We thank the two referees for their thorough and constructive comments. We agree with nearly all of their points and have revised the manuscript incorporating their suggestions. We were pleased to see that both referees' found the study to be technical sound, with mostly stylistic suggestions, and both suggested minor revisions. The most significant change is to the introduction, as both referees' requested more motivation and road mapping for the paper. Below we give detailed responses to the referees' comments.

Anonymous Referee #1

Summary. The authors present a study on suspended sediment concentration via the Rouse equation. They review existing equations and through a new compilation of suspended sediment profiles they provide an improved empirical fit for the sediment entrainment function, a fitting parameter Beta, and the Rouse number.

General Comments. Overall the paper appears technically sound and I have no major reservations with the work presented by the authors that would present it from being published following some revisions. I have some general editorial or stylistic suggestions for the authors that may improve the manuscript and I leave it to them to imple ment them or not. As the introduction currently reads the paper appears focused on providing a better empirical fit to data than past empirically based equations. Rather than focusing on previous empirical equations, why not focus on the data and allow your analysis to drive the narative. As an example, seeing all of the lines in Figure 3 is not that useful as some of them likely only differ due to differences in the datasets they were calibrated on. As is, I did not find the introduction to be any more insightful than that of Garcia and Parker (1991) other then adding a few more equations. It might be worthwhile to replace figures 2 & 3 with the concentration profiles and show the newly compiled data that is what really sets the current work apart from previous iterations.

REPLY: We thank the reviewer for their thorough and constructive comments. We agree on almost all points. We have made several edits to the introduction following the reviewer's suggestion. We now better motivate the study and discuss why entrainment is important, what we know and what are the limitations. And we also better roadmap the paper. It's not clear to us how to use the data to motivate the work, because we do not present the data until after going through the methods. One of the main motivations for the work is the inconsistency in previous work. Because of this, we decided to leave the different models on Figure 3. Reviewer # 2 seems to agree that the review of previous models is useful.

The title in this regard seems a bit misleading as this paper is primarily about sand. The gravel component is interesting, however it is not as well integrated into the manuscript and may be better as a stand alone manuscript once data is available to validate the claims. I am not necessarily suggesting that it be removed, just that from my perspective it isn't the best fit at the moment given the data limitations and scope of the rest of the paper.

REPLY: This is a good point. We understand that data on gravel is limited; however, we do present theory for gravel and compare the gravel theory to the sand data. This is a main point of our paper—to develop a theory for entrainment that may be applicable to both sand and gravel. The need for such a theory has been emphasized in the revised introduction. We present theory for gravel saltation in the Methods (Section 2.3), results (section 3.4) and discussion (Section 4.3). For these reasons we prefer to leave gravel in the title. We hope our paper will help show a path forward in future work to test the relation under the gravel regime.

Specific Comments.

Ln. 97 - Could you provide the rational for beta=1. Lines 58-76 are all about beta being less than or greater than 1, but not equal. It is fine that it is one, but please work that reasoning into the preceeding paragraphs on beta.

REPLY: This has been fixed. Betta = 1 is a common assumption, and is the simplest model.

*Table 1a. The parameter column could use a bit more explanation or consistency. As an example, Smith & McLean 1977 have t*skin & t*c in the parameter column while Van Rijn, 1984 does not, even though they are both listed in the equation.*

REPLY: This has been corrected.

Figure 3. You might consider making this figure viewable in black and white or for people with visual impairments (color blindness).

REPLY: We have changed the line thickness to further distinguish the models. Thanks for this suggestion. We also carried this through to Figure 10.

Ln. 118 - 'workers' is a bit of an odd word for researchers here.

REPLY: Changes as suggested.

Ln. 227 - missing an 'is' or substract 'that'. '...one that based on...'

REPLY: Changes as suggested.

Ln. 252 - A shields stress of a 1000 seems to be a bit far fetched for gravel. Consider that at a 10% slope for pea gravel (0.5 cm) that would require an 80 m deep flow. That isn't realistic.

REPLY: Thank you for this point. This is simply a range over which to explore the model. Fine sand and silt in mountain rivers can have shields numbers this high. They can also occur during geologically significant floods, such as the in the Missoula Floods. We list references in the main text.

Ln. 258 - It is not clear that an R2 of 0.4 is significantly better than 0.33. The *distributions of the predicted/measured (fig. 6) also do not look to be statistically distinct to make a claim of significance either. Could you instead provide some physical reasoning as to why the two parameter model is the best choice.*

REPLY: We have clarified in the text that we prefer the two parameter model because the r2 is not significantly improved with the addition of a third parameter, and that there are many three parameter models with similar misfits. That said, all models are given in Table S2 including their r2 values, so the reader can choose which ever model best fits the application.

Fig.4 - Please clarify if the following is the correct interpretation. Equation (2) is fit to the profile data where P is treated as a fitting parameter. Then P is regressed against a variety of variables in Figure 5. This could be made a bit clearer in the begining of the results section as it was not entirely clear where P comes from in Fig. 5.

REPLY: Yes, that is correct. We have added further clarification on line 270 - 275.

Fig. 7 - Could you provide a reasoning for the choice of binned data width and number of bins?

REPLY: This is an arbitrary choice for binned width. This has been added to the figure captions.

Fig. 5, 7, 8, 10, 11, 14 - Consider plotting the data as a 2D density plot as this won't obscure the majority of the data. At the moment it is hard to see what the data actually look like when they are all plotted on top of each other.

REPLY: We appreciate this point. We tried plotting the data many different ways, but decided the best way is to bin data to show the central tendency. We have added binned data to Figures 5, 8 and 14. We believe that Fig. 11 would be confusing with binned data since it is already divided into three groups. Figs. 7 and 10 already had binned data.

Ln. 288 - The previous relations (and the new ones) are all semi empirical based on limited field data, it is not surprising that by increasing the data (especially the ranges) that new model fit to these data performs better overall. I am not sure the numerous model comparisons are really a necessary component for this paper.

REPLY: We understand this point and thank you for the opportunity to clarify. Although the models are semi-empirical, they are intended to represent the physics in a generic way. One of the motivations for us to conduct this study is that the previous papers did not show comparisons between different models. This makes it unclear how the various entrainment models compare with each other. This is why we think it is important to show how the model compares to previous models.

Fig. 10 - Not clear what the solid black line that tracks the dashed black line is in panel (a).

REPLY: This was an error and has been deleted. Thank you for noting that.

Ln. 312 - Not seeing a fig. 11b.

REPLY: This typo has been fixed.

Ln. 381 - It would be worth taking a look at the recently published work by Ashley et al. (2020) in Water Resources Research on 'Estimating bedload from suspended load...'.

REPLY: Thank you for this suggestion. We have added that citation as suggested.

Ln. 385 - It looks like Ci/Ca increases as z/H approaches 0. You might show that the trend is not significant and that would justify the mean, which looks a bit skewed high, potentially by some outliers. Maybe the median would be a better parameter.

REPLY: We have added binned data that more clearly shows that the concentration data is nearly uniform with depth. We agree and have added the geometric mean value rather than the mean.

Anonymous Referee #2

Received and published: 18 February 2020

This paper presents the development of new equations for transport by suspension. The author fit the Rouse number and Entrainment parameter with a large data set, and ultimately they derive a general equation for concentration. The results seem to be very promising; nonetheless the model was calibrated but not validated. The paper is well written but I think it could be improved for clarity, especially in introduction and discussion. I propose minor revision; the authors will have no difficulty in answering the various point presented below.

Comments I found the introduction a bit confused. Instead of presenting general considerations on suspension (why it is important, what do we know, what are the limitations, what are the differences between lowland and mountain rivers. . .), you go straight in a presentation of limitations of existing mechanistic approaches through a very exhaustive literature review (congratulation for the review) and new analysis. In addition the title is a bit confusing because when mentioning "sand and gravel" we expect more consideration for suspension of coarse sand and gravel, and this aspect is not really developed (in the introduction but also in the paper where the data sets comprises fine sands only) which, in my opinion, reduces the scope of the paper to situations where suspension can freely develop from fine bed sediments. Finally, it takes time to really understand the objectives of the paper. For clarity it might have been more efficient to really explain the context and objectives in introduction and describe the equations limitations in a next part called for instance "review of the existing theory".. ? This is a suggestion, I let the authors decide how to arrange the paper, but the must improve the message in introduction.

REPLY: Thank you for raising this concern about the introduction. We have made several edits to the introduction following the reviewer's suggestion. We now better motivate the study and discuss why entrainment is important, what we know and what are the limitations. And we also better roadmap the paper as suggested.

Line 80: If Fz is the upward flux of sediment it is not clear how Fz/ws is dimensionless. Could you give the dimension each time you introduce a parameters?

REPLY: We have added dimensions to the parameters as suggested.

Line 177: Eq.7 is not usual; maybe you can give a reference or explain how it was obtained?

REPLY: We have rewritten this equation in its typical form, added a reference, and better explained how it is used. Thank you for pointing out the confusion.

Line 188: it is not straightforward: write the Shields stress with Eq11

REPLY: Thank you for pointing this out. We have added the algebra to make this substitution.

Line 199: does hiding effects make sense for sands?

