Response to reviews of "“Short communication: Rivers as lines within the landscape”

In this revised manuscript I have tried to address all the reviewer comments, and I am really grateful for the effort the two reviewers took to review my work. I include a latexdiff version at the end of the response to the reviews. Below I detail the revisions made.

Reviewer 1, Liran Goren:

First, given the fixed cell size, the differences between the cell-to-cell steepest descent routing algorithms and the node-to-node steepest descent routing algorithms might not be accurately presented. Isn't it possible to cast the cell-to-cell as a node-to-node over the complementary hexagonal graph, whose edges connect the centers of the triangles of the original grid? In this case, the differences between the two implementations lie in the grid spacing, l vs. l_s, and in the possibility to route water in 6 directions over the triangular grid with respect to 3 directions in the complementary hexagonal grid. If the number of routing directions is the critical issue, then the difference between the simulations is not the outcome of river representation (i.e., rivers with finite width with respect to rivers as lines), but of the grid shape. Specifically, using cell-to-cell hexagonal grid should be similar to node-to-node triangular grid. If this claim is true, then the comparison between these two implementations could be invalid, and as a consequence, it cannot be used to test the theoretical claim for the advantage of representing rivers as lines, which is central to the manuscript. On a side note, other LEMs such as CASCADE and DAC use line representation of rivers as well.

I agree that the difference in the model outcome is that the node-to-node representation leads to a greater number of flow directions. I have removed the previous weight I gave to “rivers as lines”, and rather discussed how to model flow that has a width that is smaller than a cell. I suggest using a node-to-node model might be a reasonable assumption, and then explore the consequences of this. In the renamed section 4 “Sub-grid scale processes” I now explicitly state that by taking the node-to-node assumption I get more directions to distribute flow, and that this is the crucial factor in getting improved resolution independence (page 7, lines 6-10, and page 8, lines 1-7).

Another issue is the observed unsteadiness of the topography only with the distributed flow routing algorithm. Goren et al., 2014 (Earth Surf. Process. Landforms) showed that a similar unsteadiness emerges in the DAC LEM that uses the steepest descent algorithm, but importantly, allows for the drainage area and the discharge in each grid node to vary continuously though time due to shifting water divides. The possibility for a continuous change in the drainage area leads to the emergence of the drainage area feedback, by which an increase in the drainage area, leads to faster channel downcutting that propagates downstream and then upstream back to the original node, promoting further drainage area increase. Since the downcutting signal propagates throughout all the tributaries, it affects all the neighboring basins, leading to ongoing “ringing” in the landscape elevation and erosion rate, namely, to unsteadiness. Goren et al., 2014 further developed the argument that this behavior is similar to the one observed in the distributed flow routing algorithm of Pelletier 2004, whereby small changes in elevation affect the local discharge (equivalent to area), which leads to further changes in elevation. This means that distributed flow routing is just one possible implementation for representing the ‘area feedback’ that is responsible for unsteadiness in numerical, experimental, and possibly natural landscapes.

While I cannot disagree that DAC creates an unsteadyness in the “steady-sate” landscape, my comparison to the model of Pelletier (2004) is pertinent because in both cases we have used a distributive algorithm. In the study of Perron et al. (2008) they explicitly state that distributed routing does not create such unsteady landscape. I have to instead agree with the study of Pelletier (2004). In light of the third comment below, I have however acknowledged that DAC is also resolution independent (see revised abstract, page 8 lines 8-14, and page 9, lines 1-5).

A third issue is the observation of resolution independency with the distributed flow algorithm. This, as well, has been documented in Goren et al. 2014 (e.g., fig 10) for the DAC LEM that implements the steepest descent algorithm. It is therefore possible that the resolution independency is the outcome of the LEM ability to represent the area feedback, while the distributed flow algorithm is just one possible implementation for it.

After testing DAC for valley spacing as a function of resolution I have to agree. It looks like the key is to account for sub-grid scale processes, either at the drainage divides or at the flow channel. I have tried to make this point clear in the abstract and in section 4. Perhaps models with mixed transport and detachment-limited laws should use both distributive flow routing and capture drainage divide migration.

Finally, the use of a diffusion equation (eq. 1) rather than an advection equation to represent incision along fluvial channels at the scale of a mountain range needs to be justified, since such an equation cannot produce knickpoints, which are a dominant feature in mountainous rivers.

I am sorry, but I don't want to model landscape evolution using the stream power law. I dislike the fact that it does not account for deposition. In section 2 I have tried to justify the choice of model equations, and be clear that indeed this model does not capture knickpoint migration (page 3, lines 2-11).

Reviewer 2: Andrew Wickert

General comments:

The idea that rivers can be approximated as lines (with appropriate parameterized widths) should be appropriate for landscapes in which the significant lateral scales are >> river widths. This can often be the case in actively-uplifting mountain ranges, where uplift and incision work together to keep valleys narrow and V-shaped, and the lack of significant catchment area leads to small rivers, validating your assumptions – in this setting. Active orogens may well be the primary type of environment where LEMs are used, but either because of or in spite of this, I am concerned that such strong wording in this paper can lead to an echo chamber effect, in turn leading to the neglect of the broader range of landscapes on Earth. Therefore, I am writing a reminder that even in erosional landscapes, there can be rivers with km-scale widths. In the USA, the upper Mississippi, upper Missouri, Hudson, and Susquehanna have widths that are ≥ observed hillslope lengths; I am considering these widths in absence of control structures. This is similarly true for the Niger, the Irrawaddy, and probably many more. Furthermore, I am considering only the rivers themselves, and not their valleys, which can often be wider and are also not considered in most landscape-evolution models. (Langston and Tucker, 2018, offer a starting point to address this piece of reality.)

This is not to say that the foundation of the idea presented here is wrong. It is simply to say that it is not right all the time. Indeed, it is a reasonable assumption in mountain ranges, in which limited drainage area leads to narrow rivers. But I find it to be important to not make the places that one tends to model and think about become, in the scientific literature, the only landscapes that exist. My second general point is on how well your model equations represent reality, which I note in the line-by-line comments and so will not restate here.

I have deleted the term “rivers as lines”. This comment however got me thinking about how wide rivers are globally. A recent study developed the Global River Widths from Landsat (GRWL) Database (Allen & Pavelsky, 2018). In this study the mean and max river widths can be plotted. It looks like the very wide rivers are an exception and not the norm. If I were to develop a generic LEM should I capture the exceptionally large rivers, or the vast majority? If I look at the distribution of mean river widths, then 75% of rivers have a mean width less than 432 m. From the maximum width, 75% of rivers are less than 1755 m. Therefore, if I wish to model landscape evolution of a large domain, the majority of rivers would have a width that is less than that of my cells. Therefore, perhaps my model assumption is valid. However it could be that the exceptionally large rivers are very important, and that can be something for the future.

I have modified the Introduction to make my line of thought clearer, I hope.

