9/6/2018

Dear Dr. Willenbring,

Thank you for considering publication in *Earth Surface Dynamics* the manuscript "Short Communication: Increasing vertical attenuation length of cosmogenic nuclide production on steep slopes negates topographic shielding corrections for catchment erosion rates". I appreciate the constructive feedback from three reviews, and have revised the manuscript to address reviewer comments, as outlined in detail below (reviewer comments are *italicized*). I hope you will find the revised manuscript ready for publication.

Sincerely,
Roman DiBiase

**Reviewer #1 comments (Anonymous)**

1) *Gosse and Phillips (2001) noted on pg 1521 that increasing effective attenuation length due to increased surface slope is exactly offset by foreshortening. Is that not also the case here?*

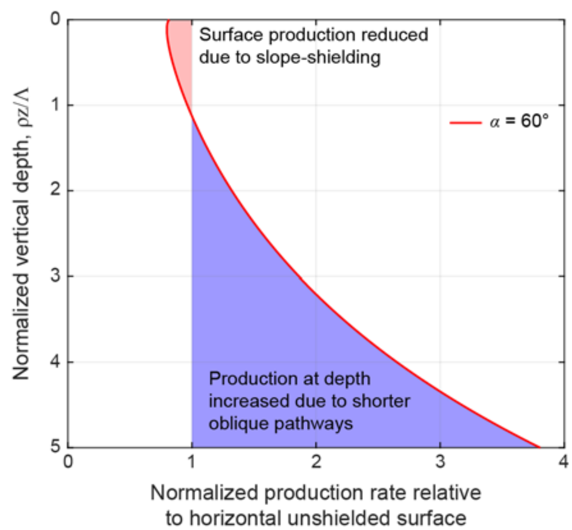
   **Response:** As noted by Dunne et al. (1999), the competing factors described by Gosse and Phillips (2001) of changing effective attenuation length and foreshortening are already accounted for in the formulation of Equation 1 in the present manuscript (Equation 11 of Dunne et al., 1999). Specifically, these geometrical effects fall out by framing the problem using the mass attenuation length for collimated radiation, $\lambda$, and directly determining the mass length, $d$, along a ray path from the surface to a given position ($x$, $z$) in the subsurface. The consequence of this explicit treatment of exponential attenuation in the ray path direction is nicely shown by Figure 5 and Equations 16 and 17 of Dunne et al. (1999), highlighting the difference between $\lambda$ and the more commonly used attenuation length, $\Lambda$.

   I also include the helpful explanation of the foreshortening issue by G. Balco in his review below:

   > "The issue of "foreshortening" commonly comes up in this discussion as a point of ambiguity because most people with an Earth science background are used to thinking about this effect in the context of remote sensing or radiative heating, in which a finite amount of incident radiation is spread out over a larger area when the incidence angle is lower. The key difference is that cosmogenic-nuclide production happens within a volume, not on a surface, so the radiation-incident-on-a-surface model is not the right way to think about this, and the treatment of foreshortening that would be used in that context is not applicable here. The clearest discussion of this issue is in Dunne, and the present paper follows Dunne and gets it right."

**2)** *It is counterintuitive that the effective shielding factor can be greater than 1. In this model, this is due to the large increase in vertical attenuation length. Previous authors have noted that attenuation path length decreases on sloped surfaces due to increasingly oblique incidence angles reducing the intensity. This discrepancy should be addressed. On a similar topic, it is not clear is how production rates were dealt with here. For a give incoming flux, increasing the attenuation path length must decrease the near surface production rate as it implies fewer collisions per mass length. The implication is that as normalized effective attenuation length increases, the normalized effective surface production rate must decrease. This would offset the effect of increasing attenuation length (requiring a topographic shielding correction again). This could be treated as equivalent to foreshortening.*

**Response:** Similar to the above response to point **1**, the effects of changing attenuation path length and "foreshortening" are already accounted for in the model. The counterintuitive result of shielding factors greater than 1 emerges entirely because of treating the problem in the vertical direction rather than normal to the surface – the cosine correction from slope-normal to vertical frame of reference quickly overcomes the shielding effect for sloped surfaces. To clarify this transition, I now include a new figure highlighting the change in normalized production rate as a function of depth for a 60 degree sloping surface (Fig. 5):




"Figure 5: Plot of normalized production rate relative to horizontal unshielded surface as a function of normalized vertical depth for a 60° slope with no additional skyline shielding. Near the surface, production rates are decreased due to slope shielding of incoming cosmic radiation; however, production rates at depth increase relative to the unshielded case due to additional radiation along shorter oblique pathways (Fig. 1c)."

**3)** *There is an important potential talking point here on how erosion/denudation is defined in cosmogenic nuclide studies. Both lowering rates (i.e. m My-1) and mass loss rates (i.e. t km-2 yr-1) tend to be based on 2D areas. This is in line with the definition of attenuation length presented here. However, it is not clear that this is the appropriate definition (of either erosion or attenuation) for the real world. A broader discussion around the implications of setting the attenuation path length to the vertical could be* *quite useful since previous authors rotate the coordinate system to determine attenuation path length perpendicular to the surface. The vertical definition makes sense since we tend to perform shielding calculations on a DEM and often define erosion as a lowering rate. However, it seems unlikely that an 80° slope would be eroding vertically. In this case, using a vertical attenuation path length would result in an artificial increase in production rate (i.e. it would appear as less shielding, as found here). The 'true' surface area in this case is also probably the 3D surface area and erosion would be spread across a larger area (essentially the foreshortening argument applied to erosion). I recognise that this is a bit circular, but it highlights the need for a clearer explanation around coordinate definitions.*

**Response:** I now include a new paragraph at the beginning of section 3 that clarifies and justifies the use of a vertical reference frame:

> "Throughout the analysis below, both the effective mass attenuation length, $\Lambda_{eff}$, and erosion rate, $E$, are defined in the vertical, rather than slope-normal direction. The vertical (with respect to the geoid) reference frame was chosen for three reasons. First, most studies report erosion rate as a vertical lowering rate and assume primarily vertical exhumation pathways. Second, treatment of slope-normal processes introduces a grid-scale dependence of erosion and shielding calculations that varies with topographic roughness (Norton and Vanacker, 2009). Third, for the case of uniform erosion rate, the resulting shielding calculations do not depend on the choice of reference frame, as long as the orientation of $\Lambda_{eff}$ and $E$ are defined similarly." (Page 4, Line 26-31)

Additionally, I added a sentence to the discussion highlighting the limitations of this approach for treating landscapes dominated by cliff retreat:

> "For steep catchments with spatially variable quartz content or erosion rate, direct calculation of shielding at depth is likely needed to calculate the spatially distributed total effective shielding parameter. In particular, shielding calculations in landscapes dominated by cliff retreat are poorly suited for treatment in a vertical reference frame (e.g., Ward and Anderson, 2011)." (Page 8, Line 26-29)

**Reviewer #2 comments (G. Balco)**

**4)** *I strongly encourage the author to check the calculations again. I don't see any reason to think anything is not correct, but I haven't attempted to fully replicate all the calculations. The main thing I haven't verified is the effect of the slope angle on the apparent attenuation length in the vertical direction – obviously, when you set the geometry up this way, in the vertical direction has to converge on infinity for a vertical cliff, so the principle is clearly correct, but it would be embarrassing to get this wrong and I suggest checking it carefully.*

**Response:** I went through and checked all calculations/equations again, and cleaned up a few typos (the code and results are unchanged). The inclusion of the Matlab code as a supplement should help others reproduce these calculations.

**5)** *It does not appear at present that the computer code used to do the calculations is included with the paper or otherwise available (or, at least, I couldn't find it via the journal web interface). This is a deficiency and the author should correct it.*

**Response:** I agree – this was an oversight on my part, and I have posted the Matlab code on GitHub and include a script to generate Figs. 3,4, and 6 as a supplement to this manuscript (referred to in the acknowledgements).