REPLY: This parameter was included by Garcia and Parker, 1991; Wright and Parker, 2004. We are not aware of an argument against hiding effects in sands.

Lines 213-214: this sentence is not really clear but is essential for understanding the methodology. I understood that you fit P with the data and compare to variables? I suggest that you develop a bit more this methodological point to insist on the absence of spurious correlation in Figure 5.

REPLY: We expanded this sentence to clarify the point.

Line 230: Because of the absence of data, the approach for gravels is purely conceptual. One can for instance question on the validity of Eq20 and 21 at high shear stress (was this aspect considered in the original paper)?

REPLY: We appreciate this concern. It is partly theoretical, but partly based on semi-empirical equations from datasets on gravel saltation. At very high shear stresses, Eq. (20) and (21) asymptote to reasonable values (Chatanantavet et al., 2013). We added these points to revised paper.

Line 278: it could be clearer to start this paragraph with : "Figure 8 plots. . ." and explain again the parameters tested. For instance what was the reference level used for Esi in Figure 8?

REPLY: This has been changed as suggested.

Line 295-297: The way it is presented seems a bit arbitrary. Could you give a reference for that?

REPLY: Yes, this is following the approach of Garcia and Parker (1991).

Lines 398-399: I suppose that the threshold is 0.015 in Eq. 26? Is there a figure where we can visualize this threshold effect?

REPLY: We believe the reviewer is referring to lines 298-299 here. Yes, we clarified in the text that the threshold is 0.015 and added the model with the threshold to Fig. 8b.

Line 301: Equation 26 is made complex to limit its maximum to 0.33. In my opinion it's too bad to lose the equation aesthetics: you could keep a simplest form and just mention Esi<=0.3

REPLY: We appreciate this point, but have followed previous work on framing this limiting value.

Line 310: Why don't you give the definitive model (used for Fig 11)?

REPLY: Good point. We have added the model to the text here and in the caption to Fig. 11.

Line 316: This very short paragraph looks like more a discussion point (a perspective) than a real result.

REPLY: We understand this point. However, because we manipulated the bedload transport equations, as discussed in the Methods, we think it is appropriate to show the results of this work in the Results. We have added text for the gravel part of the study to the introduction and methods to better integrate this work into the paper.

Discussion: It could be worth discussing the model limitations (if any). For instance I have in mind the complex interactions that may exist with coarse gravel and cobles beds in Mountain Rivers.

REPLY: We have added unknown limitations with complex topography in mountain rivers on line 498

Your model has been calibrated but not validated. A lot of data are available in the literature could they be used for validation? If not could you discuss what should (could) be done for a validation in future research.

REPLY: We have added the need for model validation on line 460.

Figure 14: This result is surprisingly good. How was measured Cai for the runs considered? And what do you obtained when comparing qb_meas and qb_cal?

REPLY: Cai is concentration at 10% of the flow depth from our large dataset used throughout the results. This has been clarified in the figure caption. Unfortunately we cannot compare the total sediment flux because we did not develop a sediment or flow velocity model.

Entrainment and suspension of sand and gravel

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Abstract. Entrainment and suspension of sand and gravel is important for the evolution of rivers, deltas, coastal areas and submarine fans. The prediction of a vertical profile of suspended sediment concentration typically consists of assessing 1) the concentration near the bed using an entrainment relation and 2) the upward vertical distribution of sediment in the water column. Considerable uncertainty exists in regard to both of these steps, and especially the near-bed concentration. Most entrainment ~~theories~~ relations have been tested against limited grain-size specific data, and no relations have been evaluated for gravel suspension, which can be important in bedrock and mountain rivers~~, as well as powerful turbidity currents~~. To address these issues, we compiled a database with suspended sediment data from natural rivers and flume experiments, taking advantage of the increasing availability of high-resolution grain-size measurements. We evaluated 1~~4~~2 dimensionless parameters that may determine entrainment and suspension relations, and applied multivariate regression analysis. A best-fit two-parameter equation ($r^2 = 0.79$) shows that near-bed entrainment, evaluated at 10% of the flow depth, ~~increases~~ decreases with the ratio of ~~skin-friction shear velocity to~~ settling velocity to skin-friction shear velocity to (~~u_{*skin}/w_{st}~~ w_{si}/u_{*skin}), as in previous relations, and increases with Froude number (Fr), possibly due to its role in determining bedload-layer concentrations. We used the Rouse equation to predict concentration upward from the reference level, and evaluated the coefficient β_i, which accounts for differences ~~between~~

~~turbulent~~in the turbulent diffusivity~~ies~~ of sediment from the parabolic eddy viscosity model used in the Rouse derivation~~and momentum~~. The best-fit relation for β_i ($r^2 = 0.40$) indicates greater relative sediment diffusivities for rivers with greater flow resistance, possibly due to bed-form induced turbulence, and ~~smaller~~ larger w_{si}~~u_{*skin}~~$/$~~w_{si}~~u_{*skin}; the latter dependence ~~effect makes the dependence of Rouse number on u_{*skin}/w_{si}~~ is nonlinear, and therefore different from standard Rousean theory. In addition, we used empirical relations for gravel saltation to show that our relation for near-bed concentration also provides good predictions for coarse-grained sediment. The new relations ~~are a significant improvement compared to previous work,~~ extend the calibrated parameter space over a wider range in sediment sizes and flow conditions as compared to previous work, and result in 95% of concentration data throughout the water column predicted within a factor of nine.

1 Introduction

Suspension of sediment by water plays a critical role in the dynamics of rivers, river deltas, shallow marine environments~~,~~ and submarine canyons and fans. For example, suspended sediment dominates the load of lowland rivers and builds land in subsiding river deltas and coastal landscapes (Ma et al., 2017; Syvitski et al., 2005). Transport of sediment on the continental shelf is dominated by suspension of mud and sand due to wave~~s~~ and current~~s~~ bottom traction (Cacchione et al., 1999; Nittrouer et al., 1986). Suspended sediment provides the negative buoyancy of turbidity currents that move sand and gravel to the deep sea. Suspension of gravel is important in large floods, such as outburst floods (Burr et al., 2009; Larsen and Lamb, 2016), and in steep mountain canyons (Hartshorn et al., 2002), where it can contribute to bedrock erosion (Lamb et al., 2008a). Suspended sediment transport also is important in landscape engineering, such as river restoration (Allison and Meselhe, 2010), fish habitat (Mutsert et al., 2017), and the capacity of dams and reservoirs (Walling, 2006). The balance between entrainment and deposition from suspension determines patterns of deposition and erosion in these environments and therefore controls landform morphology and stratigraphic evolution (Garcia and Parker, 1991; Paola and Voller, 2005). To predict suspended sediment flux across these environments, we need robust theory for the entrainment of sediment from the bed and the vertical distribution of suspended sediment in the water column.

We focus here on understanding the suspended sediment load of cohesionless grains that are entrained from the bed (i.e., suspended bed material), rather than wash load. Most models for sediment suspension are based on application of Rouse theory (Rouse, 1937; Vanoni, 1946),

$$\frac{C}{C_a} = \left[\frac{\frac{H-z}{z}}{\frac{H-a}{a}}\right]^P, \qquad (1)$$

where C [L^3/L^3] is the volumetric sediment concentration at elevation (z [L]) above the bed, C_a [dimensionless] is the reference near-bed concentration at $z = a$ [unit: L] and, H [L] is the flow depth. ~~Also~~ P denotes the dimensionless Rouse number,

$$P = \frac{w_s}{\beta \kappa u_*}, \qquad (2)$$

in which w_s [L/T] is the particle settling velocity, κ is the dimensionless von Karman constant of 0.41, u_* [L/T] is the bed shear velocity and β is a dimensionless factor that accounts for differences between turbulent diffusivity of sediment and ~~momentum~~ the parabolic eddy viscosity model used to derive the Rouse equation (e.g. Graf and Cellino, 2002). Although more sophisticated models exist, some of which abandon the Rouse theory entirely in favor of a more rigorous turbulence model (Mellor and Yamada, 1982), the Rouse equation remains a useful and tractable approach for modeling and field application (Graf and Cellino, 2002; van Rijn, 1984; Wright and Parker, 2004b). The Rouse equation ~~can be~~was derived assuming an equilibrium suspension where the upwards volumetric flux of sediment per unit area due to turbulence (F_z [L/T])~~)~~ is balanced by a downwards gravitational settling flux ($C w_s$)~~, and where F_z is parameterized using a parabolic eddy viscosity~~ (Fig. 1) (Rouse, 1937). It predicts the shape of the concentration-depth profile, with a greater gradient in concentration for larger P (Fig. 1). ~~Although more sophisticated models exist, some of which abandon the Rouse theory entirely in favor of a more rigorous turbulence model (Mellor and Yamada, 1982), calibrating β remains a useful and tractable approach for modeling and field application (Graf and Cellino, 2002; van Rijn, 1984; Wright and Parker, 2004b).~~By mass balance, the difference between entrainment, F_{za}, and settling, $C_a w_s$, fluxes per unit area near the bed (at $z = a$) controls the bed deposition rate~~By definition, the equilibrium near-bed concentration is~~

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~~equal to a dimensionless entrainment parameter, $E_s = F_{za}/w_s$ (Garcia and Parker, 1991). The entrainment parameter, E_s, can then be used to predict the upward flux of sediment in non-equilibrium conditions (Garcia and Parker, 1991). Thus, the imbalance between the upward and downward flux of sediment determines the deposition or erosion rate~~, dz_b/dt [L/T] (Parker, 1978), i.e.,

$$(1-\lambda)\frac{dz_b}{dt} = F_{za} - C_a w_s = w_s(E_s - C_a)\ , \qquad (3)$$

where λ [dimensionless] is the bed porosity ~~(Parker, 1978)~~, and $E_s \equiv F_{za}/w_s$ is a dimensionless sediment entrainment rate or entrainment parameter (Garcia and Parker, 1991). –At steady state, Equation (3) reduces to $C_a = E_s$; thus, the near-bed entrainment rate, E_s, ~~At steady state, Equation (3) reduces to $E_s = C_a$, and thus the near-bed concentration for equilibrium suspensions can be used to find E_s. As such, C_a~~ is necessary both to predict the vertical distribution of suspended sediment at steady state ~~in~~ (i.e., C_a in Eq. (1)), and the rate of erosion and deposition ~~and the transient pickup of sediment~~ for disequilibrium suspensions ~~in~~ (Eq. (3)).