My third general point, and perhaps the most important, is my line-by-line comment, "fig. 3.", below. As noted in the overview, I believe that multiple-flow-direction routing is the real answer to parameterizing sub-grid topographic and flow-routing complexity, and that the cell-centered vs. cell-edge routing is actually a separate issue. I suggest that you investigate multi-direction routing from cell to cell, as this will tell you whether the answer lies in using cells vs. edges or the SFD vs. MFD routing

I really liked this suggestion. I have added the consequences of modelleing cell-to-cell distributed flow. I find that while sediment flux output is still somewhat resolution dependent, the valley spacing is very much improved. This suggests that indeed "*multiple-flow-direction routing is the real answer to parameterizing sub-grid topographic and flow-routing complexity*". As such the text has been modified to include these model results.

Line-by-line comments:
p1,l3. And as flow in the unsaturated zone; perhaps consider lumping all subsurface flow together

I have modified the text such that the offending sentence no longer exists. I note that I now have the phrase, "Water is the primary agent of landscape erosion. There are multiple pathways within the hydrological cycle from evaporation, transpiration, and ground water flow, however for many landscapes the river network is the primary route through which water flows down slope." (page 2, lines 3-5).

p1,l4. Typically, continent > country ∼ mountain range, typically, so it might make more logical sense to reverse these. However, I am not sure why you include this, because LEMs are typically run at the mountain range or smaller scale.

Deleted. But, please do no assume LEMs are run at the scale of a mountain or smaller. This is simply not the case as there are many projects to couple mantle convection to surface processes.

p1,l12-18. Flow routing and river width are two processes that are about as separable as any become in Earth-surface processes. Flow routing occurs over the scale of a basin, is non-local, and is cumulative. River width responds to local conditions, e.g., shear stress, and is typically thought to tend towards an equilibrium – see Parker (1978), Phillips and Jerolmack (2016), and Pfeiffer et al. (2017) for some background on the latter. Therefore, I find it difficult to understand why you have gone from writing that parameterizing width is hard (yes, this is true, but also necessary if it is sub-grid) to writing that therefore we just focus on flow-routing? Mustn't we do both?

I will be honest, I have no idea why rivers and flow routing are so separate in Earth-surface processes (I am out of my specialisation here). I was simply trying to argue that river width is small, in general. To resolve this problem, I have tried to consistently refer to flow routing and not mention rivers, beyond the introduction and discussion.

p2,l4. Could you describe the reality that this equation is portraying? Because there is a gradient in water flux (i.e., depth-integrated discharge), I presume that this is a transport-limited-style system. However, the situation in which rivers can be lines, based on your initial argument, is germane to steep mountains. Could you then explain either (a) how this equation is appropriate, or (b) how it might, even if not appropriate for the physical reality, create a mathematical setting that is useful for exploring your key concept?

I have added text to justify this transport-limited model set-up (page 3, lines 2 to 11).

p2,l14 (eq. 2). This equation works dimensionally, but I am not convinced that it represents reality. In a typical river system, one would accumulate flow over the full landscape [m^3 s^{-1}]. Then, after partitioning groundwater and surface water and any losses due to ET (not so common in LEMs), one would assuming a rectangular channel with minimal wall friction for simplicity, and divide water discharge by the channel width to obtain a discharge per unit width (or "water flux") with units of m^2 s^{-1}. What you have effectively done is replaced the channel width by the distance between two cell centers. This means that the effective channel width in this case is a direct function of grid size. I think I am starting to understand why you combined flow-routing and channel width at the start of this paper, but I think that this is a distinct downgrade from actually considering channel width!

In many LEMs width is taken from the cell width. Of course I am making big assumptions for equations 2 and 3, however I am aiming at a simple method to capture surface flow. It is not my intention to model all the processes, but create a LEM that is low on complexity but not resolution dependent. That is all. I have added a few lines to explain where equation 2 comes from and to explain that they are approximations and not reality.

p3,l2. Calling cell edges "rivers" does not seem like a good idea – this terminology implies, regardless of your intent with it, that you have pre-determined the dimensions of hillslopes and the drainage density, and that rivers can meet across drainage divides.

Agreed and deleted.

p3,l3 (eq. 3). Here you are accumulating flow with l, which again implies that the geometric relationship between the edge length of an equilateral triangle and its area is proportional to any hydrologic lossiness and a realistic channel-width function. This certainly cannot be true. However, there is a linear proportionality between the cell-side length and the cell area, so this relationship – in spite of its dimensional consistency with water flux – retains a linear scaling with water discharge.

I am assuming that any segment of my flow is has water added to it as a linear function of the length of that segment. That is that the flux of water is constant and independent of the area of "land" to either side of my flow segment. Is this completely wrong? Water flux is said to be related to catchment area, $Q_w \sim A^{0.8}$ (Syvitski & Milliman, 2007). The catchment length, l, is related to area by, $l \sim A^{1/p}$, where $1.4<p< 2.0$ (Armitage et al., 2018). If I take the lower end then I get $Q_w \sim l^{1.12}$. So a linear model might not be *so* wrong?

fig. 3. As I'm sure you're aware, and I'm hoping you weren't dreading that a reviewer would ask, there is a fourth case here: cell-to-cell multi-flow-direction routing. It seems to me that the multiple flow directions, rather than a single flow direction, could well be the key component here, because it provides a mechanism to redistribute water at a sub-grid level. This is a major point, as it could completely change your conclusions – though I hope you agree with me that it validates the importance of your work in identifying grid-scale dependence, which is a first-order numerical

modeling issue that is worth solving properly!

I have added the extra model.

p6,l8. Why are you mentioning avulsions? These are typically features of depositional rivers, but in these cases, river dimensions are often not well represented by lines.

After discussions with Chris Paola I have ended up thinking of terraces and avulsions as being aspects of the same process, a channel adjusting to water flux. I have deleted “avulsions”.

p6,l9. I do not think that terraces, in general, represent changes in river flow paths, at least on the scale of a landscape. They can represent that it takes time for a river to modify its full valley, and sometimes moves across all of it, and sometimes does not. But in your initial premise of rivers being lines, all of this complexity (terraces, valleys, rivers) would be lumped together. Perhaps I am taking your initial statement too literally – but I am finding it hard now to follow a consistent thread of thoughts in terms of what is considered to be important for your story and what is not.

While the complexity of terraces is not captured in the model, it is worth noticing that the steepest descent algorithm has generated a fixed topography, with fixed valleys, while the distributed model has not. This is my only point. Terraces are evidence of a flow path that is not steady in time. Perhaps this statement is obvious, but before I got involved in the project in the Rio Bergantes, to me this point was not obvious. Hopefully with the reduced mention of “rivers” the text is less confusing.

p6,l13. While I know this is just a test case, it would be useful for my curiosity to know how you know that the Ebro’s valleys were filled during the late Pleistocene, when sealevel was low. I presume that either there has been rapid progressive incision, or that colluvial (or other) processes may have filled the valleys with classic material (in which case the same LEM rule would be applied to both intact rock and loose material), or...?

