We thank reviewer for their positive consideration, encouragement and insightful comments on our manuscript. For ease of understanding the questions are in black, and the reply is in green colour. Also, the comments are marked by green in the manuscript.

General comments:

1) I find sections 1.1-1.6 to be a great improvement. The structure of the paper, and the nature of the models being compared, is now quite a lot easier for readers to understand. Also, definitions of concepts such as cover hysteresis and clast rough/smooth have been moved closer to the beginning of the paper, which is very helpful. I have one final request to make with regard to the introduction/presentation of the models: please consider keeping consistent the order in which the models are discussed. In line ~125, the models are listed in the text. Then in sections 1.2-1.6, they are presented in a different order. Then in lines 553-555, they are listed again in an order that matches sections 1.2-1.6 but does not match the first list. Understanding this paper requires that readers keep a lot of concepts in mind at once; please try to make it as easy for them as possible by keeping the model order consistent.

We have modified line ~125 to match the rest of the paper. The models are now mentioned in order of the year they were proposed (matches section 1.2 to 1.6):

the Eulerian description models are roughly classified into four categories; the linear model proposed by Sklar and Dietrich (1998, 2004), the exponential model proposed by Turowski et al. (2007), the roughness models proposed by Nelson and Seminara (2012), Inoue et al. (2014), Johnson (2014), Zhang et al. (2015) and the probabilistic model proposed by Turowski and Hodge (2017).

2) Abstract: the results reported in the abstract are vague: “the results suggest a fit of certain models…” Consider revising the abstract to better clarify your results (for example, that the two simplest models do well on clast-rough but not clast-smooth beds). As much as we might like to believe otherwise, the abstract is all that some people will ever read!

We have changed the abstract:

Several studies have demonstrated the importance of bedrock-bed roughness on alluvial cover; furthermore, several mathematical models have also been introduced to mimic the effect bed roughness may project on alluvial cover in bedrock channels. Here, we provide an extensive review of research exploring the relationship between alluvial cover, sediment supply and bed topography of bedrock channels, describing various mathematical models used to analyse deposition of alluvium. To test one-dimensional theoretical models, we performed a series of laboratory-scale experiments with varying bed roughness under simple conditions without bar formation. Our experiments show that alluvial cover is not merely governed by increasing sediment supply, and that bed roughness is an important controlling factor of alluvial cover. A comparison between the experimental results and the five theoretical models shows that: (1) two simple models that calculate alluvial cover as a linear or exponential function of ratio of sediment supply to capacity, produce good results for rough bedrock beds but not for smoother bedrock beds; (2) two roughness models which include changes in roughness with alluviation and a model including the probability of sediment accumulation can accurately predict alluvial cover in both rough and smooth beds; (3) however, except for a model using the observed hydraulic roughness, it is necessary to adjust model parameters even in a straight channel without bars.

3) I appreciate the much-improved introduction, with its broader scope and much more complete inclusion of the recent literature. I thought hard about why I still find the revised introduction a little bit difficult to follow, and I think it’s because the purpose of a given paragraph is not always made clear in its first sentence. For example, a couple of paragraphs just start
with sentences describing what a single previous study did, so it’s hard to understand what the paragraph is trying to say up front. This is up to the authors, but the introduction might be easier to understand if each paragraph started with a more general statement. For example, the second paragraph (lines 34-50) might be easier to read if it started with the sentence (from lines 49-50) “The ratio of sediment supply to capacity controls the alluvial cover ratio, bedrock incision rate, and morphodynamics in bedrock rivers.” This way, readers immediately see the point of all the descriptions of past work.

Line 36.

4) Terminology: the paper repeatedly states that the study evaluates the “efficiency” of models. To most people I think this would imply that you are testing how quickly simulations can be run. Really what you are testing is the “applicability” (or any similar word of your choice) of each model to a benchmark set of experiments.

Line: 10, 487, 557 and others have been changed as per suggestion.

Line comments:

15: Similar to general comment #2, the last sentence of the abstract could be a little bit more specific about which models did best, and what their basic limitations are.

Please see general comment 2

95: The model of Shobe et al (2017) also includes both the cover effect and a statement of mass conservation (based on entrainment/deposition).

Included

135: This sentence is confusing; consider re-wording or deleting the second clause for clarity.

When the sediment supply is larger than the transport capacity, the bedrock eventually becomes completely covered by alluvial material and the alluvial cover ratio \( P_c \) is equal to 1.

138: typo: linearly.

Corrected

149-150: Please add a quick (~1 sentence) explanation of what you mean by this. How/why is their assumption incorrect?

We have deleted this sentence in response to a comment from the editor. Instead, added detail explanations in the section of Turowski and Hodge (2017).

177: I’ve never seen a footnote in a paper before—it might be better to just incorporate it into the text.

Moved to lines: 178-179

221-224: As with general comment #1, please just make sure that whenever you list all the models they occur in the same order.

Mentioned in order of year they were introduced.

280: “unless” should be “until,” I think.

Changed

374: consider spelling out “with respect to” or using another phrase

Changed to Alluvial cover with respect to relative roughness

375: Do you mean “for example?”

Yes. Changed: Figure 8 shows the variation in \( P_c \) with respect to relative roughness. In cases with lower initial relative roughness, for example: Gravel 50 and Net2, the relative roughness is increasing with an increase in \( P_c \).

419: Missing an “and” in the list of models.

Changed: Figure 10 shows the comparison among experimental results presented in this paper, Sklar and Dietrich’s linear model (2014) and Turowski et al.’s exponential model (2007).
This sentence uses “Run#” terminology as in the initial paper draft, whereas the authors have helpfully changed their terminology to be “Gravel#/Net#.” As such, this sentence doesn’t match up with the figure and the run names should be changed in the text.

Changed accordingly: In Figure 11, in Gravel5 and Net2 series with relatively smooth beds, a rather scarce deposition was observed when sediment supply was low, and rapid alluviation occurred when sediment supply exceeded the transport capacity of the channel i.e. the bed was suddenly completely covered by alluvium.

450: Needs a comma instead of a period.

Changed:

490-494: This sentence is confusing; consider breaking it up into multiple sentences.

Changed: When we compare the observed $k_{sb}$ with the adjusted $k_{sb}$ in the roughness models proposed by Inoue et al. (2014) and Johnson (2014), the adjusted $k_{sb}$ strongly depends on observed $k_{sb}$ in our experiments without alternate bars (Figure 14a). Whereas, the adjusted $k_{sb}$ is not dependent on the observed $k_{sb}$ in case of experiments with alternate bars conducted by Chatanantavet and Parker (2008) (Figure 14b). This suggests that bedrock roughness has a smaller effect on the alluvial cover in case of mixed alluvial – bedrock rivers with alternate bars.

546-547: Do you mean the change in alluvial cover with the sediment supply rate?

Yes. Changed: The experimental results show that the change in alluvial cover with the sediment supply rate is controlled by bedrock roughness to a great extent.

555: Again, just make sure to keep the model order constant in whichever order you think is best.

Mentioned in order of year they were introduced.

567: “into a plane two dimensional” is confusing; consider rewording.

Changed to: Although models that extended the roughness model into two-dimensional planes (e.g., Nelson and Seminara, 2012; Inoue et al., 2016) will be able to capture the bar formation in a bedrock river, these models require long calculation time.
We thank reviewer for their positive consideration, encouragement and insightful comments on our manuscript. For ease of understanding the questions are in black, and the reply is in orange colour. Also, the comments are marked by orange in the manuscript.

Section 1 covers a wide range of relevant papers, but it’s still not clear what the main points are. It might help to structure it around a series of ideas (e.g. the role of bedload, relationships between sediment supply and sediment cover, roughness), and then use the papers to explain how these ideas have been investigated. The current structure just presents one study after another, making it difficult to identify what the key ideas are, and when the argument is moving on from one idea to the next.

The same applies to most of section 1.1, with the exception of the final paragraph.

Both hydraulic and bedrock topographic roughness lengths are calculated for all beds, and shown to be related to each other (Fig 2). It is far easier to calculate the bedrock roughness length from topographic data, in contrast to calculating the hydraulic roughness from flow data. Can you comment on whether it would be possible to use the bedrock topographic length to predict flow and sediment cover, or do you need the hydraulic data to make accurate predictions?

> Thank you for your insightful comment. Yes, it is easier to calculate the bedrock roughness length from topographic data. It would be best if we could establish a method to accurately evaluate hydraulic roughness from topographic roughness. However, it is very difficult to evaluate accurately. For example, when comparing Gravel 30 and Gravel 50 in Figure 2, the topographic roughness of Gravel 50 is higher than that of Gravel 30, but hydraulic roughness of Gravel 50 is lower than that of Gravel 30. This is due to hydrological roughness height does not only depend on the topographical roughness but also on the arrangement of the unevenness as described in Line 307. Accurate prediction of hydraulic roughness requires further works, which is a challenge for the future. We added this in Discussion: Line 427

My understanding is that Johnson’s model predicts the roughness length from the topography, whereas Inoue’s model uses the measured value. Does this difference explain why Inoue’s model better predicts the data? If you substituted the measured hydraulic roughness lengths into Johnson’s model, would that improve its performance?

> No, the results of Johnson's model have not improved. Please see Figure 14a. The adjusted ksb that minimizes the RMSD of Johnson's model is smaller than the observed ksb, especially in the region where ksb is large. Therefore, RMSD becomes large when we substitute the observed ksb into Johnson's model (i.e., not improved). Also, Johnson's model cannot accurately evaluate pc on clast-rough bedrock (e.g., Gravel30 and Net 4) because pc is not greater than qbs/qbca in Johnson's model. This is due to a problem with the method of calculating sediment transport capacity, as described in Lines 480-485. We added this in Discussion.

It’s not entirely clear to me how the values of qbca are calculated for the flume data in Fig. 10 onwards. Is it using one of the initial equations, or was it measured in the flume? Furthermore, was the initial sediment feed rate calculated to be equal to the transport rate for an alluvial bed? It’s notable that for Gravel5 and Net2 the qbs/qbca values are mainly greater than one, but this is not mentioned in the text, and I think that it should be pointed out.

> 3.73 x 10^{-5} was measured in the flume with completely alluvial bed before this experiment. This value is in good agreement with the calculated value obtained from Equation 4. We added this in Experimental method Line 293

We did not directly use the qbca for plotting Figs 10, 11 and 12. In Figs 10 and 12, we changed the ratio of qbs/qbca like 0.01, 0.02, 0.03... In roughness models, it is easier to calculate qbc / qbca with a given pc. So, we changed pc at 0.01 intervals to back-calculate the qbs / qbca in Figure 11. Line 444, 451
In clast-smooth bedrock (i.e., Gravel5 and Net2), it is possible to supply more sediment flux than \( q_{bca} \) because \( q_{bcb} \) (transport capacity on completely bedrock bed) is larger than \( q_{bca} \) (transport capacity on completely alluvial bed). We mentioned this in Section 3.2. Line 341.
We thank the editor for their positive consideration, encouragement and insightful comments on our manuscript. For ease of understanding the questions are in black, and the reply is in blue colour. Also, the comments are marked by blue in the manuscript.

We were not able to understand some of the questions raised by the editor and we therefore request suggestions for improvement in the changes made by us and also for sections we were not able to modify.

1) The literature review in the beginning of the paper can still be improved. I agree with both reviewers that over most of the section, it reads like a collection of disconnected statements about various papers, without a coherent summary of the state of knowledge, a critical assessment of what we do and do not know, and a synthesis. Both reviewers make concrete suggestions about how you can improve this. Please rewrite to establish a clear line of argument, leading to a few points that go beyond a few statements about what previous authors have done.

We reclassified previous studies and added a summary in its first sentence.

2) The description of the selected models can still be improved. This is much better than in the original manuscript, but partially, the physical and rational basis of the models is still clear. With respect to the tests, the differences in model prediction could be better worked out. This aspect can also be picked in the discussion, by putting the model performance into the context of the assumptions that went into their construction (see also reviewer #2).

We added the physical aspects of the models, especially in your model. If anything is missing or incorrect, please kindly point out.