Application of Eqs. (1) – (3) requires specification of β , E_s and a, and the approach in previous work has been to identify important dimensionless variables and fit data from flume experiments and natural rivers to these variables (e.g., Smith and McLean, 1977; van Rijn, 1984; Garcia and Parker, 1991—). Owing to differences in datasets analyzed and the dimensionless variables explored, there is considerable deviation in the form of previous models and their predictions (Figs. 2 & 3, Table 1 & 2). For example, Einstein (1950), Engelund and Fredsøe (1976), van Rijn (1984) and Smith and McLean (1977) used a relation for a that depends primarily on grain diameter (D [L]), and secondarily on u_*/w_s or Shields number, $\tau_* = \tau_b/(\rho_s - \rho_w)gD$ (where τ_b [ML^{-1}T^{-2}] is bed stress, ρ_s [M/L^3] is sediment density and ρ_w [M/L^3] is fluid density). Their rationale was based on the idea that there is a well-mixed near-bed zone of bedload transport, and that suspended sediment is entrained from this zone (Fig. 1B). Thus, they developed relations for a that scale with the height of bedload saltation. In contrast, Garcia and Parker (1991) and Wright and Parker (2004b) argued for a simpler approach and used a reference height that is a small fraction of the flow depth, and they proposed $a = 0.05H$ as a useful reference height, with no dependence on D.

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~~Application of the Rouse equation requires specification of β and C_a; significant uncertainty exists in estimates of both of those parameters.~~

In practice ~~The~~the -factor β is often neglected (i.e., $\beta = 1$) under the assumption that the parabolic eddy viscosity assumption used to derive Eq. (1) represents well the turbulent transport of fine suspended sediment. $\beta < 1$ in ~~Equation~~ Eq. (2) ~~is often~~ has been attributed to damping of turbulence due to sediment-induced stratification, which alters the eddy viscosity (Einstein, 1955; Graf and Cellino, 2002; Wright and Parker, 2004a), ~~or~~and to flocculation, which ~~mainly~~ increases the settling velocity of fine-grained sediment (e.g. Bouchez et al., 2011; Droppo and Ongley, 1994). ~~Although more sophisticated models exist, some of which abandon the Rouse theory entirely in favor of a more rigorous turbulence model (Mellor and Yamada, 1982), calibrating β remains a useful and tractable approach for modeling and field application (Graf and Cellino, 2002; van Rijn, 1984; Wright and Parker, 2004b).~~ ~~Several formulas have been proposed for β, which have very different forms (Fig. 2; Table 1b). The relations of van Rijn (1984) and Graf and Cellino (2002) propose that β is a function of u_*/w_s. Wright and Parker (2004b) propose that β is a function of reference concentration divided by slope (C_a/S) and Santini et al. (2019) propose that β is a function of u_*/w_s and the ratio between flow depth and bed grain size (h/D). Wright and Parker (2004b) suggest that the dependence on C_a/S is due to sediment-induced stratification, while others do not provide a physical rationale for the parameters in their relations. Graf and Cellino (2002) have different formulas for cases with and without bedforms, and suggest that the turbulence generated by bedform roughness results in better sediment mixing and thus in a higher β. Sediment-induced stratification and aggregation reduce vertical mixing, which would imply $\beta < 1$. Nonetheless,~~ Ssome formulas (van Rijn, 1984; Santini et al., 2019) and datasets (Graf and Cellino, 2002; Lupker et al., 2011) indicate $\beta > 1$, which implies enhanced mixing of sediment relative to momentum (Table 1; Fig. 2). Graf and Cellino (2002) showed that turbulence generated by bedform roughness results in better sediment mixing and thus in a higher β. Field and flume data (Chien, 1954; Coleman, 1970; van Rijn, 1984) indicate that β is ~~often~~ greater than unity for the coarse grain-size fraction of the suspended material (Greimann and Holly Jr, 2001). Nielsen and Teakle (2004) ~~have shown~~argued that ~~with mixing length theory that~~ the

Fickian diffusion model ~~that is used~~ in the derivation of the Rouse equation is not valid for steep concentration gradients~~. Instead, for coarse sediment with a high settling velocity, the steep concentration gradient makes vertical mixing more efficient~~, resulting in $\beta > 1$. Most previous relations show a trend of decreasing β with increasing u_*/w_s (van Rijn, 1984; Graf and Cellino, 2002; Wright and Parker, 2004b) (Fig. 2a), which could be due to turbulence damping associated with high concentration suspensions. Wright and Parker (2004b) showed that β is also a function of reference concentration divided by slope (C_a/S) (Fig. 2b), which they attributed to sediment stratification. Santini et al. (2019) found that β is a function of u_*/w_s and the ratio between flow depth and bed grain size (H/D) (Fig. 2c).

Compared to β, even larger uncertainty exists~~ in~~ the dimensionless entrainment rate, E_s.~~predicting C_a. For equilibrium suspensions with steady state concentration profiles, the near-bed concentration, C_a, is typically thought to be a function of flow parameters, grain size and the availability of sediment on the bed (Garcia and Parker, 1991; McLean, 1992; Wright and Parker, 2004b). By definition, the equilibrium near-bed concentration is equal to a dimensionless entrainment parameter, $E_s = F_{za}/w_s$ (Garcia and Parker, 1991). The entrainment parameter, E_s, can then be used to predict the upward flux of sediment in non-equilibrium conditions (Garcia and Parker, 1991). Thus, the imbalance between the upward and downward flux of sediment determines the deposition or erosion rate, dz_b/dt, i.e.,~~

~~$(1-\lambda)\frac{dz_b}{dt} = F_{za} - C_a w_s = w_s(E_s - C_a)$, (3)~~

~~where λ is the bed porosity (Parker, 1978). At steady state, Equation (3) reduces to $E_s = C_a$, and thus the near-bed concentration for equilibrium suspensions can be used to find E_s. As such, C_a is necessary both to predict the vertical distribution of suspended sediment at steady state in Eq. (1), and the rate of erosion and deposition and the transient pickup of sediment for disequilibrium suspensions in Eq. (3).~~

~~Several existing formulas for C_a are applicable only to uniform bed sediment (Akiyama, 1986; Celik and Rodi, 1984; Einstein, 1950; Engelund and Fredsøe, 1976; van Rijn, 1984; Smith and Mclean, 1977). However, given the strong control of grain size on near-bed concentrations, accurate formulas likely need to make grain size specific predictions for sediment mixtures (Garcia and Parker, 1991; McLean, 1992; Wright and Parker, 2004b).~~ In previous work E_s was assessed by measuring the near-bed concentration

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for equilibrium suspensions, since $E_s = C_a$ under those conditions (i.e., $\frac{dz_b}{dt} = 0$ in Eq. 3). Previous work found that C_a was a function of bed stress and grain size (Fig. 3; Table 2) (Einstein, 1950; Engelund and Fredsoe, 1976; Smith and McLean, 1977; van Rijn, 1984; Cedik and Rodi, 1984; Akiyama and Figushima, 1986, Garcia and Parker, 1991; Wright and Parker, 2004b). While holding bed stress constant, most relations predict smaller C_a with increasing grain size across the sand regime, as expected, but some surprisingly show increasing C_a for coarse sands and gravel (Akiyama and Figushima, 1986, Garcia and Parker, 1991; Wright and Parker, 2004b) (Fig. 3a, 3c). Due to the greater weight and settling velocity of larger particles, this behavior is unrealistic and likely occurs because these coarse particles are outside of the data range used to fit the relations. Despite the importance of gravel suspension in steep mountain rivers and megafloods, data do not exists to evaluate the models in the gravel regime. All relations show increasing C_a with bed stress, as expected, but there are orders of magnitude differences in the predictions for C_a (Fig. 3b). Part of this deviation is due to differences in the reference level ($z = a$) (Fig. 3b); but even when using a common reference level ($a = 0.05H$; Fig. 3d), significant variability still exists. Some of the variance is also due to extrapolating the models beyond the range in which they were calibrated; ~~Garcia and Parker (1991) reviewed the entrainment relations that were available at that time and found that their relation and the relations of van Rijn (1984) and Smith and McLean (1977) performed best in tests against field and experimental data for sand. Since then, new entrainment relations have been introduced by McLean (1992) and Wright and Parker (2004b) (Table 1a). In order to compare existing models of C_a, we varied bed stress or grain size for each relation, while holding the other parameters constant (Fig. 3a,b). Since the equations predict concentration at different near-bed reference levels, we extrapolated, using the Rouse equation with $\beta = 1$, the predicted concentrations to a common reference level at 5% of the flow depth (Fig. 3c,d). This comparison highlights the fact that the entrainment relations differ considerably in terms of dependence on median bed grain size (Fig. 3a,c) and bed stress (Fig. 3b,d). In addition, some predictions by existing entrainment relations (Fig. 3) are unrealistic:~~ for example, van Rijn (1984) predicts concentrations greater than 100% at high shear stresses~~, and Wright and Parker (2004b) predicts that entrainment rates increase for larger grain sizes (and constant bed stress) in the gravel range, which is unlikely due to the greater weight and settling velocity of larger particles. ~~. ~~None of the existing relations have been evaluated in the gravel regime.~~

