Earth Surf. Dynam. Discuss., https://doi.org/10.5194/esurf-2018-50
Manuscript under review for journal Earth Surf. Dynam.
Discussion started: 27 June 2018

SHORT COMMUNICATION: RIVERS AS LINES WITHIN THE LANDSCAPE

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Abstract. Landscape evolution models (LEMs) aim to capture an aggregation of the processes of erosion and deposition within the Earth's surface and predict the evolving topography. A key aspect of any LEM is the algorithm chosen to route water down the surface. In nature precipitation makes its way to rivers as a surface flow and as groundwater. Furthermore, at the scale of a mountain range, country, or even continent, the width of any given river is so small relative to the scale of the landscape that it is essentially a line. Taking this abstraction as a starting point, I explore the consequences of assuming that water flows over the surface of a LEM along lines rather than over the surface area. By making this assumption and distributing the flow along the edges of the mesh cells, node-to-node, I find that the resolution dependence of the evolution of LEM is significantly reduced. Furthermore, the flow paths of water predicted by this node-to-node distributed routing algorithm is significantly closer to that observed in nature. Therefore I suggest that rivers are lines within the landscape, and we must treat them as such within LEMs that operate on a scale larger than a reach.

1 Introduction

It is known that resolution impacts landscape evolution models (LEMs) (Schoorl et al., 2000). The route of this problem is in the calculation of upstream area for the water flux term in the set of governing equations. As resolution is increased the upstream area typically changes, and this problem has lead to the addition of for example weighting terms to control the width of the river at a sub grid level (Perron et al., 2008). Yet, the addition of an arbitrary river width term is not ideal as it might influence observations such as valley spacing and response times for the landscape to recover to a perturbation. For example, landscape response time is a function of the water flux (Armitage et al., 2018), which will be influenced by the introduction of a term that controls the river width. In this contribution, I will therefore explore how flow routing effects landscape evolution.

The river network is the primary route through which water flows down slope. In any given reach of a river or stream the majority of the water flowing through it comes from the uphill end and exits through the down hill end. A smaller amount of water will enter the river from the sides through groundwater seepage or over-land run-off. This raises the question: is it reasonable for a long term landscape evolution model to have water flow over all surfaces, or to only allow water to flow down lines? Is the river network sensitive to the upstream network length or the upstream area? Building on this question I will explore how a simple LEM evolves as I change the flow routing algorithm from one that routes water from cell-to-cell down the steepest descent routing, to an algorithm that distributes flow from node-to-node.

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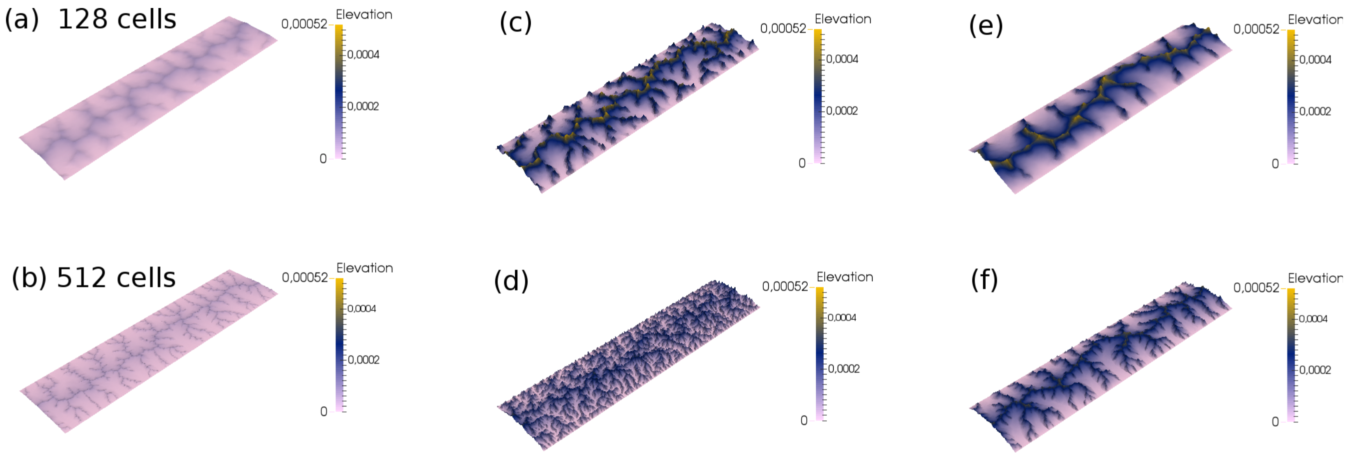