They defined \( P \) as the probability that a grain will settle on exposed bed, and used a power law dependence of \( P \) on exposed area \( (1 - P_c) \), taking the form \( P = (1 - P_c)^\omega \), here \( \omega \) is a model parameter. Similar to the exponential model (Turowski, 2007), integrating \( d(1 - P_c) = -PdM_s \),

\[
P_c = 1 - [1 + (1 - \omega)M_s]^{\frac{1}{\omega}}
\]  

(12)

They further introduced the mass conservation equation and derived the following equation.

\[
P_c = 1 - \left[ 1 + (1 - \omega)\ln \left\{ 1 - \left( -\omega \frac{M_s}{Q_{bs}} \right) (Q_{bs}/Q_{bs}) \right\} \right]^{\frac{1}{1-\omega}}
\]  

(13)

3) I would appreciate clear conclusions, if possible with recommendations of which models are suitable under which circumstances. At the moment, you provide a summary, but no conclusions. Also, a summary figure including all models and fits (e.g., the RMSE of all models plotted for the different experimental runs), and a table summarizing the goodness of fit statistics would be helpful.

We have tried to improve the conclusion part and have restructured the paper.

4) Finally, the manuscript would benefit from thorough language editing.

We tried to revise as per suggestions.

Comments by line

23 …in controlling landscape evolution and determining the morphology of rivers… changed
Nice to see that the list of references is comprehensive. Still, for readability, it may be good to limit it to some key papers at this point. Some of the references are listed in the wrong groups. Shepherd 1972 is an experimental paper and Hobley et al. 2011 a field paper. Sklar and Dietrich 1998 do not describe experiments, but develop concepts and the basics of the model published later in their 2004 based on general field observations. Please check through. Some of the references do not suggest the existence of the tools and cover effects, but build on these notions, and are therefore ill-placed in the list.

We have removed two papers (Shepherd 1972, Sklar and Dietrich 1998) from this sentence, and moved Hobley et al. 2011 to field scale study.

The notion of impacts of particles driving the erosion should be mentioned here.

Changed to: It acts as a tool by increasing the number of impacting particles and erodes the bedrock bed, known as tools effect... Changed

unclear, rewrite for clarity. Maybe there is a word confusion and you mean 'hardly' instead of 'merely'.

Changed as suggested by Reviewer 2.

Their field surveys of bedrock gorge cut by Daán River in Taiwan showed that the channel bed did not erode in the absence of coarse bedload, despite floods and available suspended sediment

showed the formation...

Corrrected

...roughness of the bedrock bed...

Corrected

...excavating a channel into natural bedrock...

Corrected

...is proportional to...

Corrected

...introduced a reach-scale...

Corrected

typos, Turowski

Corrected

you need to be more specific here on which part exactly was incorrect. The derivation of the exponential model for mass is correct, the transformation from mass to fluxes is not correct.

We deleted this sentence because because a similar explanation is in section 1.1, i.e., simple models do not conserve sediment mass.

It is unclear why this model is tested even though it is deemed to be incorrect.

We deleted this sentence. Linear and exponential models are valuable as the simplest models. Actually, these models are applicable to rough bedrock.

Grammatical problem in the sentence about Meyer-Peter and Müller. Unclear.

Changed to: Here we introduce Johnson’s model that employ’s Meyer-Peter and Müller (MPM) equation:

What does this mean that Inoue et al. directly use hydraulic roughness? Please clarify.

Changed to: The model by Inoue et al. (2014) uses the observed hydraulic roughness, but the model by Johnson (2014) calculates the hydraulic roughness from the roughness (topographic unevenness) of the bed surface.

They did not propose it, it’s a prediction of the model (for the specific assumption of equal probability of deposition on exposed and cover bed).

Changed: Please see the response to general comment (2).
212 I do not understand this statement. Similar to which other models? And similar in what way?

213 Any probability can take values between 0 and 1.

214 It may help a reader unfamiliar with that paper to explain the rationale behind this equation.

217 I am not sure where you found this, but I am pretty sure that we did not say this.

219 None of the parameters have obscure physical meaning. The meaning of the parameters is clearly explained in the paper. We do not have any observational constraints on some of the parameters, but this is something your analysis could provide. Changed to: however we are employing Equation 13 in this study as the equation has the highest flexibility of \( P \) and is likely to be able to include roughness feedbacks.

230 …sandbar conditions

233 …were formed…

234 width-depth ratios

238 This could do with some more detail. What kind of mortar? What was the sand fraction? Did you quantify the hardness in some way? Was the mortar erodible by the bedload?

A mixture of cement, fine sand and water. Sorry. We did not record the sand fraction. We measured the bed elevation before and after the experiment to investigate the alluvial thickness, and there was no difference at bedrock parts in cases with almost no cover (e.g., Gravel 5-1).

238-240 The last sentence repeats information of the previous sentence. This could be consolidated.

In order to achieve different roughness conditions, the bed in Gravel30 was embedded with gravel of particle size \( 30 \) mm, Gravel50 was embedded with \( 50 \) mm gravel, and Gravel5 was embedded with \( 5 \) mm gravel.

239 The term ‘validation’ does not fit here. Changed to: Measurement of Alluvial cover

378 The bed does not try anything. Please reformulate.

380 …when the bed…

387.392 This seems to be a description of methods to me.

390 calibrated
Inoue et al. (2014) proposed theoretical and empirical models for $\tau^{*cb}$. Equation (5) is an empirical equation and can be revised with additional experimental data.

The statement here is self-contradicting. Are they consistent or not?

The results obtained from Johnson’s model (2014) (Eq. 10) (surface-roughness model) are roughly consistent with our experimental results (i.e., $0.8 < k_s/d < 9.6$), but is inconsistent in the experimental results of Inoue et al. (2013) (i.e., $k_s/d < 0.8$).

What is ‘rather scarce deposition’? Can you make a quantitative rather than a qualitative statement?

No. We only measured the cover ratio, did not measure the deposition ratio. We deleted this.

What does ‘accurately’ mean in this context?

In good agreement.

Why is that particularly important?

We want to emphasise for the readers that the hydraulic roughness is adjusted by trial and error method.

We changed: “Because the two models do not include the 2-D effects caused by bar formation, we adjusted $k_{sb}$ in the macro-roughness model in addition to $r_{br}$ in the surface model.”

An optimization procedure is not possible?

We did not use Chi squared, but reanalyzed your model using a numerical method to minimize RMSD.

504 Actually, in this case, physically, you would need to use different $P$ functions for entrainment and deposition.

Thank you for your suggestion. We included this comment in the new text.

The mentioning of the Turowski 2020 sits a bit odd here, and the statement of the model content is misleading. Either provide some context or remove.

We removed.

The sentence gives next to no information to the reader. Bramante et al. looked at tools and cover effects in oscillatory flows and successfully fitted the Turowski and Hodge value. For context, you could cross-compare the fit values of omega. Comparison is difficult because no information about roughness is given in Bramante et al.

Fig. 12: For some of the plots, especially a, the cover function based on the exponential model may provide a better fit (eq. 39 in the Turowski and Hodge paper). This would be equivalent to the limit in which omega approaches 1.

Since Equation 39 includes time, we did not use it.

This is a side note, but I still wonder why it is not possible to evaluate the $P$-function directly. In my mind that would lead to deeper insights into the physics.

We do not know how to evaluate the $P$-function directly. We measured the ratio of alluvial cover but did not measure the deposition probability.
When sediment supply exceeds transport capacity, the bed is abruptly covered by sediment and quickly reaches a completely alleviated bed. The statement here is somewhat misleading, because a calibration of some of the model parameters to the experimental conditions is required. The macro-roughness model (Inoue et al. 2014) and surface-roughness model (Johnson, 2014) can predict the rapid alluviation and hysteresis for clast-smooth bedrock as well as the proportionate increase in alluvial cover for clast-rough bedrock.
Alluvial Cover on Bedrock Channel: Applicability of Existing Models

The Influence of Bed Roughness and Sediment Supply on Alluvial Cover in Bedrock Channels

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Abstract. Several studies have demonstrated the importance of alluvial cover; furthermore, several mathematical models have also been introduced to mimic the alluvial cover in bedrock channels. Here, we provide an extensive review of research exploring the relationship between alluvial cover, sediment supply and bed topography of bedrock channels, describing various mathematical models used to analyze deposition of alluvium. To test one-dimensional theoretical models, we performed a series of laboratory-scale experiments with varying bed roughness under simple conditions without bar formation. Our experiments show that alluvial cover is not merely governed by increasing sediment supply, and that bed roughness is an important controlling factor of alluvial cover. A comparison between the experimental results and the five theoretical models shows that: (1) two simple models that calculate alluvial cover as a linear or exponential function of the ratio of sediment supply to capacity, produce good results for rough bedrock beds but not for smoother bedrock beds; (2) two roughness models which include changes in roughness with alluviation and a model including the probability of sediment accumulation can accurately predict alluvial cover in both rough and smooth beds; (3) however, except for a model using the observed hydraulic roughness, it is necessary to adjust model parameters even in a straight channel without bars; several studies have implied towards the importance of bedrock bed roughness on alluvial cover, besides, several mathematical models have also been introduced to mimic the effect bed roughness may project on alluvial cover in bedrock channels. Here, we provide an extensive review of research exploring the relationship between alluvial cover, sediment supply and bed topography of bedrock channels, thereby, describing various mathematical models used to analyze deposition of alluvium. In the interest of analyzing the efficiency of various available mathematical models, we performed a series of laboratory-scale experiments with varying bed roughness and compared the results with various models. Our experiments show that alluvial cover is not merely governed by increasing sediment supply, and bed topography is an important controlling factor of alluvial cover. We tested five theoretical models with the experimental results and the results suggest a fit of certain models for a particular bed topography and inefficiency in predicting higher roughness topography. Three models efficiently predict the experimental observations, albeit their limitations. 

1 Introduction

Economic growth worldwide has fuelled the demand for the construction of straightened river channels, sabo dams, the collection of gravel samples for various research, etc., leading to a decline in sediment availability and alluvial bed cover. Sumner et al. (2019) reported that the straightening of the Yubari River, which was carried out to improve the drainage of farmland, caused the bedrock to be exposed and the knickpoint to migrate upstream. In addition, also construction of a dam in the upstream section of Toyohira river in Hokkaido – Japan, decreased the sediment availability to the downstream section contributing to the formation of a knickpoint (Yamaguchi et al., 2011). Sediment availability plays a very important role in controlling the landscape evolution and determining the morphology of the river over geologic time (Moore, 1926; Shepherd 1972). It has two contradicting effects on bedrock-bed known as tools and cover effect (Gilbert, 1877; Sklar and Dietrich, 1998). It acts as a tool by increasing the number of impacting particles and erodes the bedrock bed, known as tools effect. As sediment availability increases, the sediment starts settling down on the river bed providing a cover for the bedrock and reducing the erosion.
bed underneath from further erosion, known as the cover effect. In the last 20 years, various field-scale (Gilbert, 1877; Shepherd, 1972b; Turowski et al. 2008b; Turowski and Rickenmann, 2009; Johnson et al., 2010; Jansen et al., 2011; Hobley et al. 2011; Cook et al., 2013; Inoue et al., 2014; Beer and Turowski, 2015; Beer et al., 2017), laboratory-scale (Sklar and Dietrich, 1998; 2001; Chatanantavet and Parker, 2008; Finnegan et al., 2007; Johnson and Whipple, 2010; 2011; Fernandez et al., 2019; Inoue and Nelson, 2020), and theoretical and numerical studies (Hancock and Anderson, 2002; Sklar and Dietrich, 2004, 2006; Lager, 2010; Jansen et al., 2011; Nelson and Seminara, 2011, 2012; Johnson, 2014; Nelson et al., 2014; Zhang et al., 2015; Inoue et al. 2016, 2017a; Turowski and Hodge 2017; Turowski, 2018) have been suggested that sediment availability, performed for revealing the effects of tools and cover on bedrock erosion and erosional morphology. 