**6)** *Page 1, line 15, in abstract. Using 'catchment mean effective shielding factor of one' is premature here and makes no sense to the reader, because you haven't defined it yet. Of course, you're not talking about the shielding factor as usually defined ($P/P_0$), you're talking about the effect of both production and attenuation ($P\Lambda/P_0\Lambda_0$). Instead you should say something like 'for flat catchments, the effect of increasing attenuation length due to shielding offsets the effect of decreasing surface production rate, resulting in no change in surface nuclide concentrations in relation to the unshielded case.' Something like that, anyway. Clunkier but necessary to be more clear for purposes of the abstract.*

**Response:** The abstract has been reworded for clarity and now includes a definition of the total effective shielding factor:

> "The most common method for calculating topographic shielding accounts only for the reduction of nuclide production rates due to shielding at the surface, leading to catchment-mean corrections of up to 20% in steep landscapes, and makes the simplifying assumption that the effective mass attenuation length for a given nuclide production mechanism is spatially uniform. Here I evaluate the validity of this assumption using a simplified catchment geometry with mean slopes ranging from 0° to 80° to calculate the spatial variation in surface skyline shielding, effective mass attenuation length, and the total effective shielding factor, defined as the ratio of the shielded surface nuclide concentration to that of an unshielded horizontal surface. For flat catchments (i.e., uniform elevation of bounding ridgelines), the effect of increasing vertical attenuation length as a function of hillslope angle and skyline shielding exactly offsets the effect of decreasing surface production rate, indicating that no topographic shielding correction is needed when calculating catchment-mean vertical erosion rates. For dipping catchments (as characterized by a plane fit to the bounding ridgelines), the catchment-mean surface nuclide concentrations are also equal to that of an unshielded horizontal surface, except for cases of extremely steep range-front catchments, where the surface nuclide concentrations are counterintuitively higher than the unshielded case due to added production from oblique cosmic ray paths at depth." (Page 1 Line 9-20)

**7)** *Page 2, line 6. "In general shown to be robust." Actually, I am only aware of the one paper by Granger that actually validates the method against sediment fluxes. Are there others? In any case, I'm not sure this is true at all. Maybe omit this.*

**Response:** I've omitted this admittedly vague statement.

**8)** *Page 3, line 25. Why $\Lambda$ increases could be explained more clearly here. You never really say why this happens, which is twofold: for topographic shielding, you have excluded cosmic rays with oblique incidence angles that stop at relatively shallow depths (shallow* *in a vertical direction of course), leaving a higher fraction of near-vertical ray paths that stop at deeper depths, and then for sloping surfaces the vertical coordinate system means that points that are deep in the vertical coordinate aren't deep in a slope-normal direction any more.*

**Response:** In section 2, my main focus is introducing the general theoretical framework for the treatment of shielding at depth. The effect of surface slope does not enter until the model geometry setup in section 3, so I think it is perhaps premature to discuss the influence of the vertical coordinate system on $\Lambda$ up front. Instead, I explain why $\Lambda$ increases in the presentation of model results (Page 7, Line 14-26).

**9)** *Pages 3-4, end of section 2 and beginning of 3. I think it would be helpful here to introduce the simplest possible case, that is, stable nuclide, spallation only, as described above, where $N = P\Lambda/\varepsilon$. That makes it clear that increasing $\Lambda$ has an offsetting effect on reducing P. As it is you just jump right into the complicated watershed-with-manypixels case without really making the basic relationship clear.*

**Response:** This is a good idea. I now briefly summarize the simple case to highlight how $P$ and $\Lambda$ contribute to the surface nuclide concentration:

> "The importance of accounting for both changes in surface production rate, $P$, and changes in the effective mass attenuation length, $\Lambda_{eff}$, is illustrated by the analytical solution for nuclide concentration, $C$, measured on a steadily-eroding surface for a stable nuclide with an exponential decrease of production rate with depth:
>
> $$C = P\Lambda_{eff}/E \quad (5)$$
>
> where $E$ is erosion rate (g cm$^{-2}$ yr$^{-1}$) (Lal, 1991). From Eq. (5) it is clear that increasing $\Lambda_{eff}$ counters the effect of decreasing $P$ in determining the surface nuclide concentration (or alternatively for inferring erosion rate)." (Page 4, Line 9-14)

**10)** *Page 5, line 15-20. This is a little misleading as written, because of course for a flat surface the depth-dependence is not exactly exponential either, it's just a good approximation. How good depends on how you define the angular dependence of the flux. See the Argento paper.*

**Response:** Good point. I revised this section to make it clear that 1) the exponential decrease for unshielded horizontal surfaces is expected by Equation 1; and 2) any deviation from exponential production with depth will lead to inaccuracies using the analytical solution for steady erosion:

> "Although spallogenic production of cosmogenic nuclides following Eq. (1) is well-described by an exponential decrease with depth for horizontal unshielded surfaces, this is not true in general for shielded samples (Dunne et al., 1999). The effective vertical mass attenuation length, $\Lambda_{eff}(x)$, is approximated by the vertical depth below the surface at which the shielding factor is 5% of the surface shielding (i.e., 3 e-folding lengths) such that:
>
> $$S\left(x, \frac{3\Lambda_{eff}(x)}{\rho}\right) = 0.05S(x, 0). \quad (9)$$

If nuclide production as a function of depth deviates from an exponential decline, it is inaccurate to use the analytical relationship between surface sample concentration, $C(x)$ (atoms $g^{-1}$), and steady-state vertical erosion rate, $E$ (g $cm^{-2}$ $yr^{-1}$), typically applied to eroding samples" (Page 5, Line 26 to Page 6, Line 4)

**11)** *Page 7, top, and in general throughout sections 2 and 3 as well. It is rather important to understanding all this that the reader really realizes that the z coordinate is always vertical, rather than surface-normal. There is nothing actually wrong with the paper here, but I suggest reminding the reader of this more times than seems necessary at first. For example, this would be a good place to remind the reader of this.*

**Response:** Good suggestion. I have added clarification of the vertical frame of reference for depth, *z*, throughout the manuscript. I also added a paragraph at the beginning of Section 3 to emphasize this:

"Throughout the analysis below, both the effective mass attenuation length, $\Lambda_{eff}$, and erosion rate, $E$, are defined in the vertical, rather than slope-normal direction. The vertical (with respect to the geoid) reference frame was chosen for three reasons. First, most studies report erosion rate as a vertical lowering rate and assume primarily vertical exhumation pathways. Second, treatment of slope-normal processes introduces a grid-scale dependence of erosion and shielding calculations that varies with topographic roughness (Norton and Vanacker, 2009). Third, for the case of uniform erosion rate, the resulting shielding calculations do not depend on the choice of reference frame, as long as the orientation of $\Lambda_{eff}$ and $E$ are defined similarly." (Page 4, Line 26-31)

**12)** *Page 8, line 20-ish. For muons, the penetration depth is so long and the associated time-to-equilibrium is likewise so long that there is really no plausible scenario in which the steady state assumption is ever correct. Thus, calculating the surface nuclide concentration due to muon production for a given erosion rate is pretty wildly speculative to begin with. Like, for rapidly eroding catchments where a significant fraction of production is by muons, you can get almost factor-of-two differences between, for example, assuming steady-state erosion and assuming that the erosion rate sped up recently. Possibly worth pointing this out here.*

**Response:** Good suggestion. This complication is now addressed:

"…the assumption of steady lowering is likely to be increasingly inappropriate for rapidly eroding landscapes characterized by a significant contribution of muonogenic production or slowly-eroding landscapes where $^{10}Be$ concentrations integrate over glacial-interglacial climate cycles." (Page 9, Line 16-18)

**13)** *Page 8, line 30. This is funny because it is written from the perspective of people who care about steep, rapidly eroding basins, where one worries that the steady-state assumption is wrong because of stochastic landslides/slope failures. In contrast, if you are a person who cares about slowly eroding basins, instead you worry that the steady-state assumption is wrong because there is no way that erosion rates haven't been unsteady due to glacial-interglacial-scale climate changes. You should probably mention both things here.*

**Response:** Good suggestion. I revised the paragraph in question to address this complication:

> "…the assumption of steady lowering is likely to be increasingly inappropriate for rapidly eroding landscapes characterized by a significant contribution of muonogenic production or slowly-eroding landscapes where $^{10}Be$ concentrations integrate over glacial-interglacial climate cycles." (Page 9, Line 16-18)

**14)** *Page 8, line 30. One shouldn't say "should" in papers (apparently only in reviews). Just state the facts — "The analysis here shows that accounting only for surface skyline shielding yields incorrect results." Let the reader decide what to do about it. The conclusions on the next page are much better in this regard.*

**Response:** Thanks—I should have caught this earlier. Fixed:

> "Nonetheless, in all cases accounting only for surface skyline shielding (e.g., Codilean, 2006) without including its concurrent influence on the effective attenuation length yields incorrect results." (Page 9, Line 20-22)

