The study site appears to be the bottom and adjacent toe slopes of a valley that was glaciated in late Quaternary time. I write "appears to be" because I do not know the glacial geology of this region but merely note that large moraines are present elsewhere in the valley in Google Earth imagery. The equations of this paper are applicable to hillslopes dominated by soil production and colluvial and overland flow sediment transport processes and where an approximate balance between the erosion caused by those processes and rock uplift has been achieved. Glacial erosion and/or deposition is not considered, yet these are possibly the dominant processes in this study site.

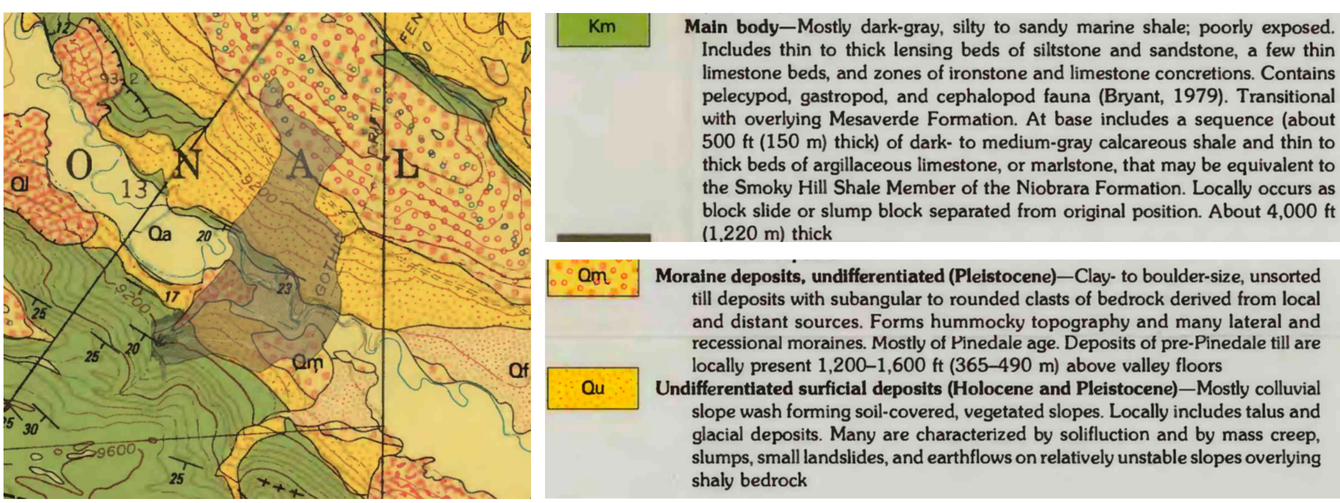
<u>*Our response:*</u>
We thank the reviewer for pointing this out. Glacial deposits are mapped throughout the watershed though they are rather isolated in that they have a limited spatial extent and are not characteristic for the depositional environment. In most boreholes that have been drilled there was no such layer. For the areas we analyzed, the amount of moraine deposits is indeed small (see the figure below). We add the following paragraph in Section 3:

> *"The last glacial advancing and retreating in the Upper Colorado River Basin is dated between 16.1 and 20.8 ka (Brugger, 2010). Glacial deposits are mapped at many locations throughout the watershed (Gaskill, 1991), but they are rather isolated and have a limited spatial extent, including in the area analysed in this study. In most boreholes that have been drilled in the studied area, the material beneath the soil layer is weathered shale (or saprolite) in the size roughly between 0.2 cm to 5 cm with light brown color (Fig. S3)."*

Reference:

Brugger, K. A.: Climate in the Southern sawatch range and Elk Mountains, Colorado, U.S.A., during the last glacial maximum: Inferences using a simple degree-day model, Arctic, Antarct. Alp. Res., 42(2), 164–178, doi:10.1657/1938-4246-42.2.164, 2010.

Gaskill, D. L.: Geologic map of the Gothic quadrangle, Gunnison County, Colorado., doi: 10.3133/gq1689, 1991.