Sediment availability strongly affects vertical bedrock incision including knickpoint propagation. Reach scale studies by Erlenbach performed by Turowski et al. (2013) showed how extreme flood events can contribute to incision by ripping off the channel’s alluvial cover. Yenites et al. (2011) studied the changes in the Peikang River in central Taiwan triggered by the thick sediment cover introduced by landslides and typhoons during the 1999 Chi-Chi earthquake. Their results showed slow or no incision in high transport capacity and low transport capacity channels. Cook et al. (2013) suggested that rapid knickpoint propagation, bedrock incision rates were dominantly controlled by the availability of bedload. Their field surveys of bedrock gorge cut by Dain River in Taiwan showed that the channel bed did not erode in the absence of coarse bedrock supply for 15 years despite floods and available suspended sediment. Izumi et al. (2016) showed that sediment transport and bedrock abrasion lead to the formation of cyclic steps, and Scheinmann et al. (2019) suggested that undulating bedforms like cyclic steps grow to become waterfalls and knickpoints.

Sediment availability also controls the width of bedrock channel. Finnegan et al. (2007) conducted laboratory-scale experiments and studied the interdependence among between incision, bed roughness and alluvial cover. Their results indicated that alluvial deposition on the bed shifted bedrock erosion to higher regions of the channel or bank of the channel, and suggested that the sediment supply rate controls the thalweg width of bedrock channels. Similar findings were noted in flume studies conducted by Johnson and Whipple (2010). They have shown the importance of alluvial cover in regulating the roughness of the bedrock bed by providing a cover for the local lows and thereby inhibiting the erosion and focusing erosion on local highs. Field observations also show that channels with higher sediment supply to capacity ratios are expected to be wider as alluvial cover shifts erosion from bed to banks of the channel (Beer et al. 2016; Turowski et al., 2008a and Whitbread et al., 2015). Inoue et al. (2016), and Inoue and Nelson (2020) showed the formation of several longitudinal grooves at low sediment supply to capacity ratio. As the sediment supply increases, one of the grooves attracts more sediment supply and progresses into a comparatively straighter, wide and shallower inner channel which further progresses into a more sinuous, deeper inner channel (Wohl and Ikeda, 1997; Shepherd and Schumm, 1974).

Some studies have credited the seasonally and climatically driven higher sediment supplies during floods to be the driving force for bedrock meander and strath terrace formation (De Vecchio et al., 2012; Hancock & Anderson, 2002). Periods of higher sediment supply promote lateral erosion and strath terrace formation, whereas periods of lower sediment supply lead to vertical erosion and steep slip-off slopes (e.g. Fuller et al. 2009; Inoue et al. 2017a). Mishra et al. (2018) showed that in the bend, lateral abrasion followed a monotonically increasing linear relationship with sediment feed rate. Fuller et al. (2016) performed laboratory-scale experiments and established the importance of bedrock roughness in determining lateral erosion rates because high roughness scatters the direction of bedload transport, increasing the frequency with which it collides with the wall.

There have been advances in theoretical and numerical methods mimicking, reproducing and predicting the morphodynamics of laboratory scale and field-scale observations. A majority of traditional bed erosion models are classified as the stream power.
and shear stress family of models (cf. Shobe et al., 2017; Turowski, 2018) (e.g., Howard, 1994; Whipple and Tucker, 1999), in which bed erosion is a function of discharge and bed-slope. These models—however, cannot describe the role of sediment in controlling the bed dynamics. Several models remedy this shortcoming by considering the tools and cover effect. Predicting tools and cover effects is essential for better understanding the bedrock landscape evolution. In this study, we review the advances of alluvial cover models in the past two decades and test several major models. Previous previous studies conducted on field or in lab have emphasized on the importance of sediment availability for determining the river will proceed laterally or vertically. Sklar and Dietrich (2001) and Scheingraber et al. (2015) performed rotary-abrasion mill experiments showing the importance of cover in controlling incision rates in bedrock channels. Channel incision occurred only when bedload tools became available.

The latest studies of alluvial cover in bedrock rivers have entered the next stage, which includes not only the effect of sediment supply-capacity ratio but also the effect of bed roughness. The latest studies of alluvial cover in bedrock rivers have entered the next stage, which includes not only the effect of sediment supply-capacity ratio but also the effect of bed roughness. In section 1.1, we introduce previous theoretical and numerical models that take into account sediment cover in bedrock channel. In sections 1.2 to 1.6, we describe in detail the governing equations of the five models dealt with in this study.

1.1 Previous Models for Sediment Cover

The sediment cover models predict cover from taking into account factors like sediment flux, roughness, discharge, grain size, etc. One of the simplest and first models to incorporate effects of sediment availability and transport capacity of the channel was introduced by Sklar and Dietrich (1998, 2004). According to the saltation-abrasion model proposed by Sklar and Dietrich (1998, 2004), the alluvial cover 

\[ P \]

increases linearly with the ratio of sediment supply to sediment transport capacity

\[ q_{sa}/q_{tc} \]

i.e. in absence of sediment supply, the alluvial cover is absent. However, when sediment supply becomes equal to or exceeds the transport capacity of the channel, the channel bed is fully covered. In contrast, in order to express the non-linear relationship between 

\[ P \]

and

\[ q_{sa}/q_{tc} \]

Turowski et al. (2007) proposed a model that considered the cover effect as an exponential function of the ratio of sediment flux to sediment transport capacity. The model uses a probabilistic argument i.e., when sediment supply is less than the capacity of the channel, grains have an equal probability of settling down over any part of the bed. Also, the deposited grains can be static or mobile. The erosion formula including the above model was able to reproduce the relationship between the sediment mass and the erosion rate observed in the rotary-abrasion mill experiment performed by Sklar and Dietrich (2001). However, subsequent experiments using straight channel pointed out a phenomenon that cannot be reproduced by the above models. Chatanantavet and Parker (2008) conducted laboratory-scale experiments in straight concrete bedrock channels with varying bedrock roughness and evaluated bedrock exposure with respect to sediment availability. In their experiments, alluvial cover increased linearly with increasing sediment supply in case of higher bed roughness, whereas in case of lower bed roughness and higher slopes, the bed shifted abruptly from being completely exposed to being completely covered. This process of the bedrock bed suddenly becoming completely alluvial from being completely exposed is known as rapid alluviation. Rapid alluviation was also observed in the laboratory scale experiment conducted by Hodge and Hoey (2016a, 2016b) in a 3D printed flume of natural stream Trout Beck, North Pennies-U.K. Their first set of experiments focused on quantifying hydraulic change with varying discharge, suggesting that hydraulic properties fluctuate more during higher discharge. Their second set of experiments
(Hodge and Hoey, 2016b) concentrated on quantifying the sediment dynamics for varying discharge and sediment supply. They supplied 4 kg and 8 kg sediment pulse to the channel and observed a similar alluvial pattern in both cases suggesting that the deposition of sediment on the bed may not only depend on the amount of sediment supplied, but may be strongly influenced by the bed topography and roughness. Inoue et al. (2014) conducted experiments by excavating a channel into natural bedrocks in Ishikari River, Asahikawa, Hokkaido – Japan. They conducted experiments with different combinations of flow discharge, sediment supply rate, grain size and roughness. Their experiments advocated that the dimensionless critical shear stress for sediment movement on bedrock is related to the roughness of the channel. Their experiments also suggested that the case when the alluvial cover is less rough/smooth than the bedrock, both an increase in alluvial cover, the relative hydraulic roughness i.e., the ratio of bedrock hydraulic roughness to moving sediment size in a mixed bedrock–alluvial bed decreases, also erosion in stress with an exposed bed is proportional to the sediment flux.

In addition, besides, the simple models described above cannot capture the sediment mass in a channel that changes due to sediment supply and runoff because they do not conserve sediment mass. These models, however, lack the statement of sediment mass conservation do not conserve sediment mass. Inoue et al. (2014) used the Exner equation to calculate alluvial thickness with respect to average grain size $d$. Their model, however, lacks the tools effect for bed erosion. Recently, Johnson (2014) and Inoue et al. (2014) proposed reach-scale physically-based models that could encompass the effects of bed roughness in addition to alluvial thickness mass conservation. Inoue et al. (2014) also conceptualised ‘Clast Rough’ and ‘Clast Smooth’ bedrock surfaces. A bedrock surface is clast rough when bedrock hydraulic roughness is greater than the alluvial bed hydraulic roughness (supplied sediment), otherwise, a surface is clast-smooth i.e. when the bedrock roughness is lower than the alluvial roughness. Inoue et al. (2014) and Johnson (2014) clarified that the areal fraction of alluvial cover exhibits a hysteresis with respect to the sediment supply and transport ratio in a clast smooth bedrock channel. They described that along with rapid alluviation, perturbations in sediment supply can also lead to rapid entrainment. Whether the bed undergoes rapid alluviation or rapid entrainment is determined by the bed condition when perturbations in sediment supply occur. If the perturbations occur on an exposed bed, it undergoes rapid alluviation, conversely, when perturbations happen on an alluviated bed, it undergoes rapid entrainment. Zhang et al. (2015) proposed a macro-roughness saltation abrasion model (MRSA) in which cover is a function of alluvial thickness and macro-roughness height. Nelson and Seminara (2012) proposed a linear stability analysis model for the formation of alternate bars on bedrock. Inoue et al. (2016) expanded Inoue et al. (2014) to allow variations in the depth and width of alluvial thickness in the channel cross-section. They further modified the numerical model (Inoue et al., 2017a) and implemented the model to observe changes in a meander bend.

Turowski and Hodge (2017) generalized the arguments presented by Turowski et al. (2007) and Turowski (2009), and proposed a reach-scale probability-based model that can deal with the evolution of cover residing on the bed and the exposed bedrock. Turowski (2018) proposed a model and linked the availability of cover in regulating the sinuosity of the channel. Shobe et al. (2017) proposed The SPACE 1.0 model for the simultaneous evolution of an alluvium layer and a bedrock bed. These models utilise the entrainment/deposition flux for sediment mass conservation. A step of model is the entrainment/position flux or Exner equation for sediment mass conservation (Turowski, 2009; Lagre, 2010; Inoue et al., 2014, 2016, 2017a; Nelson and Seminara, 2012; Hodge and Hoey, 2012; Johnson, 2014; Zhang, 2015; Turowski and Hodge, 2017; Shobe et al., 2017).

Hodge and Hoey (2012) introduced a reach-scale Cellular Automaton Model that assigned an entrainment probability to each grain. The assigned probability of each grain was decided by the number of neighbouring cells containing a grain. If five or more of total eight neighbouring cells contained a grain, the grain was considered to be a part of the cover, otherwise, it was considered an isolated grain. They suggested that rapid alluviation occurred only in cases when isolated grains were more than the cover on the bed. Also, they advised a sigmoidal relationship between $q_b/q_s$ and $1 - P_e$. Aubert et al. (2016) proposed a Discrete-Element Model where they determined $P_e$ from the velocity distribution of the grains. If the velocity of a grain is...
1/10^6 or lower than the maximum velocity, the grain settles as cover on the bedrock surface. The model, however, cannot deal with non-uniform velocity fields and hence cannot predict results for varying alluvial cover.


Except for the Lagrangian description models that track individual particles (i.e., Hodge and Hoey, 2012; Aubert et al., 2016), the Eulerian description models are roughly classified into four categories. The linear model proposed by Sklar and Dietrich (1998, 2004), the exponential model proposed by Turowski et al. (2007), the probabilistic model proposed by Turowski and Hodge (2012) and the roughness models proposed by Nelson and Seminara (2012), Inoue et al. (2014), Johnson (2014), Nelson and Seminara (2012) and Zhang et al. (2015) and the probabilistic model proposed by Turowski and Hodge (2017). In this study, we focus on a detailed study of the similarities and differences among the Eulerian description models proposed by Sklar and Dietrich (2004), Turowski et al. (2007), Inoue et al. (2014), Johnson (2014) and Turowski and Hodge (2017). These one-dimensional models have already been compared to an experiment with bars (Chatanantavet and Parker, 2008) and experiments with irregular roughness arrangement (Hodge and Hoey, 2016a, 2016b; Inoue et al., 2014), but a test in one-dimensional flow fields have not been performed. In this study, we compare the efficacy of these models from comparisons with our experimental results without bars with relatively regular roughness distribution. In addition, we apply the roughness models (Inoue et al., 2014; Johnson, 2014) to the experiments conducted by Chatanantavet and Parker (2008) in order to discuss the effect of bar formation on roughness on alluvial cover in a mixed bedrock-alluvial river with alternate bars.