**Reviewer #3 comments (D. Scherler)**

**15)** *My main point is in line with what reviewer #1 already mentioned. I think it is important to emphasize that the reference frame, in which the attenuation length increases with increasing slope angle, is vertical with respect to the geoid and not the surface itself. That basically means that we assume that all particles approaching the Earth's surface follow trajectories that are normal to the geoid. While this assumption appears reasonable for hillslope angles <30° or so, to me it appears unreasonable for very steep hillslope angles, where the described effect is most pronounced. When standing in front of a rock face that is inclined 60° or more, I guess that most people would think the rock wall retreats and not that it lowers. The resulting particle trajectories would thus be less steeply inclined with respect to the surface and the effective attenuation length would not be that large. As a result, the shielding effect would likely be significant, hence lowering the surface production rates; but there would be no counter-acting effect due to increasing attenuation length. Dylan Ward and Bob Anderson, for example, looked at steep hillslopes in glaciated landscapes and assume slope-normal trajectories (Ward and Anderson, 2010, Earth Surf. Process. Landforms 36, 495-512). I think this is an important point that needs to be better exposed in the beginning and discussed later on.*

**Response:** I now include a new paragraph at the beginning of section 3 that clarifies and justifies the use of a vertical reference frame:

> "Throughout the analysis below, both the effective mass attenuation length, $\Lambda_{eff}$, and erosion rate, $E$, are defined in the vertical, rather than slope-normal direction. The vertical (with respect to the geoid) reference frame was chosen for three reasons. First, most studies report erosion rate as a vertical lowering rate and assume primarily vertical exhumation pathways. Second, treatment of slope-normal processes introduces a grid-scale dependence of erosion and shielding calculations that varies with topographic

> roughness (Norton and Vanacker, 2009). Third, for the case of uniform erosion rate, the resulting shielding calculations do not depend on the choice of reference frame, as long as the orientation of $\Lambda_{eff}$ and $E$ are defined similarly." (Page 4, Line 26-31)

Additionally, I added a sentence to the discussion highlighting the limitations of this approach for treating landscapes dominated by cliff retreat:

> "For steep catchments with spatially variable quartz content or erosion rate, direct calculation of shielding at depth is likely needed to calculate the spatially distributed total effective shielding parameter. In particular, shielding calculations in landscapes dominated by cliff retreat are poorly suited for treatment in a vertical reference frame (e.g., Ward and Anderson, 2011)." (Page 8, Line 26-29)

**16)** *My second point is related: is it meaningful to show on Figure 4, curves for inclinations up to 80°? I would argue that there hardly exist catchments with mean hillslope angles of >40°. Such angles may exist locally, but are they relevant for the problem that you discuss? One solution could be to have the y-axis in log scaling, to emphasize the curves with angles <40°, which currently are hard to decipher. As you rightfully note in your discussion, the effect of topographic shielding is small in most cases. All the curves >40° are thus steering the readers attention towards cases that actually don't matter.*

**Response:** Although it is true that few catchments exist with slopes >50-60°, I think it is important to highlight the extreme cases to emphasize: 1) the catchment shielding correction is not simply smaller than previously assumed, but cancels out entirely for most watersheds; and 2) the spatial variability of factors that control surface nuclide concentration on steep hillslopes. I also find it helpful to better intuit the model behavior by including a wide range of slopes.

I tried changing the y-axis on the plots in Figure 4 to a logarithmic scale, but this does not actually help much the visualization as there is only a factor of 4-5 variation in the parameters being plotted.

**17)** *P2, Line 18: You cite Norton and Vanacker (2009), but you don't discuss the main point of their paper in any detail later one. I think you should, because they propose that topographic shielding measured from coarse DEMs may underestimate the actual shielding. If that were true, does it mean that, after taking different attenuation lengths into account, there might still be a net shielding effect?*

**Response:** I now include an additional citation to Norton and Vanacker in discussing the potential influence of rough topography:

> "However, while not entirely transferable to arbitrarily rough topography (e.g., Norton and Vanacker, 2009), Fig. 4c suggests that for slopes less than 40°, the total effective shielding factor does not vary significantly across the hillslope." (Page 8, Lines 25-26)

Note that the slopes measured on coarse DEMs are also typically lower than those of high-resolution DEMs, such that the increase in attenuation length will be commensurately smaller. It is not straightforward to model the effects of surface roughness, but my intuition is that these effects will cancel out for rough surfaces and lead to similar interpretations of (vertical) erosion rate.

**18)** *P3, Line 28: Probably here you could mention more explicitly the assumed particle trajectory. You actually say "vertical depth below the surface", but that's ambiguous. Vertical with respect to the surface or the geoid?.*

**Response:** This is a good point to make explicitly. At this point in the manuscript, I have not yet introduced complications associated with sloped surfaces. I add a note about vertical exhumation pathways on Page 6:

> "$t_{surface} = t_0 + \rho z_0/E$ is the time it takes for a rock parcel to travel from depth $z_0$ to the surface (assuming a vertical exhumation pathway)." (Page 6, Line 13-14)

**19)** *P4, Line 12: Mention here already if the model valley is inclined?*

**Response:** I revised this sentence to emphasize the model geometry:

> "Because the ridgelines have uniform elevation, there is no net dip to the catchment; the effect of valley inclination will be assessed in Section 3.3." (Page 5, Line 3-4)

**20)** *P5, Line 11: How good is this approximation?*

**Response:** It depends on the application, and so it difficult to state concisely here. Mainly, I use this as a way to frame the need for characterizing the effective mass attenuation length numerically according to Eq. (10).

**21)** *P8, Line 17: The factor 3 emerges only for hillslopes >80°. I think it would be better here to refer to commonly observed hillslope angles, given the title of this chapter, and not extreme cases.*

**Response:** The factor of 3 and 30% values are both for extreme cases – I added a sentence to highlight a more typical range of effective attenuation length increase due to collimation and slope-effects:

> "However, the magnitude of changes in the effective mass attenuation length due to shielding-induced collimation is at most 30% (Dunne et al., 1999), compared to the potentially factor of 3 or more increase due to shorter oblique radiation pathways on very steep slopes (Fig. 1c; Fig. 4b). For hillslope gradients commonly observed in cosmogenic nuclide studies of steep landscapes (30-40°), the increase in effective mass attenuation length due to shielding-induced collimation and slope effects are 2-5% and 6-15%, respectively (Dunne et al., 1999; Fig. 4b). The dependence of Λ on atmospheric depth, which is typically not accounted for in catchment erosion studies, is minor (<10% for extreme case of catchment with 4 km of relief (Marrero et al., 2016)) compared to the above slope effect for most landscapes." (Page 9, Line 4-11)

**22)** *Figure 5: I'm curious whether it is ok to refer to mean hillslope angles? Pixel-based hillslope angles are often measured using the steepest descent algorithm. In other words, this algorithm will give you always the maximum slope angle possible. Is that the one you want to have for inferring attenuation length effects? Or would you rather want to refer to hillslope angles measured by fitting a plane to each pixel and its surrounding neighbors, or something like this?*

**Response:** For catchment-mean hillslope angles, there is not too much difference between measuring local slope along a steepest descent path versus fitting a plane to a local neighborhood. The biggest difference in resulting values is related to the difference in the scale of measurement (i.e., calculating over 2 pixels vs. 3 pixels or more). For the case of a planar slope, the two measurements are of course equal. For the data presented in Figure 5, I suspect the difference would be imperceptible, and much smaller than issues related to DEM quality/resolution.

# Short Communication: Increasing vertical attenuation length of cosmogenic nuclide production on steep slopes negates topographic shielding corrections for catchment erosion rates

Roman A. DiBiase [1]

[1]Department of Geosciences, Pennsylvania State University, University Park, Pennsylvania, 16802, USA

*Correspondence to*: Roman A. DiBiase (rdibiase@psu.edu)

**Abstract.** Interpreting catchment-mean erosion rate from in situ produced cosmogenic $^{10}$Be concentration in stream sands requires calculating the catchment-mean $^{10}$Be surface production rate and effective mass attenuation length, both of which can vary locally due to topographic shielding and slope effects. The most common method for calculating topographic shielding accounts only for the ~~effect~~ reduction of nuclide production rates due to~~of~~ shielding at the surface, leading to catchment-mean corrections of up to 20% in steep landscapes, and makes the simplifying assumption that the effective mass attenuation length for a given nuclide production mechanism is spatially uniform. Here I evaluate the validity of this assumption using a simplified catchment geometry with mean slopes ranging from 0° to 80° to calculate the spatial variation in surface skyline shielding, effective mass attenuation length, and the total effective shielding factor~~.~~, defined as the ratio of the shielded surface nuclide concentration to that of an unshielded horizontal surface~~for catchments with mean slopes ranging from 0° to 80°~~. For flat catchments (i.e., uniform elevation of bounding ridgelines), the effect of ~~increase~~ increasing ~~in~~ vertical ~~effective~~ attenuation length as a function of hillslope angle and skyline shielding exactly offsets the effect of decreasing surface production rate, ~~leads to a catchment-mean total effective shielding factor of one, implying~~ indicating that no topographic shielding ~~factor~~ correction is needed when calculating catchment-mean vertical erosion rates. For dipping catchments (as characterized by a plane fit to the bounding ridgelines), the catchment-mean ~~total effective shielding factor is also one~~surface nuclide concentrations are also equal to that of an unshielded horizontal surface, except for cases of extremely steep range-front catchments, where the ~~shielding correction is~~surface nuclide concentrations are counterintuitively ~~greater than one~~higher than the unshielded case due to added production from oblique cosmic ray paths at depth. These results indicate that in most cases, topographic shielding corrections are inappropriate for calculating catchment-mean erosion rates, and only needed for steep catchments with non-uniform distribution of quartz and/or erosion rate. By accounting only for shielding of surface production, existing shielding approaches introduce a slope-dependent systematic error that could lead to spurious interpretations of relationships between topography and erosion rate.

