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Computing water flow through complex landscapes, Part 3: Fill-Spill-Merge: Flow routing in depression hierarchies

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Abstract. Depressions—inwardly-draining regions—are common to many landscapes. When there is sufficient moisture, depressions take the form of lakes and wetlands; otherwise, they may be dry. Hydrological flow models used in geomorphology, hydrology, planetary science, soil and water conservation, and other fields often eliminate depressions through filling or breaching; however, this can produce unrealistic results. Models that retain depressions, on the other hand, are often undesirably expensive to run. In previous work we began to address this by developing a depression hierarchy data structure to capture the full topographic complexity of depressions in a region. Here, we extend this work by presenting a Fill-Spill-Merge algorithm that utilizes our depression hierarchy to rapidly process and distribute runoff. Runoff fills depressions, which then overflow and spill into their neighbors. If both a depression and its neighbor fill, they merge. We provide a detailed explanation of the algorithm as well as results from two sample study areas. In these case studies, the algorithm runs 90–2,600× faster (with a 2,000–63,000× reduction in compute time) than the commonly-used Jacobi iteration and produces a more accurate output. Complete, well-commented, open-source code is available on Github and Zenodo.

1 Introduction

Depressions (see Lindsay (2015) for a typology) are inwardly-draining regions of a DEM that lack any outlet to an ocean or other designated base elevation. Depressions occur naturally, and can be formed by glacial erosion and/or deposition (Breckenridge and Johnson, 2009), compressional and/or extensional tectonics (Reheis, 1999; Hilley and Strecker, 2005), and cratering Cabrol and Grin (1999). They often host lakes and wetlands by retaining water locally. Depressions may themselves contain depressions. Such regions confound algorithms for geomorphological and terrain analysis, as well as those for hydrological modeling, because many such algorithms simply route water down topographic slope following the local gradient: depressions neither fill with water, nor drain.

Many hydrological models deal with the complexity of depressions by removing them. This can be done either by filling the depressions with earth so that they form a flat region of landscape (e.g. Jenson and Domingue (1988); Martz and Jong (1988)); breaching (Martz and Garbrecht, 1998) or carving them (Soille et al., 2003) so that water flows from their lowest point through

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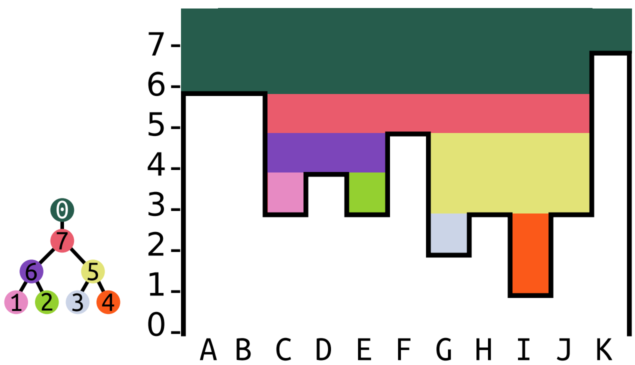

Figure 1. A single subtree of a depression hierarchy and the depression it represents. Depressions 1–4 are leaf depressions. Depression 6 is a parent depression (also termed a meta-depression) that contains depressions 1 and 2. Water from the plateau on the left above cells A and B might *fill* Depression 1 (cell C), causing it to *spill* into Depression 2 (cell E). Only when both depressions are full do they *merge* and begin filling Depression 6 (cells C, D, and E). Modified from Barnes et al. (2020).

the carved channel and onward to downstream regions; or some combination of these (Lindsay and Creed, 2005b; Schwanghart and Scherler, 2017; Soille, 2004; Lindsay, 2016). This approach is justified for situations in which spatiotemporal aspects of the analysis allow depressions to be ignored or for cases in which all depressions can be considered to be data errors (Lindsay and Creed, 2005a). Historically, many DEMs were constructed from sparse data, and small data errors produced depressions, especially in flat areas (O'Callaghan and Mark, 1984). Such an assumption is no longer justified, as improved and increasingly high-resolution data have become available (Li et al., 2011). Even coarse-resolution data are capable of resolving real-world depressions (e.g. Riddick et al., 2018; Wickert, 2016). With this in mind, new approaches are beginning to be examined, particularly in post-glacial landscapes where depressions have a significant impact on local hydrology (e.g. Lai and Anders (2018)) and therefore cannot be ignored during modeling.