1.2 Linear Model

When the value of \( q_b \), i.e. the alluvial cover ratio is 1 when the sediment supply is larger than the transport capacity, the bedrock eventually becomes completely covered by alluvial material and the alluvial cover ratio \( P_c \) is equal to 1. When \( P_c = 1 \), the alluvial cover is retained as the sediment rate deposited on the bed does not decrease as the sediment gets deposited on the bed, consequently, the bedrock is not exposed. If there is no sediment supply, the sediment deposit will disappear, and eventually, the bedrock becomes completely exposed and \( P_c \) will become equal to 0. Sklar and Dietrich (2004) linearly connected these two situations, and proposed a linear model to include the cover effect in their saltation – abrasion model:

\[
P_c = \begin{cases} 
q_{bc}/q_{bc} & \text{for } 0 \leq q_{bc}/q_{bc} \leq 1 \\
1 & \text{for } q_{bc}/q_{bc} > 1
\end{cases}
\]

where, \( P_c \) is the mean areal fraction of alluvial cover, \( q_{bc} \) and \( q_{bc} \) are the volume sediment supply rate per unit width and transport capacity, respectively.

1.3 Exponential Model

When the dimensionless mass of sediment on the bed \( M^{*} \) is increased by a small amount \( dM^{*} \), a fraction of this amount will fall on exposed bedrock and cover it. Hence, \( d(1 − P_c) = −\varphi dM^{*} \), where, \( \varphi \) is a dimensionless cover factor parameter and determines sediment deposition on covered areas for \( \varphi < 1 \) and deposition on uncovered areas for \( \varphi > 1 \). Taking the integration, \( P_c = 1 − \exp(−\varphi M^{*}) \), Turowski (2007) assumed that the \( M^{*} \) is equal to the ratio of sediment supply to capacity, and derived the following exponential model using a probabilistic argument:

\[
P_c = 1 − \exp\left(−\varphi \frac{dM^{*}}{dX}\right)
\]
1.4 Macro Roughness Model

The experimental results of Inoue et al. (2014) motivated their mathematical model formulating the interaction between alluvial cover, dimensionless critical shear stress, transport capacity and the ratio of bedrock hydraulic roughness to alluvial hydraulic roughness. They calculated the total hydraulic roughness height \( (k_r) \) as a function of alluvial cover:

\[
k_r = \begin{cases} 
(1-P)k_{sa} + (P)k_{sb} & \text{for } 0 \leq P \leq 1 \\
 k_{sa} & \text{for } P > 1
\end{cases}
\]  

(3)

where \( k_r \) is the total hydraulic roughness height of bedrock channel, \( P \) is the cover fraction calculated as proposed by Parker et al. (2013) that depends on the ratio \( \eta_0/L \) where \( \eta_0 \) is the alluvial cover thickness and \( L \) is the bedrock macro-roughness height (i.e. topographic unevenness of the bed). \( k_{sa} \) and \( k_{sb} = \frac{1}{2} [1 - (0.5) \text{ here set to } 2] \) represent the hydraulic roughness height of bedrock and alluvial bed respectively. The total transport capacity per unit width \( q_{bc} \) in Inoue et al.’s model is calculated as follows:

\[
q_{bc} = \alpha (r_c - \tau_{wc})^{1.5} \sqrt{R g d^2} \]  

(4)

\[
\tau_{wc} = 0.027 (k_s/d)^{0.75} \]  

(5)

where \( \alpha \) is a bedload transport coefficient taken as 2.66 in this study, \( r_c \) and \( \tau_{wc} \) are the dimensionless shear stress and dimensionless critical shear stress, \( R \) is the specific gravity of the sediment in water (1.65), \( g \) is the gravitational acceleration and \( d \) is the particle size. In this model, \( P_c \) is back-calculated from Equations (3), (4) and (5) under the assumption that the sediment supply rate \( q_{bc} \) and the sediment transport capacity \( q_{bc} \) are balanced in a dynamic equilibrium state (i.e., \( \partial \eta_0/\partial t = 0 \) in Exner’s mass conservation equation).

The sensitivity analysis of bedrock roughness and sediment supply rate conducted by Inoue et al. (2014) showed that for a given sediment supply, the deposition \( (P_c) \) is higher when bedrock roughness is larger. They also showed that if sediment transport capacity is larger than the transport capacity of bedrock bed the clast-smooth surface shows a sudden transition from completely exposed bedrock to completely alluvial, i.e., clast-smooth surfaces show rapid alluviation.

1.5 Surface Roughness Model

Johnson (2014) proposed a roughness model using the median diameter grain size. They also calculated the hydraulic roughness using the aerial alluvial cover fraction.

\[
k_{sa} = r_c d [1 + (k_{sa} - 1)P] \]  

(6)

where \( r_c = 2 \) is a coefficient and \( k_{sa} \) is called a non-dimensional alluvial roughness representing variations in topography.

For a fully alluviated bed, \( k_{sa} = 2d \). The bedrock hydraulic roughness \( k_{sr} = r_c \sigma_{sr} \), where \( r_c \) is a scaling parameter for bedrock roughness to grain roughness and \( \sigma_{sr} \) is the bedrock surface roughness. This method for estimating \( k_{sa} \) applies only to Johnson’s model. The method of calculating the observed value of \( k_{sa} \) is explained in section 2.3. Their model calculates bedrock shear stress using Wilcock and Crowe (2003) hiding/exposure function \( (b_v) \), modified to depend on a standard deviation of bedrock elevations and a bedrock roughness scaling parameter. Johnson (2014) calculated the total transport capacity using bedload equations proposed by Meyer-Peter and Müller (1948) and Wilcock and Crowe (2003). Here we introduce Johnson’s model that employs Meyer-Peter and Müller (MPM) equations based Johnson’s model:

\[
q_{bc} = (1 - P)q_{bc0} + (P)q_{bc1} \]  

(7)

\[
q_{bc0} = a (r_c - \tau_{wc})^{1.5} \sqrt{R g d^2} \]  

(8)

\[
q_{bc1} = a (r_c - \tau_{wc})^{1.5} \sqrt{R g d^2} \]  

(9)

\[
\tau_{wc} = \frac{\tau_{bc0} \alpha (k_{sa})^{1.5}}{\tau_{bc1} (k_{sa})^{1.5}} \]  

(10)
The models proposed by Inoue et al. (2014) and Johnson (2014) may seem rather similar in that they estimate the transport capacity of a mixed alluvial–bedrock surface. However, both models opt for different approaches when it comes to estimating hydraulic roughness. The model by Inoue et al. (2014) directly uses the observed hydraulic roughness, but the model by Johnson (2014) calculates the hydraulic roughness from the roughness (topographic unevenness) of the bed surface. The model by Inoue et al. (2014) needs measurements of observed bedrock hydraulic roughness, and the model by Johnson (2014) needs topographic bedrock roughness. In the model by Inoue et al. (2014), the macro roughness of the bed acts only when converting the alluvial layer thickness to the alluvial cover ratio. The macro roughness affects the temporal change of the alluvial cover ratio but does not affect the alluvial cover ratio in the dynamic equilibrium state. In addition, in the model by Johnson (2014), first, the sediment transport capacities for the bedrock and alluvial bed are separately calculated, then total transport capacity is estimated using $P_t$. Whereas, in the model by Inoue et al. (2014), first, the total hydraulic roughness height is calculated using $P_t$, then total transport capacity is estimated using the total hydraulic roughness.

### 1.6 Probabilistic Model

Turowski and Hodge (2017) proposed a probability-based model for prediction of cover on bedrock channels, and investigated the distribution of sediment on the bedrock. Because they mainly focused on the transformation between a point of view considering sediment masses and one considering sediment fluxes, they did not treat the interaction between the alluvial cover and the bed roughness. However, there is a possibility to capture the effects of bedrock roughness on the alluvial cover by adjusting the probability of grain entrainment and deposition included in the model. They defined $P$ as the probability that a grain will settle on the exposed bed, and used a power-law dependence of $P$ on the exposed area $(1 - P_t)$, taking the form $P = (1 - P_t)\omega_1$, here $\omega_1$ is a model parameter. Similar to the exponential model (Turowski, 2007), integrating $d(1 - P_t) = -P_t M_c d\omega_1$,

$$P_t = 1 - (1 - \omega_1)M_c 1^{1/\omega_1}$$

(12)

They further introduced the mass conservation equation and derived the following equation,

$$P_t = 1 - \left[1 + (1 - \omega_1)\ln\left(1 - 1 - e^{-M_c q_{bas}} \left(q_{bas}/q_{bas}\right)\right)\right]^{1/\omega_1}$$

(13)

where $M_c$ is the dimensionless characteristic sediment mass obtained as follows:

$$M_c = \frac{\sqrt{\pi} \tau_{cr} \left(\frac{x_1}{\tau_{cr}}\right)^{1.8}}{2n}$$

They model provides a combined linear and exponential relationship between $P_t$ and $\omega_1$. It is suggested that several factors can affect the sediment cover over a bedrock surface. Their model also provides two other analytical solutions and potentially other variables (Equation 30, 31 in Turowski and Hodge, 2017), however, we are employing Equation 12-13 in this study as the equation has the highest flexibility of $P_t$ and is likely to be able to include...
roughness feedbacks does not contain any parameter with obscure physical meaning and all the parameters can be calculated in laboratory or analytically.


2 Experimental Method

2.1 Experimental Flume

We conducted experiments to measure how sediment cover developed over surfaces of different roughnesses and different sediment fluxes. The experiments were conducted in a straight channel at the Civil Engineering Research Institute for Cold Region, Sapporo, Hokkaido, Japan. The experimental channel was 22 m long, 0.5 m wide and had a slope of 0.01. The width-depth ratio was chosen to achieve no-sandbar condition (i.e., small width-depth ratio, 6.1 to 8.3 in our experiments). Chatanantavet and Parker (2008) conducted several flume experiments with sandbar conditions (i.e., large width-depth ratio, 11 to 30 in their experiments) and suggested that the alluvial cover increases linearly to the ratio of sediment supply and transport capacity of the channel when the slope is less than 0.015. The formation of bars strongly depends on the width-depth ratio (e.g., Kuroki and Kishi, 1984; Colombini et al., 1987). Generally, neither alternate bars nor double-row bars are formed under conditions with width-depth ratios < 15.

In this study, we investigated the influence of bedrock roughness on the alluvial cover under conditions where the slope and width-depth ratios were small compared to the experiments of Chatanantavet and Parker (2008).

2.2 Bed characteristics and conditions

The channel bed consisted of hard mortar that was not eroded by the bed load supplied in this experiment. In order to achieve different roughness conditions, the beds in Gravel30 was embedded with gravel of particle size 30 mm, Gravel50 was embedded with 50 mm gravel, and Gravel5 was embedded with 5 mm gravel. We also embedded with gravel of different sizes. In Gravel30, the embedded particle size is 30 mm, in Gravel50, particle size of 50 mm is embedded and in Gravel5, 5 mm particle size is embedded.

We performed an additional 2 cases with net-installation on the riverbed. The net was made of plastic. An installed net on the riverbed can trap sediment during high flow, eventually protecting the bed from further erosion from abrading sediment (Kanazawa Mutsuura et al., 2015, in Japanese). A net of mesh size 30 mm X 30 mm was installed on the bed in Net4 and Net2. The height of the net was 4 mm and 2 mm respectively. Figure 1 shows the experimental channel bed of all 5 runs.

For each bed roughness (example: Gravel50 series), a group of experiments with varying sediment supply were performed for different time durations.

(a) Gravel30 (b) Gravel50 (c) Gravel5 (d) Net4 (e) Net2

Figure 1: Initial channel bed for each run. (a) Gravel30 is embedded with 30 mm gravel (b) Gravel50 is embedded with 50 mm gravel (c) Gravel5 is embedded with 5 mm gravel (d) Net4 is installed with a net of height 4 mm (e) Net2 is installed with a net of height 2 mm.
2.3 Measurement of observed bedrock roughness

In order to measure the initial bed roughness (before supplying sand), a water discharge of 0.03 m$^3$/s was supplied, and the water level was measured longitudinally at every 1 m at the centre of the channel. The hydraulic roughness height for bedrock ($k_a$) was calculated using Manning – Strickler relation and Manning’s velocity formula.

\[
k_a = \left[7.6 \frac{w_m}{D^{rac{1}{3}}} \right]^6
\]

\[
n_m = \frac{D}{3} S_e^{rac{1}{2}}
\]

where $w_m$ is the Manning’s roughness coefficient and $D$ is the water depth. $S_e$ is the energy gradient. Several previous studies have suggested that in bedrock rivers the Manning’s $n_o$ value can depend on the discharge (Heritage et al., 2004; Hodge and Hoy, 2016a), but in our experiments, the discharge is kept constant between the different runs.