## 1 Introduction

Measurement of in situ produced cosmogenic $^{10}$Be concentrations in stream sediments has rapidly become the primary tool for quantifying catchment-scale erosion rates over timescales of $10^3$-$10^5$ y (Brown et al., 1995; Granger et al., 1996; von Blanckenburg, 2006; Portenga and Bierman, 2011; Codilean et al., 2018). Although requiring a number of simplifying assumptions about the steadiness of erosion and sediment transport (Bierman and Steig, 1996), erosion rates determined from $^{10}$Be concentrations in stream sediments ~~have in general shown to be robust and~~ have yielded insight to a number of key questions in tectonic geomorphology regarding the sensitivity of erosion rates to spatiotemporal patterns of climate, tectonics, and rock strength (e.g., Safran et al., 2005; Binnie et al., 2007; Ouimet et al., 2009; DiBiase et al., 2010; Bookhagen and Strecker, 2012; Miller et al., 2013; Scherler et al., 2017).

In contrast to point measurements, where a clear framework exists for converting $^{10}$Be concentrations to either a surface exposure age or steady erosion rate (e.g., Balco et al., 2008; Marrero et al., 2016), the interpretation of $^{10}$Be concentrations in stream sediment requires accounting for the spatial variation in elevation, latitude, quartz content, and erosion rate throughout a watershed (Bierman and Steig, 1996; Granger and Riebe, 2014). Additionally, topographic shielding corrections that account for the reduction of cosmic radiation flux on sloped or skyline-shielded point samples (Dunne et al., 1999) are applied to varying degrees for determining catchment-mean production rates. These shielding corrections are either applied at the pixel level (e.g., Codilean, 2006), catchment level (e.g., Binnie et al., 2006), or not at all (e.g., Portenga and Bierman, 2011). Although typically small (<5%), topographic corrections can be as large as 20% for steep catchments (e.g., Norton and Vanacker, 2009). Because these corrections vary as a function of slope and relief, any systematic corrections can influence interpretations of relationships between topography and erosion rate.

The pixel-by-pixel skyline shielding algorithm of Codilean (2006) results in the largest topographic shielding corrections, and has gained popularity due to its ease of implementation in the software packages TopoToolbox (Schwanghart and Scherler, 2014) and CAIRN (Mudd et al., 2016), the latter of which was used to recalculate published $^{10}$Be-derived catchment erosion rates globally as part of the OCTOPUS compilation project (Codilean et al., 2018). A key simplification of the Codilean (2006) approach is that it accounts only for the skyline shielding of surface production, and not for the change in shielding with depth, which determines the sensitivity of the effective mass attenuation length for nuclide production as a function of surface slope and skyline shielding (Dunne et al., 1999; Gosse and Phillips, 2001). Because a change in the effective mass attenuation length will directly influence the inferred erosion rate of a sample (Lal, 1991), the full depth-integrated implications of topographic shielding must be accounted for when inferring catchment erosion rates from $^{10}$Be concentrations in stream sediments.

Here I model the shielding of incoming cosmic radiation flux responsible for spallogenic production at both the surface and at depth for a simple catchment geometry to evaluate as a function of catchment slope and relief the total effect of topographic shielding ~~factor~~ on surface nuclide concentrations and the partitioning of shielding into surface skyline shielding and changes to the effective mass attenuation length. I then apply this framework to catchments that have a net dip (i.e., dipping plane fit to boundary ridgelines) and compare calculations of total shielding to those from typical pixel-by-pixel skyline shielding corrections.

## 2 Theory

The incoming cosmic ray intensity, $I(\theta, d)$, responsible for in situ cosmogenic nuclide production by neutron spallation can be most simply described as a function of the incident ray path inclination angle above the horizon ~~of the incident ray path~~, $\theta$, and the mass ~~depth~~distance, $d$ (g cm$^{-2}$), traveled along that pathway:

$$I(\theta, d) = I_0 \sin^m \theta \, e^{-d/\lambda}, \tag{1}$$

where $I_0$ is the maximum cosmic ray intensity at the surface, $m$ is an exponent typically assumed to have a value of 2.3 (e.g., Nishiizumi et al., 1989), and $\lambda$ is the mass attenuation length (g cm$^{-2}$) for unidirectional incoming radiation (Dunne et al., 1999). The mass attenuation length for unidirectional radiation, $\lambda$, differs from the nominal mass attenuation length that describes cosmogenic nuclide production as a function of depth, $\Lambda$, due to the integration of radiation from all incident angles. Assuming $m = 2.3$, a value of $\lambda = 1.3\Lambda$ results in a close match for horizontal unshielded surfaces with exponential production profiles typical of spallation reactions (Dunne et al., 1999; Gosse and Phillips, 2001).

For a horizontal surface sample ($d = 0$), the unshielded total cosmic radiation flux, $F_0$, ~~can be~~is described by:

$$F_0 = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_0 \sin^m \theta \cos\theta \, d\theta \, d\varphi = \frac{2\pi I_0}{m+1}, \tag{2}$$

where $\varphi$ is the azimuthal angle of incoming radiation, and the term $\cos\theta$ accounts for the convergence of the spherical coordinate system. For point samples that are either at depth ($d > 0$) or have an incomplete view of the sky due to topographic shielding by thick ($d \gg \lambda$) objects, the total cosmic radiation flux, $F$, is modulated by a shielding factor, $S$~~$(\theta_0, d)$~~, such that:

$$S\,\sout{(\theta_0, d)} = \frac{F}{F_0} = \frac{m+1}{2\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=\theta_0(\varphi)}^{\pi/2} \sin^m \theta \, e^{-d(\theta,\varphi)/\lambda} \cos\theta \, d\theta \, d\varphi, \tag{3}$$

where $\theta_0(\varphi)$ is the inclination angle above the horizon of topographic obstructions in the direction $\varphi$ and $d(\theta, \varphi)$ varies as a function of both ray path azimuth and inclination angle (Dunne et al., 1999; Gosse and Phillips, 2001).

Equation (3) has two implications for interpreting exposure ages or erosion rates from cosmogenic nuclide concentrations of samples partially shielded by skyline topography ($\theta_0(\varphi) > 0$). First, skyline shielding will reduce the surface production rate of cosmogenic nuclides by a factor of $S_0$:

$$S_0 = \frac{m+1}{2\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=\theta_0(\varphi)}^{\pi/2} \sin^m \theta \cos\theta \, d\theta \, d\varphi. \tag{4}$$

Second, due to shielding of low intensity cosmic radiation below incident angles of $\theta_0(\varphi)$, the effective mass attenuation length, $\Lambda_{eff}$, will increase relative to the nominal mass attenuation length for describing cosmogenic nuclide production as a function of depth, $\Lambda$ (Dunne et al., 1999; Gosse and Phillips, 2001). For calculating surface exposure ages, only the reduction in surface production rate due to skyline shielding need be taken into account, and Eq. (4) is easily calculated for single points in the landscape (e.g., Balco et al., 2008). However, for determining erosion rates both the surface shielding and changing effective attenuation length must be accounted for, which requires solving Eq. (3) numerically as a function of vertical depth below the surface, as described in section 3 below.

The importance of accounting for both changes in surface production rate, $P$, and changes in the effective mass attenuation length, $\Lambda_{eff}$, is illustrated by the analytical solution for nuclide concentration, $C$, measured on a steadily-eroding surface for a stable nuclide with an exponential decrease of production rate with depth:

$$C = P\Lambda_{eff}/E \qquad (5)$$

where $E$ is erosion rate (g cm$^{-2}$ yr$^{-1}$) (Lal, 1991). From Eq. (5) it is clear that increasing $\Lambda_{eff}$ counters the effect of decreasing $P$ in determining the surface nuclide concentration (or alternatively for inferring erosion rate).