Figure 1. Dimensionless elevation from the landscape evolution model with different flow routing algorithms at different numerical resolutions after a dimensionless run time of 1.563×10^{-6} (5 Myr), with an aspect ratio of 8×1. (a) Cell-to-cell steepest descent routing algorithm with a resolution of 1024×128 cells. (b) The same model but with a resolution of 4096×512 cells. (c) and (d) node-to-node steepest descent routing algorithm. (e) and (f) node-to-node distributed flow routing algorithm.

2 A landscape evolution model

In this study I will assume landscape evolution can be effectively simulated with the classic set of diffusive equations described in (Smith and Bretherton, 1972):

$$\frac{\partial z}{\partial t} = \nabla \left[\left(\kappa + c q_w^n \right) \nabla z \right] + U \quad (1)$$

where κ is a linear diffusion coefficient, c is the fluvial diffusion coefficient, q_w is the water flux, n is the water flux exponent, and U is uplift. This equation is solved with a finite element scheme written using Python and the FEniCS libraries (I will call the code "fLEM", see Code Availability). The equations are solved on a Delaunay mesh, where the mesh is made up of predominantly equilateral triangles with an opening angle of 60°. Model boundary conditions are initially of fixed elevation on the sides normal to the x-axis and zero gradient on the sides in normal to the y-axis. The model aspect ratio is 1 to 8 (see Figure 1). Uplift is fixed at $U = 10^{-4}\,\mathrm{m\,yr^{-1}}$, the linear diffusion coefficient is $\kappa = 1\,\mathrm{m^2\,yr^{-1}}$, the fluvial diffusion coefficient is $c = 10^{-4}\,(\mathrm{m^2\,yr^{-1}})^{\mathrm{n}-1}$, and the water flux exponent is $n = 1.5$.

Water can be routed from cell-to-cell, where precipitation is collected over the area of each cell, sent downwards, and accumulates. In this cell-to-cell configuration the water flux has units of length squared per unit time and is given by:

$$q_w[\mathrm{cell}] = \frac{\alpha a}{l_s}, \quad (2)$$

where α is precipitation rate, a is the cell area, and ls is the length from cell center to cell center down the steepest slope (Figure 2). I assume a precipitation rate of $\alpha = 1\,\mathrm{m\,yr^{-1}}$. However, as discussed, water can also be assumed to only flow down the edges of each cell, from node-to-node. In this case water collects down all the edges, is sent downwards, and accumulates.

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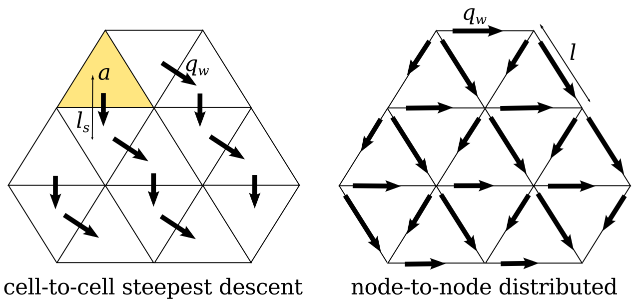

Figure 2. Diagram of flow routing from cell-to-cell down the steepest descent and a node-to-node routing down all slopes weighted by the relative gradient.

The water flux term does not know about the area of each cell, and the precipitation rate includes additions of water from the sides of the edges (*rivers*). Water flux again has units of length squared per unit time:

$$q_w[\text{node}] = \alpha l, \tag{3}$$

where l is the length of the edge of that joins the up-slope node to the down-slope node (Figure 2). For both the cell-to-cell and node-to-node methods the flow can then be routed down the steepest slope of descent. For the node-to-node method of routing water, I also route water down all slopes weighted by the relative gradient of each slope. This therefore gives three routing algorithms, (1) cell-to-cell steepest descent, (2) node-to-node steepest descent, and (3) node-to-node distributed.

