

## Wickert

We thank the reviewer for his comments and suggestions. In an effort to diffuse debate around the precise characterization of gravel-bedded rivers as purely threshold channels, we have rephrased our conclusion to read, "We propose that all alluvial rivers, regardless of their bed material grain size, organize their hydraulic geometry such that they cluster around the threshold of motion for the most resistant material — the structural component of the channel that is most difficult to mobilize." We hope that by using the word "cluster" instead of "are", we do not come across as in disagreement with previous papers that have investigated order 1 deviations from the threshold of motion in gravel bedded rivers. We have included a sentence in the introduction that defines near threshold as an order 1 multiple of the threshold of motion and far-above threshold as an order 10-100 multiplier of the threshold of motion. We have furthermore included a sentence in the introduction that acknowledges previous work that has been done on order 1 deviation from the threshold of motion.

Use `citep` instead of `citet` with extra parentheses. Please do this. Or maybe you were just using `cite`? Whichever way it is, please just use BibTeX properly. Thank you for the catch. Sorry about this persistent error

...The former would require just a sentence noting that they are not looking at environments of rapid uplift, and therefore are not evaluating the explanation put forward by Pfeiffer and co-authors. The latter would require a more extensive explanation. In my opinion, it is better to look at the publication of the two papers by Phillips and Jerolmack (2016) and Pfeiffer et al. (2017) as a challenge of how to look deeper into the data and validate whether or not the discrepancy exists, how to improve our measurement abilities, how to critically evaluate our assumptions, and (if necessary) how to unify theory.

We thank the reviewer for his suggestion and have included the following sentence acknowledging the variability around the threshold channel clustering: "Several studies have presented evidence that sediment supply and bank vegetation may drive gravel channels further above threshold (Pfeiffer et al., 2017; Millar and Quick, 1998). Values for Shields stress in gravel-bed rivers reported for a wide range of environments, however, rarely exceed 2-3 times critical."

p.11,l.31-32: Would you like to discuss some of the reasons for the low frequency of channels with 1-10 mm grain size? In particular, do you think that this may have to do with the crystal size / granule break-down problem, or possibly be connected to the transition between cohesion-dominated banks and particle-weight dominated banks that makes these grains either difficult to move or whisked away in a larger-clast gravel-bed river? This is of course ignoring arguments for equal mobility...

The authors state that this point is not important to this paper. However, there remains a fundamental question: does the bimodal transport state exist as the result of a bimodal input, a bimodal filter within the system, or both? The authors seem to argue for the middle answer, but do not mention that it may instead be, for example, the result of a bimodal input to which the internal response of the system that the discuss herein is more a response and less a driver.

We thank the reviewer for reminding us of this previous criticism. There have been numerous papers that have been published on this subject and we don't believe, given the data and results presented in this paper, that we have much to add on this subject. We believe that we implicitly address this issue with figure 5 that compares the bimodality in shields stress to the bimodality in grain size.

In the authors' response to Reviewer 3 on the use of a constant Chezy friction factor, I have some open questions. First, how did they calculate the friction factor in the response? This is not stated. Second, friction will change with the presence of bedforms. The authors explicitly do not address form drag. This is OK by me since they state this clearly. However, I question at this point whether a constant Chezy friction factor selection (0.1) is to look for deviations from a particular relationship, or if it is, as the authors state, that they simply do not care about reducing scatter. Could you please clarify this?

We calculated the friction factor using  $u = Cf\sqrt{H_{bf}S}$ . This equation is a statement of how much of the body force acting on the fluid is translated into flow velocity. This is a consequence of form drag at all scales: grain, bed form, bar form, meanders, etc. In showing the friction factor plots in the previous review, we wanted to make the point that there is no strong trend with any other variable; therefore, changes in friction factor cannot explain the first-order hydraulic geometry scaling relationships. The only use of a constant friction factor is in the calculation of the threshold channel curve, which is only used as a reference line to examine systematic deviations from it. Chezy friction factor could explain some of the second-order variance in the data, but not the deviation for the trend lines for hydraulic geometry from the threshold prediction.

p. 1, line 15: (I did not know this during the first review round sorry to bring it up only in round 2) Lacey was actually the first to generate a power-law relationship for hydraulic geometry. A nice review is by Savenije (2003): "The width of a bankfull channel; Lacey's formula explained". Thank you for the correction. We have corrected the opening sentence to read, "Almost 100 years ago, Lacey (1930) proposed an empirical relationship relating the width of an alluvial river to its water discharge. Leopold and Maddock (1953) built upon this to derive hydraulic scaling relations for bankfull channel geometry of

alluvial rivers.”

p. 1, lines 20-21: I am not going to hold you to this because you are trying to generalize (as is appropriate here), but I will note that there is significant scatter in the power-law hydraulic geometry relationships, and would suggest that the scales of the scatter vs. the strengths of the relationships are appropriately acknowledged.

We thank the reviewer for his comment, and with all due respect to the reviewer, we believe that the graph clearly an approximately 1 order of magnitude of scatter over approximately 12 orders of magnitude range of data is a strong enough case to warrant the generalization that the hydraulic scaling trends are robust

p. 2, line 5: in a rectangular channel (again, I missed this the first time around, but is easy to note)

Noted, and has been included.

p. 8, lines 27-29: Phillips and Jerolmack put significant effort into measuring  $\tau_{*c}$  and compiling measurements of it along gravel-bed rivers in the mostly tectonically-inactive mountain west of the USA and in Puerto Rico. (I know they included a river in the Oregon Coast Range, but study the low-relief portion of the river.) Their study excludes data from rapidly-uplifting landscapes required to address the Pfeiffer et al. (2017) argument as well as some of the references brought forth by Reviewer 3 on non-threshold behavior. Therefore, the authors have described only part of the sum of the knowledge, and have put forward a statement about threshold behavior based only on this. The cautionary note here is that it is easy to say “most rivers do this” but hard to say that “all rivers do this”. The difference between “most” and “all” could hold some important information into the forcings and response. I appreciate that these may be beyond the scope of this paper, but I find it important to avoid such blanket statements and partial referencing of the literature on a particular problem that can result in a disjointed scientific literature.

With all due respect to the reviewer, this comment is most pertinent to Phillips and Jerolmack in which order 1 magnitude deviations from the threshold of motion discussed. This current paper is only about the separation of near-threshold (order 1 multiple of threshold) channels and far above threshold (order 10-100 multiple of threshold) channels. The results of this paper do not lean on Phillips and Jerolmack, nor are these order 1 deviations from threshold pertinent to the results presented in this paper. For clarification to readers, we have modified the following sentence to define what we mean by “near threshold” and “far-above threshold”: “We address these questions by re-analysis of existing data. We revisit the global data compilations of Li et al. and Trampusch et al. and argue that natural rivers appear to exhibit bi-modal transport states

corresponding to near threshold (order 1 multiplier of threshold) and far-above threshold (order 10-100 multiplier of threshold).”

p. 12, lines 1-2: The Kean and Smith reference does not support the statement about mud, vegetation, and erosion thresholds. (I missed this as well in round 1 because I had not yet read this paper, and therefore did not realize that it was mis-cited.)

Thank you for the catch and we apologize for this error. We have changed the sentence and have included more appropriate citations: ”We do not consider vegetation explicitly; however, we note that numerous studies have analyzed the effects of vegetation on erosion thresholds (Micheli and Kirchner, 2002; Abernethy and Rutherford, 2001).

p. 12, lines 32-33: Your paper is a really nice piece of work about the channel width transition between sand- and gravel-bed rivers. You do not address the question at all about all alluvial rivers being near-threshold (see above comments)

Sorry, we don’t quite understand the reviewer’s criticism. The data from Singer (2010) was used in this paper to illustrate the the bimodality that is present in the global datasets is also reflected in an individual river. The focus of this paper has always been the bimodality that exists across all alluvial rivers and an attempt to identify attractors of these two modes.

p. 14, line 3: Contrary to my last comment, this statement is still at odds with Millar and Quick (although their range of  $\tau_{*b} / \tau_{*c}$  is less than that reported by Pfeiffer et al.). I have gone back and checked this following Lamb et al. (2008), and my calculated  $\tau_{*b} / \tau_{*c}$  values range from 0.6 to 2.6.

