spatial resolution is increased with increasing gradient. The numerical model is bench-marked against the analytically solution for the case where  $np = 2$ .

20 The steady state solutions are plotted in the case that  $\delta = p = \sqrt{2}$  and for reference the stream power model solution with  $m = 0.5$  and  $p = \sqrt{2}$  (Figure 15). Such a value of  $p$  assumes that  $h \sim 0.7$ , which is towards the higher end for observed Hack exponents and that the river catchment is very elongate. When plotting the logarithm of the model slope against drainage area (Figure 15b), where area is given by  $a = x^p/k_w$  and assuming  $k_w = 1$ , for the simple stream power law derived here the gradient  ~~$\beta = -m$~~   $\theta = -m$ . The value of the dimensional constant  $k$  has no impact on the gradient as expected. The ~~sediment~~  
 25 transport model likewise creates river long profiles that have on average a negative curvature. For small values of  $x$  there is however a region of positive curvature where  $\kappa > ck_w\alpha^\delta L^{\delta p}$ . For the slope area analysis this leads in there being a positive gradient in the trend for small catchment areas. This relationship subsequently has a negative slope for larger catchments. The point of inflection is dependent on the value of  $D_e$ , where for smaller values of  $\kappa$  the region of positive gradient is reduced.



**Figure 15.** (a) Steady state profiles of elevation against down-system length, and (b) the slope of the profile plotted against the drainage area assuming that area  $a = x^p$  where  $p = 1/h = \sqrt{2}$  and  $h$  is the Hack exponent. Solid-Dashed lines are for the stream power law (Equation 12) with  $m = 0.5$ , and  $k = 10^{-4}$ ,  $10^{-3.5}$ , and  $10^{-3}$ . The dashed-solid lines are for the sediment-transport model (Equation 8 with  $n = \sqrt{2}$ ,  $\kappa = 10^{-3}$ , 1 and  $10^3 \text{ m}^2 \text{ yr}^{-1}$ , and  $c = 10^{-6}$ ,  $10^{-5.5}$ , and  $10^{-5}$ ).



**Figure 16.** (a) Steady state elevation and (b) slope-area relationship for the numerical solution to 1-D sediment transport (Equation 8), where the area,  $a$ , is taken to be related to distance  $x$  by  $a = x^p$  where  $p = 1/h$  and  $h$  is the Hack exponent. Red lines are for the case where  $n = 1$ , blue lines for  $n = 1.5$  and black lines for  $n = 2$ . Solid lines are for  $h = 0.7$ . Dashed lines are for  $h = 0.6$ .

There is therefore a critical catchment area that is dependent on the diffusive term  $\kappa$ . After this critical point the slope area relationship becomes negative. At distances down-system, where the upstream area is greater than this critical area, the gradient  $\beta = -0.88$ .  $\beta$  is insensitive to the coefficient  $c$  as would be expected.

The range of gradients found for river catchments for this type of slope area analysis, usually referred to as concavity, generally lies within the range  $\beta = -0.35$  to  $-0.70$  (Snyder et al., 2000) (Snyder et al., 2000; Wobus et al., 2006). It is trivial to find the values of  $m$  for the steady state solution to the stream power law such that fits such values of  $\beta$ . To

**Table 3.** Gradient,  $\beta\theta$ , of the slope vs. area trend at steady state for 1-D sediment transport (Equation 8, Figure 16).

$\delta$	$p = 1.40$		$p = 1.67$		$p = 2.00$	
	$c$	$\beta\theta$	$c$	$\beta\theta$	$c$	$\beta\theta$
1	$10^{-6}$	-0.50	$10^{-5}$	-0.40	$10^{-4}$	-0.30
1.5	$10^{-8}$	-1.01	$10^{-7}$	-0.91	$10^{-6}$	-0.81
2	$10^{-10}$	-1.51	$10^{-9}$	-1.41	$10^{-8}$	-1.31

further explore how  $\beta\theta$  depends on  $\delta$  and  $p$  within the ~~sediment~~-transport model we solve Equation 8 numerically for  $\delta = 1, 1.5$  and  $2$  while keeping  $h = 0.7$  or  $0.6$  (Figure 16). The result is that  $\beta\theta$  varies from  $-0.3$  for the case of  $\delta = 1$  to  $-1.31$  for  $\delta = 2$ . The values of the gradient for the slope area analysis for  $1.4 < p < 2$ , where we are assuming  $d = 1$  and hence  $p = 1/h$ , are displayed in Table ~~??3~~. For the ~~sediment~~-transport model the slope is dependent on both  $\delta$  and  $p$ .

