

Validity of the river system representation

The conventional morphodynamic model of a river reach at short and intermediate scale, usually consists of the standard one-dimensional partial differential equations (de St. Venant, Exner and Hirano with one or more active layers) and the annexed semi-empirical algebraic equations that describe the water flow and the sediment motion in the stream and in the bottom, complemented by the necessary boundary conditions. To simulate the evolution of a river system, this set of equations should in principle be applied to the hydrographic network of the river, together with a time-dependent and space-dependent predictor of water and sediment production (by surface- and mass erosion) entering the streams from the basin slopes.

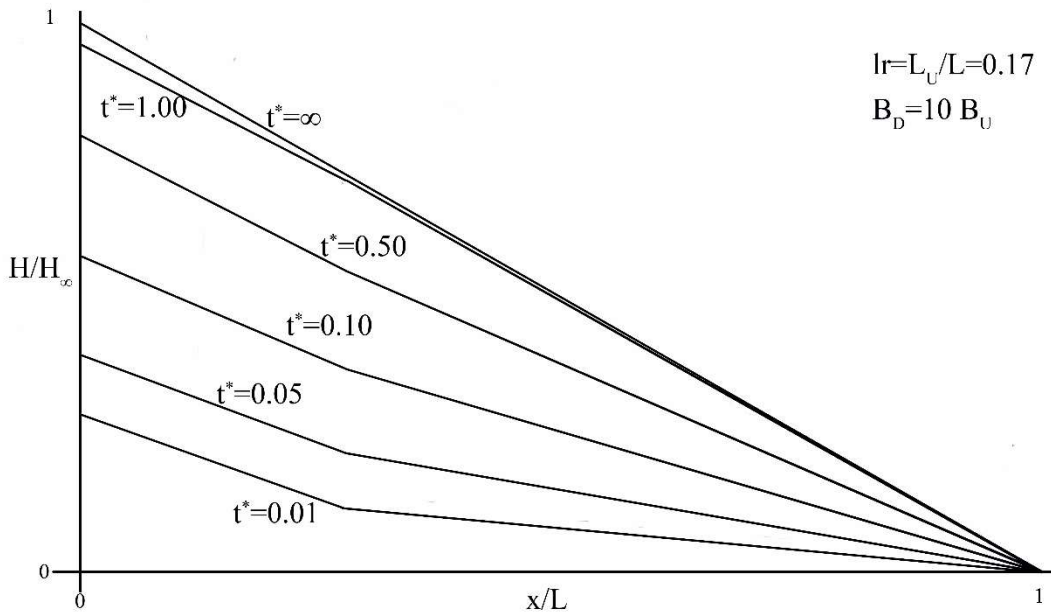
To reduce the computational effort, the partial differential equations are often simplified (Fasolato et al., 2011) and the model schematization limited to the largest branches of the hydrographic network. Moreover, if the analysis is devoted to long-term simulations, the partial differential equations are properly averaged (to include algebraically the effects of the shorter time-scales) over pluriannual time-steps, as mentioned in the previous section. Despite all these expedients for reducing the numerical complexity, however, the computational effort remains largely high. Even more important, similar models are still too much detailed to highlight the salient aspects of the long-term morphodynamics for broad categories of river basins. For this reason, we decided to go further with the process of aggregating and averaging the basin model and yet preserving the essential peculiarities of each river system.

First, following a procedure widely shared by both hydraulic engineers and geomorphologists, we aggregated the tree-shaped hydrographic network into a number of reaches connected in series and representing the different parts of the main watercourse. A second important simplification consists in integrating the 1D partial differential equations over the length of each reach. In this way, these partial differential equations are transformed into ordinary differential equations, while the physical description of the basin is expressed in terms of “concentrated” (0D) parameters, much more concise and manageable than the “distributed” (1D) parameters. The limit-case of aggregating the entire hydrographic network into one single reach of uniform width fed from upstream corresponds, in fact, to the case of the “sediment fed flume” mentioned by the Reviewer. Although very synthetic, the sediment fed flume scheme has been utilized by several researchers (often assuming uniform grain size material) for analysing the reaction of rivers to different types of perturbation. These applications (both with 1D and 0D approach) show the relevance of the overall parameter “morphological dispersion” closely related to the “response time” of the river (Paola et al, 1992 b; Castelltort and van Driessche, 2003; Gupta, 2007; Di Silvio and Nones, 2014 etc.).

The 0D single-reach single-grainsize scheme, however, is, by definition, unable to investigate the specific aspects we are interested in: the general tendency of rivers to display a concave profile and a grainsize fining in the downstream direction. To analyze this behaviour, adding the minimum complication to the single reach scheme, we applied a 0D two-reach two-grainsize scheme. This approach, distinguishing an upland and a lowland segment of the basin, permits in fact to portray a vast variety of river systems. As indicated in Section 2.2 of the paper, the partition between the two segments is made in such a way as to mimic the basic structure of the hydrographic network and to put into account the planimetric characteristics of each reach (length and width) of the valley, inferred from any geographical data base.