In order to compare the hydraulic roughness height and the riverbed-surface unevenness height, the riverbed height before water flow was measured along a 1-metre length (12 m to 13 m) with a laser sand gauge. The measurements were taken longitudinally at every 5 mm. The measurements were taken at three points: 0.15 m away from the right wall, the centre of the channel, and 0.15 m away from the left wall. The standard deviation representing the topographic roughness $n_m$ was obtained by subtracting the mean slope from the riverbed elevation and then calculating the standard deviation of the remaining elevations (Johnson and Whipple, 2010).

2.4 Measurement of dimensionless critical shear stress on bedrock

To measure the dimensionless critical shear stress of grains on completely bedrock portion, i.e. $\tau_{cb}$. 30 gravels of 5 mm diameter each, were placed on the flume floor at intervals of 10 cm or more to make sure that there was no shielding effect between the gravels (there was shielding effect due to unevenness of the bedrock). Next, water flow was supplied at a flow discharge that no gravel moved, and was slowly increased to a flow discharge at which all the gravels moved. The water level and the number of gravels displaced were measured and recorded for each flow discharge. These measurements were performed for all the 5 bedrock surfaces.

We calculated the dimensionless shear stress $\tau_{cb}(= D S_e/R D)$, where $R$ is the specific gravity of the submerged sediment (1.65). We defined the critical shear stress was $\tau_{cb}$ is the weighted average of $\tau$, using the number of displaced gravels.

2.5 Measurement of Validation of an Alluvial cover

In order to perform the main set of experiments, different amounts of gravel (5 mm, hereafter called as sediment) was supplied manually at a constant rate while the flow rate was kept constant at 0.03 m$^3$/s. The alluvial cover ratio was measured once equilibrium state was achieved. Once the areal fraction became stable in qualitative observations and the variation of hydraulic roughness of mixed alluvium – bedrock bed $k_a$ calculated from the observed water depth $D$ decreased despite sediment being supplied, we considered that the experiment has reached its equilibrium state. Equilibrium conditions were achieved after 2–4 hours of sediment supply. The sediment supply volumes and other experimental conditions for various cases are provided in Table 1. Each run has multiple cases, each with different sediment supply and time duration. Each case was performed until the $P_c$ became constant. The gravels were supplied from Run-0 of no sediment to Run-4–5 of completely alluvial cover. The Run-0 with no sediment supply in each run represents the bedrock-roughness measurement experiment explained in section 2.3.

For each roughness condition, initially, we supplied sediment at the rate of 3.73x10$^{-4}$ m$^3$/s and observed the evolution of $P_c$. A sediment supply rate of 3.73x10$^{-4}$ m$^3$/s is used as it was measured in the flume with complete alluvium bed and it is in good
agreement with the calculated value obtained from Equation 4. If \( P_c \approx 1 \), the sediment supply was approximately reduced by 1.5 times in the subsequent run, and then the sediment supply was further reduced to 2 times and 4 times in subsequent runs (example: Gravel30, Gravel50 and Net4). In roughness conditions where sediment supply of \( 3.73 \times 10^{-5} \) m\(^2\)/s resulted in \( P_c \approx 0 \), the sediment supply was increased by 1.25 or 1.5 times and 2 times in the subsequent runs (example: Gravel5 and Net2).

However, for ease of understanding, we will present each experimental run in ascending order of sediment supply rate.

Equilibrium conditions were achieved after 2-4 hours of sediment supply. The alluvial cover was calculated at the end of the experiment, using black and white photographs of the flume by taking the ratio of the number of pixels. The dark/black colour represented sediment cover while white represented exposed bedrock. The water level was measured and recorded every hour at the centre of the channel, to calculate the hydraulic roughness during and at the end of the experiment. The cross-sectional profile of the channel bed was measured with a laser sand gauge at longitudinal intervals of 1 m from 10 m to 15 m from the downstream end before and after each run. We calculated the alluvial thickness from the difference between the two data.

Bedrock topography with alluvial cover was also measured with a laser sand gauge. Since bedrock topography without alluvial cover has been measured in section 2.3, we can calculate the alluvial thickness from the difference of the two data.
Table 1: Experimental Conditions. $k_0$ represents the hydraulic roughness height of purely bedrock bed. $k_0/d$ is the relative roughness. $q_s$ represents sediment supply rate. $P_c$ is the alluvial cover. $D$ is the water depth. $U$ is the depth-averaged velocity. $F_r$ is the Froude number. $k/d$ is the ratio of hydraulic roughness heights to grain size.

<table>
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<tr>
<th>Run</th>
<th>$k_0$ (mm)</th>
<th>$k_0/d$</th>
<th>$q_s$ ($m^3/s$)</th>
<th>Time (hour)</th>
<th>$P_c$</th>
<th>$D$</th>
<th>$U$</th>
<th>$F_r$</th>
<th>$k/d$</th>
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<td>9.6</td>
<td>0.00</td>
<td>0.25</td>
<td>0.00</td>
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<td>0.74</td>
<td>0.82</td>
<td>9.6</td>
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<td>0.73</td>
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<td>4.00</td>
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<td>0.082</td>
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<td>6.9</td>
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<td>0.082</td>
<td>0.74</td>
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<td>0.73</td>
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<td>0.95</td>
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<td>0.063</td>
<td>1.00</td>
<td>1.30</td>
<td>1.1</td>
</tr>
<tr>
<td>Gravel15-3</td>
<td>3.8</td>
<td>0.8</td>
<td>2.80</td>
<td>4.00</td>
<td>0.91</td>
<td>0.063</td>
<td>0.96</td>
<td>1.23</td>
<td>2.0</td>
</tr>
<tr>
<td>Net4-0</td>
<td>36.3</td>
<td>7.3</td>
<td>0.00</td>
<td>0.25</td>
<td>0.00</td>
<td>0.077</td>
<td>0.78</td>
<td>0.90</td>
<td>7.3</td>
</tr>
<tr>
<td>Net4-1</td>
<td>36.3</td>
<td>7.3</td>
<td>0.93</td>
<td>4.00</td>
<td>0.46</td>
<td>0.079</td>
<td>0.76</td>
<td>0.87</td>
<td>4.2</td>
</tr>
<tr>
<td>Net4-2</td>
<td>36.3</td>
<td>7.3</td>
<td>1.87</td>
<td>4.00</td>
<td>0.62</td>
<td>0.079</td>
<td>0.76</td>
<td>0.87</td>
<td>4.1</td>
</tr>
<tr>
<td>Net4-3</td>
<td>36.3</td>
<td>7.3</td>
<td>2.80</td>
<td>4.00</td>
<td>0.81</td>
<td>0.079</td>
<td>0.76</td>
<td>0.86</td>
<td>3.6</td>
</tr>
<tr>
<td>Net4-4</td>
<td>36.3</td>
<td>7.3</td>
<td>3.73</td>
<td>5.00</td>
<td>0.99</td>
<td>0.078</td>
<td>0.77</td>
<td>0.89</td>
<td>3.2</td>
</tr>
<tr>
<td>Net2-0</td>
<td>9.6</td>
<td>1.9</td>
<td>0.00</td>
<td>0.25</td>
<td>0.00</td>
<td>0.068</td>
<td>0.88</td>
<td>1.08</td>
<td>1.9</td>
</tr>
<tr>
<td>Net2-1</td>
<td>9.6</td>
<td>1.9</td>
<td>0.93</td>
<td>4.00</td>
<td>0.06</td>
<td>0.068</td>
<td>0.88</td>
<td>1.08</td>
<td>1.9</td>
</tr>
<tr>
<td>Net2-2</td>
<td>9.6</td>
<td>1.9</td>
<td>1.87</td>
<td>4.00</td>
<td>1.00</td>
<td>0.068</td>
<td>0.88</td>
<td>1.07</td>
<td>2.4</td>
</tr>
<tr>
<td>Net2-3</td>
<td>9.6</td>
<td>1.9</td>
<td>2.80</td>
<td>4.00</td>
<td>1.00</td>
<td>0.068</td>
<td>0.88</td>
<td>1.07</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Here, $k_0$ represents the hydraulic roughness height of purely bedrock bed. $k_0/d$ is the relative roughness of the bedrock bed. $q_s$ represents sediment supply rate. $P_c$ is the alluvial cover. $D$ is the water depth. $U$ is the depth-averaged velocity. $F_r$ is the Froude number. $k/d$ is the ratio of hydraulic roughness heights to grain size. $F_r = u/(gD)^{0.5}$.
3 Experimental results

3.1 Initial topographic roughness and hydraulic roughness

Figure 2 shows the relationship between the hydraulic roughness height of bedrock bed $k_{sb}$ and the topographic roughness height of bedrock bed $\sigma_{br}$. This figure suggests that Gravel30 with 30 mm sized embedded gravel, has the largest hydraulic roughness and Gravel5 with 5 mm sized embedded gravel has the lowest hydraulic roughness. Gravel50 embedded with 50 mm gravel has large topographical roughness error bars for the reason that, the large gravels were embedded randomly in the bed, resulting in unintended longitudinal spatial variation in the unevenness of the channel bed. The error bars here represent the minima, average and maxima of the calculated standard deviation of measurements taken along the left wall, centre and right wall of the channel, as mentioned in section 2.3. Although the hydraulic roughness tends to increase with an increase in topographical roughness, it has a large variation. This variation is due to the fact that the hydrological roughness height does not only depend on the topographical roughness but also on the arrangement of the unevenness.

Figure 2: Relationship between initial bed hydraulic roughness height and topographic roughness height. The black circles in the image represent the average values measured on the three data collection lines, and the error bars represent the minimum and maximum value.
3.2 Relative roughness of the bedrock bed, sediment supply and alluvial cover

Figure 3 shows the channel bed after the experiments of the Gravel30 series (Gravel30-1, Gravel30-2, Gravel30-3 and Gravel30-4) with the highest relative roughness of the bedrock bed \( k_{sb}/d \). Figure 4 shows the channel bed after the experiments of the Gravel5 series (Gravel5-1, Gravel5-2, Gravel5-3) which has the lowest relative roughness of the bedrock bed. In these two figures, we can compare Gravel30-4 and Gravel5-1 with equal sediment supply rates. The bed in Gravel30-4 is completely covered with sediment whereas the bed in Gravel5-1 has almost no accumulated sediment on the bed. Figure 5 shows the relationship between alluvial-cover fraction \( P_c \) and sediment supply per unit width \( q_{bs} \). The value of \( P_c \) is 1 for a completely covered channel and 0 for a completely exposed bedrock bed. Moreover, in Gravel30 series, Gravel50 series and Net4 series with high relative roughness of the bedrock bed \( k_{sb}/d \) (ratio of the hydraulic roughness height of bedrock bed \( k_{sb} \) to the grain size \( d \)), \( P_c \) is roughly proportional to the sediment supply rate \( q_{bs} \). However, in Gravel5 series and Net2 series, which have lower \( k_{sb}/d \) (relative roughness of the bedrock bed), \( P_c \) shows hardly any increase when \( q_{bs} \) is low (Gravel5-0, Gravel5-1, Gravel5-2, Net2-0, Net2-1) and when sediment supply \( (q_{bs}) \) increases (Gravel5-3, Net2-2), the bedrock suddenly transitions to completely alluvial bed. In clast-smooth bedrock (i.e., Gravel5 and Net2), it is possible to supply more sediment flux than \( q_{bs} \) because \( q_{bca} \) (transport capacity on completely bedrock bed) is larger than \( q_{bca} \) (transport capacity on a completely alluvial bed).