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~~The existing relations also vary in their choice of the reference height, $z = a$. For example, Einstein (1950), Engelund and Fredsøe (1976), van Rijn (1984) and Smith and McLean (1977) used a relation for a that depends primarily on grain size (D), and secondarily on u_*/w_s or Shields number, $\tau_* = \tau_b/(\rho_s - \rho_w)gD$ (where τ_b is bed stress, ρ_s is sediment density and ρ_w is fluid density) on the basis that these formulas capture the height of the near bed saltation layer. In contrast, Garcia and Parker (1991) and Wright and Parker (2004b) used a height that is a small fraction of the flow depth, and they proposed $a = 0.05H$ as a useful reference height, with no dependence on D.~~

W~~W~~e revisited the problem of sediment entrainment and suspension of cohesionless bed sediment by compiling a large database of sediment-size specific data for bed-sediment mixtures, testing existing relations against the database, and proposing improved relations for E_s and P. Several existing formulas for E_s are applicable only to uniform sized bed sediment (Akiyama, 1986; Celik and Rodi, 1984; Einstein, 1950; Engelund and Fredsøe, 1976; van Rijn, 1984; Smith and Mclean, 1977) (Table 2). However, given the strong control of grain size on near-bed concentrations, accurate formulas likely need to make grain-size specific predictions for sediment mixtures (Garcia and Parker, 1991; McLean, 1992; Wright and Parker, 2004b). Our database ~~is a significant improvement compared to data used in~~takes advantage of ~~past studies due to development of~~ high resolution grain-size measurements using laser diffraction~~, which is now commonly used in field and laboratory studies~~ (Lupker et al., 2011; Gitto et al., 2017; Haught et al., 2017; Santini et al., 2019), which~~. These grain-size measurements~~ allow a single concentration profile to be separated into many grain-size specific concentration profiles~~ and for the parameterization to be tested over a wide range of parameter space~~. Our dataset also contains a wide range ~~of median~~ grain sizes, extending into the silt regime (median bed sizes range from 44 μm to 517 μm), and expands on the number of field measurements compared to previous efforts. ~~-~~We are unaware of studies on C_a for gravel, but ~~workers~~ data does exist for ~~have measured s~~saltation heights, velocities and bedload fluxes for gravel, and calibrated relations for these variables also exist (Chatanantavet et al., 2013; Sklar and Dietrich, 2004). To attempt to better constrain suspension of very coarse sand and gravel, ~~W~~we used existing semi-empirical saltation theory for gravel to ~~existing relations for gravel and Rouse theory to~~ check for consistency between sand suspension data and what might be expected for near-bed gravel concentrations.

2 Methods

2.1 Suspended sediment ~~data~~profiles

Based on previous theory, we searched for available datasets from rivers and flume experiments that had suspended sediment profiles ($C(z)$), depth-averaged flow velocity (U), flow depth (H), channel bed slope (S) ~~and~~ bed material grain-size (D), and the grain-size distribution of the bed material and suspended sediment samples (Table 3; Table S1). Some of the experimental studies used a narrow grain-size distribution, and, like previous work~~ers~~, we assumed that the sediment distribution from these studies was uniform. Many of the older datasets were ~~were also~~ used in empirical regressions from previous relations (e.g. Garcia and Parker, 1991; van Rijn, 1984; Wright and Parker, 2004b). In addition, we used a river dataset from the Yellow River (Moodie, 2019), which provides a fine-grained end member. In total, our database contains 180 concentration profiles from 8 rivers and 62 profiles from 6 different experimental studies. We analyzed only the grain fractions coarser than 62.5 µm (i.e., sand). The mud fraction was present on the bed only in small amounts, and following previous work, mud was assumed to require a different approach, potentially due to supply limitation (Garcia, 2008), cohesion or flocculation.

Grain-size distributions in older studies were determined from sieve analysis of bed material and suspended sediment samples. The more recent studies used laser diffraction techniques, which have the advantage that a larger number of grain-size classes can be distinguished. We calculated the grain-size specific suspended sediment concentration (C_i) using

$$C_i = f_i C_{tot}, \tag{4}$$

where f_i [dimensionless] is the mass fraction of grains of the i-th size and C_{tot} is the total suspended sediment concentration for all sizes. In addition, we computed the D_{50} (median grain size) and D_{84} of the bed material using linear interpolation between the logarithm of D and the cumulative size distribution.

Concentration profiles in the database typically contain 3 to 8 measurements in the vertical dimension. The Rouse profile was fitted to the profile data for each grain-size class in log-transformed space using linear least squares to find P_i (Fig. 4a). Confidence bounds (68%; 1 σ) for the fitted coefficients were obtained using the inverse R factor from QR decomposition of the Jacobian. Data were excluded from further steps in the analysis if the ratio between the upper and lower bound of the confidence interval was greater than 10 or smaller than 0.01, as these data do not follow a Rouse relation for unknown reasons (e.g., measurement error), and would appear as sparse outliers. Of the data ~~points~~ analyzed, 201 points (15%) were excluded based on these criteria. Some studies (Bennett et al., 1998; Muste et al., 2005) have shown that P (or β) can vary over the flow depth, but this effect cannot be incorporated into our approach; instead we found one value of P_i that best fit the concentration profile for each grain size class.

We used the Rouse equation (Eq. 1) ~~written~~ for each grain-size class~~,~~ to extrapolate or interpolate the concentration to a reference level at 10% of the flow depth, i.e., we set $a = 0.1H$ and ~~(~~C_{ai}~~)~~ $= C_i(z = a)$.~~.~~ Extrapolation to very near the bed can be difficult due to large concentration gradients and poorly constrained near-bed processes such as interactions with bedforms. However, a reference level that is too far away from the bed may poorly capture the exchange of sediment with the bed. Previous researchers used reference levels that were either at some fraction of the flow depth (Akiyama, 1986; Celik and Rodi, 1984; Garcia and Parker, 1991; Wright and Parker, 2004b) or that were related to the bed roughness height (Einstein, 1950; Engelund and Fredsøe, 1976; Garcia and Parker, 1991; Smith and Mclean, 1977) (Table 2). We explored the collapse of the entrainment data for both types of reference levels. For each reference level, we fit our preferred and best-fitting two-parameter entrainment relation (as described in the Results) to the data with u_{*skin}/w_{si} as its first parameter and Froude number as the second parameter. We found that a reference level at a fraction of the flow depth gave a better collapse of the entrainment data than a reference level related to the saltation layer height. Furthermore, we also tested different flow-depth fractions and found that the fit improved, in a least-squared sense, as the reference level moved to a larger fraction of the flow depth (Fig. 4B). However, there is little change in r^2 once the reference level is higher than ~10% of the flow depth. Therefore, we used a reference level at 10% of the flow depth for all results shown below.

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For sediment mixtures, the grain-size specific near-bed concentration is partially controlled by the fraction of each grain-size class in the surface bed material. To account for this effect, Garcia and Parker (1991) introduced an entrainment rate (E_{si}) for each grain-size class that is linearly weighted by the fraction of that material in the bed:

$$E_{si} = \frac{C_{ai}}{F_{bi}}, \tag{5}$$

where C_{ai} is the near-bed concentration of that grain-size class and F_{bi} is the mass fraction of bed material that falls in that grain-size class. For uniform sediment, the entrainment rate (E_{si}) is equivalent to the near-bed concentration (C_{ai}).