## 3 Topographic shielding model for a simplified catchment geometry

### ~~3.1 Simplified catchment geometry and model setup~~

For stream sediment samples that require calculating cosmogenic nuclide production rates across an entire catchment, solving Eq. (3) as a function of depth is presently too~~impractically~~ computationally intensive to be practical. Consequently, numerical implementations of topographic shielding calculations at the catchment scale make the simplifying assumption that $\Lambda_{eff} = \Lambda$, and thus $S = S_0$ (Codilean, 2006; Schwanghart and Scherler, 2014; Mudd et al., 2016), accounting only for the effect of decreasing surface production rate, $P$. ~~.~~Here I use a simplified catchment geometry to solve Eq. (3) and calculate directly the impact of topographic shielding and surface slope on interpretations of catchment erosion rates from cosmogenic nuclide concentrations in stream sediments. For simplicity, I assume that cosmogenic nuclides are produced only by neutron spallation (i.e., $\Lambda$ = 160 g cm$^{-2}$) and that the erosion rate~~, $E$,~~ is high enough that radioactive decay is negligible (i.e., $E$ > 0.01 g cm$^{-2}$ yr$^{-1}$ for $^{10}$Be).

Throughout the analysis below, both the effective mass attenuation length, $\Lambda_{eff}$, and erosion rate, $E$, are defined in the vertical, rather than slope-normal direction. The vertical (with respect to the geoid) reference frame was chosen for three reasons. First, most studies report erosion rate as a vertical lowering rate and assume primarily vertical exhumation pathways. Second, treatment of slope-normal processes introduces a grid-scale dependence of erosion and shielding calculations that varies with topographic roughness (Norton and Vanacker, 2009). Third, for the case of uniform erosion rate, the resulting shielding calculations do not depend on the choice of reference frame, as long as the orientation of $\Lambda_{eff}$ and $E$ are defined similarly.

### 3.1 Simplified catchment geometry and model setup

Catchment geometry is simplified as an infinitely long v-shaped valley with width $2L_h$ and uniform hillslope angle $\alpha$ (Fig. 1). Because the ridgelines have uniform elevation, there is no net dip to the catchment~~,~~ ; the effect of ~~which~~ valley inclination will be ~~explored~~ assessed in Section 3.3. At a horizontal distance from the ridgeline $x$ and vertical depth below the surface $z$, the shielding factor, $S(x, z)$, ~~can be~~is defined as:

$$S(x,z) = \frac{m+1}{2\pi}\int_{\varphi=0}^{2\pi}\int_{\theta=\theta_0(x,L_h,z,\varphi,\alpha)}^{\pi/2} \sin^m\theta\, e^{-d(z,\rho,\theta,\gamma(\alpha,\varphi))/\lambda}\cos\theta\, d\theta\, d\varphi, \qquad (~~5~~6)$$

where $\rho$ is rock density~~, here assumed to be 2.7 g cm$^{-3}$,~~ and $\gamma$ is the apparent dip of the hillslope in the azimuthal direction $\varphi$ (Fig. 1b). ~~The mass attenuation length for unidirectional radiation, $\lambda$, differs from the nominal mass attenuation length that describes cosmogenic nuclide production as a function of depth, $\Lambda$, due to the integration of radiation from all incident angles. Assuming $m = 2.3$, a value of $\lambda = 1.3\Lambda$ results in a close match for horizontal unshielded surfaces with exponential production profiles typical of spallation reactions (Dunne et al., 1999; Gosse and Phillips, 2001).~~ The inclination angle integration limit, $\theta_0$, is a function of topographic skyline shielding inclination, and can be determined geometrically (Fig. 1) as:

$$\tan\theta_0 = \begin{cases} \frac{(x\tan\alpha+z)\cos\varphi}{2L_h-x}, & 0 \le \varphi < \frac{\pi}{2} \\ -\tan\alpha\cos\varphi\,\cancel{\theta} - \frac{z}{x}\cos\varphi, & \frac{\pi}{2} \le \varphi \le \pi \end{cases}. \qquad (~~6~~7)$$

The apparent dip, $\gamma$, can be derived from the model geometry in Fig. 1 as:

$$\tan\gamma = -\tan\alpha\cos\varphi, \qquad (~~7~~8)$$

and the mass distance traveled through rock by a given incident ray as:

$$d = \frac{\rho z\cos\gamma}{\sin(\theta-\gamma)}. \qquad (~~8~~9)$$

Equation (5) was solved numerically for a series of hillslopes over a grid of $(x/L_h = [0,1];\ \rho z/\Lambda = [0,40])$ with horizontal spacing $dx = L_h/500$ and vertical spacing $dz = \Lambda/500\rho$. To characterize mean slope controls on the ~~total~~ shielding factor, $S(x, z)$, the above calculation was applied to nine hillslopes with mean slope, $\alpha$, ranging from 0-80° in 10° increments. Because $L_h >> \Lambda/\rho$ for most natural landscapes, the resulting distribution of shielding factors is independent of hillslope scale.

### 3.2 Calculation of shielding parameters from model results

After applying Eq. (~~5~~6) to a hillslope, it is straightforward to calculate the surface skyline shielding component, $S_0(x) = S(x, 0)$. This skyline shielding component should match the topographic shielding factor determined from the algorithm of

Codilean (2006), so for comparison this parameter was calculated at each pixel in the model catchment using TopoToolbox (Schwanghart and Scherler, 2014). Two additional parameters were calculated at each slope position using Eq. (5): the effective ~~(~~vertical~~)~~ mass attenuation length, $\Lambda_{eff}(x)$, and the total effective shielding factor, $C_{eff}(x)$.

Although spallogenic production of cosmogenic nuclides following Eq. (1) is well-described by an exponential decrease with depth for horizontal unshielded surfaces, this is not true in general for shielded samples (Dunne et al., 1999). ~~Thus, while not exactly equivalent, the~~ The effective vertical mass attenuation length, $\Lambda_{eff}(x)$, ~~can be~~is approximated by the vertical depth below the surface at which the shielding factor is 5% of the surface shielding (i.e., 3 e-folding lengths) such that:

$$S\left(x, \frac{3\Lambda_{eff}(x)}{\rho}\right) = 0.05S(x,0).$$ (~~9~~10)

~~As a consequence of the non-exponential decrease in shielding factor with depth~~If nuclide production as a function of depth deviates from an exponential decline, it is inaccurate to use the analytical relationship between surface sample concentration, $C(x)$ (atoms g$^{-1}$), and steady-state vertical erosion rate, $E$ (g cm$^{-2}$ yr$^{-1}$), typically applied to eroding samples:

$$C(x) = \frac{S(x)P_0(x)\Lambda_{\mathrm{eff}}(x)}{E},$$ (~~10~~11)

where $P_0(x)$ is the unshielded surface production rate, corrected for latitude and air pressure (Lal, 1991). Equation (~~10~~11) derives from integrating the path history of a particle being exhumed vertically at a steady rate $E$ and emerging at the surface with an accumulated nuclide concentration $C(x)$:

$$C(x) = P_0(x)\int_{t_0}^{t_{surface}} S(x, z(t))dt,$$ (~~11~~12)

which can be parameterized in terms of vertical depth below the surface, $z$, according to:

$$C(x) = \frac{P_0(x)}{E/\rho}\int_0^{z_0} S(x,z)dz,$$ (~~12~~13)

where the depth of a rock parcel below the surface $z_0$ at time $t_0$ is deep enough such that there is no cosmogenic nuclide production ($z_0 = 40\Lambda/\rho$ for the calculations below) and $t_{surface} = t_0 + \rho z_0/E$ is the time it takes for a rock parcel to travel from depth $z_0$ to the surface (assuming a vertical exhumation pathway). Because there is no analytical solution for Eq. (~~12~~13), the integral needs to be solved numerically. A total effective shielding factor, $C_{eff}(x)$, acts as a correction factor to interpret local erosion rate from a sample concentration, defined by:

$$C_{eff}(x) = \frac{C_{shielded}(x)}{C_{unshielded}(x)} = \frac{\sum_{z=0}^{z=z_0} S(x,z)}{\sum_{z=0}^{z=z_0} S'(x,z)},$$ (~~13~~14)

where $\sum_{z=0}^{z=z_0} S'(x,z)$ is the integrated shielding depth profile for the case $\alpha = 0$ (i.e., no slope or skyline shielding), and $C_{eff}(x)$ does not depend on spatial variations in latitude or air pressure corrections. Finally, a mean effective shielding factor, $\overline{C}_{eff}$, ~~can be~~is defined for the whole hillslope as:

$$\overline{C}_{eff} = \frac{1}{L_h}\sum_{x=0}^{x=L_h} C_{eff}(x),$$ (~~14~~15)

which is equivalent to the catchment-mean shielding factor for the simplified valley geometry shown in Fig. 1.