![Figure 3: Bedrock exposure in Gravel30 series at the end of the experiment. Initial bed had 30mm embedded particles. White bed represents exposed bedrock. Dark bed represents sediment covered bed.](image-url)
Figure 4: Bedrock exposure in Gravel5 series at the end of the experiment. *The initial* bed had 5mm embedded particles.

Figure 5: Variation in alluvial cover fraction ($P_c$) with sediment supply.

Commented [r61]: Is there a Net2 data point missing? It should also have $P_c = 1$ at $q_b = 5.6$. 

24
3.3 Relationship between gravel layer thickness and alluvial cover fraction

As explained in Section 1.5, the ratio of alluvial thickness \( \eta_a \) to macro-roughness \( L \) affects the temporal change of the alluvial cover ratio but does not affect the alluvial cover ratio in the dynamic equilibrium state. Thus, \( \eta_a/L \) is not used in the model comparison in this study. However, we experimentally investigate \( \eta_a/L \) because various numerical and theoretical models have employed alluvial cover as a function of relative alluvial thickness (Zhang et al., 2015; Inoue et al., 2014; Parker et al., 2013; Tanaka and Izumi, 2013; Nelson and Seminara, 2012).

\[
P_c = \begin{cases} 
\frac{\eta_a}{L} & \text{for} \ 0 \leq \frac{\eta_a}{L} \leq 1 \\
1 & \text{for} \ \frac{\eta_a}{L} > 1 
\end{cases}
\] (15)

Here, \( \eta_a \) is the average thickness of the alluvial layer, \( L \) is the macro-roughness height of the bedrock bed. Parker et al. (2013) define \( L \) as the macroscopic asperity height of rough bedrock rivers \( L \approx 2\sigma_b \). Tanaka and Izumi (2013) and Nelson and Seminara (2012) define \( L \) as the surface unevenness of alluvial deposits on smooth bedrock rivers \( L \approx d \). In this study, we define \( L = 2\sigma_b + d \) so that it can cope with both smooth and rough bedrocks. Figure 6 shows the relationship between relative gravel layer thickness \( \eta_a/L \) and alluvial cover ratio. The figure confirms that the alluvial cover ratio of the experimental result can be efficiently evaluated by Equation (15).

Commented [r62]: R2: I'm not entirely sure what this first sentence means - temporal change with what?
Commented [r63]: R2: predicted
Commented [r64]: R2: I know that I raised this in my previous review, but it would be useful to clarify here that when calculating average depth you are dividing by the total flume area, not just the area of the sediment patches.

Figure 6: Relationship between relative gravel layer thickness and alluvial cover. The black line represents the 1:1 line.
3.4 Time series change of relative roughness

Figure 7 shows the change in relative roughness in a mixed alluvial-bedrock channel i.e. $k_s/d$-with time in Gravel30 and Gravel5 series. The red and blue points lines in Figure 7 show the variation in alluvial cover fraction after water supply in Gravel30 and Gravel5 series, respectively.

In Run 4 (Gravel30) series with a higher initial relative roughness, relative roughness decreased due to the increase in alluvial deposition and cover. In Run 3 (Gravel5) series which has a lower initial relative roughness, relative roughness increased due to the increase in alluvial deposition and cover.

The relative roughness approaches 2 for both Gravel30-4 and Gravel5-3, in which the alluvial cover fraction approaches 1. This value is almost the same as the relative roughness of flat gravel bed (about 1 to 4 times the particle size, generally about 2 times). This confirms that with an increase in alluvial cover, the relative roughness of the bed is determined by the gravel size.

3.5 Alluvial cover w.r.t relative roughness

Figure 8 shows the variation in $P_c$ with respect to relative roughness. In cases with lower initial relative roughness, for example Gravel50 and Net2, the relative roughness is increasing with an increase in $P_c$. Whereas, in cases with higher initial relative roughness, Gravel30, Gravel5 and Net4, an increase in $P_c$ reduces the relative roughness. Besides, irrespective of the initial relative roughness, the bed tries to become completely alluvial as $P_c \approx 1$. Furthermore, irrespective of the initial hydraulic relative roughness, an increase in $P_c$ force each and its relative roughness condition (every experimental series) to achieve a similar stabilized roughness value (i.e. 1 to 4). Also several studies in the past have suggested that when the bed consists of a uniform grain size and also comprises of bedload consisting of uniform and same size grains as the cover, the hydraulic roughness height $k_s$ for such a gravel bed is 1 to 4 times the grain diameter $d$ (Inoue et al., 2014; Kamphuis, 1974; Parker, 1991) which is also the case in our experiments as shown in Figure 8.
4 Discussion and Comparison of the Existing Models with Experimental Results

4.1 Relationship between gravel layer thickness and alluvial cover fraction

As explained in Section 1.5, the ratio of the alluvial thickness $\eta_a$ to macro-roughness $L$ temporal change of the alluvial cover ratio but does not affect the alluvial cover ratio in the dynamic equilibrium state. Thus $\eta_a/L$ is not used in the model comparison in this study. However, we experimentally investigate $\eta_a/L$ because various numerical and theoretical models have predicted alluvial cover as a function of relative alluvial thickness (Zhang et al., 2015; Inoue et al., 2014; Parker et al., 2013; Tanaka and Izumi, 2013; Nelson and Seminara, 2012).

\[ P_c = \begin{cases} \frac{\eta_a}{L} & \text{for } 0 \leq \frac{\eta_a}{L} \leq 1 \\ 1 & \text{for } \frac{\eta_a}{L} > 1 \end{cases} \] (15)

Here, $\eta_a$ is the average thickness of the alluvial layer calculated from the total flume area instead of just the area of sediment patches, $L$ is the macro-roughness height of the bedrock bed. Parker et al. (2013) define $L$ as the macroscopic asperity height of rough bedrock rivers $L_b (\approx 2\sigma_{br})$. Tanaka and Izumi (2013) and Nelson and Seminara (2012) define $L$ as the surface unevenness of alluvial deposits on smooth bedrock river $L_a (\approx d)$. In this study, we define $L = 2\sigma_{br} + d$ so that it can cope with both smooth and rough bedrocks. Figure 8 shows the relationship between relative gravel layer thickness $\eta_a/L$ and alluvial cover ratio. The figure confirms that the alluvial cover ratio of the experimental result can be efficiently evaluated by Equation (15).
Figure 8: Relationship between relative gravel layer thickness and alluvial cover. The black line represents the 1:1 line.
4.1.2 Calibrating $k_{sb}$ and $r_{br}$

For the purpose of model comparisons with experimental results, we need to first calibrate Johnson’s model parameters $k_{sb}$ and $r_{br}$ to minimize RMSD (root mean square deviation) of cover between experimental data and the model. When $k_{br} = 1$, it means the alluvial hydraulic roughness is proportional to the grain size and is independent of the cover fraction. For our calculations, we have used $k_{br} = 4$ as applied in Johnson (2015). We also calibrated the exponential model’s parameter $c$ (Turowski et al. 2007). Table 2 provides the calibration values for $r_{br}$ and $c$ for comparison of the model with our experimental results. $M^*$ controls the onset time of alluvial cover where higher $M^*$ means earlier onset of alluvial cover i.e., follows a nearly linear model. Adjusting $c$ controls the deposition on uncovered bed (decreases for $c > 1$).

### Table 2: $r_{br}$ and $c$ values for comparison with experimental results

<table>
<thead>
<tr>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed $k_{sb}$ (mm)</td>
<td>2.7</td>
<td>4.0</td>
<td>2.2</td>
<td>4.0</td>
</tr>
<tr>
<td>Observed $r_{br}$ (mm)</td>
<td>3.2</td>
<td>3.1</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Adjusted $r_{br}$ (Johnson 2014)</td>
<td>1.0</td>
<td>0.6</td>
<td>1.1</td>
<td>0.6</td>
</tr>
<tr>
<td>Adjusted $c$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

4.2 Relative Roughness of the bedrock bed and dimensionless critical shear stress

Figure 9 shows the relationship between the ratio of the hydraulic roughness height of bedrock bed $k_{sb}$ to the grain size $d$ ($k_{sb}/d$ : referred to as the relative roughness of the bedrock bed in section 3.2) and the dimensionless critical shear stress over bedrock $r_{br}$. In this figure, we compare the results obtained from Inoue et al. (2014) (Eq 5) Johnson (2011) (Eq. 10) and from Johnson (2014) (Eq. 10) Johnson et al. (2014) (Eq. 5) i.e. surface roughness model and macro roughness model respectively. The Figure also compares with the experimental results in this study of experimental results of Inoue et al. (2013) (the same channel and grain size as this study, but with a smoother bedrock bed) and Inoue et al. (2014) (the channel excavated in Ishikari river, details are provided in section 1.1).

According to Figure 9, the non-dimensional critical shear stress depends on the relative roughness of the bedrock bed to the power of 0.6. Besides, the results obtained from Eq. (5) of the macro-roughness model are not compatible with the experimental results in the region where relative roughness of the bedrock bed is small. In this study, we used the power approximation shown below instead of Eq. (5) in the macro roughness model by Inoue et al. (2014):

$$r_{br} = 0.03(k/d)^{0.6}$$

(16)

Likewise, the results obtained from Johnson’s model (2014) (Eq. 10) (surface-roughness model) are roughly consistent with our experimental results (i.e., $0.8 < k/d < 9.6$), but the model is inconsistent when the roughness is low with the experimental results of Inoue et al. (2013) (i.e., $k/d < 0.8$).

The model could benefit from an accurate prediction method for hydraulic roughness on bedrock topographic roughness. It is easier to obtain the bedrock topographic roughness instead of obtaining hydraulic roughness data from the flow data. Hence, accurate prediction of hydraulic roughness will not only take into account the bedrock topographic roughness but also the macro-roughness of gravel.

**Diagram Comment:**
- Commented [r74]: R2. Explain that this figure also includes some data from Inoue et al 2013 & 2014, and explain how these other data were measured.
- Commented [r75]: Both hydraulic and bedrock topographic roughness lengths are calculated for all beds, and shown to be related to each other (Fig 2). It is far easier to calculate the bedrock roughness length from topographic data, in contrast to calculating the hydraulic roughness from flow data. Can you comment on whether it would be possible to use the bedrock topographic length to predict flow and sediment cover, or do you need the hydraulic data to make accurate predictions?
- Formatted: Font color: Auto
Figure 6: Relationship between relative roughness of the bedrock bed and dimensionless critical shear stress. The black squares show the results of this experiment, the white circles show the results of an investigation using the bedrock of Ishikari River in 2011 (Inoue et al., 2014), the grey rhombus represents a smooth aquifer floor (Inoue and Ito, 2013 (in Japanese)), the grey line shows the power approximation of all the experimental results. The dotted line shows the results from Eq. 5 proposed by Inoue et al. (2014). The black double dotted lines show the results obtained by Eq. 10 (Johnson, 2014). The grain size (d) in the case of Inoue et al., (2013) is 5mm, and Inoue et al., 2014 used gravels sized: 12mm and 28mm.

4.33 Predicting experimental results of alluvial cover ratio using the models

For the purpose of model comparisons with experimental results, we first calibrate the model parameters included in the exponential model, the surface roughness model and the probabilistic model to minimize RMSD (root mean square deviation) of cover between experimental data and the model. We do not calibrate the linear and macro-roughness models as they do not include model parameters.