Indirect measurements of soil thickness can be useful in augmenting measurement of soil thickness in excavated soil pits, but it is inadvisable to use such methods in isolation. Augering and penetration methods are generally considered to be minimum values in rocky soils because the auger or penetrometer can be stopped by gravel. The study would be strengthened by reporting observations from soil pits at representative locations so that the nature of the soil-saprolite contact and the presence/absence of any glacial tills in the study site can be ascertained.

<u>*Our response:*</u>

We acknowledge that measuring soil thickness at numerous locations is not simple and error-free. At this study site, for both auger and CPT measurements, we obtained measurements more than once at sampling sites where the results were suspicious. In addition, at sampling sites where the auger or CPT were stopped at a relatively shallow depth, we measured multiple times at nearby spots within about 1m diameter. Further, we tested the accuracy of the CPT measurement and found that the CPT i) shows largest change in resistance when entering weathered bedrock, and ii) is stopped very sharply only in the presence of a boulder, in which case the resistance is so strong that the measurement was deemed suspicious and repeated nearby. Because the CPT may not clearly identify the potential presence of moraine deposits, we also visually inspected the soil and saprolite materials extracted by the auger (see field images). We believe our measurement is relatively accurate and efficient, which provides a consistent assessment of soil thickness over space in comparison to other existing methods. We include the following images in the supplementary information and revise the last paragraph in Section 3 in the main text:

> *"At this study site, we used and compared both auger and CPT measurements to estimate soil thickness. The CPT measurements provide a vertical profile of soil resistance for a soil column. We tested the accuracy of the CPT measurement and found that the CPT i) shows largest change in resistance when entering weathered bedrock, and ii) can be stopped very sharply only in the presence of a boulder, in which case the resistance is so strong that the measurement was deemed suspicious and repeated nearby. Because the CPT may not clearly identify the potential presence of moraine deposits, we also visually inspected the soil and saprolite materials extracted by the auger. From the auger, the transition zone from soil to the saprolite or bedrock is based on the material size and color of retrieved samples (Fig. S3). When the auger reaches the bedrock shale, it cannot penetrate easily. We believe our measurement is relatively accurate and efficient, which provides a consistent assessment of soil thickness over space in comparison to other existing methods. Figures 1b-1e show the relationship between soil thickness estimated from auger, CPT, and local elevation. There is a high variation in soil thickness from local to hillslope scales. To fully take advantage of all the sampling data, we used auger data to fit values for CPT (Fig. S4) since more locations are sampled from auger drilling than CPT. The CPT and auger data are mostly in agreement. For soil thicknesses less than ~0.5 m, the CPT data are slightly higher than the auger, and for soil thickness larger than ~0.5 m, the CPT data are slightly lower."*



The authors state that the study site is in a soil-thickness steady state condition but Fig. S2 shows that this not to be the case. That figure shows the average soil thickness in a simulation increasingly steadily at the time (20 ky) when a steady state condition has purportedly been achieved. In stating that this figure demonstrates soil thickness steady state, I presume that the authors are referring to the fact that the rate of soil production is in decline. However, the rate of soil production is still far from zero and the rate of soil production for the simple case of a horizontal surface will result in a similar decline even as the soil thickness approaches infinity given sufficient time. Moreover, the average soil thickness should not be used to infer steady state because decreasing soil thickness in one area may be balanced out by increasing soil thickness elsewhere.