3 The effect of model resolution

At a low model resolution, 1023×128 cells, all three methods of flow routing give similar landscape morphology after 5 Myr of model evolution (Figure 1). However, elevations are significantly lower for the cell-to-cell flow routing model as the water flux term operates across the cells rather than on individual node points (Figure 1a and b). As the resolution is increased to 4096×512 cells, the landscape morphology starts to diverge (Figure 1). In the cell-to-cell routing algorithm the landscape shows more small scale branching, as previously discussed by (Braun and Sambridge, 1997) (Figure 1a and b). In the node-to-node steepest descent algorithm, the increase in resolution has lead to significant branching of the valleys, which is clearly visible when the water flux is plotted (Figure 1c and d). For the node-to-node distributed algorithm, the morphology and distribution of water flux are similar for both the low and high resolution (Figure 1e and f), yet as with the cell-to-cell, the increased resolution leads to some increased branching of the network.

To understand better how increasing resolution impacts the model evolution the total sediment flux eroded from the model domain is plotted against time, and the final valley spacing is calculated (Figure 3). To calculate the valley spacing I take horizontal swaths of the spatial distribution of water flux. For each swath profile a peak finding algorithm (Negri and Vestri, 2017) is used to find the distance from peak to peak in water flux. This distance is then averaged over the hundred swath profiles and over ten model runs to give a mean valley wavelength and a 96th percentile, which are plotted in a boxplot (Figure 3).

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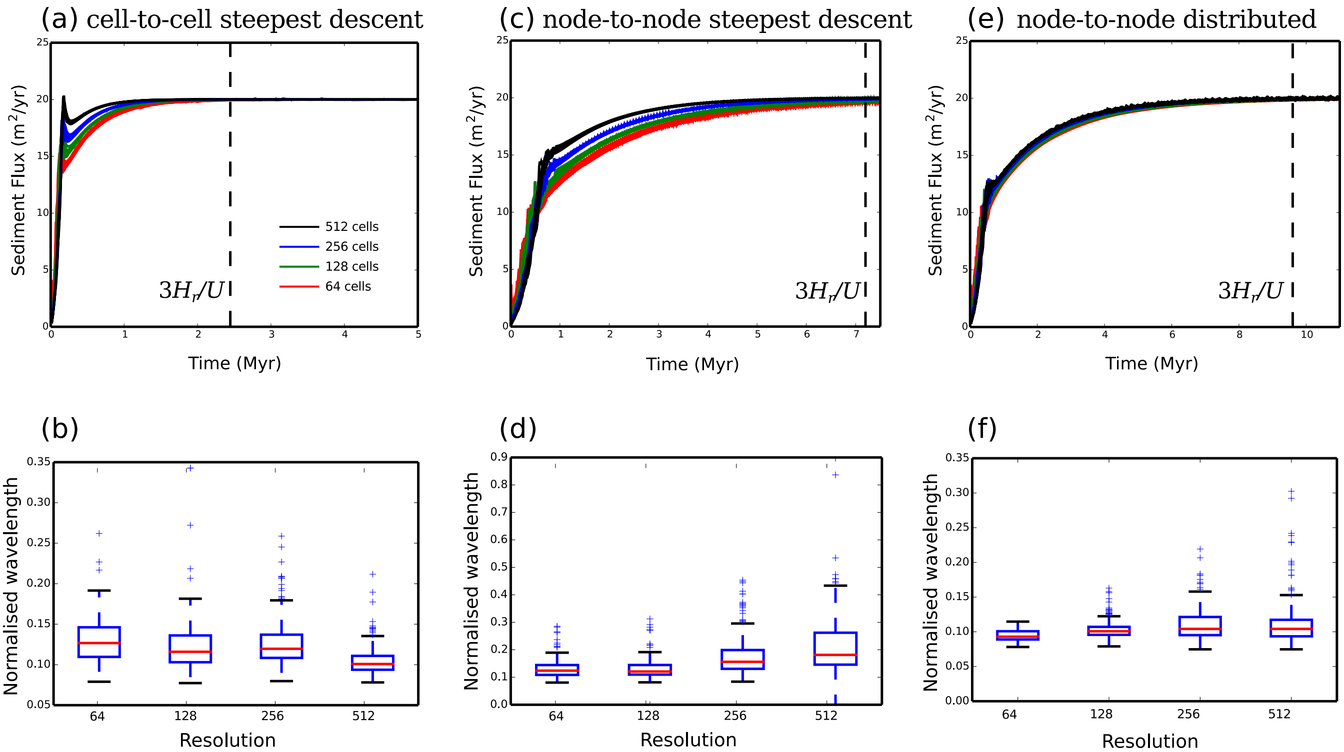