The parameter \( \sigma \) in the exponential model means implies that the probability of sediment deposition in uncovered areas (Turowski et al., 2007) can vary with the roughness of the bedrock. The parameter \( k_{sw} \) in the surface roughness model (Johnson, 2014) represents the change in alluvial roughness that varies with the cover. When \( k_{sw} = 1 \), it means the alluvial hydraulic roughness is proportional to the grain diameter size and is independent of the cover fraction. For our calculations, we have used \( k_{sw} = 4 \) as applied in Johnson (2014). The parameter \( r_{br} \) in the surface roughness model is used to calculate the hydraulic bedrock roughness \( k_{sb} \) from the topographic roughness \( \sigma_{br} \). This value can be back-calculated from the experimental results (Fig. 2), but using the back-calculated value (i.e., using the observed \( k_{sb} \) instead of the calculated \( k_{sb} \)) did not minimize the RMSD of cover. Hence we adjusted \( r_{br} \) to minimize the RMSD of cover. The parameter \( \omega \) is introduced to express the relation between the deposition probability and the cover ratio exponentially, and can vary with bedrock roughness. The parameter \( M_0^* \) in this is introduced to express the relation between the deposition probability and the cover ratio exponentially, and can vary with bedrock roughness. The parameter \( M_0^* \) represents the dimensionless value of sediment mass at sediment transport capacity and vary with bedrock roughness. Although this parameter is calculated from Equation 14, the experimental results could not be reproduced only by adjusting \( \omega \). Hence we adjusted both \( \omega \) and \( M_0^* \) by trial and error. Table 2 provides the calibration values.
Figure 10 shows the comparison among experimental results presented in this paper. Sklar and Dietrich’s linear model (2014) and Turowski et al.‘s exponential model (2007) in order to calculate the model results for Figure 10 and Figure 12, we altered the ratio of $d_{50}$/bedrock bedclast by 0.01, 0.02, 0.03 and so on. Figure 10 suggests that the linear model is generally applicable to rough bed with relative roughness of the bedrock bed $b_c/d_{50}$ of 2 or more, but not to smooth bed with relative roughness of the bedrock bed $b_c/d_{50}$ less than 2 (Run 4, Gravel30, Run 5, Gravel50 and Run Net4). As suggested by Inoue et al. (2014), in this study, the “clast smooth bed” refers to the bed with roughness less than the roughness of supplied gravel $d_{50}/b_{c,s}$ and “clast rough bed” stands for the bed with roughness more that the roughness of the supplied gravel $d_{50}/b_{c,s}$. The exponential model is also more suitable for a clast rough bed.

Figure 11 shows the comparison of our observed experimental values with Inoue et al.’s macro-roughness model (2014) and Johnson’s surface-roughness model (2015). In roughness models, $q_{bca}$ is calculated with a given $P$ at intervals of 0.01. It shows that the macro-roughness model proposed by Inoue et al. (2014) can predict the increasing alluvial cover for cases with high relative roughness of the bedrock $b_c/d_{50}$ as well as the rapid alluviation and hysteresis (green shaded region) for cases with lower relative roughness of the bedrock bed (Run 4, Gravel30 and Run Net4), without adjusting the roughness (explained in the following paragraph). The surface-roughness model proposed by Johnson (2014) also shows good agreement in predictions of alluvial cover and rapid alluviation and hysteresis if $b_{c,a}$ and $b_{c,b}$ are adjusted.

As mentioned earlier, the major difference between the macro-roughness model (Inoue et al., 2014) and the surface-roughness model (Johnson, 2014) is the way the transport capacity is calculated. In the case of the surface-roughness model (Johnson, 2014), first, the transport capacities for bedrock $q_{bca}$ and alluvial bed $q_{bca}$ are separately calculated, then the total transport capacity $q_{bca}$ is calculated for a range of cover fractions ($P$). Hence, in cases when $\tau_{c,s} < \tau_s < \tau_{c,b}$, the transport capacity over bedrock portion $q_{bca} = 0$ and thereby the bedrock roughness hardly affects the alluvial cover fraction which can also be the reason for inconsistence between the surface-roughness model (Johnson, 2014) results and experimental study for Gravel30 and Net4 in Figure 11. Whereas, in the case of the macro-roughness model (Inoue et al., 2014), the critical shear stress takes

| Table 2: $r_b$ and $\varphi$ values for comparison with experimental results. |
|---|---|---|---|---|
| **Observed** | **Adjusted** | **Calculated $k_b$** | **Adjusted $\varphi$** | **Adjusted $M_k$** |
| $k_b$ (mm) | $r_b$ | $(k_b=1)$ | $\omega$ | $(k_b=1)$ |
| **Observed** | $k_b$ (mm) | $r_b$ | $(k_b=1)$ | $\omega$ | $(k_b=1)$ |
| Run 4, Gravel30 | 5.2 | 3.7 | 3.0 | 2.2 | 3.1 | 2.7 |
| Run 5, Gravel30 | 4.8 | 3.2 | 3.0 | 2.2 | 3.1 | 2.7 |
| Run 4, Gravel50 | 3.8 | 1.1 | 3.0 | 2.2 | 0.7 | 2.7 |
| Run 5, Gravel50 | 3.6 | 2.5 | 4.6 | 2.2 | 0.5 | 2.7 |
| Run Net4 | 9.6 | 1.8 | 2.6 | 9.4 | 0.9 | 2.7 |
into account the value of total hydraulic roughness, which depends on cover fraction, alluvial hydraulic roughness and bedrock hydraulic roughness. Hence, even when $r_b$ is smaller than $r_{cb}$, the bedrock roughness tends to affect the cover fraction. The macro-roughness model (Inoue et al., 2014) is more capable of dealing with clast-rough surfaces.

Figure 12 shows the comparison of experimental results with Turowski and Hodge’s probabilistic model (2017). $h_{obs}$ are the ratio of $q_{bs}/q_{bca}$ by 0.01, 0.02, 0.03 and so on. The model produces favourable results following some parameter adjustments. Because the probabilistic model (Turowski and Hodge, 2017) does not consider the effect of bedrock roughness on entrainment and deposition, the model parameter $\omega$ and characteristic sediment mass $M_0$ needs to be adjusted by trial and error. The value of $\omega$ can be as high as 1000–94 or 28000 for runs with rapid alluviation hysteresis, whereas it is as low as ~0.7 for other runs. As explained in Turowski and Hodge (2017), $M_0$ means surface area of alluvial cover i.e. follows a roughly linear model. Adjusting $\omega$ controls the deposition on uncovered bed (equation for $\omega=1$).

In Figure 11, in Run 3-Gravel5 and Run 5-Net5 series with relatively smooth beds, a rather scarce deposition was observed when sediment supply was low and rapid alluviation occurred because the transport capacity over bedrock $q_{bca}$ is larger than that over alluvial bed $q_{bcr}$. When sediment supply exceeded the transport capacity of the channel i.e. the bed was suddenly completely covered by alluvium. The reverse-line slopes produced by macro-roughness and surface-roughness models depict similar hysteresis relationship between alluvial cover and sediment supply. i.e. sediment deposition occurs only for a certain range of sediment supply. The shaded portion shows that, as $q_{bs}/q_{bca}$ increases, the cover does not increase unless it reaches a threshold ($q_{bca}/q_{bca}$ > 1), i.e. sediment supply rate transport capacity over exposed bed is higher than transport capacity over fully covered bed, after which the cover increases abruptly, showing rapid alluviation. The green shaded portion between however, is unstable between $P_b = 0$ and $P_b = 1$, i.e. it shows the hysteresis of rapid alluviation and rapid entrainment. As long as $q_{bs}/q_{bca} < 1$, the value of $q_{bs}/q_{bca}$ will increase until it reaches 1, however, if $q_{bs}/q_{bca}$ becomes smaller than $q_{bca}$, $P_b$ will decrease until $P_b = 0$, i.e. rapid entrainment). For the bed to become alluviated again, $q_{bs}/q_{bca}$ must reach a condition where $q_{bs}/q_{bca} > 1$, in which case rapid alluviation will happen again. This phenomenon has also been observed in sufficiently deep channels, for slopes greater than 0.015 by Chatanantavej and Parker (2008). Hodge and Hoey (2016b) also suggested a similar relationship between sediment cover and sediment supply. However, our study shows that rapid alluviation occurs irrespective of the slope steepness, if roughness of the bed is less than the roughness of supplied gravel, i.e. when relative roughness of the bedrock bed is less than 2.

In a channel without bars and with a relatively regular roughness distribution (i.e., a channel close to a one-dimensional flow field), the macro-roughness model (Inoue et al., 2014) is the most suitable because it can predict alluvial cover ratio without adjusting the parameters. When the observation of hydraulic roughness is difficult, it is useful to obtain the hydraulic roughness from the topographical roughness like the surface roughness model (Johnson, 2014). However, accurate prediction of hydraulic roughness should not only take into account the bedrock topographic roughness but also the arrangement of bed unevenness.

For example, in Figure 2, the topographic roughness of Gravel50 is higher than that of Gravel30, but hydraulic roughness of Gravel50 is lower than that of Gravel20. Ferguson et al. (2019) argued that the standard deviation of exposed bed is an effective way of roughness estimation, however, their finding is for a relatively smooth bedrock it needs further research on appropriating scales. Also, in order to deploy models on field-scale, they must take into account bank-roughness and its effects on shear stress and other hydraulic parameters (Ferguson et al., 2019). Prediction of hydraulic roughness from topographic roughness requires further work.

Another solution is to use the probabilistic model (Turowski and Hodge, 2017). The probabilistic model proposed by Turowski and Hodge (2017) could reproduce experimental results but the model needed optimisation adjustment of $\omega$ and $M_0$ to minimize the RMSE by trial and error, especially for cases involving rapid alluviation. Small $\omega$ means that the deposition probability gradually decreases with increasing alluvial cover, in contrast, large $\omega$ means that the deposition probability rapidly approaches zero with increasing alluvial cover. The model however, does not emulate the hysteresis for
clast-smooth beds. In this case, we may need to use different probability functions for entrainment and deposition. In addition, \( M_f \) calculated physically from Equation (14) is 0.04 (alluvial bed) to 0.06 (smoothest bedrock, i.e., Gravel5) in this experiment, which is significantly different from the adjusted \( M_f \). Because the model does not include the effects of bed roughness yet, further alterations to take into account the effect of the probability of grain entrainment and deposition can greatly extend the applicability of the model to natural bedrock rivers. Taking into account the spatial variability in the tools effect (laboratory experiments by Bramante et al., 2020) will also take the models closer to field-scale studies. In this case, how to link \( \omega \) and \( M_f \) with topographical roughness is a future issue. As mentioned earlier, the major difference between the macro-roughness model (Inoue et al., 2014) and surface roughness model (Johnson, 2014) is the way the transport capacity is calculated. In case of the surface roughness model (Johnson, 2014), first, the transport capacities for bedrock \( (c_{bc}) \) and alluvial bed \( (c_{bc}) \) are separately calculated, then the total transport capacity \( c_{bc} \) is calculated for a range of cover fractions \( (c_f) \). Hence, in cases when \( c_{bc} \leq c_f \leq c_{bc} \), the transport capacity over bedrock portion \( c_{bc} = 0 \) and thereby the bedrock roughness hardly affects the alluvial cover fraction which can also be the reason for inconsistency between the surface roughness model (Johnson, 2014) results and experimental study for Gravel30 and Net4. Runs 1 and 2 in Figure 11 and RA2 Slope = 0.0115 in Figure 13. Whereas, in the case of macro-roughness model (Inoue et al., 2014), the critical shear stress takes into account the value of total hydraulic roughness, which depends on cover fraction, alluvial hydraulic roughness and bedrock hydraulic roughness. Hence, even when \( c_{bc} \) is small, the bedrock roughness tends to affect the cover fraction. This macro-roughness model (Inoue et al., 2014) is more efficient at modeling problems with clast-smooth surfaces.

### 4.4 The Effects of Bar formation on Alluvial Cover

For investigating the influence of bed roughness and bar formation on the alluvial cover, we bedrock channel with alternate bars, we also compared the experimental results of Chatanantavet and Parker (2008) with the model results of the physically based models including interaction between roughness and alluvial cover (i.e., Inoue et al., 2014; Johnson, 2014). Chatanantavet and Parker (2008) conducted experiments in a metallic straight channel with three different types of bedrock surfaces namely Longitudinal Grooves (LG), Random Abrasion Type 1 (RA1) and Random Abrasion Type 2 (RA2), where RA1 is smoother than RA2. They performed various cases for each type with varying slope range of 0.0115 – 0.03. They also varied the sediment supply rate and grain size (2mm and 7mm). The major difference between their experiment and our experiments is the width-depth ratio. The width-depth ratios of our experiments were 6.1 – 8.3, as a result, alternate bars usually do not develop. Although we can see alternate alluvial patches in Figure 5, their thickness was less than 1 cm, and the patches did not progress to alternate bars with large wave height.