Our response:

We thank the reviewer's comment on this issue. We agree that the average soil thickness is not appropriate to infer the steady state condition. At depositional areas, the soil gradually accumulates on the land surface, and meanwhile, the soil formatting slows at the bottom, therefore, the soil thickness is supposed to gradually increase (Dietrich et al, 1995). Due to the complexity of soil deposits, such as expansion or compression of soils, we consider that using an empirical relationship is appropriate for the soil thickness at depositional areas. At erosional sites, the erosion from the land surface can be balanced out by the soil formation from the bottom. Therefore, we only apply the mass conservation method in erosional 2-D grid cells (see answer to the next question below). We also extend the simulation to a longer period than in the previous simulation to see if the model can reach steady state. Even though it takes almost 25 ka, which is longer than the last great glacial history that ends about 16-20 ka; we believe it is still acceptable because (1) the elevation data that we input in the model is the current lidar DEM rather than the historical topography, and (2) the parameters are from field calibration which follows ambient soil thickness rather than the one when the glacial period ended. The Figure S5 has been updated as:

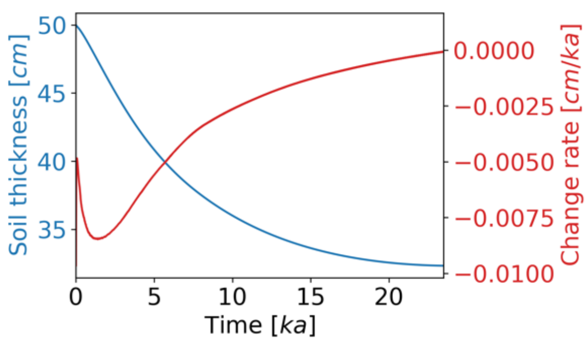


Figure S5: Spatial mean values of erosional sites of soil thickness evolution over time. The initial soil thickness is 0.5 m, time step is 1 year, and the initial elevation is the current DEM data. The boundary condition is Neumann boundary condition, the surface transport fluxes around the edge is zero. The time step is 1 yr, and the diffusion coefficient is m^2/yr for the north-facing hillslope and m^2/yr for the south-facing hillslope.

The revised sentences in Section 4.3 are:

"*At erosional sites, the erosion from the land surface can be balanced out by the soil formation from the bottom, therefore, the soil thickness may reach a steady state condition. By coupling soil thickness with landscape evolution, we found that the soil thickness reaches a dynamic steady-state after approximately 25 kyr at this study site (Fig. S5), which is consistent with other studies in mountainous areas (Dietrich et al., 1995; Vanwalleghem et al., 2013). This implies that the current soil thickness in the East River Watershed may have already reached a steady-state condition since the last glacial legacy. Here, we only focus on the steady state condition at erosional sites because they are where we apply the mass conservation equation. For depositional sites, the soil gradually accumulates on the land surface;*

and meanwhile, the soil weathers slowly at the bottom; therefore, the soil thickness is supposed to continuously increase (Dietrich et al, 1995). Due to the complexity of soil depositional environments, such as expansion or compression of soils, we consider that using an empirical relationship is appropriate for the soil thickness at depositional areas. "

Dietrich, W. E., Reiss, R., Hsu, M. and Montgomery, D. R.: A process-based model for colluvial soil depth and shallow landsliding using digital elevation data, Hydrol. Process., 9, 383–400, 1995.

Vanwalleghem, T., Stockmann, U., Minasny, B. and Mcbratney, A. B.: A quantitative model for integrating landscape evolution and soil formation, J. Geophys. Res. Earth Surf., 118(November 2011), 331–347, doi:10.1029/2011JF002296, 2013.

The authors state (line 35) that the mass conservation method can return no finite soil thickness but this has not been demonstrated. Without such a demonstration application of the Patton method seems premature. If the model is applicable to the study site and correctly parameterized it should return a finite soil thickness value. Perhaps the model was unable to achieve such a steady state because the actual study site is not in steady state (e.g. recently glaciated) or because the authors used an inappropriate method for computing discharge by overland flow (d8 or steepest descent, as shown in Fig 1.)). In any case the paper would be strengthened by applying the model to the study site with appropriate assumptions first (see Pelletier et al., JGR, 2011 for examples of soil production and transport modeling without a steady state assumption) and then using the Patton method if and only if the model can be demonstrated to produce no adequate solution for any reasonable set of parameter values.