### 3.3 Approximation for dipping catchments

Although the above framework accounts for variations in catchment relief and hillslope angle, $\alpha$, in all cases there is no net dip to the entire catchment (i.e. ridgeline elevations are uniform), which is not the case for natural watersheds. To simplify the geometry of a dipping catchment, I use a similar approach as Binnie et al. (2006) to model the catchment as a plane fit through the bounding ridgelines with dip $\beta$. I focus on two end-member cases, using examples from the San Gabriel Mountains, California, USA for illustration (Fig. 2). First, for an "interior" catchment that is tributary to a larger valley within a mountain range, the catchment will have a net shielding similar to the geometry of the hillslope in Fig. 1. Consequently, the shielding geometry can be approximated by Eq. (~~5~~6)-(~~8~~9) with $\alpha = \beta$. For the case of an "exterior" catchment that has a net dip $\beta$ but no opposing skyline shielding, Eq. (~~6~~7) becomes:

$$\tan\theta_0 = \begin{cases} 0, \; 0 \le \varphi < \frac{\pi}{2} \\ -\tan\alpha\cos\theta - \frac{z}{x}\cos\varphi, \; \frac{\pi}{2} \le \varphi \le \pi \end{cases}. \qquad (\sout{15}16)$$

For both examples, I compared the ~~catchment~~ catchment-mean shielding factor, $\overline{C}_{eff}$, to the mean surface skyline shielding factor, $\overline{S}_0$, as calculated using the commonly applied topographic shielding algorithm of Codilean (2006) in TopoToolbox (Schwanghart and Scherler, 2014).

## 4 Model results

For the catchment geometry shown in Figure 1, the local shielding factor, $S(x, z)$, decreases with increasing depth, $z$, distance downslope, $x$, and increasing slope, $\alpha$ (Fig. 3). The surface skyline shielding factor, $S_0(x)$, decreases with distance downslope, $x$, and increasing hillslope angle, $\alpha$, with the greatest shielding occurring in the valley bottoms of steep catchments (Fig. 4a). For the case $\alpha$ = 80°, comparison of $S_0(x)$ with the topographic shielding algorithm of Codilean (2006) shows that the two are equivalent.

The normalized effective attenuation length, $\Lambda_{eff}/\Lambda$, decreases as a function of distance downslope and increases with increasing hillslope angle (Fig. 4b). Although for low slopes cosmogenic nuclide production is concentrated at depths of $\rho z/\Lambda = [0, 3]$, for very steep slopes production rates at depth can be greater than those of flat landscapes despite lower surface production rates (Fig. ~~3~~5). This effect emerges in part due to the increased effective attenuation length for collimated radiation in skyline-shielded samples (up to a factor of 1.3—Dunne et al., 1999; Gosse and Phillips, 2001), but mainly because on steep slopes a point at vertical depth $z$ below the surface is receiving incident radiation from oblique pathways that can be much shorter than those overhead (Fig. 1c). Consequently, there is an additional radiation flux that increases the effective ~~(~~vertical~~)~~

mass attenuation length, $\Lambda_{eff}$, an effect that is most pronounced near ridgelines ($x/L_h < \sim 0.4$) where skyline shielding is minimized (Fig. 3, 4b).

The combined effect of the decrease in surface production (Fig. 4a) and the increase in effective attenuation length (Fig. 4b) leads to a pattern whereby the total effective shielding factor, $C_{eff}(x)$, is greater than one along the upper portion of hillslopes and less than one along the lower portion of hillslopes near the valley bottom (Fig. 4c). Although for steep slopes ($\alpha > 60°$) there may be considerable variation in shielding depending on slope position, the mean effective shielding parameter, $\overline{C}_{eff}$, is unity for all cases (Fig. ~~5a~~6a).

For the case of dipping catchments (Fig. 2), the sensitivity of the mean effective shielding parameter to catchment dip, $\beta$, depends on whether catchments are "interior" (i.e., shielded by an opposing catchment) or "exterior" (i.e., no external skyline shielding). For "interior" catchments, the shielding calculations are identical to the analysis above, and thus $\overline{C}_{eff}$ is again unity for all cases (Fig. ~~5a~~6a). For "exterior" catchments, the increase in effective attenuation length at steep slopes due to shorter oblique radiation pathways (Fig. 1c) is larger than the decrease in surface production due to skyline shielding, and $\overline{C}_{eff}$ is greater than one (Fig. ~~5b~~6b). However, for all but the most extreme catchment dips ($\beta \leq 40°$), $\overline{C}_{eff}$ is effectively one (within 1%).

For the two example catchments in the San Gabriel Mountains (Fig. 2), the mean total effective shielding factor, $\overline{C}_{eff}$, is 1.00, despite steep catchment dips ($\beta$ = 17° and 32°) and high mean surface skyline shielding, $\overline{S_0}$ ($\overline{S_0}$ = 0.87 and 0.84 as calculated by the Codilean (2006) algorithm) (Fig. ~~5a~~6a).

## 5 Implications for interpreting catchment erosion rates from $^{10}Be$ concentrations in stream sediment

The above results indicate that no correction factor for topographic shielding is needed to infer catchment-mean erosion rate from $^{10}Be$ concentrations in stream sands for most cases, as long as the assumptions of spatially uniform quartz content and steady uniform erosion rate are valid. Only in the extreme case of an "exterior" catchment with mean dip $\beta > 40°$ will such corrections be necessary. Although the approach of calculating only the surface skyline shielding component of the total effective shielding factor is appropriate for calculating surface exposure ages, neglecting the slope and shielding controls on the effective mass attenuation length leads to a systematic under-prediction of the actual erosion rate. The magnitude of this under-prediction increases with increasing catchment mean slope, as highlighted by a ~~recent~~ compilation of catchment erosion rates from steep catchments in the Himalaya and Eastern Tibetan Plateau (red data points, Fig. 5a).

For catchments with spatially variable quartz content or erosion rate, a spatially distributed total effective shielding factor, $C_{eff}$, must be calculated at each pixel. While Although calculating the surface skyline shielding component is straightforward (Codilean, 2006), solving Eq. (3) at depth for arbitrary catchment geometries is presently too computationally intensive to be practical. However, while not entirely transferable to arbitrarily rough topography (e.g., Norton and Vanacker, 2009), Fig. 4c suggests that for slopes less than 40°, the total effective shielding factor does not vary significantly across the hillslope. For steep catchments with spatially variable quartz content or erosion rate, direct calculation of shielding at depth is likely needed to calculate the spatially distributed total effective shielding parameter. In particular, shielding calculations in landscapes dominated by cliff retreat are poorly suited for treatment in a vertical reference frame (e.g., Ward and Anderson, 2011).

The modeling approach above assumes a simplified angular distribution of cosmic radiation flux (Eq. (1)) and accounts only for cosmogenic nuclide production via spallation. In actuality, the cosmic radiation flux does not go to zero at the horizon, and becomes increasingly collimated (higher $m$) with increasing atmospheric depth (Argento et al, 2015). Thus, the sensitivity of the effective mass attenuation length to shielding will increase with increasing elevation. However, the magnitude of changes in the effective mass attenuation length due to shielding-induced collimation is at most 30% (Dunne et al., 1999), compared to the potentially factor of 3 or more increase due to slope effects (i.e., shorter oblique radiation pathways on very steep slopes; (Fig. 1c; Fig. 4b). For hillslope gradients commonly observed in cosmogenic nuclide studies of steep landscapes (30-40°), the increase in effective mass attenuation length due to shielding-induced collimation and slope effects are 2-5% and 6-15%, respectively (Dunne et al., 1999; Fig. 4b). Similarly, tThe dependence of $\Lambda$ on atmospheric depth, which is typically not accounted for in catchment erosion studies, is minor (<10% for extreme case of catchment with 4 km of relief (Marrero et al., 2016)) compared to the above slope effect for most landscapes. Treatment of cosmogenic nuclide production by muons is less constrained than spallogenic production, but the angular distribution of production by muons is likely similar to that for spallation reactions and also sensitive to latitude and atmospheric depth (Heisinger et al., 2002a; 2002b).