We acknowledge the reviewer’s point, however we must point out that for every point noted to be above the threshold of motion, there is one below the threshold of motion. This is why in the paper we use the term ”cluster” to describe the threshold of motion as an attractor. We have included a sentence in the paper to define near-threshold as an order 1 multiplier of threshold and far above threshold to be an order 10-100 multiplier of threshold.

Conclusions (in general): it is possible that some material in the second half of your conclusions could go in your discussion.

We feel that the material in the conclusion is necessary because we want to present the threshold-limiting material hypothesis and acknowledge that there is still a lot of work to be done to validate or invalidate it. If the reviewer believes that the conclusion is too long, we can remove the sentence ”Consideration of the slope- or grain-size-dependence of the critical Shields stress shows that alluvial rivers are bi-modal in terms of transport stage and bed-material grain

size, and that these modes correspond generally (but not always) to bed-load gravel rivers and suspension sand rivers.” as this is covered in the Discussion section.

## Chartrand

We thank the reviewer for his comments and suggestions and we have made our best effort to address them.

General comment that it was difficult to quickly determine grain size trends in your Figures related to sandy vs. gravelly beds. Without changing your color mapping, I think it would help readers if you used one symbol type for sandy beds and a different symbol type for gravel beds. The suggestion stems from the fact that throughout your paper you use the words sandy and gravel, or derivatives of the words, and it sure would help in reviewing the Figures to have these sizes jump out

We thank the reviewer for his suggestion, however we believe that it would be better practice to treat the data as a continuum and allow patterns of distinction to emerge organically.

Figure 1 – I had a very hard time seeing the cross-stream profile sketched in plane with the top of the figure. It would be much easier to see the profile if you rotated it up, and sketched it above the image from bank to bank. In this configuration it would sit perpendicular to the top plane of the graphic and inspection for the reader would be simple.

The angled perspective was chosen to be able to showcase the lateral change in the stress profile.

Figure 2 – I spent about ten minutes trying to figure out which symbols represent sand size sediments and which represent gravel. I could guess... then when I reached Figure 3 I saw your helpful colorbar for grain size. I suspect this is accidentally omitted from Figure 2? Why begin the panels with W\*H; seems more natural to begin with W or H, then present W\*H. This links to comments below. Last, is the light blue line a measure of error for the seepage data? If so it is not indicated in the caption.

We thank the reviewer for this catch and apologize for the confusion. There was a mixup uploading the figures to the latex document. The colorbar has been added and a statement of the meaning of the cyan line.

Figure 3 – Second to last sentence in the caption is missing from other Figures. Either move this note to Figure 2 where it is first of relevance, or include wherever it is relevant.

Thank you for the suggestion. We have moved it to the caption of figure 2.

Figure 4 – Caption calls out one point’s color (cyan) but not the other (red). Keep it consistent.

Thank you for the catch, we have made the correction.

Figure 5 – Panel D: how about color coding the bars to help readers relate to your other plots? Or place a vertical lines in the plot at  $10^{-3}$  and  $10^{-2}$  to highlight your separation of sand beds vs. gravel beds? The suggestion is focused on making it easier for the readers to link information between plots. You may want to indicate units for the x-axis in the Figure caption.

Thank you for the suggestions. We have changed the labels on the x-axis to be more consistent with the other plots. We would prefer not to colorcode each bar as we think having a big colorful graph would detract from the goal of this figure which is to have a simple histogram that clearly shows a bimodal distribution to the reader, which might not be as apparent in the scatter graphs presented earlier.

Figure 6 – This Figure is quite small in the PDF; I am not sure what the published size may be, but as presented it was hard to see the details you discuss. It is a nice presentation of Singer’s (2010) data. Last sentence in your caption you use the phrase “far above threshold”. Can you quantify or characterize this more precisely (i.e. an order of magnitude...)? As stated it is broad brushed and detracts from your work.

Thank you for the suggestion. In response to both reviewers, we have now modified a sentence in introduction to explain that by near threshold, we mean an order 1 deviation from the threshold of motion, and by far-above threshold, we mean an order 10-100 deviation from the threshold of motion. Sorry, don’t know what happened regarding the size of the image. We have enlarged it.

As a suggestion, begin Section 3 with “ Figure 2 shows...”. Then work through the results, weaving in Equation 7 where appropriate. The first sentence of the section is not needed.

With all due respect to the reviewer, we believe that our way better links the broader idea of regime theory and hydraulic geometry to the data.

Page 1 - Line 13: I read the sentence leading up to the ending a few times. I struggled with the last phrase. Tentative evidence seems tricky as a concept.

It is more straightforward to just state that you present a set of results which supports your hypothesis or idea. Which certainly seems to be the case from my read of the paper.

Thank you for the suggestion, however we believe that "tentative evidence", while admittedly not the strongest of phrases, is the best descriptor of what this paper provides. The work leading up to this paper has lead us to the threshold-limiting material hypothesis, the validity of which is indicated by the data presented in this paper but not directly supported. Thus, we believe that the use of the phrase "tentative evidence" is most appropriate.

Page 2 – Line 1:  $Q_*$  presents a huge range of parameter space, ranging over 14 orders of magnitude in Figure 2. From the formulation the range depends on how the flow magnitude compares to the  $D_{50}$  raised to a power 5. For grain sizes from 0.0001 to 0.1 m, the square root of this term ranges in magnitude from approximately  $1E - 10$  to 0.003. This may be more an observation, but it might be helpful to point this out because it is pretty uncommon to see a parameter space of 14 orders of magnitude in the associated literature. I can think of only a few, and they come from the same group as the authors.

Thank you for the suggestion, however in dimensional space, the range is already 10 orders of magnitude, and given that we present the equation that non-dimensionalizes discharge by grain size before presenting this graph, we believe that the large range of the data is explained.

Line 12: ... "the addition of" is not necessary  
Removed

Lines 19 – 27: These sentences are long and hard to follow. Edit for clarity. Here are some suggestions. Line 22: ... Suggest: "This line of thinking links with the second branch of regime theory established by Parker (1978a). Parker (1978a) solved..." Lines 26-27: Suggest: "...rivers that demonstrate that bedload dominated gravel-bedded rivers are slightly offset from a threshold channel..."

Thank you. We have made the suggested changes.

Page 3 – Lines 1 – 15: These sentences are also long and hard to follow. Edit for clarity. Here are some suggestions. Line 1: I don't understand the first part of the first sentence. Suggest: "The last branch of regime theory suggests that alluvial rivers optimize their geometry to maximize flow resistance and hence minimize the boundary fluid shear stress."

We mean to explain that there are multiple types of "regime theory" that exist at the moment and are practiced with little to no interaction with one

another. That is why the word "parallel" was chosen.

Lines 3 – 7: Break into two sentences.

Done

Line 9: This is a particularly key sentence for your argument, and with respect to comments by Maartin Kleinhans. As written it is hard to understand. Suggest: "This paper highlights the bedload-transport state transition between gravel-bedded river segments explained by Parkers theory, and sand-bedded river segments which do not fit within Parkers theory."

With all due respect to the reviewer, we would prefer to leave the sentence as is because Parker's theory is shown to be applicable to rivers with bed and banks made up of uniform, non-cohesive material, regardless of grain size.

Line 13: "...is the that..." the and that should be reversed.

Thanks for the catch.

Lines 16 - 18: Break into two sentences

With all due respect to the reviewer, we believe that, given that this is one idea, we believe it is best kept as a single sentence.

Line 17: Naïve? This word distracts from your point.

We have removed it.

Lines 24 - 29: I have read these sentences several times. Where exactly are these results presented? I reviewed both papers by Metivier et al. and it is not obvious to me how these sentences fit within the Metivier et al. papers. Please clarify.

We just mean to say that gravel-bedded rivers follow the threshold prediction with a slight offset and sand-bedded rivers follow threshold prediction with a larger offset

Page 5 – Lines 8 – 9: Second part of the first sentence is not needed.

We have removed the second half of that sentence.

Lines 24 – 26: Item (1) is hard to understand as written.

With all due respect to the reviewer, we believe this wording is the best way



to get the point across.

Page 6 – Line 12: No indent needed. Is “simplicity” the best word? Seems like you used their values for comparison sake.

We have removed the indent. We would prefer to keep the term simplicity, because we are not doing a comparison between our results and those presented by the IPGP group, rather we are just using their framework to present data in this paper.