<u>Our response:</u>

We agree that soil production and transport rate modeling do not require a steady-state assumption. For the mass conservation equation, if we know the initial value of soil thickness and proper boundary conditions, then the steady-state assumption is unnecessary because the mass conservation equation is a partial different equation that can be solved numerically. The steady-state assumption is only needed for the soil thickness estimation in the assumption that the regolith production balances the physical erosion, as used in other studies (Pelletier and Rasmussen, 2009; Pelletier et al, 2011; Dietrich et al, 1995).

The equation to calculate soil thickness, h:

$$\frac{\rho_r}{\rho_s}\frac{1}{cos\theta_s}P_o e^{-hcos\theta/h_o} - \nabla\cdot q_d - \nabla\cdot q_s = 0 \qquad (*)$$

can be rearranged as:

$$\frac{\rho_r}{\rho_s}\frac{1}{cos\theta_s}P_o e^{-hcos\theta/h_o} = \nabla\cdot q_d + \nabla\cdot q_s \qquad (**)$$

Because the soil production rate is always larger than or equal to zero under any conditions, expressed as $\frac{\rho_r}{\rho_s}\frac{1}{cos\theta_s}P_o e^{-hcos\theta/h_o} \geq 0$,

from here, the only way for the soil thickness 'h' to have a real number is that $\nabla\cdot q_d + \nabla\cdot q_s > 0$.

In a landscape evolution model, the values calculated by $\nabla\cdot q_d + \nabla\cdot q_s$ on each 2-D grid cell can be either positive, meaning erosional process; or negative, meaning a depositional process. In depositional cells where $\nabla \cdot q_d + \nabla \cdot q_s < 0$, then there is no real number for 'h' in equation (**). For this reason, we apply Patton's method on cells whichever undergoes depositional process.

We compute our overland flow following the shallow water overland flow equation as shown in Equation 4: $\frac{\partial H_w}{\partial t} = \frac{\partial}{\partial x}\left(D_h \frac{\partial H_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_h \frac{\partial H_w}{\partial y}\right)$. The details of this equation can be found in Lal (1998) and Yan et al (2019).

We deleted "... *can return no finite soil thickness*..." and revised the sentence in the updated manuscript:

> "*…there is no real number for soil thickness if* $\nabla \cdot q_d + \nabla \cdot q_s < 0$..."

We include the following in Section 2.1.4 in the revised manuscript:

> *" **2.1.4 Combine the mass conservation method with the empirical method***
> *For the mass conservation equation, the steady-state assumption is needed for the soil thickness estimation in the assumption that the regolith production balances the physical erosion, as used in other studies (Pelletier and Rasmussen, 2009; Pelletier et al, 2011; Dietrich et al, 1995). Therefore, the mass conservation method with the steady-state assumption can be used to solve the soil thickness at erosional sites but has limitations at depositional sites (Eqn. 7, Dietrich et al, 1995). Patton's method is better adapted to depositional sites. However, it can provide negative values of soil thickness at zones with high negative-curvature values where erosion is the main process. Also, in a low gradient and divergent area, if the soil transport rate is assumed as a linear relationship with curvature (i.e.,* $\nabla \cdot q_d = -K_d \nabla \cdot \nabla\eta$, *and* $\nabla \cdot q_s = 0.0$*), then Equation 7 can be further simplified in that the soil thickness has a natural logarithm relationship with curvature (i.e.,* $h = -m * ln(\nabla \cdot \nabla\eta) + C$, *where* m *and* C *are constant parameters that can be calibrated from field sampling data). However, Patton's method (Patton et al., 2018) always assumes a linear relation. This may be why his empirical relationship does not work very well in the erosional areas. However, this can be compensated for by using the mass-conservation method."*

Reference:

Lal, A. M. W.: Performance comparison of overland flow algorithms, J. Hydraul. Eng., 124(4), 342–349, doi:10.1061/(ASCE)0733-9429(1998)124:4(342), 1998.

Dietrich, W. E., Reiss, R., Hsu, M. and Montgomery, D. R.: A process-based model for colluvial soil depth and shallow landsliding using digital elevation data, Hydrol. Process., 9, 383–400, 1995.