I have just returned from a second field trip to the Rio Bergantes catchment. It is clear to me now that my colleagues are not certain what the valleys were filled with. The whole region was at one time a endorheic basin. Once the River Ebro eroded through the coastal ranges, this ended and there was significant incision. This occurred at between 10 and 8 Ma. Subsequently the region has been adjusting to the loss in regional loading. Elevations are of 500 m, so sea level change is likely not a factor here.

In a future study the very simple LEM developed here will be compared with complex process based LEMs such as LAPSUS and CAESAR. It is my hope that one simple continuum equation will perform as well as the process based LEMs. I might be proven wrong, but for now my only intention is to solve the question of resolution dependence and to demonstrate that distributive routing is preferable to steepest descent. I hope that this is OK.

SHORT COMMUNICATION: ~~RIVERS~~ FLOW AS DISTRIBUTED LINES WITHIN THE LANDSCAPE

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Abstract. Landscape evolution models (LEMs) aim to capture an aggregation of the processes of erosion and deposition within the Earth's surface and predict the evolving topography. Over long time-scales, i.e. greater than one million years, the computational cost is such that numerical resolution is coarse and all small-scale properties of the transport of material cannot be captured. A key aspect of ~~any~~ therefore of such a long time-scale LEM is the algorithm chosen to route water down the surface. ~~In nature precipitation makes its way to rivers as a surface flow and as groundwater. Furthermore, at the scale of a mountain range, country, or even continent, the width of any given river is so small relative to the scale of the landscape that it is essentially a line. Taking this abstraction as a starting point,~~ I explore the consequences of ~~assuming that~~ two end-member assumptions of how water flows over the surface of a LEM~~along lines rather than over the surface area. By making this assumption and distributing the~~ , either the steepest descent or distributed down all down-slope surfaces, on model sediment flux and valley spacing. I find that by distributing flow along the edges of the mesh cells, node-to-node, ~~I find that~~ the resolution dependence of the evolution of LEM is significantly reduced. Furthermore, the flow paths of water predicted by this node-to-node distributed routing algorithm is significantly closer to that observed in nature. ~~Therefore I suggest that rivers are lines within the landscape, and we must treat them as such within LEMs that operate on a scale larger than a reach~~This reflects the observation that river channels are not necessarily fixed in space, and a distributive flow captures the sub-grid scale processes that create non-steady flow paths. Likewise, drainage divides are not fixed in time. By comparing results between the distributive transport-limited LEM and the stream power model "Divide And Capture", which was developed to capture the sub-grid migration of drainage divides, I find that in both cases the approximation for sub-grid scaled processes leads to resolution independent valley spacing. I would therefore suggest that LEMs need to accurately capture processes at a sub-grid scale to accurately model the Earths surface over long time-scales.

1 Introduction

It is known that resolution impacts landscape evolution models (LEMs) (Schoorl et al., 2000). The ~~route of this problem is in the calculation of upstream area for the water flux term in the set of governing equations. As resolution is increased the upstream area typically changes, and this problem has lead to the addition of for example weighting terms to control the width of the river at a sub grid level (Perron et al., 2008) . Yet,~~ resolution dependence of LEMs is caused by how run-off is routed down the model surface. It has been demonstrated that the outcome of distributing flow down all slopes, or simply allowing flow to descent down the steepest slope, gives different outcomes for landscape evolution models (Schoorl et al., 2000; Pelletier, 2004) . It has been noted that landscape potentially has a characteristic wavelength for the spacing of valleys (Perron et al., 2008) . Therefore, a landscape evolution model should be able to reproduce such regular topographic features independently of the model resolution. For a model of channelised flow it was however found that the routing of run-off lead to a resolution dependence in the valley spacing, which could be overcome by the addition of an arbitrary river width term is not ideal as it might influence observations such as valley spacing and response times for the landscape to recover to a perturbation. For example, landscape response time is a function of the water flux (Armitage et al., 2018) , which will be influenced by the introduction of a term that controls the river width. In this contribution, I will therefore explore how flow routing effects landscape evolution. a parameterised flow width that was less than the numerical grid spacing (Perron et al., 2008) .

The There is a potential problem with parameterising the flow width to be fixed at a sub grid level. The response time of LEMs to a change in external forcing is strongly dependent of the surface run-off (Armitage et al., 2018) . This means that the model response time becomes likewise dependent on the chosen flow width. Ideally the LEM would be independent of grid resolution without introducing a predefined length scale that impacts the model response.

Water is the primary agent of landscape erosion. There are multiple pathways within the hydrological cycle from evaporation, transpiration, and ground water flow, however for many landscapes the river network is the primary route through which water flows down slope. In any given reach of a river or stream the majority of the water flowing through it comes from the uphill end and exits through the down hill end. A smaller amount of water will enter the river from the sides through groundwater seepage or over-land run-off. This raises the question: is it reasonable for a long term landscape evolution model to have water flow over all surfaces, or to only allow water to flow down lines? Is the river network sensitive to the upstream network length or the upstream area? Building on this question I will Mean river width varies from 5 km to a few meters (**?**) . The very wide rivers, greater than 1 km are however outliers within this global data set, with the median of the distribution of mean river width being 124 m and the upper quartile at 432 m (Figure 1). In LEMs developed for understanding long-term landscape evolution the large time scales necessitate large spatial scales, where a single grid cell can be a kilometer wide or more (Temme et al., 2017) . A spatial resolution of cells larger than a few meters becomes necessary when modelling at the scale of a continent (e.g. **?**) . This means that flow has a width at a subgrid level.

If the width of the flow path for run-off is narrower than can be reasonably modelled, then can the flow paths be treated as lines, from model node-to-node (Figure 2), where water collects along these lines? To explore this idea and understand LEM sensitivity to resolution, I wish to explore how a simple LEM evolves as I change the flow routing algorithm from one that routes water from under four scenarios (Figure 2): (1) simple steepest descent routing from cell area to cell area, (2) a distributed flow version of this cell-to-cell down the algorithm, (3) a node-to-node steepest descent routing, to an algorithm that distributes flow from and (4) a node-to-node distributed routing algorithm.

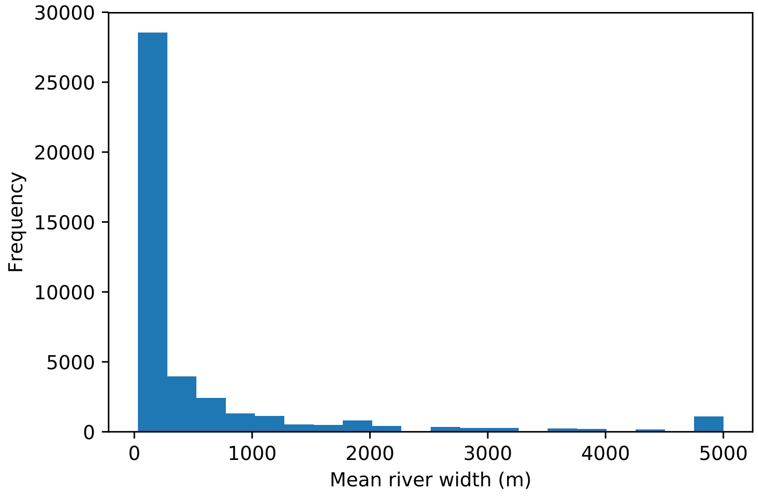

Figure 1. Distribution of mean river width taken from the Global River Widths from Landsat (GRWL) Database ? .