Page 7 – Lines 2 – 4: There are other explanations beside a long timescale. Bed slope locally could adjust more readily (i.e. characterized by a shorter response time scale) than bank position, for example. Since you are plotting point values which reflect a range of length scales, my quick review of your data indicates you have a mix of length representations. I do not dispute the perspective of profile adjustment at the basin or many reach scale over relatively long times; but your data do solely reflect these conditions.

We acknowledge the possibility that locally this can happen. However, it has been reported from field data and models of alluvial river profiles that regrading the slope of an alluvial river happens slower. Generally, everybody reports scatter in slopes and we are just referring to the scatter that is reported.

Page 8 – Line 1: Naively? This word distracts from your point.

We have removed the word “naively”

Lines 2 – 3:  $Q_*$  and  $W * H$  are normalized by grain size. I don’t understand your grain size point as a result.

What we mean is that there is no offset from the central trend for fine particles in the upper range, as we see in plots of dimensionless width and depth.

Lines 8 – 9: The first sentence is confusing, and the second does not add much. Consider deleting both.

With all due respect to the reviewer, we believe it is important to explain why we look at shields stress instead of just geometry to understand the transport condition.

Line 23: Don’t need conspicuous. The data position in the plot says it all.

We have removed the word "conspicuous".

Lines 23 – 24: Last sentence not needed. It only distracts from your message. We have removed this sentence.

Line 31: I think you mean to reference Figure 5, not Figure 3. The bimodality is evident in both figures 3 and 5.

Page 10 – Line 1: I don't know where this result is presented. Thanks for the catch. We have removed the sentence

Line 15: You discuss results which you do not present. Please show the results or delete the last sentence.

The lack of bimodality in slope or depth is implicit in the relatively consistent fluid shear stress across the GST shown in Figure 6.

Page 11 – Lines 15 – 20: Here are examples of how you use parenthetical structure to make many points at the same time. Please break the thoughts up and present the material in a manner that is easier to follow.

With all due respect to the reviewer, each of these sentences makes exactly 1 point each. We don't know how we can make it easier to follow.

Page 12 – Lines 1 – 5: Where is the material of the last sentence presented? I have no idea, but want to know.  
van Rijn (2016) as referenced in equation 6.

Lines 7 – 22: I struggled with the main point of this paragraph. What is your main message and how does it link to the paragraphs around it? I could not piece it together.

This paragraph builds off of the previous one that states that the shields stress is not a relevant parameter for cohesive materials. This paragraph explains why and provides examples of critical shear stresses for cohesive substrates.

Page 13 – Line 2: There is less data in the 1-10 mm range to  $\mu$  or  $\delta$ , but does it really represent a paucity of data?

"Paucity" to us is only meant as the dictionary definition. We are agnostic

as to the origins, however that has been looked into by a number of people.

Lines 14 – 16: The last sentence doesn't really fit in with the paragraph. I think you need to link it better

With all due respect to the reviewer, the purpose of that last sentence is to relate the broad topic of the paragraph (dynamics vs. statics) to the data presented in this paper. In the paragraph, we discuss how there are central trends in the face of large variability that become apparent when enough data is able to average out the noise. By presenting potential sources of that noise, we hope to give readers a better understanding of our differentiation between signal and noise.

# Evidence of, and a Proposed Explanation for, Bimodal Transport States in Alluvial Rivers

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**Abstract.** Gravel-bedded rivers organize their bankfull channel geometry and grain size such that shear stress is close to the threshold of motion. Sand-bedded rivers on the other hand typically maintain bankfull fluid stresses far in excess of threshold, a condition for which there is no satisfactory understanding. A fundamental question arises: Are bed-load (gravel-bedded) and suspension (sand-bedded) rivers two distinct equilibrium states, or do alluvial rivers exhibit a continuum of transport regimes as some have recently suggested? We address this question in two ways: (1) re-analysis of global channel geometry datasets, with consideration of the dependence of critical shear stress upon site-specific characteristics (e.g. slope and grain size); and (2) examination of a longitudinal river profile as it transits from gravel to sand-bedded. Data reveal that the transport state of alluvial river-bed sediments is bimodal, showing either near-threshold or suspension conditions, and that these regimes correspond to the respective bimodal peaks of gravel and sand that comprise natural river-bed sediments. Sand readily forms near-threshold channels in the laboratory and some field settings, however, indicating that another factor, such as bank cohesion, must be responsible for maintaining suspension channels. We hypothesize that alluvial rivers adjust their geometry to the threshold-limiting bed and bank material — which for gravel-bedded rivers is gravel, but for sand-bedded rivers is mud (if present) — and present tentative evidence for this idea.

## 1 Introduction

Almost 100 years ago, Lacey (1930) proposed an empirical relationship relating the width of an alluvial river to its water discharge. Leopold and Maddock (1953) built upon this to derive the hydraulic scaling relations for bankfull channel geometry of alluvial rivers. Decades of research since have added geographic (Parker et al., 2007; Richards, 1987) and morphologic (e.g., braided vs. meandering, Gaurav et al. (2015); Métivier et al. (2016)) variety to data compilations, and recognized the importance of vegetation and geologic controls that were not originally considered (e.g., Huang and Nanson (1998); Schwendel et al. (2015); Kleinhans et al. (2015); Nanson and Young (1981); Ferguson (1987)). Yet the original findings are robust: bankfull width ( $W_{bf}$ ), bankfull depth ( $H_{bf}$ ) and slope ( $S$ ) scale as power-law functions of bankfull discharge ( $Q_{bf}$ ) with little variation in the exponents (Parker et al., 2007), suggesting a simple and common organizing principle for alluvial rivers. Cast in dimensionless form following Métivier et al. (2016) and Andrews (1984), with  $Q_* = Q_{bf} / \sqrt{RgD_{50}^5}$  where  $D_{50}$  is the

river-bed median grain size,  $R$  is the particle submerged specific gravity, and  $g$  is gravity, the often-called “regime equations” read:

$$\begin{aligned} W_{bf}/D_{50} &= \alpha_W Q_*^{\beta_W} \\ H_{bf}/D_{50} &= \alpha_H Q_*^{\beta_H} \\ S &= \alpha_S Q_*^{\beta_S} \end{aligned} \quad (1)$$

where  $\alpha$  and  $\beta$  are dimensionless parameters. The theoretical underpinning of the regime equations (1) is both well known and elusive; it is the equilibrium channel geometry problem (Leopold and Maddock, 1953). **Considering fluid mass conservation in a rectangular channel:**

$$Q_{bf} = u_{bf} H_{bf} W_{bf}, \quad (2)$$

and friction via a Chezy-type relation:

$$u_{bf} = \sqrt{g H_{bf} S} / C_f, \quad (3)$$

where  $u_{bf}$  and  $C_f$  are average bankfull flow velocity and friction factor, respectively, we obtain two relations among the governing hydraulic variables. If  $Q_{bf}$ ,  $D_{50}$  and  $C_f$  are specified (as is typical), one still requires an additional relation among the parameters to close the set of equations and derive equation 1 (Métivier et al., 2017).

“Regime theory” is the application of these agreed upon relationships with one additional threshold channel based-assumption to allow for closure. There are three dominant branches of regime theory, each with their own form of a threshold channel closure assumption that separate regime theory into three distinct schools of thought: 1) assume that river are canals, and thus threshold channels; 2) assume that the transport regime is purely bedload and solve the 2-D flow field to balance fluid shear stress and particle weight at the edge of the channel, while simultaneously allowing for transport at the center; 3) assume that the river undergoes an optimization process that maximizes friction in order to reduce fluid shear stress, ultimately resulting in a threshold channel.