FlowFill (Callaghan and Wickert, 2019) began to combat this problem by routing water across landscapes in a way that conserved water volume, creating flow-routing surfaces that could still contain real depressions. Under reasonable runoff conditions, their results show landscapes that still contain depressions and disrupted flow routes. The FlowFill method iteratively routes water from higher to lower terrain. As depressions fill, they pose an extreme challenge to such a method: since water seeks a level surface, the surface of a filled depression must eventually become flat and any fluid flowing onto the surface diffuses across it. Even for moderately-sized surfaces it can take many iterations for a solver to reach steady state; we provide a theoretical analysis of this in Section 4. Runtimes for FlowFill ranged from seconds to days: large datasets quickly became unwieldy. Of those examples tested by Callaghan and Wickert (2019), the slowest was a dataset of 4,176,000 cells which took approximately 33 hours for FlowFill to process. In contrast, the Fill-Spill-Merge algorithm presented here fills a similarly-sized dataset in 8.7 s.

To achieve this, we developed a data structure—the *depression hierarchy*—which represents the topologic and geographic structure of depressions. In an accompanying paper, we provide details concerning the depression hierarchy and its construc-

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tion (Barnes et al., 2020). In this paper, we explain how the depression hierarchy can be leveraged to accelerate hydrological models using a paradigm we call *Fill-Spill-Merge*.

2 Using The Depression Hierarchy

Depressions can themselves contain depressions, as shown in Figure 1. A depression hierarchy (DH) is a forest of binary trees, as shown in Figure 2a, that represents the relationships between depressions (Figure 2a–d). Each node in the DH represents a depression. Nodes higher in the DH are depressions that themselves contain depressions; we term these *meta-depressions*. A node in the DH can have several classifications:

- **Parent**: A node, such as #10 in Figure 2, that represents a meta-depression, and whose topological descendants therefore also form depressions.
- **Child**: A depression, such as both #10 and #1 in Figure 2, that geographically and topologically exists within the meta-depression formed by its parent.
- **Leaf**: A depression, such as #1 and #2 in Figure 2, that has no children. The leaves of the binary trees represent the smallest, most deeply-nested depressions. If a landscape were initially devoid of water, then water flowing down slopes would begin to collect in some subset of these leaf depressions before it would begin to fill their parent depressions.
- **Root**: A depression, such as #0 in Figure 2, that has no parent. This term may also refer to any node that is used as the starting point for a traversal that only considers the node and its descendants.
- **Descendant**: A child of a given parent, or the child of a child of that parent, and so on. In Figure 2, #1, #2, #3, and #10 are all descendants of #12.
- **Sibling**: Every node has either no children (leaf nodes) or two children. Nodes which share a parent are siblings. In Figure 2, #1 and #2 are siblings, as are #4 and #5.

As depressions fill, their water surfaces eventually reach a *spill elevation* (Figure 2e) at which they overflow into neighboring depressions. During this spilling, water flows from a depression into a geographically neighboring leaf depression, topologically connected by a *geolink*. The spill elevations in Figure 1 are the highest points of each band of color.