The long-term evolution of the system is provided by eq. 10 of the paper, when initial and boundary conditions are prescribed. If the boundary conditions (equivalent waterflow $Q(t)$, sediment input $G(t)$ and input grainsize composition $\alpha_G(t)$) remain constant, the system will evolve extremely slowly from a flat hypothetical initial condition (orogeny) towards the already mentioned

“equilibrium conditions”, as shown in the Figure reported here.



The evolving profiles shown in the Figure are expressed in non-dimensional terms as they represent the solution of eq.12, namely the non-dimensional formulation of eq.10. Eq.12 indicates that the long-term evolution of the system towards a uniform slope I_∞ and a uniform bed composition β_∞ can be described by only *five independent non-dimensional parameters* (br , lr , α_G , I_∞ and β_∞), which incorporate all the relevant morphometric quantities describing the river system and its boundary conditions.

Namely:

- B , constant river width (due to the assumption of constant Q) measured near the mouth;
- B_U and B_D , averaged valley widths (allowing the river wandering), respectively for the upstream and downstream reach;
- L_U and L_D , length of the main watercourse (properly defined) for the upstream and downstream reach;
- Q , constant (equivalent) waterflow, accounting for the hydrological variations at flood and seasonal scale;
- G , constant long-term sediment input (surface and mass erosion) from the basin slopes;
- α_G , grainsize sediment composition of the sediment input.

Note that, by assuming any reasonable sediment transport formula for a sediment mixture (e.g. eq. 3 of the paper), the prescribed boundary conditions Q , G and α_G univocally define the equilibrium slope I_∞ and the equilibrium bed composition β_∞ of the river. In this way, the non-dimensional quantities I_∞ and β_∞ appearing in eq. 12 constitute a proxy of the prescribed equivalent waterflow Q , not explicitly included in the same equations.

The five independent non-dimensional parameters, moreover, identify together another important quantity; namely, the “filling time” $T_{fill} = V_\infty/G$, which is the only fundamental parameter in the one-reach one-grainsize scheme.

In the present model, the filling time (although defined in a somewhat more complex way, see eq.13 of the paper) is still the scaling quantity of the long-term time t , but is not anymore *proportional to the response time* of the river system as in the case of the one-reach one-grainsize scheme. In the two-reach, two-grainsize model, the five parameters br , lr , α_G , I_∞ and β_∞ define indirectly the rapidity of reaction and, more in general, the long-term behaviour of the system.

The evolution of a certain river system, characterized by the five non-dimensional parameters, is represented in the Figure by a series of profiles at different values of the non-dimensional time t^* . A similar representation may also be obtained for the bottom composition of the two reaches. From these representations, however, one cannot immediately perceive how evolve the quantities we are more interested in, namely the overall *concavity* and *fining* of the system.

The model tests reported in the paper show that both the non-dimensional concavity $X(t^*)$ and fining $\Phi(t^*)$ (by definition equal to zero for $t^*=0$ and $t^*=\infty$) tend to present a persistent maximum value within an intermediate range of t^* ($0.1 < t^* < 10$), depending upon the five parameters characterizing the river basin. In any case, it seems reasonable to presume that maximum values for $X(t^*)$ and $\Phi(t^*)$ correspond more or less to the “quasi-equilibrium configuration” of the present rivers. We plan therefore to verify the validity of this hypothesis by simulating the long-time evolution at geological scale of several real rivers, in a wide range of morphoclimatic conditions, i.e. size, hydrology and lithology of the basin.

References

Castelltort, S., Van Den Driessche, J.: How plausible are high-frequency sediment supply-driven cycles in the stratigraphic record? *Sedimentary Geology* 157, 3-13, 2003.

Di Silvio, G., and Nones, M.: Morphodynamic reaction of a schematic river to sediment input changes: Analytical approaches. *Geomorphology*, 215, 74-82, doi: 10.1016/j.geomorph.2013.05.021, 2014.

Fasolato, G., Ronco, P., Langendoen, E. J., and Di Silvio, G.: Validity of Uniform Flow Hypothesis in One-Dimensional Morphodynamics Models. *Journal of Hydraulic Engineering*, 137(2), 183–195, doi: 10.1061/(ASCE)HY.1943-7900.0000291, 2011.

Gupta, A.: *Large Rivers: Geomorphology and Management*. eds. A. Gupta, Wiley, 2007

Paola, C., Heller, P. L. and Angevine, C. L.: The large-scale dynamics of grain-size variation in alluvial basins, 1: Theory, *Basin Research*, 4, 73–90, 1992b.