2 A landscape evolution model

In this study I will assume landscape evolution can be effectively simulated with the classic set of diffusive equations described in (Smith and Bretherton, 1972):

$$\frac{\partial z}{\partial t} = \nabla\left[\left(\kappa + c q_w^n\right)\nabla z\right] + U \tag{1}$$

where κ is a linear diffusion coefficient, c is the fluvial diffusion coefficient, q_w is the water flux, n is the water flux exponent, and U is uplift. This equation is heuristic concentrative-diffusive equation is capable of generating realistic landscape morphology, with the slope-area relationships commonly observed (Simpson and Schlunegger, 2003; Armitage et al., 2018) . Strictly it assumes that there is always a layer of material to be transported by surface run-off, and as such it can be classed as a transport-limited model. Furthermore, it cannot capture processes such as knickpoint migration, but it does however account for both erosion and deposition, and is therefore appropriate for modelling landscape evolution beyond mountain ranges and into the depositional setting (see models such as DIONISOS; Granjeon and Joseph, 1999).

Equation 1 is solved with a finite element scheme written using Python and the FEniCS libraries (I will call the code “fLEM”, see Code Availability). The equations are solved on a Delaunay mesh, where the mesh is made up of predominantly equilateral triangles with an opening angle of 60°. Model boundary conditions are initially of fixed elevation on the sides normal to the x-axis and zero gradient on the sides in normal to the y-axis. The model aspect ratio is 1 to 8 (see Figure ??). 4. Uplift is fixed at $U = 10^{-4}\,\mathrm{m\,yr^{-1}}$, the linear diffusion coefficient is $\kappa = 1\,\mathrm{m^2\,yr^{-1}}$, the fluvial diffusion coefficient is $c = 10^{-4}\,(\mathrm{m^2\,yr^{-1}})^{\mathrm{n}-1}$, and the water flux exponent is $n = 1.5$.

~~Dimensionless elevation from the landscape evolution model with different flow routing algorithms at different numerical resolutions after a dimensionless run time of 1.563×10^{-6} (5Myr), with an aspect ratio of 8×1. (a) Cell-to-cell steepest descent routing algorithm with a resolution of 1024×128 cells. (b) The same model but with a resolution of 4096×512 cells. (c) and (d) node-to-node steepest descent routing algorithm. (e) and (f) node-to-node distributed flow routing algorithm.~~

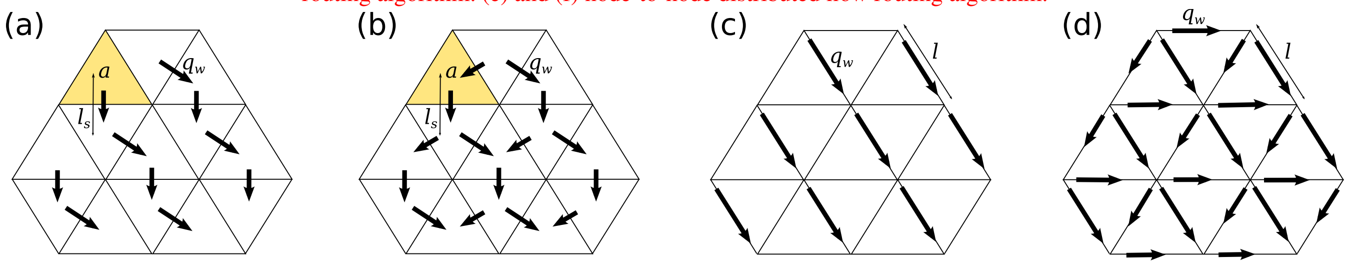

Figure 2. Diagram of flow routing from cell-to-cell down the steepest descent and a node-to-node routing down all slopes weighted by the relative gradient.

Water can be routed from cell-to-cell, where precipitation is collected over the area of each cell, sent downwards, and accumulates. In this cell-to-cell configuration the water flux has units of length squared per unit time and is given by:

$$q_w[\text{cell}] = \frac{\alpha a}{l_s}, \tag{2}$$

where α is precipitation rate, a is the cell area, and ls is the length from cell center to cell center down the steepest slope (Figure 2~~). I assume a precipitation rate of $\alpha = 1\,\mathrm{m\,yr^{-1}}$~~a and b). This gives a water flux per unit length, which has the advantage of not having to explicitly state the sub-grid width of the flow (Simpson and Schlunegger, 2003) . However, ~~as discussed, water can also be assumed to only flow down the edges of each cell, from node-to-node. In this case water collects down all the edges, is sent downwards, and accumulates. The water flux term does not know about the area of each cell , and the precipitation rate includes additions of water from the sides of the edges (rivers). Water flux again has units of length squared per unit time:~~ implicitly this implies that the flow is over the width of a cell. An alternative is to route water from node to node along cell edges. I assume that along the length of the cell edge water can be added to the flow line, assuming that the input is linearly related to the length of the flow line,

$$q_w[\text{node}] = \alpha l, \tag{3}$$

where l is the length of the edge ~~of~~ that joins the up-slope node to the down-slope node (Figure 2~~).~~ c and d). This means that the cell area is ignored and instead water enters the low path uniformly along its length. Both equations 2 and3 do not attempt to capture the interaction between water flux and river width, rather these are two methods to approximate run-off within a coarse numerical grid. For both the cell-to-cell and node-to-node methods the flow can then be routed down the steepest slope of descent ~~. For the node-to-node method of routing water, I also route water down all slopes~~ or weighted by the relative gradient ~~of each slope. This therefore gives three routing algorithms, (1) cell-to-cell steepest descent, (2) node-to-node steepest~~

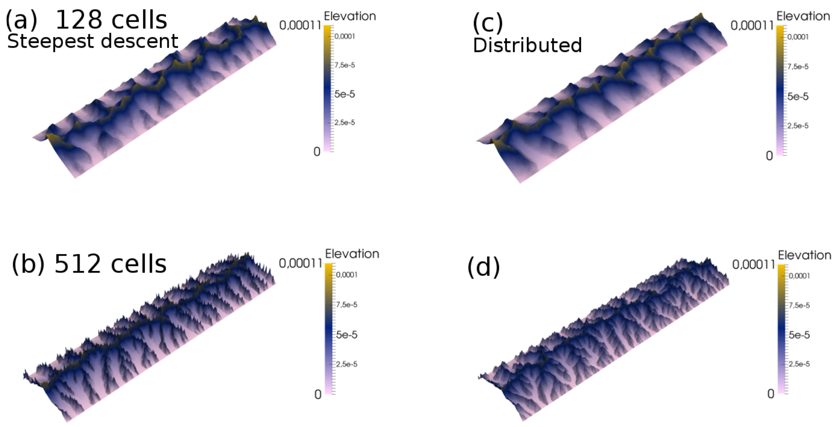

Figure 3. Dimensionless elevation from the cell-to-cell flow routing landscape evolution model with different flow routing algorithms at different numerical resolutions after a dimensionless run time of 1.563×10^{-6} (5 Myr), with an aspect ratio of 1×4. (a) Cell-to-cell steepest descent routing algorithm with a resolution of 128×512 cells. (b) The same model but with a resolution of 512×2048 cells. (c) and (d) cell-to-cell distributed flow routing algorithm.