Figure 3. Dimensional sediment flux that exits the model domain and box whisker plots of the dimensionless valley-to-valley wavelength for each model for different resolutions, where the numeber of cells along the y-axis is shown. (a) sediment flux and (b) valley-to-valley wavelength for the cell-to-cell steepest slope of descent routing algorithm. (c) sediment flux and (d) valley-to-valley wavelength for the node-to-node steepest slope of descent routing algorithm. (e) sediment flux and (f) valley-to-valley wavelength for the node-to-node distributive routing algorithm. The dashed line in parts a, c, and e, marks the time at which erosion balances uplift, given by $t \geq 3H_r/U$ where H_r is the relief height and U is the uplift rate (Howard, 1994).

For the cell-to-cell routing it can be seen that the evolution of the model is resolution dependent, as the wind-up time reduces as resolution is increased from 64 to 512 cells along the y-axis (Figure 3a). Furthermore, the mean valley spacing reduces with increasing resolution (Figure 3b). This behavior is not ideal, as it means that model behavior to perturbations in forcing might become resolution dependent. The node-to-node steepest descent routing algorithm is no better. In this case wind up time is resolution dependent, and the valley spacing increases with increasing resolution (Figure 3c and d). For the node-to-node steepest descent routing, at a resolution of 256 cells along the y-axis or less there is a instability in the sediment flux output. This is due to the flow tipping between adjacent nodes due to small differences in relative elevation after each time iteration. This unstable behavior disappears for the higher resolution of 512 cells along the y-axis (Figure 3c).

It is only when flow is distributed from node-to-node that the LEM becomes significantly less resolution dependent (Figure 3e and f). For the node-to-node distributed algorithm the time evolution of sediment flux is similar for all resolutions, and the

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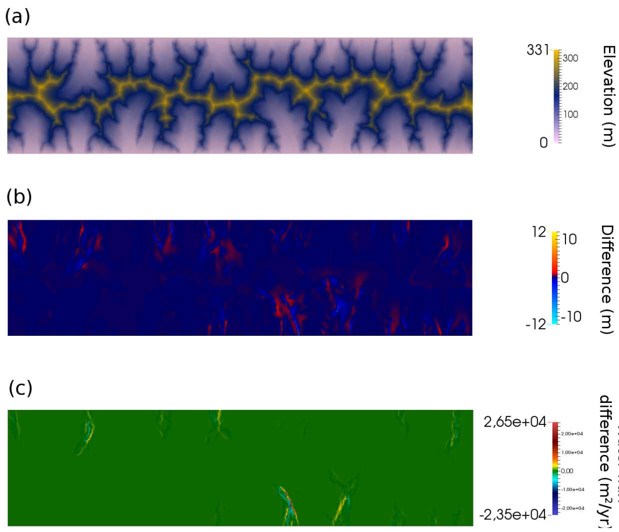

Figure 4. Final steady state of a example model run for the distributed node-to-node flow routing algorithm. (a) Final model elevation where the domain is 800 km long by 100 km wide and uplift is fixed at $U = 10^{-4}\,\mathrm{m\,yr^{-1}}$, the linear diffusion coefficient is $\kappa = 1\,\mathrm{m^2\,yr^{-1}}$, the fluvial diffusion coefficient is $c = 10^{-4}\,(\mathrm{m^2\,yr^{-1}})^{\mathrm{n}-1}$, and the water flux exponent is $n = 1.5$. (b) Difference in elevation between the last two model time steps, where the time step duration is 100 yrs. (c) Difference in water flux between the last two model time steps.

mean valley spacing is much more similar as resolution is increased. For the distributed flow routing the steady state sediment flux is not completely stable (Figure 3e). This is due to the migration of the flow across the valley floors created within the model topography (Figure 4). Even once a balance has been achieved between erosion and uplift, small lateral changes in elevation can be seen to create a negative to positive change in elevation of a few meters between time iterations, where the time step is 100 yrs (Figure 4b). This is associated with an equivalent change in water flux (Figure 4c).