2.2 Independent parameters and profile fitting ~~approach~~

Here we review the independent parameters that we evaluated for dependencies with dimensionless entrainment rate, E_{si}, and for β_i in the Rouse number. The primary group of parameters describes the ratio between bed stress and grain size or grain settling velocity. These parameters include the ratio between shear velocity and settling velocity (u_*/w_s), where we evaluated the total shear velocity as,

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$$u_* = \sqrt{\tau_b/\rho_w} = \sqrt{gHS}, \tag{6}$$

assuming steady, uniform unidirectional flow. Others have proposed that entrainment depends on the skin-friction portion of the total shear stress, u_{*skin} ~~(minus the portion due to form drag)~~, rather that the total shear stress. To estimate u_{*skin}, ~~, and w~~we used the Manning-Strickler relation ~~to calculate the skin-friction shear velocity as~~,

$$\frac{U}{u_{*skin}} \cancel{u_{*skin}} = 8.1 \left(\frac{H_{sk}\cancel{U}}{k_s\cancel{8.1}}\right)^{\cancel{0.8}1/6} \cancel{(gSk_s)^{0.1}}, \tag{7}$$

where $k_s = 3\ D_{84}$ is the grain roughness on the bed, H_{sk} is the depth due to skin friction and $u_{*skin} = \sqrt{gH_{sk}S}$ (e.g., Wright and Parker, 2004b). To calculate the particle settling velocity, we followed Ferguson and Church (2004) for each grain-size class,

$$w_{si} = \frac{RgD_i{}^2}{C_1\nu + \left(0.75C_2RgD_i{}^3\right)^{0.5}}, \tag{8}$$

in which $R = (\rho_s - \rho_w)/\rho_w$ is the submerged specific density of sediment, ν is the kinematic viscosity of the fluid, C_1 =18 and C_2 = 1 are constants set for natural sediment, D_i is the grain diameter within the size class of interest. -Another parameter that relates to the ratio between bed stress and gravity acting on the grains is the Shields number,

$$\tau_* = \frac{\tau_b}{(\rho_s - \rho)gD_i}, \tag{9}$$

and we again assumed steady, uniform flow to find $\tau_b = \rho gHS$. Similar to the shear velocity, it is also possible to calculate a Shields number for the skin-friction component of the total shear stress,

$$\tau_{*skin} = \frac{\tau_{skin}}{(\rho_s - \rho)gD_i}, \tag{10}$$

where $\tau_{skin} = \rho u_{*skin}^2$ by definition. Shields numbers can be rewritten in terms of u_*/w_{si} through use of a particle drag coefficient,

$$C_d = \frac{RgD_i}{w_{si}^2}, \tag{11}$$

which we also evaluated (i.e., $\tau_* = \frac{\tau_b}{(\rho_s - \rho)gD_i} = \left(\frac{u_*}{w_{si}}\right)^2 \frac{1}{C_d}$).~~.~~

The next group of parameters describes dimensionless particle sizes, including the particle Reynolds number,

$$R_p = \frac{u_*D_i}{\nu}. \tag{12}$$

Likewise, this parameter can also be calculated with the skin-friction component of the shear velocity,

$$R_{p,skin} = \frac{u_{*skin}D_i}{\nu}. \tag{13}$$

A particle Reynold number can be defined without shear velocity as,

$$Re_p = \frac{\sqrt{RgD_i}D_i}{\nu}. \tag{14}$$

For sediment mixtures, the relative particle size might play a role due to hiding and exposure effects (Garcia and Parker, 1991; Wright and Parker, 2004b); this effect can be captured with $\frac{D_i}{D_{50}}$~~$\left(\frac{D_i}{D_{50}}\right)$~~. Sediment-induced density stratification can decrease entrainment by damping near-bed turbulence, and this effect is thought to be most important in deep, low gradient rivers (Wright and Parker, 2004b). Wright and Parker (2004b) proposed that the ratio of near-bed concentration to bed slope is a good predictor for stratification, $\frac{C_a}{S}$, where they used C_a at 5% of the flow depth. Large, low gradient rivers also have small Froude numbers and low bed slopes, so we evaluated Froude number and slope as additional parameters. Froude number was calculated as:

$$Fr = \frac{U}{\sqrt{gH}}. \qquad (15)$$

The entrainment rate could also be affected by turbulence or changes to the boundary layer from bed roughness or bedforms, which tend to correlate with a flow resistance friction coefficient (e.g., Engelund and Hansen, 1967),

$$C_f = \frac{u_*^2}{U^2}.$$

(16)

We also evaluated H/D_{50} as a proxy for flow resistance due to grain roughness.

In order to find relations that explain the variation in ~~the data for~~our best fit E_{si} and P_i ~~β~~values from the vertical concentration profiles, we regressed the E_{si} and P_i values ~~data~~ against the 1~~4~~2 ~~independent~~ variables described above (Eq. 6 – 16). In some applications, like reconstructing flow conditions from sedimentary strata, it is useful to have an entrainment relation that depends on u_{*skin}, while for ~~most~~ forward modeling a relation based on u_* is preferred. The two shear velocities are highly correlated; therefore, we explored two versions of the fit relations using either ~~-~~u_* or u_{*skin}, but not both at the same time. Because the Rouse parameter, ~~(~~P_i~~,)~~by definition depends inversely on u_*/w_{si} (or u_{*skin}/w_{si}) (Eq. 2), we found the best fit relations for P_i ~~found best fit relations for P_i~~ rather than β_i t~~t~~o avoid spurious correlation. We then solved ~~, and then solved~~ for the equivalent relation for β_i using the~~ose~~

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~~relation~~definition of~~s~~ P_i ~~and~~ (Eq. ~~(~~2). ~~Some studies (Bennett et al., 1998; Muste et al., 2005) have shown that β can vary considerable over the flow depth, but this effect cannot be incorporated in the Rouse solution; instead we find one value of β_i that best fits the concentration profile for each grain size class.~~ We started the analysis by testing all models with one explanatory variable and ranked the models according to the coefficient of determination from linear least squares regression (r^2) evaluated in log-log space. Next, a second parameter was ~~added~~ used in addition to first best-fitting parameter and the resulting two-parameter models were ranked according to r^2. The procedure was repeated with additional parameters until the increase in r^2 was smaller than 0.04. ~~-~~For the fitting of multi-parameter models, we varied the exponents on each parameter in the model simultaneously to find the combination of exponents that yielded the best fit. This approach gave a higher r^2 compared to the stepwise approach used in previous work (e.g., Garcia and Parker, 1991) of first fitting the dominant variable and then fitting the secondary variables to the residuals. In addition, we tested fitting with the York method (Table S2), which gives less weight to data with large errors (York, 1968), but found only minor differences and so all results presented use the simpler linear least-squares method. All parameters were used to evaluate relations for E_{si} and β_i~~β~~, except for D_i/D_{50} ~~$\left(\frac{D_i}{D}\right)$~~, which we ~~only~~ used for only E_{si} since it is relevant for particle-particle interactions ~~in~~at the bed surface. In the results we report two versions of the best fitting one, two and three parameter models: one version that is based on the total bed shear stress and one version that is based on the skin-friction component of the bed shear stress. Model fits using all possible combinations of the input parameters are given in Table S2.

2.3 Comparison to theory for gravel

Although gravel suspension is important in bedrock and steep mountain rivers, and during mega~~large~~ floods, we are not aware of datasets of near-bed concentration ~~suspension~~ in the gravel range. Following previous work (McLean, 1992; Lamb et al., 2008a), our approach was to derive the near-bed concentration in the bedload layer, and then use Rouse theory to predict that concentration at $0.1H$ to compare with the sand dataset~~s~~ (e.g., Fig. 1B).

The near-bed volumetric concentration within the bedload layer can be calculated by continuity as

$$C_b = \frac{q_b}{(H_b U_b)}, \qquad (17)$$

where q_b [L^2/T] is the volumetric bedload flux per unit width, H_b [L] is the bedload layer thickness and U_b [L/T] is the bedload velocity. Most relations for bedload flux take the form

$$\frac{q_b}{\sqrt{(RgD^3)}} = a(\tau_* - \tau_{*c})^b, \qquad (18)$$

where a and b are empirical constants, which we set to a = 5.7 and b = 1.5 (Fernandez Luque and van Beek, 1976), and τ_{*c} is the critical Shields number at initial motion, which we set to Lamb et al.~~,~~ (2008b)

$$\tau_{*c} = 0.15S^{0.25}. \qquad (19)$$

The bedload layer height and velocity were determined from Chatanantavet et al. (2013). ~~-~~They compiled a large dataset of gravel saltation observations~~studies~~ and found a good fit with the following relations:

$$\frac{U_b}{U} = 0.6 \quad , \qquad (20)$$

$$\frac{H_b}{H} = 0.6\left(Fr\left(\frac{D_i}{H}\right)^2\right)^{0.3}. \qquad (21)$$

~~Note that others have proposed that U_b and H_b depend on τ_* rather than U (Sklar and Dietrich, 2004), but Chatanantavet et al. (2013) found a better collapse using flow velocity as a scaling parameter.~~

Equations (17) – (21) were combined with a flow resistance relation (Eq. 7, assuming no form drag) and C_d = 0.7 for gravel (Lamb et al., 2017) to calculate C_b in the bedload layer using a numerical iterative scheme. Manipulating the equations revealed that C_b is ~~as~~ a function of only τ_* and Fr ~~using an iterative procedure~~. To ~~predict~~ calculate C_a for gravel, ~~$= C$ ($z = 0.1H$),~~ we extrapolated the concentration profile (Fig. 1b) from the top of the saltation layer to 0.1H using the Rouse equation (Eq. 1). To obtain the Rouse number, we used our best-fit one-parameter model that uses total shear velocity (u_*) (see Table 4). We then used a wide range of input parameters (0.1 < Fr < 1 and ~~) and (~~1 < τ_* < 1000)~~,~~ relevant to suspension of gravel in mountain rivers and large floods~~,~~ to predict a range of expected values of C_a for gravel. Although Eqs. (20) and (21) have not been tested for high Shields numbers in the suspension regime, they