5     Clearly there exists a combination of  $\delta p$  that is equally capable of fitting the observed river long profile. Furthermore, for the ~~sediment~~-transport model the slope is a function of the Hack exponent  $h$  (and therefore  $p$ ) and the choice of  $\delta$ . This because of the diffusivity term that leads to positive curvature and rounded 1-D profiles (Figure 15b and 16). The magnitude of the water flux term within the transport equation (Equation 8) is dependent on how much water the river network captures, which is in turn a function of how elongated the catchment is. ~~For this reason, the slope-area relationship is sensitive to the river network geometry for the diffusive case, while for the simple stream power relationship it is not.~~

10     The positive slope area relationship for the transport model for small catchment ~~area~~areas, see Figure 15b and 16b, has been previously explored in Willgoose et al. (1991). ~~In the study of Willgoose et al. (1991) the governing equations were however of a significantly greater complexity. For realistic catchment areas, for the sediment transport model the~~The gradient of the ~~slope area analysis (concavity)~~relationship between slope and catchment area is dominantly a function of the exponent  $\delta$   
15     within Equation 6. The value of this exponent ~~has been assumed to be~~is likely within the range of  $1 < \delta < 2$  depending on the bed-load transport law assumed (Armitage et al., 2013). If the observations of trunk river slope against catchment area are representative of a landscape at steady state, then for the smaller range of  $1 < \delta \leq 1.5$ , a realistic catchment topography can be generated.

**Appendix B: Idealized linear 2-D models**

20     **A1 Erosion by Sediment Transport**

~~We explore how an idealized landscape evolves under uniform uplift at a rate of  $0.1 \text{ mm yr}^{-1}$  for the case in which erosion is dependent on sediment transport. The initial condition is of a flat surface with a small amount of noise added to create a roughness. The boundary conditions are of fixed elevation at the left and right sides, and of no flow at the sides. The sediment flux output is displayed in Figure 2. Six models have been run without a change in precipitation to find the steady~~

state topography. The models explored are first a set of three with varying  $\delta$  and constant  $c$ , i.e.;  $\delta = 1.1, 1.3$  and  $1.5$  with  $c = 10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$  (Figure 2a), and a set of three where  $\delta$  and  $c$  co-vary, i.e.;  $\delta = 1.1$  with  $c = 10^{-2} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$ ,  $\delta = 1.3$  with  $c = 10^{-3} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$ , and  $\delta = 1.5$  with  $c = 10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$  (Figure 2b).

Sediment flux out of the model domain for the sediment transport model for models where (a)  $\delta = 1.1, 1.3$  and  $1.5$ ,  $\kappa = 10^{-2}$  and  $c = 10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$ , and (b)  $\delta = 1.1, 1.3$  and  $1.5$ ,  $\kappa = 10^{-2}$  and  $c = 10^{-2}, 10^{-3}$ , and  $10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$ .

When the transport coefficient  $c$  is the same for the three values of the exponent  $\delta$  the model wind-up time increases with decreasing  $\delta$ , and takes several million years where  $\delta < 1.5$  (Figure 2a). Steady state sediment flux is greater for increasing  $\delta$  when  $c$  is kept constant. The dimensions (units) of  $c$  depend on  $\delta$  which means that the value of the coefficient  $c$  must be adjusted when  $\delta$  is changed to yield the same unit erosion rate per water flux, regardless of  $\delta$  (see Armitage et al., 2013). Consequently, when  $c$  is suitably adjusted the model can reach a steady state in a similar time for all three values of  $\delta$  (Figure 2b).

(a) Steady state topography, after 10Myr, for the sediment transport model where  $\delta = 1.5$  and  $c = 10^{-4} (\text{m}^2 \text{yr}^{-1})^{1-\delta}$ . (b) Slope area relationship for sediment transport model for  $\delta = 1.3$  and  $\delta = 1.5$ .