Overall, the effect of topographic shielding corrections on interpreting catchment erosion rates is small compared to typical assumptions inherent to detrital cosmogenic nuclide methods. In particular, the assumption of steady lowering is likely to be increasingly inappropriate for rapidly eroding landscapes characterized by a significant contribution of muonogenic production or slowly-eroding landscapes where $^{10}Be$ concentrations integrate over glacial-interglacial climate cycles. steep Steep landscapes characterized by stochastic mass wasting present additional complications (Niemi et al., 2005; Yanites et al., 2009), requiringan effect that requires the non-trivial calculation of spatially distributed shielding parameters for an arbitrary catchment geometry. Nonetheless, in all cases accounting only for surface skyline shielding (e.g., Codilean, 2006) without including its concurrent influence on the effective attenuation length should be avoidedyields incorrect results.

## 6 Conclusions

The simplified model presented here for catchment-scale topographic shielding of incoming cosmic radiation highlights the two competing effects of slope and skyline shielding. As catchment relief increases, surface production rates are reduced due to increased skyline shielding. However, for shielded samples radiation is increasingly collimated, and for sloped surfaces oblique radiation pathways increase nuclide production at depth. Both of these effects lead to deeper effective vertical mass attenuation lengths, which offset the reduction in surface production when inferring erosion rates from cosmogenic nuclide concentrations. At the catchment scale, the mean total effective shielding factor is one for a large range of catchment geometries, suggesting that topographic shielding corrections for catchment samples are generally not needed, and that applying commonly used topographic shielding algorithms leads to underestimation of true erosion rates by up to 20%. Although these corrections are typically small compared to other methodological uncertainties, they vary systematically with slope and relief. Consequently, misapplication of shielding correction factors could influence interpretations of relationships between topography and erosion rate.

## Competing interests

The author declares no conflict of interest.

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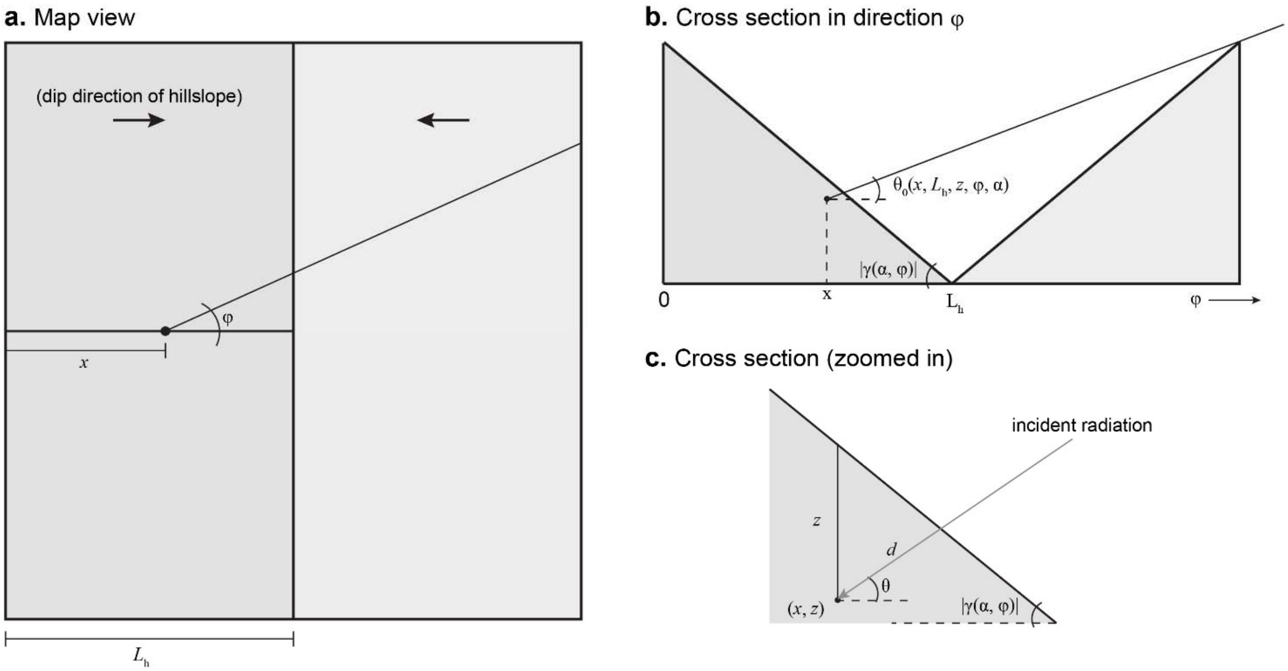



**Figure 1: Model catchment setup, showing (a) map view, (b) cross section along azimuthal angle $\varphi$ (note that $|\gamma|$~~$\gamma$~~ $= \alpha$ for $\varphi = 0$), and (c) close up of hillslope cross section.**

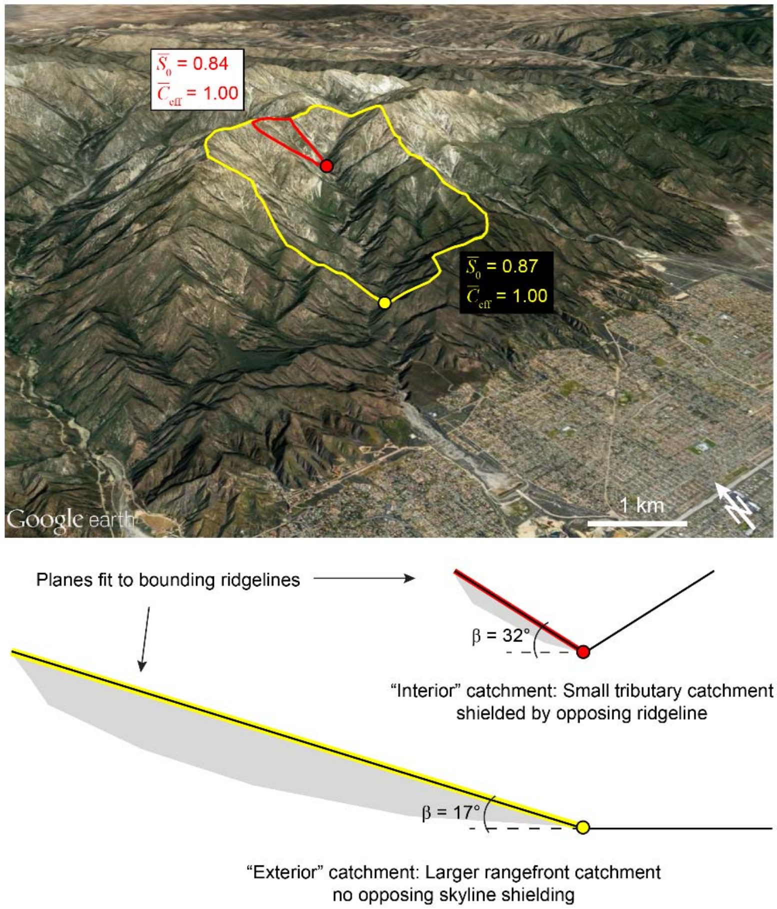



**Figure 2: Dipping catchment shielding geometry, illustrated using example from the San Gabriel Mountains, California, USA. Image is centered on 34.20°N, 117.61°W. Colored lines indicate planes fit through bounding ridgelines dipping at angle $\boldsymbol{\beta}$. $\overline{S_0}$ indicates mean surface skyline shielding parameter calculated using algorithm of Codilean (2006), and $\overline{C}_{eff}$ indicates the mean total effective shielding factor calculated from the simplified catchment geometry.**