The first school of thought is based upon work done to calculate the shape of a stable canal for which the bed material is at the threshold of motion (Glover and Florey, 1951). This work has been extended to natural rivers by Henderson (1961), and offers an explanation for observations of alluvial river width relating to the water discharge (Henderson, 1961; Andrews, 1984; Métivier et al., 2017). This line of thinking links well in with the second branch of a regime theory which as was established by Parker (1978a). Parker (1978a) solved the 2-D stress field to show that, for a pure bedload river, the channel is at the threshold of motion for the material at the banks and slightly above the threshold of motion in the center, allowing for the river to transport sediment, while at the same time maintaining a stable and consistent width. **This model is supported by both global compilations of data and case studies of individual rivers that demonstrate that bedload-dominanted gravel-bedded**

rivers are slightly offset from a threshold channel (Phillips and Jerolmack, 2016; Gaurav et al., 2015; Métivier et al., 2016). Several studies have presented evidence that sediment supply and bank vegetation may drive gravel channels further above threshold (Pfeiffer et al., 2017; Millar and Quick, 1998). Values for Shields stress in gravel-bed rivers reported for a wide range of environments, however, rarely exceed 2-3 times critical.

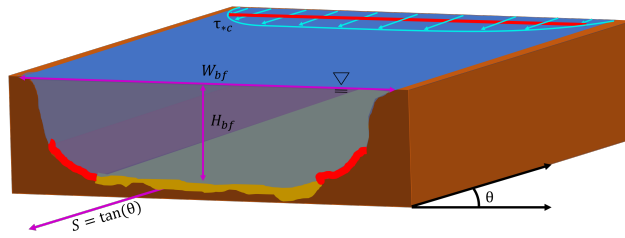
5 Parallel to this grain size-dependent channel geometry is the concept of optimization which assumes that rivers seek a threshold channel condition by maximizing the flow resistance within the channel to minimize the fluid shear stress (Eaton et al., 2004; Eaton and Church, 2007). The rational regime theory put forward by Eaton attempts to infer the importance of bank strength given deviations away from the threshold condition that is posited by optimality theory (Eaton et al., 2004; Eaton and Church, 2007). However, these relationships are predominantly calibrated on coarse-grained rivers where research has  
 10 shown that the influence of cohesive mud is a minor control on the erodibility compared to the weight of the gravel (Kothyari and Jain, 2008). What distinguishes our work from this work is that we extend the concept of Parker’s threshold channel model into the space occupied by fine-grained rivers by the suggestion that river channel geometries, and their subsequent sediment transport state are either controlled by the erodibility of their beds or their banks. This paper shows the transition from rivers that can be explained entirely by Parker’s theory (i.e. channel beds and banks composed of uniform material transported entirely in  
 15 bedload) to channels that cannot. For natural rivers, this transition most frequently occurs at the transition from a gravel-bedded to a sand-bedded condition. This transition coincides with the point at which bed material becomes small enough such that the cohesion of channel banks should become important. What we show is that the sediment transport state is bimodal because grain size is bimodal; the coarser gravel mode is more difficult to entrain than any cohesive bank material, while the finer sand mode is easier to entrain than any cohesive bank material (if present).

20 As nicely summarized in a recent series of papers (Métivier et al., 2016; Gaurav et al., 2015; Métivier et al., 2017), a useful starting point for the equilibrium channel geometry problem is to consider what we call here the “ground state” in which no sediment transport occurs. In this situation, which may be achieved in a laboratory experiment with a constant  $Q_{bf}$  and no sediment feed, the river organizes such that the boundary shear stress everywhere along the channel cross section is at the threshold of motion (Métivier et al., 2017). Accordingly, the local and width-averaged bankfull Shields stress should be at the  
 25 critical value,  $\tau_{*bf} = \tau_{*c}$ , and may be estimated assuming normal flow as:

$$\tau_{*bf} = \frac{H_{bf}S}{RD_{50}} \quad (4)$$

where  $R = 1.65$  is the assumed relative submerged grain density. Setting equation 4 equal to  $\tau_{*c}$  provides the necessary closure to determine channel geometry, as first illustrated by Lacey (Lacey, 1930) who solved for the shape of a canal. Of course, natural rivers are not canals; they transport sediment, which requires that their formative Shields stress be larger than critical.

30 Compilations of channel geometry and Shields stress, using global datasets, reveal that alluvial rivers naturally break out into two classes: gravel-bed rivers ( $D_{50} > 10mm$ ) in which  $1 \leq \tau_{*bf}/\tau_{*c} \leq 2$ , and sand-bed rivers ( $D_{50} < 1mm$ ) with  $\tau_{*bf}/\tau_{*c} \gg 1$ . The scaling exponents (equation 1) for both classes are similar and in reasonable agreement with predictions from “Lacey’s



**Figure 1.** Schematic cross-section of a sand-bedded, alluvial river with different bed vs. bank material, under bankfull flood conditions. Here  $W_{bf}$ ,  $H_{bf}$ , and  $S$  are bankfull width, bankfull depth and channel gradient at the cross-section, respectively. Cyan lines at surface illustrate horizontal stress profile across the channel. Red lines along channel bottom indicate toe of the river bank — i.e., the intersection of bed and bank material. Red line intersecting the cyan velocity profile indicates the threshold stress of the threshold-limiting material, illustrating that the bank toe is at threshold while Shields stress in the channel center is slightly in excess of threshold.

law”, however, the coefficients are different from each other and the threshold channel (Métivier et al., 2017, 2016; Gaurav et al., 2015).

Parker (1978a) provided the first generalization of the threshold channel theory to gravel-bed rivers, which transport sediment as bed load. He recognized that stable river banks are incompatible with transport; the transverse slope drives a net flux away from the bank, leading to erosion and channel widening. The solution to the so-called “stable-channel paradox” (Parker, 1978b) lies in the lateral (cross-stream) gradient in bed stress — flow velocity and depth increase with distance away from the bank. An equilibrium channel may therefore be constructed that is marginally above threshold in the center but at threshold on the banks. Parker (1978a) predicted  $\tau_{*bf}/\tau_{*c} \approx 1.2$  for equilibrium bed-load rivers, in agreement with observations of natural gravel-bed rivers (Paola et al., 1992; Parker et al., 1998; Dade and Friend, 1998; Parker et al., 2007; Phillips and Jerolmack, 2016) and laboratory experiments (Ikeda et al., 1988; Pitlick et al., 2013; Reitz et al., 2014). In terms of the regime equations 1, gravel-bed rivers thus follow expectations from the threshold theory but with a slight offset due to their higher bankfull Shields stress (Métivier et al., 2017). Parker (1978b) also realized that sandy (suspension) rivers cannot behave in a similar manner, in that boundary stresses even at the channel margins would be above threshold leading to erosion. In order to counter slope-driven bank erosion, Parker (1978b) and subsequent researchers (Ikeda and Nishimura, 1985; Ikeda et al., 1988; Wilkerson and Parker, 2010) proposed that lateral diffusion of suspended sediment outward from the channel center could compensate for inward bed-load sediment transport from the banks. While physically reasonable, suspension channel theories have not provided a satisfactory description of sandy river channel geometry. At present there is no accepted model for the equilibrium geometry of river channels far above threshold.

In the absence of a theory, subsequent research has focused on examining trends drawn from compilations of data on channel hydraulic geometry and bankfull discharge. Examination of gravel-sand transitions along downstream river profiles indicates that the mode of bed-material transport may switch abruptly from near-threshold (gravel-bedded) to suspension (sand-bedded) (Miller et al., 2014; Venditti et al., 2015, 2010; Singer, 2010, 2008; Blom et al., 2017), and hydraulic considerations have suggested that susceptibility to suspension increases rapidly as grain size decreases across the gravel to sand range (Lamb

and Venditti, 2016). On the other hand, recent compilations of global data sets have been used to suggest that rivers exhibit a continuum of transport states — from near threshold through to full suspension — and that bankfull Shields stress varies smoothly with grain size, slope and particle Reynolds number (Parker et al., 2007; Wilkerson and Parker, 2010; Li et al., 2015; Trampus et al., 2014)). Importantly, this new presentation of the data suggests that there is no range in phase space where rivers cluster near the ground state of a constant threshold Shields stress (Fig. 4). Phillips and Jerolmack (2016) found, however, that gravel-bed rivers do indeed cluster close to the threshold of motion — if the dependence of threshold upon site-specific characteristics (e.g. slope or grain size (Lamb et al., 2008; van Rijn, 2016)) is explicitly accounted for. Moreover, while previous data compilations found that bankfull Shields stress increases systematically with decreasing grain size (Li et al., 2015; Trampus et al., 2014), one may readily find data that contradict this trend. Channels formed by seepage erosion in sand (Devauchelle et al., 2011; Marra et al., 2015) are observed to transport sand as bedload and, like gravel bedload rivers, cluster approximately at the threshold of motion. Similarly, sand-bedded rivers in laboratory experiments also form near-threshold channels (Reitz et al., 2014; Métivier et al., 2016; Federici and Paola, 2003).