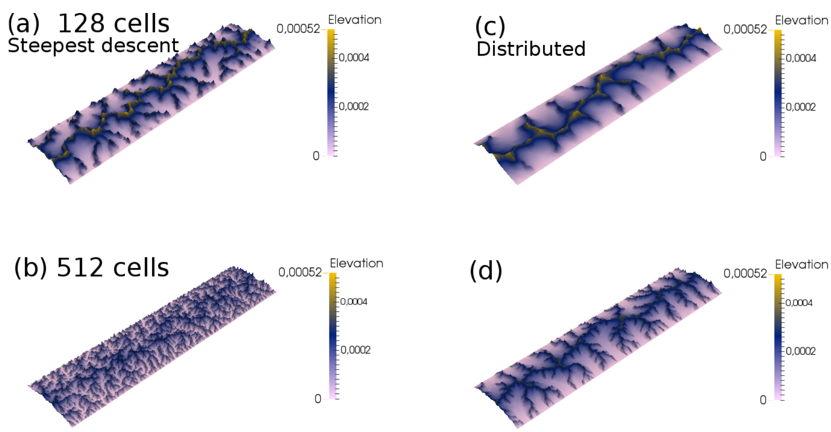

Figure 4. Dimensionless elevation from the node-to-node flow routing landscape evolution model with different flow routing algorithms at different numerical resolutions after a dimensionless run time of 1.563×10^{-6} (5 Myr), with an aspect ratio of 1×4. (a) Node-to-node steepest descent routing algorithm with a resolution of 128×512 cells. (b) The same model but with a resolution of 512×2048 cells. (c) and (d) node-to-node distributed flow routing algorithm.

~~descent, and (3) node-to-node distributed~~(e.g. Schoorl et al., 2000) . I run the numerical model with a uniform precipitation rate of $\alpha = 1\,\mathrm{m\,yr^{-1}}$.

3 The effect of model resolution

At a low model resolution, 1023×128 cells, all ~~three~~ four methods of flow routing give similar landscape morphology after 5 Myr of model evolution (Figure ~~??~~3 and 4). However, elevations are significantly lower for the cell-to-cell flow routing model as the water flux term operates across the cells rather than on individual node points (Figure ~~??a and b~~3 and 4). As the resolution is increased to ~~4096×512~~ 512×2048 cells, the landscape morphology starts to diverge~~(Figure ??)~~. In the cell-to-cell routing algorithm the landscape shows more small scale branching, as previously discussed by (Braun and Sambridge, 1997) (Figure ~~??a and~~ 3b and c). For the steepest descent algorithm it can be seen that the high resolution model has multiple peaks along the ridges (Figure 3b). ~~In~~ This roughness to the topography is removed if the flow is distributed down slope from cell to cell (Figure 3d).

For the node-to-node steepest descent algorithm, the increase in resolution has lead to significant branching of the valleys, which is clearly visible when the water flux is plotted (Figure ~~??c and d~~4a and b). For the node-to-node distributed algorithm, the morphology and distribution of water flux are similar for both the low and high resolution (Figure ~~??e and f~~4c and d), yet as with the cell-to-cell, ~~the~~ increased resolution leads to ~~some~~ increased branching of the network. The two distributed models give a smoother topography, as by distributing flow local carving of the landscape is reduced.

To understand better how increasing resolution impacts the model evolution the total sediment flux eroded from the model domain is plotted against time, and the final valley spacing is calculated (Figure ~~??~~5 and 6). To calculate the valley spacing I take horizontal swaths of the spatial distribution of water flux. For each swath profile a peak finding algorithm (Negri and Vestri, 2017) is used to find the distance from peak to peak in water flux. This distance is then averaged over the hundred swath profiles and over ten model runs to give ~~a mean valley wavelength and a 96th percentile, which are plotted in a boxplot (Figure ??~~the minimum, lower quartile, median, upper quartile, and maximum valley wavelength (Figure 5 and 6).

For the cell-to-cell steepest descent routing it can be seen that the evolution of the model is resolution dependent, as the wind-up time reduces as resolution is increased from 64 to 512 cells along the y-axis (Figure ~~??~~5a). Furthermore, the mean valley spacing reduces with increasing resolution (Figure ~~??~~5b). This behavior is not ideal, as it means that model behavior to perturbations in forcing might become resolution dependent. For the distributed algorithm wind-up times remain resolution dependent, while the mean valley spacing is similar for the four different resolutions (Figure 5c and d).

The node-to-node steepest descent routing algorithm is no better than the cell-to-cell steepest descent. In this case wind up time is resolution dependent, and the valley spacing increases with increasing resolution (Figure ~~??c and d~~6a and b). For the node-to-node steepest descent routing, at a resolution of 256 cells or less along the y-axis ~~or less there is a~~ there is an instability in the sediment flux output. This is due to the flow tipping between adjacent nodes due to small differences in relative elevation after each time iteration. This unstable behavior disappears for the higher resolution of 512 cells along the y-axis (Figure ~~??c~~6a).

It is only when flow is distributed from node-to-node that the LEM becomes significantly less resolution dependent (Figure ~~??e and f~~6c and d). For the node-to-node distributed algorithm the time evolution of sediment flux is similar for all resolutions, and the ~~mean~~ valley spacing is ~~much more~~ similar as resolution is increased. For the distributed flow routing the steady state