Changing the flow routing algorithm changes the model wind up time. This is because the rate at which the network grows and the water flux increases is effected by the choice of flow routing. The response time of the model is proportional to the water flux raised to the power n (Armitage et al., 2018). Therefore, if the drainage network forms rapidly, as is the case for cell-to-cell steepest descent, then the model wind-up is more rapid. For the node-to-node routing, it takes longer for the network to grow (Figure 3). Furthermore, the distributed flow routing model is the slowest to evolve to a steady state, where the total sediment flux is balanced by the uplift (Figure 3). I have chosen to focus on $n = 1.5$ as this value previously gave more realistic slope-area relationships at steady state (Armitage et al., 2018). However, it is interesting to note that growth of the network is a function of both the routing algorithm and the value of n.

4 Steady state but not steady topography

In experiments of sediment transport it has been noted that when the catchment outlet is fixed in time, the landscape does not achieve a steady fixed topography (Hasbergen and Paola, 2000). It has been previously suggested that this behavior can be

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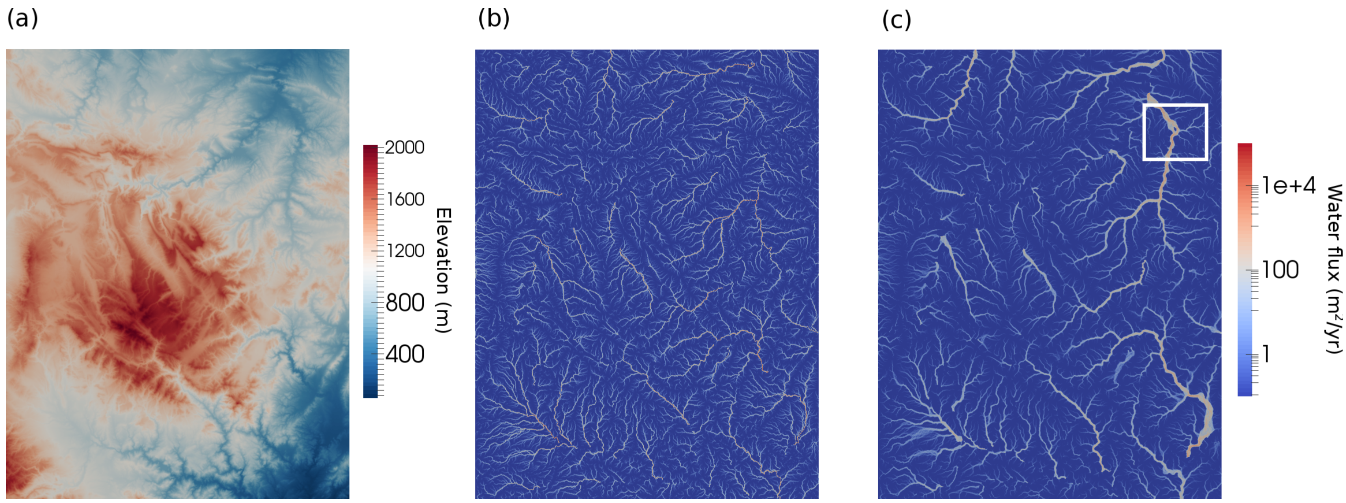

Figure 5. Application of the cell-to-cell steepest slope of descent and node-node distributed algorithms to a palaeo-DEM (digital elevation model). (a) Palaeo-DEM created from ASTER data of the Ebro region of Spain. (b) Water flux after 20 kyrs of model evolution assuming steepest slope of descent with a model resolution of 1024×1024 cells. Uplift is assumed to be very small, at $10^{-5}\,\mathrm{m\,yr^{-1}}$, with a precipitation rate held constant at $0.1\,\mathrm{m\,yr^{-1}}$. (c) Water flux for after 20 kyrs for a model assuming the node-to-node distributed flow routing. The White box in the top right highlights a region of Rio Bergantes catchment where the river is known to have shifted course during the Holocene.

replicated within a LEM by introducing a distributed routing algorithm (Pelletier, 2004). This modeling result has however been challenged by for example Perron et al. (2008), where it has been suggested that distributive flow routing algorithms in fact create a fixed topography at steady state. My model, however, is in agreement with the initial findings of Pelletier (2004). It has been previously noted that a distributed flow routing will give more diffuse valley bottoms compared to the steepest slope of descent (Freeman, 1991). If landscapes are indeed never steady, then perhaps this unsteady nature is due to the diffuse sediment transport across wide flood plains, which feeds up into the drainage basins. It is, after all, within the valley floor that the distributed flow routing is the most unsteady (Figure 4c).