We subsequently analyze the topography for the relationship between trunk river slope and drainage area, Figure 3, using Topotoolbox2 (Schwanghart and Scherler, 2014). For the case where  $\delta = 1.5$  the scaling between channel slopes and catchment drainage areas, slope area gradient,  $\beta$  is equal to  $-0.42$ , and for  $\delta = 1.3$ ,  $\beta$  is equal to  $-0.23$  (Figure 3b). The same value is calculated using the spatial transformation described within (Perron and Royden, 2012), commonly referred to as  $\chi$ -plots (Table 1). Given the reduction in  $\beta$  from  $\delta = 1.5$  to  $1.3$ , we did not analyze the case for  $\delta = 1.1$  as the slope area relationship will clearly lie below the observed range ( $0.3 < \beta < 0.7$ ; e.g. Whipple and Tucker, 2002; Tucker and Whipple, 2002). Therefore, for river networks defined by routing water down the steepest slope of descent, the sediment transport model can create catchment morphologies that have a concavity similar to that observed in nature if  $\delta \sim 1.5$ .

Slope area relationship for trunk streams sediment transport  $k_S$   $\beta$   $\delta = 1.3$   $0.86$   $-0.23$   $\delta = 1.5$   $1.76$   $-0.42$  stream power  $m = 0.3$   $0.95$   $-0.29$   $m = 0.5$   $6.52$   $-0.46$   $m = 0.7$   $71.42$   $-0.68$

## A1 Comparison to Erosion by Stream Power

Sediment flux out of the model domain for the sediment transport model for models where (a)  $m = 0.3, 0.5$  and  $0.7$ , and  $k = 10^{-5} (\text{m}^2 \text{yr}^{-1})^{1-m}$ , and (b)  $m = 0.3, 0.5$  and  $0.7$ , and  $k = 10^{-4}, 10^{-5}$ , and  $10^{-6} \text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ .

In order to provide a comparison for the morphology of the sediment transport model we return to the widely used stream power model. We explore how the stream power model evolves to a steady state for a range for the coefficient  $k$  and the exponent  $m$ .

The landscape derived from the stream power model, equation 11, evolves towards a steady state with a slightly different behaviour in comparison to the sediment transport model (Figure 4). As before we run six models where in this case the first set of three are  $m = 0.3, 0.5$  and  $0.7$  with  $k = 10^{-5} \text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$  (Figure 4a). The second set of three are of  $m = 0.3$  with  $k = 10^{-4} \text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ ,  $m = 0.5$  with  $k = 10^{-5} \text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$ , and  $m = 0.7$  with  $k = 10^{-6} \text{m}^{-1} (\text{m}^2 \text{yr}^{-1})^{1-m}$  (Figure 4b). This range of  $m$  is chosen as it spans the range of observed concavities within catchments. As with the sediment

transport model the coefficient  $k$  can be adjusted along with  $m$  as they are related, where increasing  $k$  reduces the model wind-up time (Figure 4). Decreasing the exponent  $m$  increases the timescale taken to reach a steady state (Figure 4)a, however by varying  $k$  by a factor of 100 steady state the sediment flux is reached within 3Myrs for the three values of  $m$  (Figure 4)b.

(a) Steady state topography, after 10Myr, for the sediment transport model where  $m = 0.5$  and  $k = 10^{-5} \text{ m}^{-1} (\text{m}^2 \text{ yr}^{-1})^{1-m}$ .

5 (b) Slope-area relationship for sediment transport model for  $m = 0.3, 0.5$  and  $0.7$ .

Following the previous examples, we analyze the topography for the relationship between trunk river slope and drainage area, Figure 5. Both the sediment transport model and the stream power model can create landscapes with similar slope-area relationships (Table 1). For both models, the value of the intercept  $k_s$  and the gradient  $\beta$  are of similar magnitudes. Absolute elevation for the model shown in Figure 5a is higher than the transport limited example due to the larger value of  $k$  relative to  $e$ . However, importantly, both models can create similar landscape morphologies at steady state.

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