Pelletier, J. D. and Rasmussen, C.: Geomorphically based predictive mapping of soil thickness in upland watersheds, Water Resour. Res., 45(9), 1–15, doi:10.1029/2008WR007319, 2009.

Pelletier, J. D., et al. (2011), Calibration and testing of upland hillslope evolution models in a dated landscape: Banco Bonito, New Mexico, J. Geophys. Res., 116, F04004, doi:10.1029/2011JF001976.

Yan, Q., Le, P. V. V, Woo, D. K., Hou, T., Filley, T. and Kumar, P.: Three-Dimensional Modeling of the Coevolution of Landscape and Soil Organic Carbon, Water Resour. Res., 55(2), 1218–1241, doi:10.1029/2018WR023634, 2019.

Some examples of unclear or incorrect methodology, incorrect units, parameter values that are very different from the literature, etc.:

<u>*Our response:*</u>

We appreciate the reviewer's suggestion and address the questions in the following one-by-one.

1) please state how eqn. (7) was solved to determine h; this is the heart of the modeling and it is difficult to evaluate the paper without any mention of the solution method,

<u>*Our response:*</u>

In Equation 7:

$$\frac{\rho_r}{\rho_s}\frac{1}{cos\theta_s}P_o e^{-hcos\theta/h_o} - \nabla \cdot q_d - \nabla \cdot q_s = 0$$

By rearranging equation 7, we get

$$h = -\frac{h_o}{cos\, cos\, \theta} ln\, ln \left[\frac{cos\theta_s(\nabla \cdot q_d + \nabla \cdot q_s)}{P_o}\frac{\rho_s}{\rho_r}\right]$$

From equation 3a and 3b:

$$\nabla \cdot q_s = \frac{q_{s,out} - \Sigma q_{s,in}}{d_s}$$

$$q_s = K_s H_w^{\alpha} S^{\beta}$$

From equation 2:

$$q_d = -\frac{K_d \nabla\eta}{1 - \left(\frac{\nabla\eta}{S_c}\right)^2}$$

$$\nabla \cdot q_d = -\nabla \cdot \frac{K_d \nabla\eta}{1 - \left(\frac{\nabla\eta}{S_c}\right)^2}$$

In our method, $\nabla \cdot q_d$ and $\nabla \cdot q_s$ are independent of soil thickness h. We add the following after equation 7:

"*The soil thickness value h can be directly solved here because $\nabla \cdot q_d$ and $\nabla \cdot q_s$ are independent of soil thickness*".

2) the drainage map in Fig. S1 suggests that D8 or steepest descent is used to determine sediment flux by overland flow; this method is incapable of modeling discharge by overland flow; D-infinity or another multiple flow direction routing algorithm must be used,

<u>*Our response:*</u>

Regarding the overland flow, as explained above, we use the diffusive overland flow equation ($\frac{\partial H_w}{\partial t} = \frac{\partial}{\partial x}\left(D_h \frac{\partial H_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_h \frac{\partial H_w}{\partial y}\right)$), which is Equation 4 in the manuscript. We only use ArcGIS (the D8 algorithm) for the purpose to delineate the boundary of our study site in Fig S1.

3) the h_0 value of 0.1-0.125 m is much smaller than other studies; h_0 typically ranges from 0.2-0.5 m, see papers by Heimsath et al.;

<u>*Our response:*</u>

We appreciate that the reviewer pointed this out. The values were typed wrongly. It should have been 0.2 and 0.18 for the north-facing and south-facing hillslope, respectively. The values have been updated in Table 1.