**We are left with three questions that will be considered in this paper.** First, how do rivers transition from near-threshold to suspension states? Second, is the near-threshold channel an attractor, or merely a limiting state? And third, how do suspension rivers maintain an equilibrium channel geometry? **We address these questions by re-analysis of existing data. We revisit the global data compilations of Li et al. (2015) and Trampus et al. (2014), and argue that natural rivers appear to exhibit bi-modal transport states corresponding to near threshold (order 1 multiplier of threshold) and far-above threshold (order 10-100 multiplier of threshold).** We also show that this bi-modal behavior is exhibited within a single river profile transiting the gravel to sand transition. These results lend credence to the hypothesis first put forward by Lane (1937) and then Schumm (1960): Alluvial rivers adjust their geometry to the threshold-limiting bed and bank material. It follows that sand-bed rivers may be suspension channels if their banks are composed of more resistant material (Church, 2006), e.g., cohesive sediment and/or vegetation. Gravel rivers, on the other hand, should be less sensitive to bank composition due to the relatively high threshold stress for entrainment of coarse grains (Schumm, 1960).

## 2 Data Sources

The large, global datasets utilized in this paper are identical those used by Trampus et al. (2014) and Li et al. (2015). They were subsequently combined with a longitudinal profile from the Sacramento River (Singer, 2010), river channel cross sections on the Kosi Megafan (Gaurav et al., 2015), and channels formed by seepage erosion in the Apalachicola ravines in Florida (Devauchelle et al., 2011) and in a laboratory (Reitz et al., 2014). This combination of data allows for the following comparisons between localized examples and global trends in river channel characteristics: 1) how changes in hydraulic geometry and sediment transport regime that a single river experiences across a gravel-sand transition compare to exhibited global trends in hydraulic geometry and Shields stress; and 2) how rivers that originate in sandy substrates with little cohesion compare in terms of hydraulic geometry and sediment transport regime to channels with gravel beds. All data used in this analysis (Li



et al., 2015; Trampus et al., 2014; Singer, 2010; Gaurav et al., 2015; Devauchelle et al., 2011; Reitz et al., 2014) are available as supplementary material and include bankfull estimates of width, depth, slope, grain size, and discharge.

Our re-analysis requires that we estimate the critical Shields stress for incipient motion,  $\tau_c^*$ , for each data point. Determination of  $\tau_c^*$  is a notorious problem (Buffington and Montgomery, 1997; Mueller et al., 2005; Lamb et al., 2008; van Rijn, 2016) and, despite the best efforts of researchers, no theory can reliably predict values for the field. Nevertheless, there is strong field and laboratory evidence that  $\tau_c^*$  varies with site-dependent characteristics, such as slope (Mueller et al., 2005; Lamb et al., 2008; Phillips and Jerolmack, 2014, 2016) and grain size (Shields, 1936; van Rijn, 2016). In this study we use and compare the empirically-determined slope-dependent relation of Lamb et al. (2008):

$$\tau_c^* = 0.15S^{0.25}, \quad (5)$$

to the Shields-curve fit of van Rijn (2016):

$$\tau_{*c} = \frac{0.3}{1 + D_*} + 0.055(1 - e^{-0.02D_*}) \quad (6)$$

where  $D_* = (Rg)^{1/3}D_{50}/\nu^{2/3}$  is dimensionless grain size and  $\nu$  is kinematic viscosity. We note that our findings change little if we use the linear slope-dependent relation of Mueller and Pitlick (2005) instead of equation 5.

### 3 Hydraulic Geometry Scaling Revisited

We first examine hydraulic geometry scaling as suggested by the regime equations 1. For comparison, we also compute the expectations for a threshold channel following Métivier et al. (Métivier et al., 2016; Gaurav et al., 2015):

$$\begin{aligned} \frac{W_{bf}}{D_{50}} &= \left[ \frac{\pi}{\sqrt{\mu}} (\tau_{*c})^{-1/4} \sqrt{\frac{3C_f}{2^{3/2}K[1/2]}} \right] Q_*^{1/2}; \\ \frac{H_{bf}}{D_{50}} &= \left[ \frac{\sqrt{\mu}}{\pi} (\tau_{*c})^{-1/4} \sqrt{\frac{3\sqrt{2}C_f}{K[1/2]}} \right] Q_*^{1/2}; \\ S &= \left[ (\sqrt{\mu}\tau_{*c})^{5/4} \sqrt{\frac{2^{3/2}K[1/2]}{3C_f}} \right] Q_*^{-1/2}. \end{aligned} \quad (7)$$

For simplicity, we choose values for the following coefficients to be identical to those reported in Métivier et al. (2016): Chezy friction factor  $C_f \approx 0.1$ , Coulomb friction coefficient  $\mu \approx 0.7$ , and  $K[1/2] \approx 1.85$ . These values could be manipulated to enhance their fit to data if desired, but this exercise is not performed here. We treat  $\tau_{*c}$  in two ways: (1) assuming a constant critical Shields stress with a representative gravel-bed river value  $\tau_{*c} = 0.03$  as in Métivier et al. (2016); and (2) using the slope and grain size dependent critical values from equations 5 and 6, respectively.

To first order, gravel- and sand-bedded rivers could be described by a single continuous power-law relation for dimensionless channel width  $W_{bf}/D_{50}$  as a function of  $Q_*$ . A second-order feature is present, however, in the high  $Q_*$  limit; a subset of

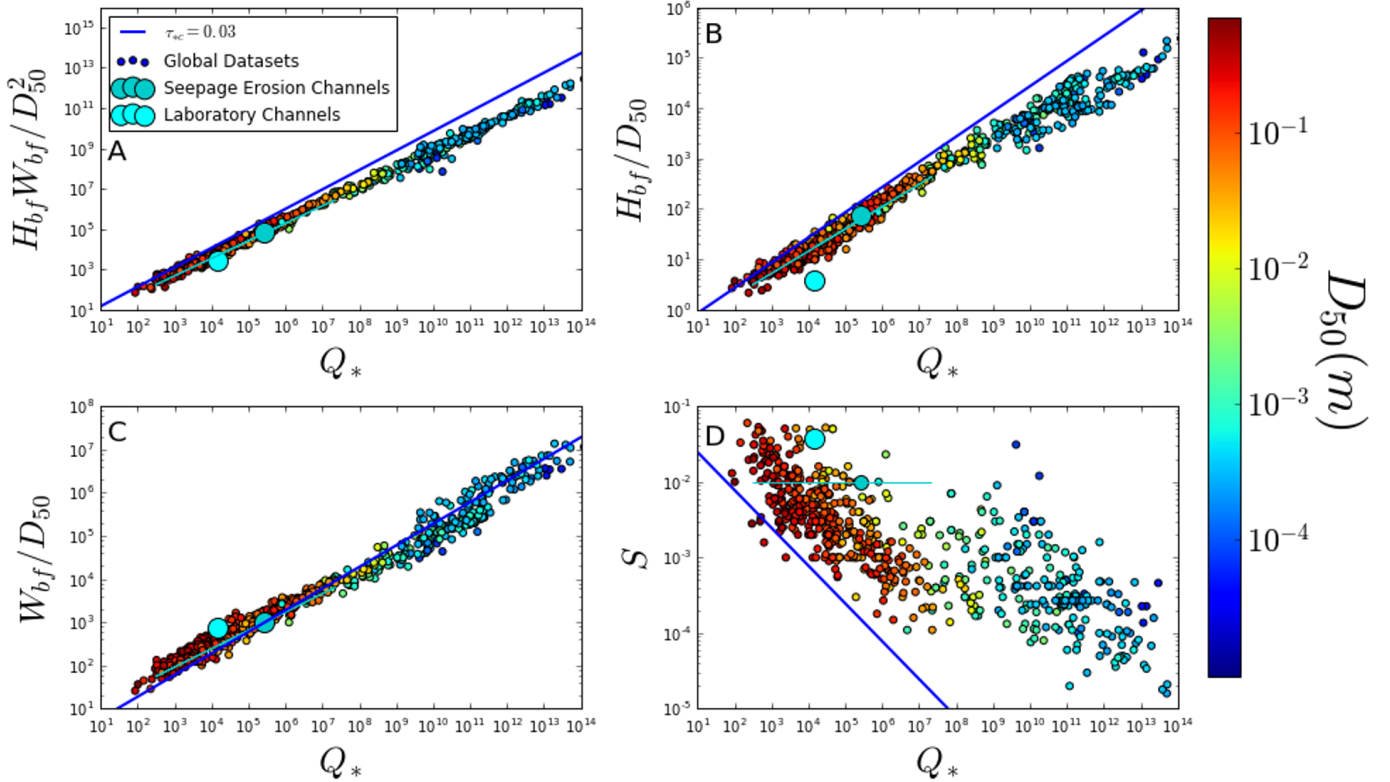
sand-bed streams show an upward offset from the general trend (Fig. 2). Dimensionless channel depth  $H_{bf}/D_{50}$  shows similar behavior, except that the high- $Q_*$  sandy streams show a downward rather than upward offset. In general, gravel-bed rivers are close to threshold predictions while sand-bed streams depart more significantly, similar to earlier findings by Métivier et al. (Métivier et al., 2016; Gaurav et al., 2015). Both constant and slope-dependent threshold channel predictions capture the general trends, but predict a systematically steeper scaling exponent than is exhibited by the data. Slope has a behavior that is distinctly different from width and depth; sand-bedded rivers in general display a large offset from the gravel-bedded river trend, and a correspondingly large offset from threshold channel predictions (Fig. 2). Slope exhibits more scatter than channel geometry, a common pattern in river data compilations that likely reflects the long timescale associated with slope adjustment (Métivier et al., 2016; Gaurav et al., 2015). Note that, for all variables, the sandy seepage erosion channels in Florida generally plot with the gravel-bedded river data showing that sand-bedded rivers do not necessarily behave differently from gravel-bedded ones.