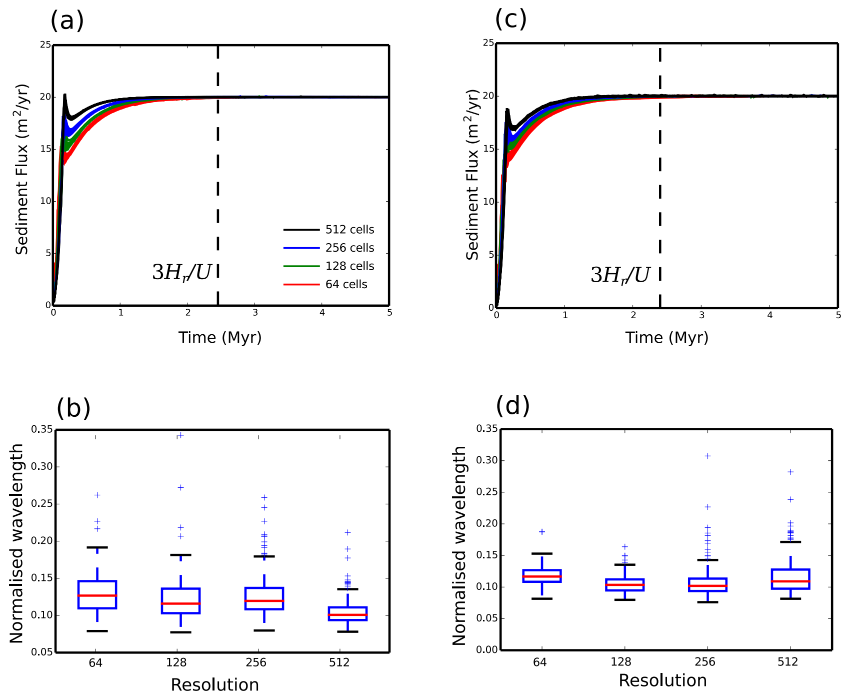

Figure 5. Dimensional sediment flux that exits the model domain and box whisker plots of the dimensionless valley-to-valley wavelength for each model for different resolutions, where the ~~numeber~~ number of cells along the y-axis is shown. (a) sediment flux and (b) valley-to-valley wavelength for the cell-to-cell steepest slope of descent routing algorithm. (c) sediment flux and (d) valley-to-valley wavelength for the ~~node-to-node steepest slope of descent routing algorithm. (e) sediment flux and (f) valley-to-valley wavelength for the node-to-node~~ cell-to-cell distributive routing algorithm. The dashed line in parts a, c, and e, marks the time at which erosion balances uplift, given by $t \geq 3H_r/U$ where H_r is the relief height and U is the uplift rate (Howard, 1994).

sediment flux is not completely stable (Figure ~~??e~~6c). This is due to the migration of the flow across the valley floors created within the model topography (Figure 7). Even once a balance has been achieved between erosion and uplift, small lateral changes in elevation can be seen to create a negative to positive change in elevation of a few meters between time iterations, where the time step is 100 yrs (Figure 7b). This is associated with an equivalent change in water flux (Figure 7c).

Changing the flow routing algorithm changes the model wind up time. This is because the rate at which the network grows and the water flux increases is effected by the choice of flow routing. The response time of the model is proportional to the water flux raised to the power n (Armitage et al., 2018). Therefore, if the drainage network forms rapidly, as is the case for cell-to-cell ~~steepest descent~~routing, then the model wind-up is more rapid. For the node-to-node routing, it takes longer for the network to grow (Figure ~~??~~5). Furthermore, the distributed flow routing model is the slowest to evolve to a steady state, where the total sediment flux is balanced by the uplift (Figure ~~??~~6). I have chosen to focus on $n = 1.5$ as this value previously gave more realistic slope-area relationships at steady state (Armitage et al., 2018). However, it is interesting to note that growth of the network is a function of both the routing algorithm and the value of n.

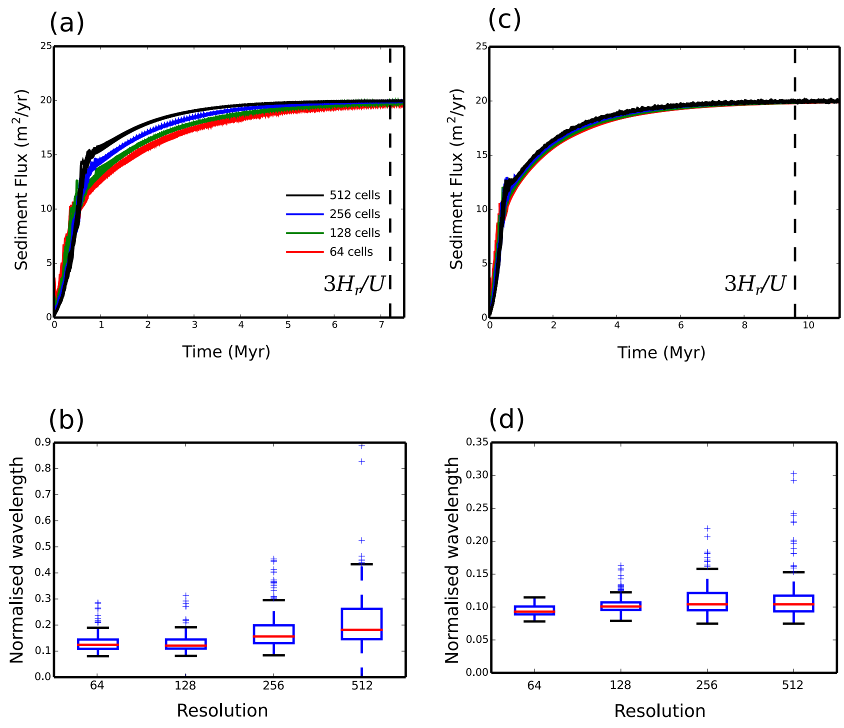

Figure 6. Dimensional sediment flux that exits the model domain and box whisker plots of the dimensionless valley-to-valley wavelength for each model for different resolutions, where the number of cells along the y-axis is shown. (a) sediment flux and (b) the node-to-node steepest slope of descent routing algorithm. (c) sediment flux and (d) valley-to-valley wavelength for the node-to-node distributive routing algorithm. The dashed line in parts a, and c, marks the time at which erosion balances uplift, given by $t \geq 3H_r/U$ where H_r is the relief height and U is the uplift rate (Howard, 1994) .

4 Sub-grid scale processes

The model that has the least resolution dependence is the node-to-node distributed flow (Figure 4 c and d, and 6c and d). The difference between this model and the other three is that this version has the maximum possible flow directions available within my set up. By treating flow paths as lines within the numerical grid, from any node there are 6 paths, which is twice as many as in the cell-to-cell distributed model. This means that there is greater distribution of the flow, and a reduced localising of flow paths within the node-to-node distributed model. For steepest descent increasing resolution however leads to multiple branches (Figure 3b and 4b).

The grid cells in the models presented are large. At the highest resolution, 512 by 2048 cells, the width of each triangle is of the order of 200 m if I was modelling a landscape 100 km wide. The model is therefore some approximation of local processes that give rise to the large scale landscape. By distributing flow the model is in a sense approximating for the hydrological processes that operate on a sub-grid scale that give rise to the river network. The assumption of steepest descent is however too strong, and the sub-grid scale processes are ignored.