In nature we observe that river networks are not fixed in space and time, rather various processes, such as avulsions, lead to changing flow directions. Observations such as terraces attest to the changing paths of river flow. To further explore how realistic the cell-to-cell steepest descent and node-to-node distributive algorithms are I compare how the flow of water is predicted to evolve after a 20 kyr interval. The initial condition is a palaeo-DEM generated from ASTER data from the Ebro Basin, Spain (Figure 5a). The river valleys have been filled, and the landscape has been smoothed, in an attempt to approximate this landscape in the late Pleistocene. This landscape is then allowed to evolve assuming a uniform uplift of $10^{-5}\,\mathrm{m\,yr^{-1}}$ and a precipitation rate held constant at $0.1\,\mathrm{m\,yr^{-1}}$. I assume that $c = 10^{-5}\,(\mathrm{m^2\,yr^{-1}})^{\mathrm{n}-1}$, $\kappa = 10^{-1}\,\mathrm{m^2\,yr^{-1}}$, and $n = 1.5$. Under these conditions the landscape is left to evolve for 20 kyrs (Figure 5) with zero gradient boundaries on the east, west and southern sides, and fixed elevation on the northern boundary.

The initial condition is derived from a real landscape, and as the model allows for deposition in regions of low slope, both model routing algorithms do not create drainage patterns that fully connect to the boundaries (Figure 5b and c). This problem of too much deposition within in regions of low slope, such that the water flux does not reach the model boundaries, can be overcome with the application of a “carving” algorithm. As for example applied within TTLEM, a minima imposition can be used to make sure rivers keep on flowing down through regios of low slope Campforts et al. (2017). Such an additional algorithm will however effect how the network grows within the model, so for this example, I have left the routing algorithm to drain internally.

Despite this imperfection, the internal drainage patterns still prove to be insightful. The cell-to-cell steepest descent algorithm creates single paths for the flow of water (Figure 5b). After the 20 kyr duration it is observed that high water flux is concentrated within the deep valleys. The node-to-node distributed algorithm creates multiple flow paths that exit the mountain valleys and migrate onto the flood plains (Figure 5c). Field studies of the Rio Bergantes have found that this catchment has experienced periods of significant sediment reworking, potentially related to climatic change (Whitfield et al., 2013). The region outlined with the white box in Figure 5c shows evidence of terrace formation related to lateral movement of the Rio Bergantes during the Holocene (Whitfield et al., 2013). In particular, where the flow paths create a small island (see Figure 5c, center of the white box), there is evidence from terrace deposits that the course of the Rio Bergantes has flipped from the eastern to the western side of this island. The cell-to-cell steepest descent cannot create this observed behavior. Therefore, as well as creating landscape evolution that is not resolution dependent, the distributive algorithm creates landscape evolution that is, relative to the steepest descent, closer to that observed in nature.

5 Conclusions

In the study of the evolution of the Earth surface we are increasingly turning to models that attempt to capture the complexities of surface processes. It is however clear that many LEMs are resolution dependent (Schoorl et al., 2000). The source of this resolution dependence is the numerical methods that we employ to route surface water. Unless we model landscape evolution at a spatial scale that is smaller than an individual river, then we must somehow approximate this flow. By assuming that rivers are lines within the model mesh, and by distributing flow down these lines, the LEM is no longer resolution dependent. Furthermore the model evolution is closer to what we observe. Therefore, I would strongly suggest that for LEMs that operate at a scale larger than the resolution of a river, if we treat rivers as lines within the landscape, we must use distributed flow routing.

Acknowledgments

This work was inspired from a series of meetings in organized by the Facsimile working group and from a visit to the Rio Bergantes catchment in Spain in October 2017. John Armitage is funded through the French Agence National de la Recherche, Accueil de Chercheurs de Haut Niveau call, grant “InterRift”.

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Code availability

The code fLEM is available from the following repository https://bitbucket.org/johnjarmitage/flem/.

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