Figure 13 shows the comparison among the two models and Chatanantavet and Parker’s experiment (2008). The experimental conditions are taken from Table 1 of Chatanantavet and Parker (2008). Figure 13a represents runs 2-C1 to 2-C4, Figure 13b represents runs 2-E1 to 2-E3, Figure 13c represents 3-A1 to 3-A5, Figure 13d represents runs 3-B1 to 3-B5, Figure 13e represents runs 1-B1 to 1-B4 (Chatanantavet and Parker 2008, Table 1). Because the two models do not include the 2-D effects caused by bar formation, we adjusted \( k_{ap} \) in the macro-roughness model in addition to \( k_{ro} \) in the surface model. In the case of the surface-roughness model, \( k_{ap} = 4 \) is used, the bedrock surface roughness required for calculations is taken as mentioned in Table 1 Johnson (2014), \( k_{ro} \) is adjusted to minimize RMSD of cover between experiments and the model. In the case of the macro-roughness model by Inoue et al. (2014), \( k_{ap} \) is adjusted to minimize RMSD of cover. The two models can accurately predict the cover fraction and rapid alluviation for the experimental study conducted by Chatanantavet and Parker (2008). However, the adjusted roughnesses were significantly different from the observed value.
In the case of Chatanantavet and Parker’s experiment, \( k_{sb} \) ~ 0.4 mm to 3.5 mm (Chatanantavet and Parker 2008, Table 1), whereas, in Johnson’s surface-roughness model (2014), \( k_{sb} \) (= \( r_s r_o \sigma_r \)) can be as much as 13 – 27 mm. Also, in the case of Inoue et al.’s macro-roughness model \( k_{sb}, k_{ra} \) is adjusted to 32 – 53 mm (Table 3).

In Table 3, when we compared the observed \( k_{sb} \) with the adjusted \( k_{sb} \) in the roughness models proposed by Inoue et al. (2014) and Johnson (2014), the adjusted \( k_{sb} \) strongly depends on observed \( k_{sb} \) in our experiments without alternate bars (Figure 14a). Whereas, the adjusted \( k_{sb} \) is not dependent on the observed \( k_{sb} \) in case of experiments with alternate bars conducted by Chatanantavet and Parker (2008) (Figure 14b). This suggests that bedrock roughness has a smaller effect on the alluvial cover in case of mixed alluvial–bedrock rivers with alternate bars. In such rivers, the bed slope may affect the alluvial cover fraction (Figure 14c) because bar formation process depends on the slope as well as the width-depth ratio (e.g., Kuroki and Kishi, 1984). The roughness models are adjusted to produce the experimental results with alternate bars by fine-tuning \( r_s \) and \( k_{br} \) values which must be determined by trial and error method. While this method can be applicable to laboratory-scale experiments, the model calibration is unfeasible for a large-scale channel or natural rivers. In general, the formation of alternate bars is barely reproduced with a one-dimensional model as introduced in this study. In the future, research to incorporate the effects of bars into a one-dimensional model, or analysis using a two-dimensional planar model (e.g., Nelson and Seminara, 2012; Inoue et al., 2016, 2017) is expected.

Also, in order to deploy models on field-scale, they must take into account bank roughness and its effects on shear stress and other hydraulic parameters.

<table>
<thead>
<tr>
<th>Type</th>
<th>Slope</th>
<th>( k_{br} ) (mm)</th>
<th>( \sigma_{er} ) (mm)</th>
<th>Adjusted ( k_{ra} ) for the macro-roughness model (mm)</th>
<th>Adjusted ( r_s ) for the surface-roughness model ( k_{br}=4 )</th>
<th>Calculated ( k_{sb} ) in the surface-roughness model (mm, ( k_{wc}=r_s r_o \sigma_r ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG</td>
<td>0.02</td>
<td>0.4</td>
<td>6.7</td>
<td>42.0</td>
<td>1.8</td>
<td>24.1</td>
</tr>
<tr>
<td>RA1</td>
<td>0.016</td>
<td>0.4</td>
<td>2.4</td>
<td>42.0</td>
<td>5.3</td>
<td>25.4</td>
</tr>
<tr>
<td></td>
<td>0.03</td>
<td>0.4</td>
<td>2.4</td>
<td>53.0</td>
<td>5.7</td>
<td>27.4</td>
</tr>
<tr>
<td>RA2</td>
<td>0.0115</td>
<td>3.5</td>
<td>2.7</td>
<td>32.0</td>
<td>2.5</td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>3.5</td>
<td>2.7</td>
<td>45.0</td>
<td>4.3</td>
<td>23.2</td>
</tr>
</tbody>
</table>

Another point of interest is, as noted earlier, surface-roughness model (Johnson, 2014) uses the predicted value of roughness whereas macro-roughness model (Inoue et al., 2014) uses the measured values and this difference may be thought to cause the better fitting results in case of macro-roughness model (Inoue et al., 2014). However, in Figure 14b, it can be seen that the adjusted \( k_{ra} \) that minimizes the RMSD of surface-roughness model (Johnson, 2014) is smaller than the observed \( k_{ra} \), especially in the section where \( k_{sb} \) is large. Therefore, RMSD becomes large when we substitute the observed \( k_{ra} \) into Johnson’s model.
The models could benefit with an accurate prediction method for hydraulic roughness or bedrock topographic roughness. It is easier to obtain the bedrock topographic roughness instead of obtaining hydraulic roughness data from the flow data. However, accurate prediction of hydraulic roughness will not only take into account the bedrock topographic roughness but also the arrangement of bed unevenness. For example, in Figure 2, the topographic roughness of Gravel 50 is higher than that of Gravel 30, but hydraulic roughness of Gravel 50 is lower than that of Gravel 30. This is since the hydrological roughness height does not only depend on the topographical roughness but also on the arrangement of the unevenness of the bed as explained in section 3.1. Accurate prediction of hydraulic roughness requires further work.
Figure 7.12: Comparison of our experimental results, linear model by Sklar and Dietrich (2004) and exponential model by Turowski et al. (2007).
Figure S11: Comparison of our experimental results with roughness models by Inoue et al. (2014) and Johnson (2014). The \( r_{br} \) for the surface roughness model and the \( \phi \) for the exponential model are adjusted to minimize RMSD of the alluvial cover (see Table 2). Note that there is no adjustment of \( k_{sb} \) in macro-roughness model.

Commented [r90]: maybe clarify that you did not adjust \( r_{br} \) for the macro roughness model.
Figure 5.4: Comparison of our experimental results with the probabilistic model proposed by Turowski and Hodge (2017).
Figure 10: Comparison of the experimental results (Chatanantavet and Parker, 2008) with the macro roughness model (Inoue et al., 2014) and the surface roughness model (Johnson, 2014). RA1, RA2 and LG represent the type of bedrock surface in the experiments conducted by Chatanantavet and Parker (2008); RA1 is Random Abrasion type 1, RA2 is Random Abrasion type 2 and LG is Longitudinal grooves, respectively. The $r_{bc}$ for the surface roughness model and the $k_{sb}$ for macro roughness model are adjusted to minimize RMSD of the alluvial cover (see Table 3).
Figure 11.4: (a) Comparison between adjusted and observed hydraulic roughness height of bedrock bed for our experiments. $k_{sb}$ for the macro roughness model is equal to the observed values because there was no need for adjustment. (b) Comparison between adjusted and observed hydraulic roughness height of bedrock bed for the experiments conducted by Chatanantavet and Parker (2008). (c) Sensitivity of adjusted $k_{sb}$ to bed-slope $S$ for experiments conducted by Chatanantavet and Parker (2008). Note: The black dotted line is the 1:1 line.
5 Summary

Here we provide a review of models and studies focused at discovering the interaction between alluvial cover and bed roughness. For evaluating the previous models, we conducted laboratory-scale experiments with multiple runs of varying bed roughness and sediment supply. The experimental results show that the change in alluvial cover rate is controlled by bedrock roughness to a great extent. When the bedrock hydraulic roughness is higher than the hydraulic roughness of the alluvial bed (i.e., clast-rough bedrock), the alluvial cover increases proportionately with the increase in sediment supply and then reaches an equilibrium state. However, in cases where bedrock roughness is lower than the roughness of the alluvial bed (i.e., clast-smooth bedrock), the deposition is insignificant unless sediment supply exceeds the transport capacity of the bedrock bed.

We have also implemented the previous models for alluvial cover, i.e., the linear model proposed by Sklar and Dietrich (2004), the exponential model by Turowski et al. (2007), the macro-roughness model by Inoue et al. (2014), the surface-roughness model by Johnson (2014) and the probabilistic model by Turowski and Hodge (2017) in order to predict the experimental results. The linear model and exponential model are inefficient for cases with a clast-smooth bedrock specifically, they cannot predict the rapid alluviation. The macro-roughness model (Inoue et al. 2014) and surface-roughness model (Johnson, 2014) can efficiently predict the rapid alluviation and hysteresis for clast-smooth bedrock as well as the proportionate increase in alluvial cover for clast-rough bedrock. Although the macro-roughness model (Inoue et al. 2014) was able to reproduce the observed alluvial cover ratio without adjusting the parameters, the surface-roughness model needs parameter adjustments. In particular, the macro-roughness model (Inoue et al. 2014) was able to reproduce the observed alluvial cover ratios without adjusting the parameters. The probabilistic model by Turowski and Hodge (2017) also needs parameter adjustments to make it sensitive to dynamic cover or rapid alluviation in clast-smooth bed, however, it does not reproduce the hysteresis.

Connecting model parameters with roughness parameters is an exciting challenge in the future. We also tested the macro-roughness model (Inoue et al. 2014) and surface-roughness model (Johnson, 2014) for their capability to predict the experimental results observed by Chatanantavet and Parker (2008), in which the bedrock surface has alluvial alternate bar formations. Both models required significant parameter adjustments to reproduce the alluvial cover fraction. The two models do not include the 2-D effects caused by variable alluvial deposition and formation of bars on bedrock. Although models that extended the roughness model into a plane two-dimensional planes (e.g., Nelson and Seminara, 2012; Inoue et al., 2016) will be able to capture the bar formation in a bedrock river, these models require long time for simulation and calculation time.

Building a simpler model that can predict alluvial cover fraction with bar formation represents another exciting challenge in the future which contributes to a better understanding of long-time evolution of natural bedrock channel.

Author Contribution: Both authors contributed equally to the manuscript.

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Notations:

\[ \alpha \] bedload transport coefficient
$h_r$  exposure function by Johnson (2014)
$d$  particle size (m)
$D$  water depth (m)
$g$  gravitational acceleration (9.81 m/s$^2$)
$k_x$  hydraulic roughness height (m)
$k_{sa}$  hydraulic roughness height of purely alluvial bed (m)
$k_{sb}$  hydraulic roughness height of purely bedrock bed (m)
$k_{sd}$  dimensionless alluvial roughness
$k$  Karman constant
$l$  flume length (m)
$L$  macro-roughness height of bedrock bed (m)
$M_0^*$  dimensionless sediment mass
$n_m$  Manning’s roughness coefficient (m$^{1/3}$/s)
$\eta_a$  average thickness of alluvial layer (m)
$P_c$  mean areal fraction of alluvial cover
$\varphi$  cover factor proposed by Turowski et al. (2007)
$q_{sa}$  sediment supply rate per unit width (m$^2$/s)
$q_{bc}$  transport capacity per unit width (m$^3$/s)
$q_{bcu}$  transport capacity per unit width for sediment moving on purely alluvial bed (m$^3$/s)
$q_{bcb}$  transport capacity per unit width for sediment moving on purely bedrock bed (m$^3$/s)
$Q$  water discharge (m$^3$/s)
$r_d$  scaling coefficient for $d$ and hydraulic roughness length
$r_{br}$  fitting parameter that scales bedrock roughness to $d$
$R$  specific gravity of sediment in water (1.68)
$S$  Bed slope
$S_e$  energy gradient
$\tau_*$  dimensionless shear stress
$\tau_{sc}$  dimensionless critical shear stress
$\tau_{scu}$  dimensionless critical shear stress for grains on purely alluvial bed
$\tau_{scb}$  dimensionless critical shear stress for grains on purely bedrock bed
$U$  depth averaged velocity (m/s)
$w$  flume width (m)
$\omega$  Exponent by Turowski and Hodge (2017)
$\sigma_{br}$  topographic roughness height of purely bedrock bed (m)
References