One interesting finding is that the product of dimensionless width and depth, i.e., dimensionless channel cross-sectional area, shows the tightest relation to  $Q_*$  and no offset between gravel- and sand-bed channels. This is noteworthy considering that width and depth plots show considerable scatter, so one might expect that their product would exhibit more scatter if the variability was due to random noise or error. This suggests that rivers systematically increase their cross-sectional area  $A$  to accommodate increasing discharge — regardless of grain size and transport stage; in other words,  $A$  is primarily controlled by hydraulics alone (indeed flow resistance, and hence flow velocity  $u_{bf}$ , is approximately independent of channel aspect ratio for values  $W_{bf}/H_{bf} > 10$  (Guo and Julien, 2005) that are typical of natural rivers). How changes in  $A$  are partitioned into width vs. depth, however, may depend on bed/bank substrate and sediment transport conditions.

#### 20 4 Bimodality in the Transport States of Global Datasets

As the name implies, hydraulic geometry scaling does not consider the transport state of sediment within channels. A simple way to do so is consideration of the bankfull Shields stress  $\tau_{*bf}$ . Earlier global compilations of river data suggested that transport states were bimodal, with gravel-bed rivers clustering around a Shields stress close to the critical value ( $\tau_{*bf} \sim 10^{-2}$ ) and sand-bed rivers clustering around a much higher value ( $\tau_{*bf} \sim 10^0$ ) (Paola et al., 1992; Parker et al., 1998; Dade and Friend, 1998). Indeed, we see compelling evidence for this bimodality across a range of slopes and grain sizes in our global compilation (Fig. 3). There are clear deviations from this trend, however; the sandy Florida seepage channels (Devauchelle et al., 2011) and sandy laboratory experimental rivers of Reitz et al. (2014) both plot in the range of phase space otherwise occupied by gravel-bed rivers. What these channels have in common is that they are small, sand-bedded rivers with bank material that is similar in composition to the bed (i.e., sandy).

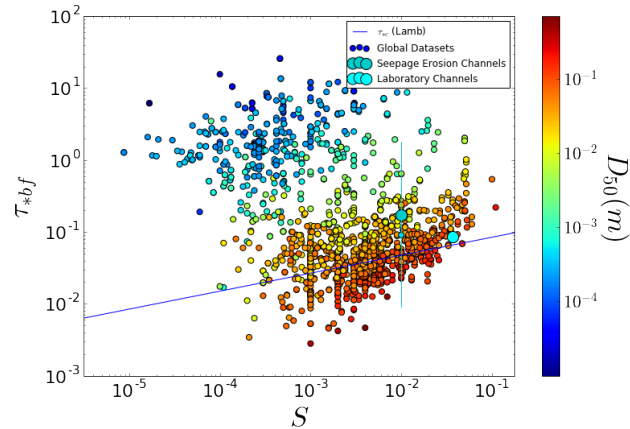
30 The case for a continuum of transport states was made more recently by Li et al. (2015), who showed that  $\tau_{*bf}$  is inversely proportional to dimensionless grain size  $D_*$  and scales with roughly the square root of  $S$ . They presented a similarity collapse for the data with a best-fit relation  $\tau_{*bf}/S^{0.53} = 1220D_*^{-1}$ , and a similar result was found by Trampus et al. (2014). Li et al. (2015) concluded that the notion of a constant formative Shields stress for either gravel- or sand-bedded channels was not



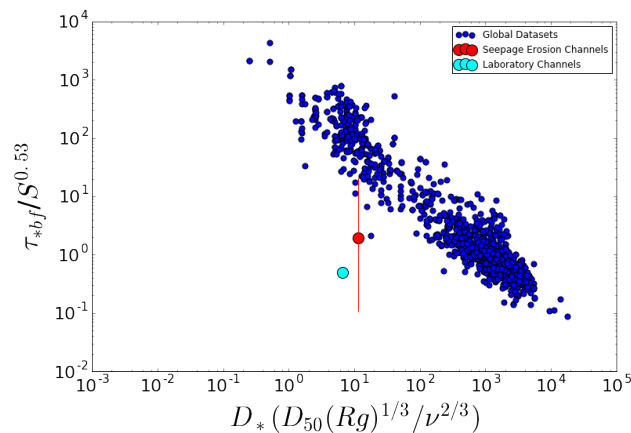
**Figure 2.** Dimensionless hydraulic geometry scaling for rivers in the global data set. (A) Cross-section area shows a tight relation with discharge across the entire range of data. (B) Depth and (C) width follow similar first-order trends for gravel vs. sand bed rivers, but with some offset between these groups. (D) Slope separates sand and gravel rivers. Blue line shows exceptions from the threshold equations (7) assuming a constant reference Shields stress for simplicity. We note that the fit does not improve if grain-size or slope dependent threshold predictions are used instead. Larger points illustrates the mean of multiple measurements taken along a single longitudinal profile. Cyan error bars represent the range of data, and are used because the original study reported only one value for slope and for grain size for all cross sections (Devauchelle et al., 2011).

supported by the data. We reproduce the figure of Li et al. (2015) here, where the addition of new data (discussed in the previous section) generally supports the similarity collapse (Fig. 4). The sandy Florida seepage channels and experimental rivers, however, fall off of this trend.

By assessing transport stage using bankfull Shields stress alone, previous authors either explicitly (Parker et al., 1998, 2007; Wilkerson and Parker, 2010) or implicitly (Li et al., 2015; Trampush et al., 2014) assumed that the critical Shields stress was constant. A recent study by Phillips and Jerolmack (2016), however, showed that, when site-specific variations in  $\tau_{*c}$  are taken into account, gravel-bedded rivers exhibit a bankfull Shields stress that is close to the threshold value. We consider transport stage as  $\tau_{*bf}/\tau_{*c}$ . To test for the influence of variations in  $\tau_{*c}$ , we examine the distributions of Shields stress and transport stage

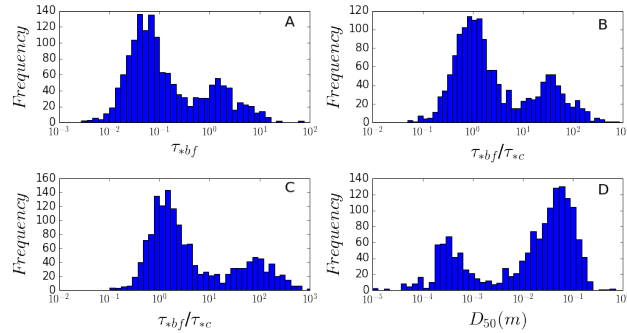


**Figure 3.** Bankfull Shields stress as a function of stream gradient. Coarse-grained rivers exhibit low Shields stresses with a moderate dependence on slope, that roughly follows but is offset from the slope-dependent relation of Lamb et al. (2008) for critical Shields stress (solid line). Fine-grained rivers cluster well in excess of the threshold of motion. River channels originating in sandy substrates found in the natural (Devauchelle et al., 2011) or laboratory (Reitz et al., 2014) environments are shown to be in the Shields stress space typically populated by gravel-bedded rivers.