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have reasonable limiting values ($U_b = 0.6U$, $H_b = H$) and provide a starting place to compare sand entrainment and gravel saltation theories. ~~.~~

3 Results

3.1 Rouse number, P_i

Equation (1) was fit to the concentration profile data where P_i was treated as a fitting parameter. Then P_i was regressed against the hypothesized controlling variables (Eqs. 11-16). Figure 5 and Table 4 show the results for P_i including our best fitting one-, two- and three~~-~~-parameter models~~ for Rouse number~~. The variable with the most explanatory power is u_*/w_{si} ($P_i = (u_*/w_{si})^{-0.45}$; $r^2 = 0.33$). ~~predictions of t~~The best-fit two-parameter model ~~are~~that includes C_f is ~~significantly~~ better than the predictions of the one-parameter model (~~$r^2 = 0.33$ vs~~ $r^2 = 0.40$). ~~Using a Rouse number with a constant $\beta = 0.94$ also provides a reasonably good fit and an improvement over several more involved relations (Fig. 5).~~ Going from a two-parameter model to a three-parameter model brings a smaller improvement (~~$r^2 = 0.40$ vs~~ $r^2 = 0.43$)~~.~~. Therefore, we recommend the two-parameter model for combined accuracy and simplicity:

$$P_i = 0.145\left(\frac{u_{*skin}}{w_{si}}\right)^{-0.46} C_f^{-0.3}, \qquad (22)$$

although there are a number of models using different variables that yield similar r^2 values (Table S2), and some combinations might be preferred over others depending on the application. ~~The relation~~Eq. (22) indicates that sediment is better mixed in the water column with larger u_{*skin}/w_{si} and with larger bed roughness coefficient, ~~(~~C_f~~)~~. Equation (22) performs well compared to previous relations, as is shown by a boxplot of measured-to-predicted ratios (Fig. 6). For the best-fit two-parameter model, the measured-to-predicted ratio falls between 0.74 and 1.29 for 50% of the data. Using a Rouse number with a constant $\beta = 0.94$ also provides a reasonably good fit and an improvement over several more involved relations (Fig. 6).

~~This ~~E~~e~~quation (22) can be rewritten for β_i ~~by combining~~using Eq. (2)~~ and (22)~~, and by assuming that u_* in Eq. (2) is actually u_{*skin}, as

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$$\beta_i = 17.24\left(\frac{u_{*skin}}{w_{si}}\right)^{-0.54} C_f^{0.3},$$

(23)

~~Equation (22) performs well compared to previous relations, as is shown by a boxplot of measured-to-predicted ratios (Fig. 6). For the best-fit two-parameter model, the measured-to-predicted ratio falls between 0.74 and 1.29 for 50% of the data.~~

Because some of the dimensional quantities appear in multiple dimensional variables, and because the dimensional variables are not necessarily independent from each other, we tested for spurious correlations by rearranging the two-parameter relation for β_i to isolate the dimensional dependencies on grain size and skin-friction shear velocity (Fig. 7). The data and our model show a decrease in suspended sediment mixing in the water column with larger grain sizes~~ (Fig 7a)~~, ~~which makes physical sense~~as expected (Fig 7a). Previous relations show the same trend, but the relations of van Rijn (1994) and Graf and Cellino (2002) have stronger dependencies in the sand regime~~ that are not consistent with the data~~. Suspended sediments are better mixed with increasing skin-friction shear velocity (Fig. 7b)~~, as expected~~. T~~However, our relation and t~~he data suggest that P_i varies proportionally to ~~∼~~$u_*^{-0.4}$ whereas standard Rouse theory (Eq. 2) indicates that P_i is proportional to u_*^{-1}.

3.2 ~~Near-bed entrainment parameter~~Dimensionless entrainment rate, E_{si}

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Results for the best fit dimensionless entrainment rate is shown in Figure 8 and Table 5, and all possible variable combinations are given in Table S1. T~~Of all relations tested (Table 5; Table S2), t~~he following one-parameter relation gives the best fit with the data for ~~near-bed entrainment parameter~~E_{si} (r^2 = 0.61) (Fig. 8a):

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$$E_{si} = 4.23\times10^{-5}\left(\frac{u_{*skin}}{w_{si}}\right)^{1.94}.$$ (2~~5~~4)

~~Figure 8a shows that Eq. (24) explains a significant part of the variation in the data (r^2 = 0.61). Next, we tested if an entrainment relation with two variables improved the fit.~~ Froude number was the most significant second parameter:

$$E_{si} = 4.74\times10^{-4}\left(\frac{u_{*skin}}{w_{si}}\right)^{1.77} Fr^{1.18}$$,~~.~~ (25)

~~This two-parameter relation~~which has a significantly better fit ($r^2 = 0.79$) ~~with the data~~ than the best fitting one-parameter relation (Fig. 8b). Addition of a third variable to the model gives little further improvement of the fit (Fig. 8c; $r^2 = 0.80$), and many of the variables used as a third parameter give a similar level of improvement (Table S1). ~~With~~ Using Eq. (25), ~~the majority (~~80%~~)~~ of the entrainment data is predicted within a factor of 3 (Fig. 8b; Fig. 9).

Along with our proposed new relation (Eq. 25), we also compared the dataset against the previous relations (Fig. 9) ~~presented by Garcia and Parker (1991) and Wright and Parker (2004b)~~. The boxplots in Figure 9 highlights that some relations systematically ~~underpredict~~under predict (Wright and Parker, 2004b) or ~~overpredict~~over predict entrainment rate (Garcia and Parker, 1991). In addition, previous relations have a larger spread in measured-to-predicted ratios than Eq. (25).

To check for spurious correlation in the dimensionless variables, we rearranged our two-parameter entrainment relation (Eq. 25) to isolate the dependencies of entrainment rate on grain size (Fig. 10a) and skin-friction shear velocity (Fig. 10b). Our relation indicates that entrainment depends on grain size to the -3.1 power in the sand range, whereas previous relations suggest a much weaker dependence. ~~On the other hand, c~~Compared to previous work~~relations~~, our relation also suggests a relatively weak dependence on skin-friction shear ~~veloci~~velocity ~~ty~~ ($E_{si} \propto u_{*skin}^{1.77}$).

Similar to Garcia and Parker (1991) and Wright and Parker (2004b), we modified ~~our~~ equation (25) such that the predicted entrainment rate is limited at 0.3, as total suspended sediment concentrations greater than ~~that~~ 30% by volume are not physically reasonable for dilute, turbulent flows. In addition, a threshold (0.015), best fit by eye, was added to the entrainment relation because the concentration data fall~~s~~ below the trend of the regression relation at the lower flow strengths (Fig. 8b), ~~possibly due to~~suggesting a threshold of significant sediment entrainment at the reference level. The resulting equation has the following form (Fig. 8b):

$$E_{si} = \frac{4.74\times10^{-4}\left(\left(\frac{u^*_{skin}}{w_s}\right)^{1.5} Fr - 0.015\right)^{1.18}}{1+3\left(4.74\times10^{-4}\left(\left(\frac{u^*_{skin}}{w_s}\right)^{1.5} Fr - 0.015\right)^{1.18}\right)}. \tag{26a}$$

The equivalent formula for the best-fit two parameter model without a form drag correction (Table 5) is

$$E_{si} = \frac{7.04\times10^{-4}\left(\left(\frac{u^*}{w_s}\right)^{0.945} Fr-0.05\right)^{1.81}}{1+3\left(7.04\times10^{-4}\left(\left(\frac{u^*}{w_s}\right)^{0.945} Fr-0.05\right)^{1.81}\right)} \qquad (26b)$$

3.3 Predicting sediment concentration

In Sections 3.1 and 3.2 we found the best-fit models for the dimensionless entrainment rate (E_{si}) and Rouse number (P_i). However, ultimately, we want to predict sediment concentration throughout the water column, and the best-fit models for (E_{si}) and (P_i) do not necessarily combine to yield the best-fit model for sediment concentration~~,~~ owing to non-linearity in E_{si}, P_i and the Rouse equation (Eq. 1). Here we used different combinations of our preferred one-, two- and three-parameter models for entrainment (E_{si}) and Rouse number (P_i) to predict the grain-size specific concentrations at each data point in the water column for all our entries in the database, and assessed model performance (Table 6).