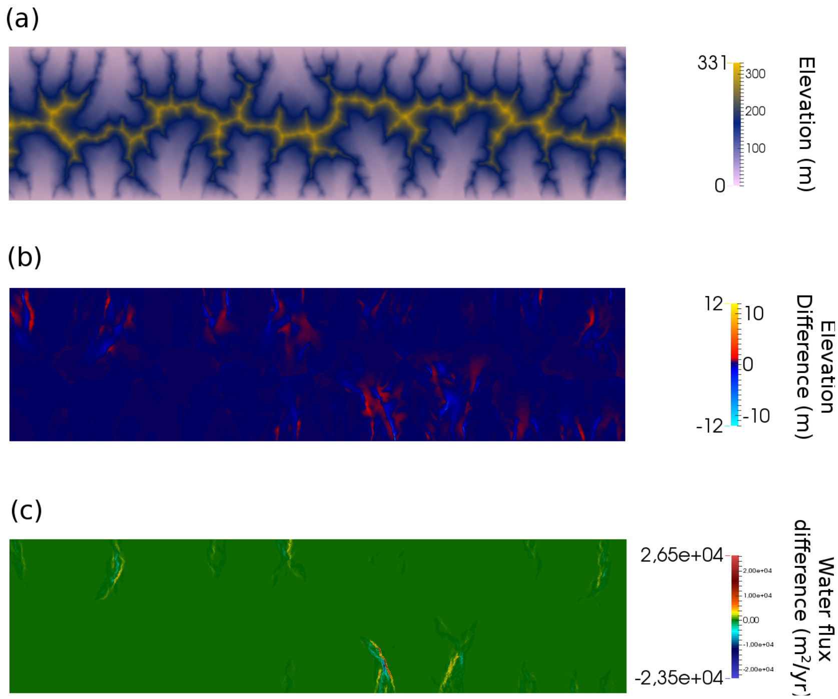

Figure 7. Final steady state of a example model run for the distributed node-to-node flow routing algorithm. (a) Final model elevation where the domain is 800 km long by 100 km wide and uplift is fixed at $U = 10^{-4}\,\mathrm{m\,yr^{-1}}$, the linear diffusion coefficient is $\kappa = 1\,\mathrm{m^2\,yr^{-1}}$, the fluvial diffusion coefficient is $c = 10^{-4}\,(\mathrm{m^2\,yr^{-1}})^{\mathrm{n}-1}$, and the water flux exponent is $n = 1.5$. (b) Difference in elevation between the last two model time steps, where the time step duration is 100 yrs. (c) Difference in water flux between the last two model time steps.

Another key sub-grid scale process is the migration of drainage divides. A drainage divide is the opposite of the flow path, as it separates the valleys. The numerical model Divide And Capture (DAC) was developed to explore if by using an analytical solution to the stream power law, the sub-grid scale migration of drainage divides could be captured (**?**) . DAC therefore uses a variant of a stream power law model, and like the model developed here, DAC uses a triangular grid, but routes flow down the steepest route of descent. By exploring how model resolution impacts the main drainage divide, it was demonstrated that the inclusion of a sub-grid level calculation for water divides is crucial to remove otherwise spurious results (**?**) .

By using the same setup of a domain of 1×4 aspect ratio, uplift at $0.1\,\mathrm{mm\,yr^{-1}}$, precipitation rate of $1\,\mathrm{m\,yr^{-1}}$, I have explored how valley spacing varies as a function of resolution in the DAC model. DAC uses an adaptive mesh, therefore the settings on how the re-meshing occurs needed to be altered to achieve an increase in the number of cells. By comparing two models at a different resolution, 23172 cells compared to 93734, it can be seen that the median wavelength is very similar (Figure 8).

The implication of the results I present here, and from the development of DAC, is that processes at a sub-grid level are of a crucial importance to model stability, and hence great care must be taken in generating reduced complexity LEMs. At a small spatial and temporal scale, the landscape evolution model CAESAR-LISFLOOD (**?**) has been tested for different

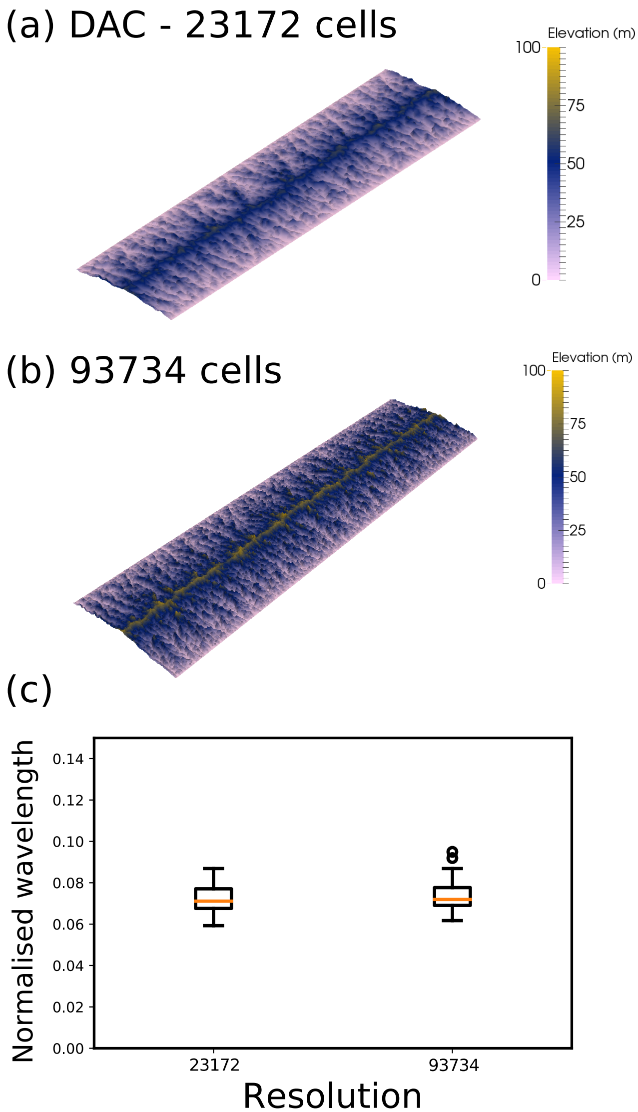

Figure 8. Comparison of two model results using Divide And Capture (DAC; **?**) at different resolutions. (a) Model steady state for an initial resolution of 51 by 204 cells, which after adaptive re-meshing increases to 23172 cells. (b) Model steady state for an initial resolution of 101 by 404 cells, which after adaptive re-meshing increases to 93734 cells. (c) Comparison of the wavelength of valleys for the two models, taken from twenty swaths 1.25 km wide from the left hand boundary (see code availability for python scripts and DAC input files).