**Figure 4.** A re-creation of the diagram from Li et al. (2015) that makes the case for a continuum of sediment transport regimes. Additional data have been added to the diagram from an additional global dataset (Trampus et al., 2014), and various longitudinal profiles (Singer, 2010; Gaurav et al., 2015; Devauchelle et al., 2011). **Clear deviations from the trend are demonstrated by river channels formed by seepage erosion in sand (with mean and error bars same as in Fig. 3), and channels formed in sand in laboratory experiments (Reitz et al., 2014) that are represented by the larger red and cyan points, respectively.**

where for the latter  $\tau_{*c}$  is estimated from either slope or grain size following equations 5 and 6. The Shields stress distribution is bimodal (Fig. 3). This bimodality becomes slightly more evident in the distributions of transport stage, though there is little difference between the results using the two different estimates for  $\tau_{*c}$  (Fig. 5 B, C). The bimodality in Shields stress and



**Figure 5.** Distributions of (A) Shields stress, (B) Transport stage estimated using grain size (van Rijn, 2016), (C) Transport stage estimated using slope (Lamb et al., 2008), and (D) median river-bed grain size. All distributions are bimodal, with near-threshold gravel rivers and far-above threshold sandy rivers.

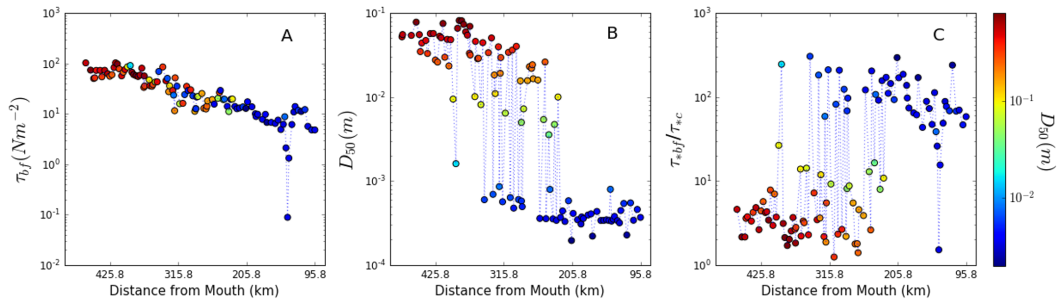
transport stage is mirrored by a comparable bimodality in river-bed grain size (Fig. 5 D). These findings revive the possibility of a constant transport-stage condition that is either close to or far above threshold, but also show that river-bed grain size is insufficient to predict transport stage as threshold sand-bed rivers may readily be found.

## 5 Bimodality in Transport Stage along a Longitudinal River Profile

- 5 The global dataset reveals an apparent dichotomy of transport states that generally (but not always) correspond to sand- or gravel-bedded rivers, but the nature of this dichotomy may be partially obscured by confounding variables among disparate river systems that are not accounted for. A useful complementary approach is to examine the longitudinal profile of a single river as it transits from gravel- to sand-bedded. We utilize data collected by Singer (2010) in his study of the gravel-sand transition of the Sacramento River. We can see that Shields stress is slightly in excess of critical for the gravel-bed portion of the river, and far above critical for the sandy portion (Figure 6). In the gravel-sand transition we observe a flickering between these two distinct states, that is indicative of patchiness of bed materials (Singer, 2010). The fluid shear stress gradually declines downstream (Fig. 6 A), and width decreases across the gravel-sand transition but only modestly (Singer, 2010). Bed-sediment size changes abruptly, showing that the large variations in transport stage are overwhelmingly driven by the grain-size pattern (Figure 6 B). In summary, the Sacramento River shows the same bimodal behavior as the global dataset, in terms of transport stage and grain size. Other factors such as slope or hydraulic geometry do not show this bimodality.
- 10
- 15

## 6 Discussion

It has long been suggested that bank composition influences the hydraulic geometry of rivers, under the premise that effective bank cohesion (silt/clay or vegetation) increases the threshold shear stress which leads to narrower and deeper channels. The evidence from gravel-bed rivers is that the cohesive effect is significant but modest; bank strength changes of up to two orders



**Figure 6.** Longitudinal profile data from the Sacramento River up to approximately 500km upstream of the river mouth (Singer, 2010). (A) Fluid shear stress decreases gradually and continuously across the gravel-sand transition. (B) Grain size behavior downstream is bimodal, changing rapidly from gravel to sand. (C) Shields stress shows abrupt transition from near-threshold to far above threshold across the gravel-sand transition.

of magnitude correspond to differences in width of 2-3 times (e.g., Andrews (1984); Millar and Quick (1993); Millar and Quick (1998); Huang and Warner (1995); Huang and Nanson (1998)). Though there are far fewer studies on sand-bed alluvial rivers, the limited data indicate that the influence of bank cohesion may be larger in these systems (Kleinhans et al., 2015, 2014). The classic study by Fisk (1944) of the Mississippi River showed major narrowing and deepening as the river moved from

5 sandy to clay-rich alluvium, while Schumm (1960; 1963) demonstrated that channel aspect ratio ( $W_{bf}/H_{bf}$ ) was inversely proportional to the percent silt-clay (a proxy for cohesion) in the bed and banks of sand-bed rivers. Interestingly, he found no correlation between aspect ratio and percent silt-clay for gravel-bed rivers (Schumm, 1960). More recent studies on deltaic and tidal channels have also shown that bank strength strongly influences channel geometry (Kleinhans et al., 2009; Edmonds and Slingerland, 2010).

10 Lane (1937) and Schumm (1960) argued that channels initially cutting into alluvium should widen “until the resistance of the banks to scour prevents it” (Schumm, 1960). We rephrase this idea to posit a more specific hypothesis: Alluvial rivers, on average, organize their geometry such that the fluid shear stress at the toe of the bank is at the threshold of motion for the bankfull flow (Fig. 1). We consider the bank toe because (1) this is the zone of maximum fluid stress on the bank, and (2) bank-toe erosion is required to undermine upper bank materials. While slumping and block failures may strongly influence

15 the rate of bank erosion, with important consequences for river dynamics such as meandering (Parker et al., 2011), these processes likely have little effect on average channel size. For rivers in which the bed and the bank toe are made of the same material — such as laboratory experiments, and some natural channels in non-cohesive sediments — we expect to recover the near-threshold “bed-load river” channel predicted by the Parker (1978a) model. For the more common case of rivers having a

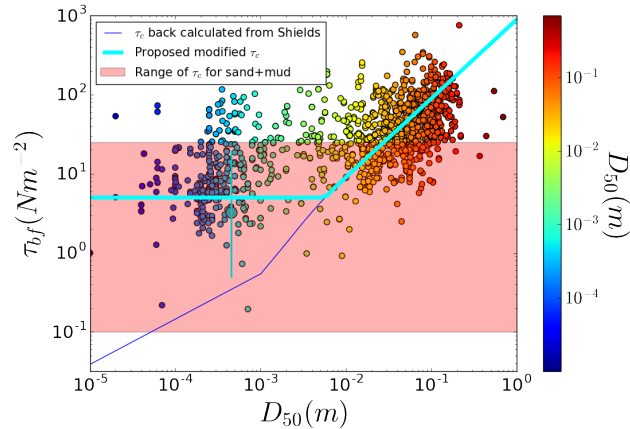
20 bank-toe composition that is different from the bed — typically cohesive and/or vegetated banks — we propose that alluvial rivers adjust their geometry to the threshold-limiting material. Thus, in order to maintain a “suspension river” like most natural sand-bed channels, the banks must be composed of cohesive sediment with a significantly higher entrainment threshold than

the bed material. Indeed, Church (2006), noted that sand-bed channels often have silt-clay banks that experience little to no deformation, while channel-bed sands are completely suspended.