The concentration predictions improve as more parameters are added to the entrainment model, whereas a Rouse model with more than one parameter makes the predictions worse (Table 6). Our preferred model for ~~depth-averaged~~ concentration throughout the water depth uses the~~a~~ two-parameter model to predict the entrainment rate ($E_{si} = 4.74 \times 10^{-4} \left(\frac{u_{*skin}}{w_{si}}\right)^{1.77} Fr^{1.18}$) and the~~a~~ one-parameter model for the Rouse number ($P_i = (u_*/w_{si})^{-0.45}$)(Fig 11~~b~~). Such a model gives significantly better predictions than the most basic formulation ~~(Fig. 11)~~ that uses one-parameter models for entrainment and Rouse number (r^2 of 0.87 versus 0.65; Table 6). The goodness of fit ~~the concentration predictions~~ also is fairly constant over the height of the flow;~~.~~ ~~T~~the ~~predictions from the~~ upper 1/3 of the flow ha~~ve~~s a slightly lower r^2 (0.80) than the predictions from the lowest 1/3 of the flow ($r^2 = 0.89$) (Fig. 11). Ninety five percent of the data are predicted within a factor of 9.

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3.4 Extension to gravel

The suspended sediment data that we used to calibrate the entrainment relation covers material in the sand range. To evaluate how our entrainment relation performs for coarser suspended sediment, we used the empirical saltation equations for gravel to infer bedload-layer concentrations, C_b, and interpolated these to the reference level (0.1H) to infer C_a for gravel (Section 2.3). Importantly, the gravel concentrations at the reference height can be predicted from the saltation model (Section 2.3) using only the independent parameters of Fr and τ_*, similar to our best fitting two-parameter entrainment model. To compare to gravel, we assumed a uniform sediment size and used the version of our best-fit two parameter model that does not include a form-drag correction (Eq. 26b). This was done because the gravel saltation studies typically were performed over a planar bed, whereas form drag likely played a significant role in the sand data sets owing to dunes.~~ (Eq. 25).~~ The marked parameter space on Fig~~.~~ure 12 shows the expected range of C_b and C_a for a wide range of model input parameters for gravel: $0.1 < Fr < 1$ and $1 < \tau_* < 1000$. Predicted concentrations in the gravel ~~saltation~~ bedload layer are up to several orders of magnitude higher than the predictions from our entrainment relation at 10% of the flow depth (Fig. 12). However, due to the rapid decrease of sediment concentration away from the bed predicted by the Rouse profile, the concentration inferred at 0.1H for gravel overlaps with the empirical relation for sand, implying that Eq. (26b~~5~~) might also be a good predictor of gravel entrainment.

4 Discussion

4.1 Physical ~~R~~rationale for m~~M~~odel d~~D~~ependencies

~~Our preferred model for β_i is the result of regression against a large dataset.~~ The explanatory variables in the models for P_i and β_i are u_{*skin}/w_{si} and C_f.~~ , and t~~The dependency on these~~se~~ parameters could reflect processes that result in the turbulent diffusivity for suspended sediment being different from the parabolic relation used in the derivation of the Rouse profile~~different effects that cause a difference between the eddy viscosity and diffusivity of sediment~~ (e.g., Bennett et al., 1998). Compared to standard Rouse theory, in which~~ where~~ the Rouse parameter is inversely dependent on u_{*skin}/w_{si} (or u_*/w_{si}), our results indicate a significant non-linearity. Our one-parameter model indicates that concentration profiles are better mixed than standard theory for small u_*/w_{si} and are more stratified for larger u_*/w_{si}, with a transition point at about $u_*/w_{si} = 7$, corresponding to $\beta_i = 1$. Sediment-induced stratification is often cited (Winterwerp, 2006; Wright and Parker, 2004b, 2004a) as a factor that decreases mixing of sediment (i.e.

β < 1). This effect is particularly important when absolute concentration is high, and may help explain why our best-fit model is more stratified (i.e. β < 1) than standard Rousean theory for large u_*/w_{si}. An alternative hypothesis by Nielsen and Teakle (2004) is that ~~a negative dependence of β on u_*/w_s can be the result of a mixing length effect. This effect is related to the fact~~ for steep concentration gradients, the size of the turbulent eddies is large relative to the mean height of sediment in the flow. Under these circumstances, ~~Fickian diffusion, which is assumed in the Rouse derivation, is no longer appropriate to describe turbulent mixing of sediment. Instead, the~~ large eddies might more effectively mix sediment that is concentrated close to the bed. This ~~effect~~ process may explain the better mixed concentration profiles we observed (i.e., β_i > 1), as compared to standard theory, at small u_*/w_{si} when near-bed concentration gradients are large.

Our relation ~~also~~ implies that β_i correlates positively with the flow friction coefficient C_f. This dependency could ~~, which could~~ be from bedforms; bedforms increase C_f due to form drag (Engelund and Hansen, 1967), and they may also increase the vertical mixing of sediment by deflecting transport paths up the stoss side of ~~the bedform~~dunes and mixing suspended sediment in the turbulent wake in the lee of ~~the bedform~~dunes. Similarly, Santini et al. (2019) found that β correlate~~s~~d positively with *H/D* (Fig. 2c), which is another measure for bed roughness in flows without bedforms. In agreement, Graf and Cellino (2002) reviewed a number of experimental studies and found that β < 1 for ~~all e~~experiments without bedforms and β > 1 for ~~all~~ experiments with bedforms. Flow resistance (C_f) also can ~~also~~ be smaller in flows with sediment-induced stratification, which ~~also~~ correlates with smaller β_i (e.g., Wright and Parker, 2004a).

Our new empirical relation for sediment entrainment suggests that only u_{*skin}/w_{si} (or u_*/w_{si}) and *Fr* are needed to predict entrainment rate, similar to the forward model we developed for the suspension~~entrainment~~ of gravel (Section 2.3). The ratio u_{*skin}/w_{si} describes the fluid forces relative to gravitational settling; similar parameters have appeared in all previous relations that we reviewed (Table ~~1a~~2). The reason for the increase in entrainment with Froude number is less clear. A small Froude number implies a deep and low-gradient flow, and Froude numbers are typically smaller in natural rivers compared to flume experiments. Wright and Parker (2004b) introduced an entrainment relation with a bed slope dependency and argued that entrainment in large low sloping rivers is reduced due to stratification effects. Sediment-induced stratification causing damping of ~~near-bed~~ turbulence might be the cause of the Fr dependency in our relation. Regardless, we found a better fit with the data using Fr than using $\frac{C_a}{S}$ or S (Table S2), the parameters suggested by Wright and Parker (2004b). Froude number also might ~~also~~ influence the size and shape of bedforms (Vanoni, 1974), which can affect boundary layer dynamics and near-bed turbulence, and surface waves. The Froude number is the ratio between inertial to gravitational forces, or more formally the ratio of a unidirectional flow velocity to the celerity of a shallow water wave (i.e., $Fr = U / \sqrt{gH}$). However, we do not think these are the reasons why the dependence of E_{si} on Fr emerges in our analysis; rather, ~~However,~~ our forward model (~~as discussed in~~ Section 2~~4~~.3) suggests that ~~, Fr~~ it emerges as a controlling parameter ~~in our forward model~~ because of ~~its~~ the role of U and H in determining bedload layer concentrations, which is discussed in more detail in Section 4.3.

Many of the dimensionless parameters we evaluated correlate with each other in rivers. While u_{*skin}/w_{si} or u_*/w_{si} were consistently the dominant variables, several of the possible secondary variables had ~~nearly equivalent~~similar explanatory power as the ones given in our preferred models. For ~~near-bed~~sediment entrainment, some of the more highly ranked ~~secondary~~ variables are C_a/S and ~~include D,~~ τ_{*skin} in one-parameter models and H/D_{50}, S and D_i/D_{50} in two parameter models~~and R_p;~~; and for Rouse number they include ~~D,~~ $R_{p,skin}$ and τ_* for one-parameter models, and Re_p and C_a/S in two parameter models (Table S2). In addition, there is some systematic deviation between datasets for different rivers (Fig. 8), and different parameters and exponents might better minimize residuals at specific locations. ~~Some previous entrainment relations (Garcia and Parker, 1991; Wright and Parker, 2004b) contain an additional parameter, D_i/D_{50}, that accounts for hiding and exposure effects due to nonuniform bed material. We did not find a significant improvement of fit with such a parameter.~~ While our model has been fit to a large dataset, it has not been validated with independent data. More data is clearly needed on

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grain-size specific concentration profiles for equilibrium suspensions under a wide range of u_*/w_{si}, *Fr* and particle sizes, including coarse sand and gravel.

4.2 Predicting sediment concentration below the reference level

Combining the predictions for~~of~~ the grain-size specific reference concentration and Rouse parameter allows for calculation of the grain-size specific sediment concentration throughout the water column (Fig. 11). Our relation shows that the grain-size specific sediment concentration at any given elevation can be predicted to relatively high accuracy ($r^2 = 0.87$) using the preferred combination of a two-parameter entrainment ~~parameter~~ relation and the one-parameter Rouse number relation (Section 3.3). However, it is unclear how to evaluate the sediment concentration below the reference level, which could constitute a significant portion of the sediment load. Bedload fluxes are notoriously difficult to estimate; further, it is unclear if the region below the reference level is entirely bedload, or if it is bedload and suspended load (e.g., Fig. 1b). Ashley et al. (2020) showed how bedload sediment fluxes can be estimated from discharge-averaged suspended sand concentrations, but their method does not predict the concentration profile. One approach might be to assume that sediment concentration is uniform below the reference level at $C_i = C_{ai}$, as might be the case in a~~the~~ well-mixed bedload layer (e.g., McLean, 1992) (Fig. 1b). However, this assumption is inconsistent with our analysis of the saltation equations, which shows that bedload layers should have greater concentrations and be less than 10% of the flow depth (Fig. 12), especially for sand in deep flows. A~~n~~ second ~~alternative~~ approach is to use the Rouse profile to extrapolate towards the bed or towards the top of the bedload layer; however, this approach is also problematic because the Rouse profile predicts infinitely large concentrations at the bed.