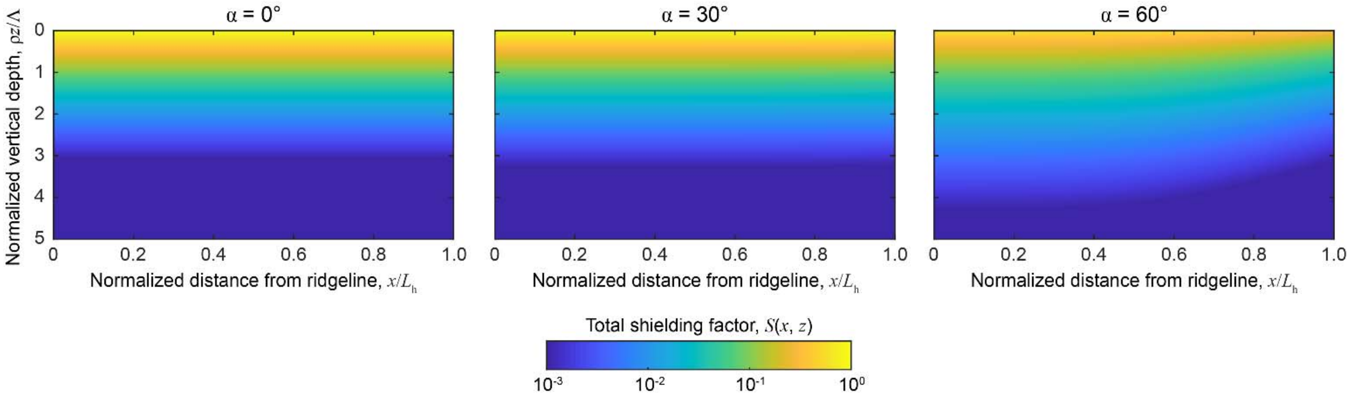



**Figure 3: Total shielding factor, $S(x, z)$, as a function of normalized vertical depth and distance from ridgeline for varying hillslope angle, $\alpha$.**

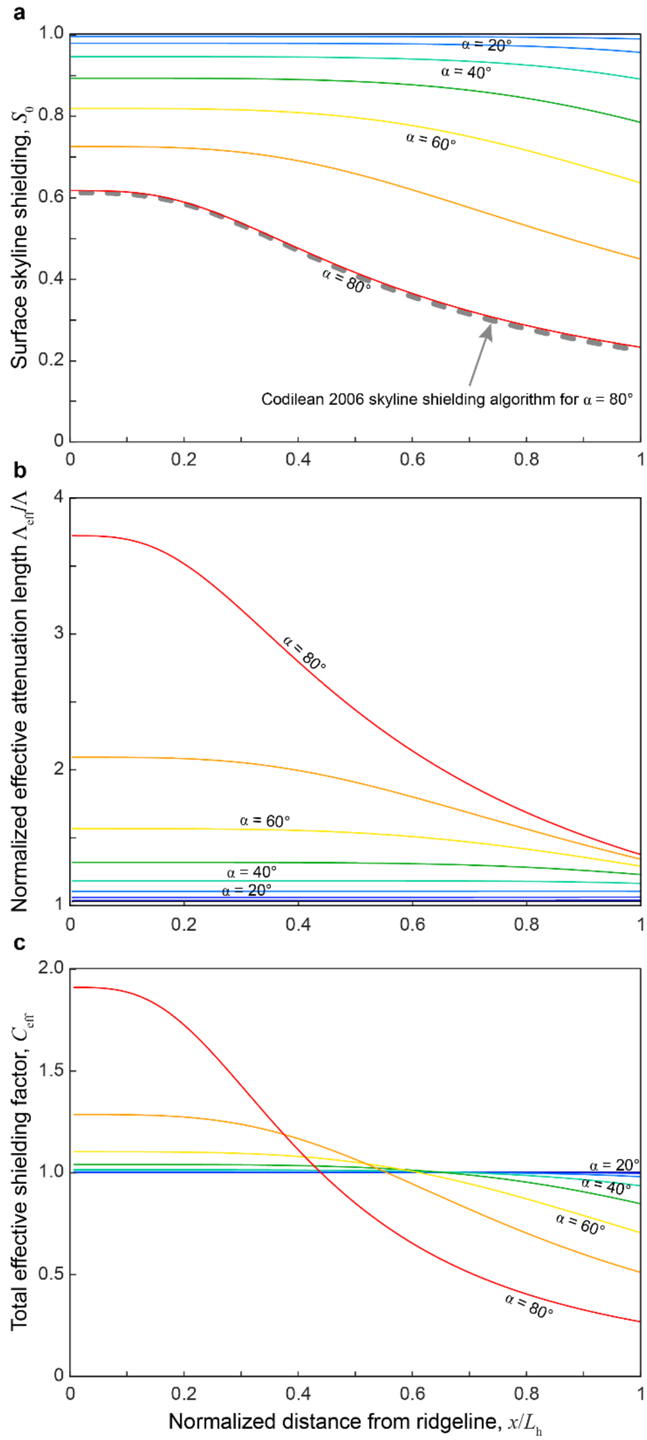
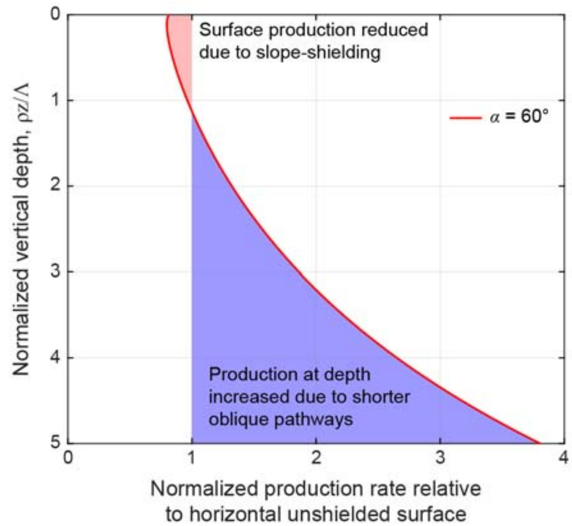



**Figure 4: Plots of (a) surface skyline shielding factor (b) normalized effective vertical attenuation length, and (c) total effective shielding factor as a function of distance from ridgeline for model runs with α = 0-80°. Dashed line in (a) indicates topographic shielding calculation using algorithm of Codilean (2006) applied to a digital elevation model of the case $\alpha$ = 80°.**




5 Figure 5: Plot of normalized production rate relative to horizontal unshielded surface as a function of normalized vertical depth for a 60° slope with no additional skyline shielding. Near the surface, production rates are decreased due to slope shielding of incoming cosmic radiation; however, production rates at depth increase relative to the unshielded case due to additional radiation along shorter oblique pathways (Fig. 1c).

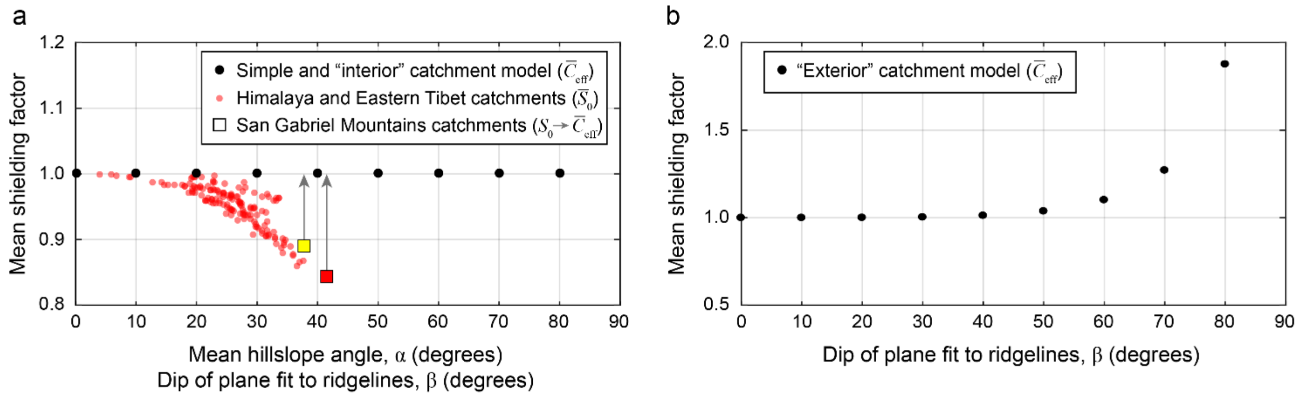



**Figure ~~5~~6: Plots showing mean total shielding factor, $\overline{C}_{eff}$, for (a) simple horizontal catchment case (Fig. 1) for varying mean hillslope angle, $\alpha$, which is equivalent to the "interior" dipping catchment case as a function of catchment dip, $\beta$ (Fig. 2), and (b), the mean total shielding factor, $\overline{C}_{eff}$, for the "exterior" dipping catchment model as a function of catchment dip, $\beta$ (Fig. 2). Red points in (a) indicate relationship between the mean surface skyline shielding factor, $\overline{S}_0$, as a function of mean hillslope angle for compilation of catchment $^{10}$Be data in the Himalaya and Eastern Tibet as reported by Scherler et al. (2017). Red and yellow squares indicate mean surface skyline factor, $\overline{S}_0$, calculated for example catchments from San Gabriel Mountains (Fig. 2). Arrows indicate difference between mean surface skyline shielding factor and mean total shielding factor, $\overline{C}_{eff}$.**