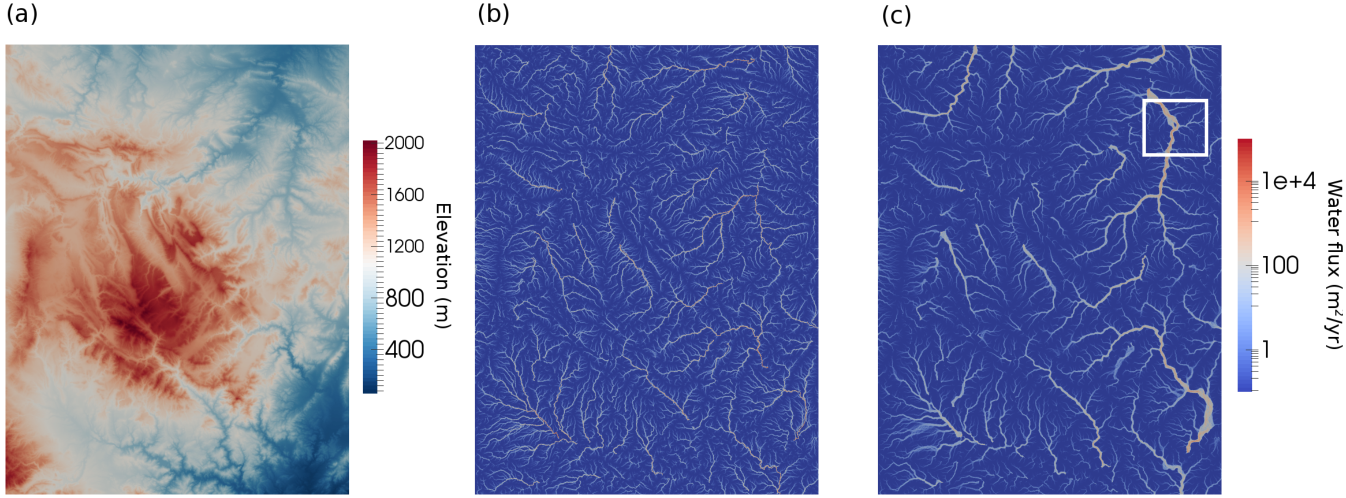

Figure 9. Application of the cell-to-cell steepest slope of descent and node-node distributed algorithms to a palaeo-DEM (digital elevation model). (a) Palaeo-DEM created from ASTER data of the Ebro region of Spain. (b) Water flux after 20 kyrs of model evolution assuming steepest slope of descent with a model resolution of 1024×1024 cells. Uplift is assumed to be very small, at $10^{-5}\,\mathrm{m\,yr^{-1}}$, with a precipitation rate held constant at $0.1\,\mathrm{m\,yr^{-1}}$. (c) Water flux for after 20 kyrs for a model assuming the node-to-node distributed flow routing. The White box in the top right highlights a region of Rio Bergantes catchment where the river is known to have shifted course during the Holocene.

resolutions, and has been found to converge to the same solution for at increased resolution. CEASAR-LISFLOOD uses a version of the shallow water equations to solve for river flow, and therefore operates on a resolution that is smaller than the width of an individual channel. Such a high resolution model however cannot be run over periods greater than several millennia (e.g. Couthard and van der Weil, 2013) . Therefore to explore how landscape evolves over millions of years I suggest we must distribute flow across the model domain to avoid the unreasonable localisation of flow.

5 Steady state but not steady topography

In experiments of sediment transport it has been noted that when the catchment outlet is fixed in time, the landscape does not achieve a steady fixed topography (Hasbergen and Paola, 2000). It has been previously suggested that this behavior can be replicated within a LEM by introducing a distributed routing algorithm (Pelletier, 2004). This modeling result has however been challenged by for example Perron et al. (2008), where it has been suggested that distributive flow routing algorithms in fact create a fixed topography at steady state. My model, however, is in agreement with the initial findings of Pelletier (2004). It has been previously noted that a distributed flow routing will give more diffuse valley bottoms compared to the steepest slope of descent (Freeman, 1991). If landscapes are indeed never steady, then perhaps this unsteady nature is due to the diffuse sediment transport across wide flood plains, which feeds up into the drainage basins. It is, after all, within the valley floor that the distributed flow routing is the most unsteady (Figure 7c).

In nature we observe that river networks are not fixed in space and time, rather various processes , such as avulsions, lead to changing flow directions. Observations such as terraces attest to the changing paths of river flow. To further explore how realistic the cell-to-cell steepest descent and node-to-node distributive algorithms, are I compare how the flow of water is predicted to evolve after a 20 kyr interval. The initial condition is a palaeo-DEM generated from ASTER data from the Ebro Basin, Spain (Figure 9a). The river valleys have been filled, and the landscape has been smoothed, in an attempt to approximate this landscape in the late Pleistocene. This landscape is then allowed to evolve assuming a uniform uplift of $10^{-5}\,\mathrm{m\,yr^{-1}}$ and a precipitation rate held constant at $0.1\,\mathrm{m\,yr^{-1}}$. I assume that $c = 10^{-5}\,(\mathrm{m^2\,yr^{-1}})^{\mathrm{n}-1}$, $\kappa = 10^{-1}\,\mathrm{m^2\,yr^{-1}}$, and $n = 1.5$. Under these conditions the landscape is left to evolve for 20 kyrs (Figure 9) with zero gradient boundaries on the east, west and southern sides, and fixed elevation on the northern boundary.

The initial condition is derived from a real landscape, and as the model allows for deposition in regions of low slope, both model routing algorithms do not create drainage patterns that fully connect to the boundaries (Figure 9b and c). This problem of too much deposition within in regions of low slope, such that the water flux does not reach the model boundaries, can be overcome with the application of a "carving" algorithm. As for example applied within TTLEM, a minima imposition can be used to make sure rivers keep on flowing down through regios regions of low slope Campforts et al. (2017). Such an additional algorithm will however effect how the network grows within the model, so for this example, I have left the routing algorithm to drain internally.

Despite this imperfection, the internal drainage patterns still prove to be insightful. The cell-to-cell steepest descent algorithm creates single paths for the flow of water (Figure 9b). After the 20 kyr duration it is observed that high water flux is concentrated within the deep valleys. The node-to-node distributed algorithm creates multiple flow paths that exit the mountain valleys and migrate onto the flood plains (Figure 9c). Field studies of the Rio Bergantes have found that this catchment has experienced periods of significant sediment reworking, potentially related to climatic change (Whitfield et al., 2013). The region outlined with the white box in Figure 9c shows evidence of terrace formation related to lateral movement of the Rio Bergantes during the Holocene (Whitfield et al., 2013). In particular, where the flow paths create a small island (see Figure 9c, center of the white box), there is evidence from terrace deposits that the course of the Rio Bergantes has flipped from the eastern to the western side of this island. The cell-to-cell steepest descent cannot create this observed behavior. Therefore, as well as creating landscape evolution that is not resolution dependent, the distributive algorithm creates landscape evolution that is, relative to the steepest descent, closer to that observed in nature.

6 Conclusions

In the study of the evolution of the Earth surface we are increasingly turning to models that attempt to capture the complexities of surface processes. It is however clear that many LEMs are resolution dependent (Schoorl et al., 2000). The source of this resolution dependence is the numerical methods that we employ to route surface water. Unless we model landscape evolution at a spatial scale that is smaller than an individual river, then we must somehow approximate this flow. By assuming that rivers are treating flow from node-to-node, lines within the model mesh, and by distributing flow down these lines, the LEM developed here is no longer resolution dependent. Furthermore the model evolution is closer to what we observe. Therefore, I would strongly suggest that for LEMs that operate at a scale larger than the resolution of a river ~~, if we treat rivers as lines within the landscape,~~ we must use distributed flow routing.

Acknowledgments

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Code availability

The code fLEM is available from the following repository https://bitbucket.org/johnjarmitage/flem/. The valley wavelength Python script and DAC input files are available from the following repository https://bitbucket.org/johnjarmitage/dac-scripts/. DAC was developed by Liran Goren, see https://gitlab.ethz.ch/esd_public/DAC_release/wikis/home.

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