We agree that among a series of Heimsath's papers and Dietrich et al (1995), h_o ranges from 0.2-0.5 in the form of $e^{(-\frac{h}{h_o})}$. One possible reason that our value is slightly smaller than this range is that we defined h as the distance along the norm direction to the land surface, which give $e^{-hcos\theta/h_o}$, where θ is the slope of the land surface in degree (Pelletier and Rasmussen, 2009). In our study site, $cos\theta$ ranges from 0.77-1.0; and among the sampling sites, $cos\theta$ ranges from 0.85-0.98. The value of h_o is calibrated based on the sampling data. Specifically, $\frac{cos\theta}{h_o} = \frac{1}{h_o/cos\theta}$, and $h_o/cos\theta$ gives the range between 0.18/0.77 (=0.20) to 0.2/0.98 (=0.23).

References:

Heimsath, A. M., Dietrich, W. E., Nishiizumi, K. and Finkel, R. C.: The soil production function and landscape equilibrium, Nature, 388(July), 358–361, 1997.

Heimsath, A. M., Chappell, J., Dietrich, W. E., Nishiizumi, K. and Finkel, R. C.: Soil production on a retreating escarpment in southeastern Australia, Geology, 28(9), 787–790, doi:10.1130/0091-7613(2000)28<787:SPOARE>2.0.CO;2, 2000.

Heimsath, A. M., Furbish, D. J. and Dietrich, W. E.: The illusion of diffusion: Field evidence for depth-dependent sediment transport, Geology, 33(12), 949–952, doi:10.1130/G21868.1, 2005.

Dietrich, W. E., Reiss, R., Hsu, M. and Montgomery, D. R.: A process-based model for colluvial soil depth and shallow landsliding using digital elevation data, Hydrol. Process., 9, 383–400, 1995.

Pelletier, J. D. and Rasmussen, C.: Geomorphically based predictive mapping of soil thickness in upland watersheds, Water Resour. Res., 45(9), 1–15, doi:10.1029/2008WR007319, 2009.

4) what is the relation of Bp to P_0? only P_0 appears in the equations yet only Bp appears in Fig. 3; if these are related by the bulk density ratio, as I would have thought, why is one larger on the n-facing side while the other is larger on the s-facing side?

Our response:

We appreciate that the reviewer pointed this mistake. In the older version, $B_p = \frac{\rho_r}{\rho_s} P_o$, where ρ_r and ρ_s are rock and soil bulk density, respectively. B_p is used for the purpose of simplicity for calculation and thus should not be considered as an extra parameter in this work. In Figure 3, it is actually P_o not B_p. In the revised manuscript, we delete B_p from Table 1, and revise Figure 3 accordingly.

5) values of alpha and beta are not reported,

Our response:

We thank the reviewer for pointing this mistake. The values and references for the beta have been added in the revised manuscript. Because beta and alpha share the same value, we use alpha only. The revised equation and texts are shown below:

$$q_s = K_s(H_w S)^{\beta} \tag{3b}$$

where K_s is the soil erodibility coefficient $[L^{2-\alpha}/T]$, S is the slope along flow direction [-], and β is an empirical constant for surface erosion, where $\beta = 1.68$ (Papanicolaou et al., 2015, Yan et al., 2019)."

Yan, Q., Le, P. V. V., Woo, D. K., Hou, T., Filley, T. R. and Kumar, P.: 3-D modeling of the co-evolution of landscape and soil organic carbon, Water Resour. Res., doi:10.1029/2018WR023634, 2019.

Papanicolaou, A. N., Wacha, K. M., Abban, B. K., Wilson, C. G., Hatfield, J. L., Stanier, C. O. and Filley, T. R.: From soilscapes to landscapes: A landscape-oriented approach to simulate soil organic carbon dynamics in intensively managed landscapes, J. Geophys. Res. Biogeosciences, 120, 979–988, doi:10.1002/2014JG002802, 2015.

6) the parameter a has incorrect units (must be L^2 since the product of a and curvature (units of 1/L) results in units of L).

Our response:

We agree and thank the reviewer's comment. We have revised the unit as "m^2" in Table 1.

7) In Fig. 7, what is the meaning of negative soil transport rate?

Our response:

The negative value means erosion, the higher value means faster rate. We have revised the caption in the updated manuscript and shown below:

> "Positive values of transport rate represent deposition, and negative values represent erosion."