Some of our datasets had concentration measurements below the reference level (Fig. 13). The data are scattered, but the binned data points suggest a nearly constant concentration with elevation below the reference level~~For 22% of the measurements below the reference level, the concentration exceeds two times the reference concentration, but there is not a clear trend of increasing concentration below the reference level. For lack of a better approach, we suggest treating the sediment concentration as constant below the reference level, and the average of our data indicates a concentration of~~. For lack of a better method, we propose to use the geometric mean of this data as C_i = 1.~~9~~66C_{ai} to approximate the concentration-depth profile below the reference level ~~in that layer~~ (Fig. 1~~4~~3).

4.3 Extension to gravel and bedload-layer theory

The transport of sand and gravel are often modeled using different empirical formulas, which hinders modeling of systems of mixed gravel-sand transport (Wilcock and Crowe, 2003) and gravel-sand transitions (Paola et al., 1992; Lamb and Venditti, 2016~~; Paola et al., 1992~~). Gravel also can be in suspension in bedrock and steep mountain rivers, and during large floods (Hartshorn et al., 2002; Larsen and Lamb, 2016), and it would be useful to have an entrainment relation to model sediment transport and bedrock erosion in these settings (Lamb et al., 2008a; Scheingross et al., 2014). Unfortunately sand and gravel are often treated separately, which limits our ability to develop a unified sediment transport theory. Our entrainment relation for sand matches expectations from gravel saltation models (Fig. 12), suggesting that the entrainment relation may be used for sand or gravel, or mixtures of the two. However, we currently lack data of gravel suspension profiles, and developing methods to acquire such data should be the focus of future efforts. It's also unclear how entrainment is complicated by bimodal mixtures (Wilcock and Crowe, 2003) and complex flow hydraulics in steep rivers with large roughness that can significantly affect lift forces and near-bed turbulence (Lamb et al., 2017).

The good fit between the modeled gravel concentrations and the measured sand data for near-bed sediment concentration suggests that the bedload layer equations (Section 2.3) might also be used to generate a forward model for near-bed sediment concentration that works for sand and gravel systems, similarly to previous efforts (e.g., McLean, 1992). To evaluate this possibility, we used the saltation equations (Eqs.17-21) to calculate sediment concentrations within the bedload layer for conditions corresponding to our dataset entries of sand bedded rivers. To extend equation (18) to grain-size mixtures, we let $D = D_i$ and $\tau_{*c} = \tau_{*ci}$ and used a hiding function (Parker et al., 1982) so that

$$\tau_{*ci} = \tau_{*c50}\left(\frac{D_i}{D_{50}}\right)^{-\gamma}, \qquad (27)$$

where τ_{*c50} is the critical Shields number for the median sized bed sediment. Equation (27~~19~~) accounts for hiding of smaller grains in between larger grains, which renders the smaller grains less mobile, and

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Formatted: Font: 12 pt, Italic, Subscript the exposure of larger grains into the flow, which renders them more mobile. ~~-~~For $\gamma = 1$, all grains in a bed mixture move at the same bed stress, while $\gamma = 0$, the critical stress for motion is proportional to grain weight. Gravel bedded rivers typically have $\gamma = 0.9$ (Parker, 1990). For sand, we evaluated the critical Shields number following Brownlie (1981)

$$\tau_{*c50} = 0.5\left[0.22\, Re_p^{-0.6} + 0.06 * 10^{\left(-7.7 Re_p^{-0.6}\right)}\right], \qquad (28)$$

rather than Eq. (19), which is intended for gravel only. We then calculated the concentrations at 10% of the flow depth using the Rouse equation (Eq. 1) with the best-fit one-parameter model for the Rouse number. Although the saltation equations in Section 2.3 were calibrated for gravel, similar relations also have ~~are also~~ been used for sand (e.g., Lamb et al., 2008a).

Surprisingly, the near-bed sand concentration for the entries in our database (Section 2.1) are relatively accurately predicted by the bedload forward model without any parameter fitting (Fig. 14b). In fact, the predictions by the bedload-layer forward model are only slightly worse than the predictions by our preferred two-parameter entrainment model (Fig. 14a) that was fit to the data (r^2 of 0.68 vs 0.87). ~~The model might likely be improved by accounting for bedform induced form drag.~~ Importantly, the forward model yields the same controlling parameters as the empirical model, namely Shields number, Froude number and the bed grain-size distribution. The forward model suggests that the dependence of E_{si} on u_{*skin}/w_{si} (or u_*/w_{si}) is predominantly because larger u_{*skin}/w_{si} correlates with larger τ_* and larger bedload-layer concentrations (Eq. 17 and 18), as well as more efficient mixing of the bedload-layer sediment up to the reference height (Eq. 1). Less intuitive is the dependence of E_{si} on Froude number. In the forward model, this dependence is because larger *Fr*, for the same τ_* correlates with larger D_i/H (which can be shown by manipulating Eq. 7). In turn, larger D_i/H with constant τ_* correlates with greater bedload fluxes ($q_b \propto D_i^{3/2}$ in Eq. 18), smaller bedload layer heights (due to smaller H in Eq. 21) and slower bedload layer velocities (due to smaller U in Eq. 20), all of which increase sediment concentration in the bedload layer (Eq. 17). Thus, the forward model indicates that the *Fr*-dependency on E_{si} emerges because bedload layer dynamics depend on U and H, and explanations for this dependency that rely on stratification, bedforms or surface waves are not necessary. The bedload-layer model, while slightly more complicated to implement, may provide a more robust solution when working outside of parameter space

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used to derive the empirical model, since it has a more physical basis. For example, the forward model can explicitly account for gravity and other physical properties of the sediment and fluid. In addition, the bedload-layer model allows for a more mechanistic link between the bedload and suspended load, and avoids uncertainty in how to evaluate the sediment concentration below the reference level. The model might likely be improved by accounting for bedform induced form drag, especially for sand-bedded rivers. That said, more accurate predictions are still achieved with the empirical entrainment relation.

5 Conclusions

We proposed new empirical models for the entrainment of bed material into suspension and for the shape of the concentration profile governed by the Rouse parameter~~above a near-bed reference level~~. The models were obtained by regression against suspended sediment data from eight different rivers and six experimental studies. The data cover a wide range of bed material grain sizes (44- 517 µm) and flow depths (0.06-32 m) and include grain-size specific data with up to 60 size classes. Our ~~empirical~~ analysis of this data suggests that near-bed sediment concentration increases with the ratio between shear velocity and settling velocity (u_{*skin}/w_s or u_*/w_{si}) and Froude number—~~—~~both parameters also emerge as the key controlling variables in a forward model based on bedload layer concentrations. A parameter such as u_*/w_{si}, which represents the ratio of fluid force to particle settling, was also present in previous relations~~, and its relevance reflects the competing effects of turbulent entrainment and particle settling~~. The Froude number dependence is less clear~~intuitive~~; it could be due to stronger sediment-induced stratification in large low-Fr rivers, but our forward model ~~suggests~~ indicates that it emerges because ~~because of Fr controls on~~flow velocity and flow depth impact bedload layer concentrations. Our preferred Rouse parameter model for the shape of the concentration profile suggests that sediment concentration is better mixed in the water column with larger u_{*skin}/w_{si} and larger bed friction coefficient, ~~(C_f)~~. The Rouse number is not inversely proportional to u_*/w_{si}, unlike standard Rousean theory, indicating that sediment is more stratified than expected with $u_*/w_{si} > \sim 7$ and ~~more well~~better -mixed than expected with $u_*/w_{si} < \sim 7$, possibly due to the competing effects of sediment-induced stratification when absolute concentrations are large (corresponding to large u_*/w_{si}) and enhanced turbulent mixing when concentration gradients are steep (corresponding to small u_*/w_{si}). The dependence of Rouse number on bed friction coefficient might result from increased turbulence close to the bed in rivers with large bed roughness or bedforms. We also demonstrated that near-bed concentrations can be accurately predicted with saltation equations that have been tested previously for gravel, suggesting a unified framework to model sand and gravel transport in rivers.

Competing interests

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The authors declare that they have no conflict of interest.

Data availability

All data used in the analysis are provided in Table S1 of the Supplementary Materials. Model results for all possible variable combinations are given in Table S2.

Author Contributions

MPL and GP conceived the study. JdL compiled data and led data analysis with input from MPL. MPL and JdL wrote the initial manuscript. AJM, DH, JGV and JAN supplied suspended sediment data and contributed to the final manuscript.

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Supplementary materials

Table S1 – Contains all the suspended sediment data that was used to find the empirical relations.

Table S2 – All entrainment and Rouse number models ranked according to goodness of fit as indicated by r^2.

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