Unfortunately, reported measurements of hydraulic channel geometry rarely include information about bank materials. To test the threshold-limiting idea indirectly, we consider the relative mobility of bed and bank materials as a function of grain size. **We do not consider vegetation explicitly; however, we note that numerous studies have analyzed the effects of vegetation on erosion thresholds (Micheli and Kirchner, 2002; Abernethy and Rutherford, 2001).** It is important to point out that Shields stress is not the relevant parameter for cohesive materials, where particle weight does not adequately describe resistance to motion. Dimensional fluid threshold stress is usually reported in studies involving cohesive sediment. Considering non-cohesive materials and neglecting slope effects, the threshold fluid stress determined from the Shields curve increases monotonically with increasing grain size following the relation presented in equation 6.

Cohesion becomes significant for particles that are silt-sized and smaller due to surface charge effects, which increases the threshold for entrainment compared to predictions from the Shields curve (Kemper et al., 1987; Kothyari and Jain, 2008). As a result, sand is the most easily entrained material: larger particles are harder to move due to their mass, while smaller particles are harder to move due to cohesion. Of course, most stream banks are composed of mixtures of cohesive and non-cohesive sediments. The threshold entrainment stress for sand increases rapidly with increasing fraction of clay and silt, with reported increases of up to two orders of magnitude for clay-rich river banks (Kothyari and Jain, 2008). For gravel particles of order centimeter and larger, however, the entrainment stress varies little with the addition of clay and silt (Kothyari and Jain, 2008). Taken together, we naively expect that rivers with bed sediment  $D_{50} > 10^{-2}m$  should organize to a threshold shear stress that is slightly in excess of the threshold predicted by the Shields curve. For natural rivers with bed sediment smaller than about a centimeter, cohesive sediments (if present) will lead to channel banks with entrainment thresholds that are larger than predicted by the Shields curve. The minimum threshold fluid stress for a sand-bed river is  $\tau_b \sim 0.1N/m^2$  based on the Shields curve. Without knowledge of bank materials in the data used here, we use results from a systematic study that examined the influence of silt-clay content on the erosion threshold of natural sandy river banks. Julian and Torres (2006) reported a maximum stress of  $\tau_b \approx 25N/m^2$  for banks composed entirely of silt and clay. For typical banks with silt-clay fractions of a few tens of percent, and/or moderate vegetation coverage, a typical value for the critical stress is  $\tau_b \approx 5N/m^2$  (Julian and Torres, 2006; Tal and Paola, 2007; Braudrick et al., 2009).

Turning to the global dataset, we compare the bankfull shear stress  $\tau_{bf}$  to bed-sediment grain size  $D_{50}$  for all rivers (Fig. 7). While there is significant scatter, we notice a general pattern in the data; sand-bed rivers show no relation between bankfull shear stress and bed-sediment grain size, while gravel-bed rivers exhibit increasing shear stress with grain size. Projecting the threshold stress based on the Shield curve onto the data, we see that gravel-bed rivers generally follow the curve while sandy rivers plot significantly above it. The range of  $\tau_{bf}$  for sandy rivers overlaps with, and is slightly offset from, the range of threshold stresses reported for sand-mud mixtures (Fig. 7). The “typical value” of  $\tau_b = 5N/m^2$  runs through the middle of the sandy rivers. The threshold-limiting material may be assessed by comparing the threshold stress of mud-sand mixtures to the threshold stress determined from the Shields curve; we see that most rivers with  $D_{50} > 1cm$  are limited by gravel mobility, while most rivers with  $D_{50} < 1mm$  are limited by bank mobility (if cohesive sediment is present).



**Figure 7.** Potential adjustment of river-bed shear stress to the threshold-limiting material for the global data. The rising line from left to right indicates expected critical shear stress determined from grain size based on the Shields curve fit of van Rijn (2016). Gravel-bed rivers generally fall along this line, but sandy rivers generally plot significantly above it. Flat line shows a reference critical shear stress for the middle of the range of sand-mud mixtures. Cyan line indicates the trace of the threshold-limiting stress. For rivers with bed sediment grain sizes smaller than about a millimeter, we expect bank material to be threshold limiting; for gravel-bed rivers, the bed is expected to be threshold limiting.

The above trends provide tentative, albeit equivocal, support for the hypothesis that all alluvial rivers are near-threshold channels adjusted to the threshold-limiting material. For the case of gravel-bed rivers, this corresponds to a transport stage close to one for the bed material at bankfull. For sand-bed rivers with cohesive banks, we expect the transport stage of bed material to be roughly the ratio of the bank to bed entrainment thresholds, which could be in the range  $10^0 \leq \tau_{*bf}/\tau_{*c} < 10^3$ .

5 Because sand has the lowest threshold, and most natural river banks contain some cohesive materials, transport stage for sandy rivers is typically much greater than 1 leading to suspension channels. Given the paucity of alluvial river-beds with median grain sizes between 1 mm and 10 mm — the range where we expect cohesive banks to become important — these factors give rise to a bi-modal distribution of transport stage. In terms of hydraulic geometry, data indicate that cross-sectional area is controlled primarily by hydraulic conveyance as it has a very tight relation with bankfull discharge for all rivers. The partitioning of this

10 area into width and depth appears to be related to the threshold constraint imposed by bank-toe material.

We close this section with a brief but important aside on the distinction between hydraulic geometry and dynamics. The idea that all alluvial rivers are near threshold may at first seem incompatible with the intrinsic and incessant dynamics we observe: widening/narrowing, meandering, sorting, and bed/bar form evolution. In this context the (near-)threshold channel geometry is the statistically-expected behavior in a dynamic, stochastic system — analogous to a mean bed-load flux, or Reynolds averaging

15 in fluid mechanics — that does not represent system behavior at any particular instant (Furbish et al., 2016). The experimental findings of Reitz et al. (2014) make this point well: “Although individual channels in the braided river are constantly changing shape through scour and fill, these appear to be fluctuations around a robust [near-threshold] geometry that becomes apparent when many individual channel geometries are averaged together.” Some of the scatter in hydraulic geometry scaling plots may



be due to a variety of factors such as: influences from vegetation, localized/temporary imbalances between the rate of floodplain formation and bank failure, and partial submergence of grains in the flow.

## 7 Conclusions

We propose that all alluvial rivers, regardless of their bed material grain size, organize their hydraulic geometry such that they cluster around the threshold of motion for the most resistant material — the structural component of the channel that is most difficult to mobilize. For coarse-grained rivers, the threshold-limiting material is the gravel that comprises the bed and bank toe. In contrast, the threshold-limiting material in sand-bedded rivers is not the bed material, but the cohesive mixture of mud and sand (and vegetation) that makes up the toe of the river bank. Thus, we posit that it is the difference in entrainment threshold between the non-cohesive bed and cohesive banks that facilitates suspended-sediment transport in sandy rivers. We expect that, in very fine-grained mud channels, the threshold-limiting material is the mud that makes up both the bed and the bank toe. Consideration of the slope- or grain-size-dependence of the critical Shields stress shows that alluvial rivers are bi-modal in terms of transport stage and bed-material grain size, and that these modes correspond generally (but not always) to bed-load gravel rivers and suspension sand rivers. We acknowledge, however, that other factors unaccounted for in our simple analysis must also play a role. For example, form drag due to roughness on multiple scales (grains, bed forms, bars, meanders) can drastically change the effective bed stress (Kean and Smith, 2006). We suspect that proper accounting of flow resistance would reveal a stronger signal of near-threshold organization. Of course, determination of the entrainment threshold at the bank toe is needed to provide direct confirmation of the hypothesis we propose here. Experiments have qualitatively demonstrated the influence of cohesion on channel geometry (Kothyari and Jain, 2008; Tal and Paola, 2007; Braudrick et al., 2009), but a systematic examination of channel shape as a function of increasing cohesion in sand-mud mixtures is necessary to demonstrate the viability of the threshold-limiting hypothesis.

*Author contributions.* K.B.J.D performed the research and analyzed the data, D.J.J. supervised the research, and both authors wrote the paper.

*Competing interests.* The authors declare no conflict of interest.

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