Each node in the DH is associated with several properties:

- **Depression volume**: This is the *total* volume of water that the depression, including all of its descendants, can contain before spilling over.
- **Water volume**: This is the *total* volume of water *actually being stored* in the depression. A parent depression will have a non-zero water volume only if all of its children are completely full and the parent itself contains some additional volume of water. In this case, the water volume will be the sum of the water volumes of the children and the additional

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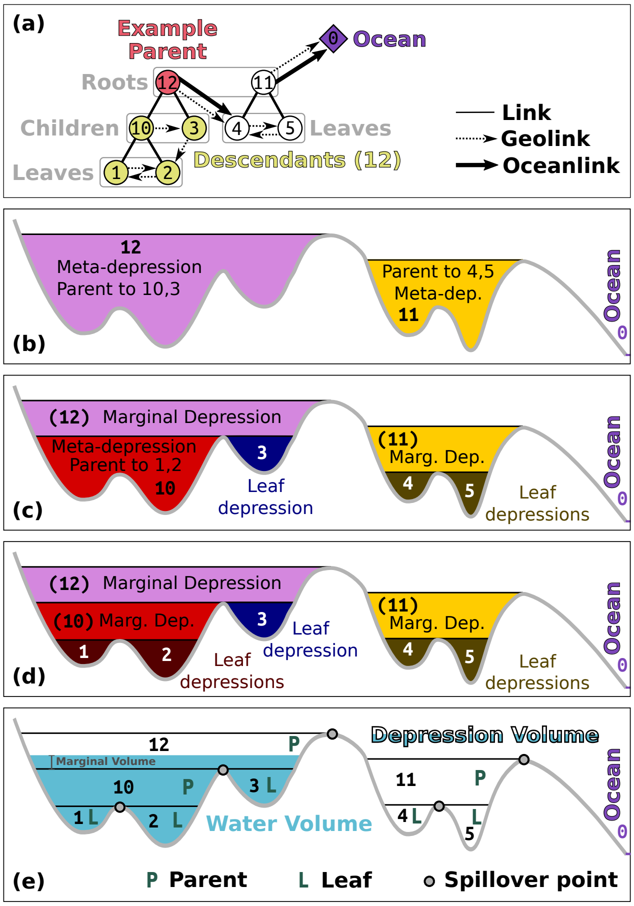

Figure 2. Terminology for the depression hierarchy and water flow through it. The depression hierarchy shown here is drawn from the left hand side of Figure 1 from the companion paper by Barnes et al. (2020). **(a)** Topology. A *parent* and its *descendants* are associated with depressions (b–d). Direct descendants are called *children*. *Leaves* are the terminal members of the depression hierarchy; they have no children and represent simple depressions (i.e., those that are not meta-depressions). Members of a single *binary tree* are joined in their hierarchy through *links*; directional links that represent water-spillover directions between geospatially adjacent depressions are called *geolinks*. Flow from one binary tree into another and towards the ocean follows the *oceanlinks*. Though only one binary tree is shown, the ocean may be the parent to an arbitrarily large *forest* of binary trees. **(b)** Parents in the hierarchy form *meta-depressions* — depressions that encompass other depressions. **(c)** These meta-depressions contain *leaf depressions* — depressions that themselves contain no depressions. These are associated with leaves in the depression hierarchy. Meta-depression 12 also contains another meta-depression, 10. The regions of Depressions 11 and 12 that lie above their child depressions are termed "marginal depressions". **(d)** Meta-depression 10 contains leaf depressions 1 and 2. **(e)** Water flow in the depression hierarchy. Water first fills *leaf depressions* before flooding into neighboring *depressions*. Once a depression and its neighbor are completely filled, their *parent* begins to flood. The *depression volume* is the full geometric volume of the depression. The *water volume*, nautrally, is the volume of water within a given depression. The *marginal volume* is the volume of water partially filling the top-level meta-depression; appropriately spreading this water across the landscape is the topic of Section 3.3.

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margin of water contained within the parent (i.e., the "marginal volume" indicated on Figure 2). Parents whose children are not all filled with water will have a water volume equal to zero. In this way, we can use this property to determine which portions of the DH are fully or partially filled, and which are the highest water-containing nodes in any of the binary trees.

- **Geolink**: When a depression spills, its water passes into the subtree rooted by its sibling. However, in a full model of flow, the water would move downslope from the spill cell into whichever leaf depression of the sibling is geographically proximal to the spill cell. *Geolinks* are pointers from depressions higher in the DH to the leaf depressions that receive their water if they overflow. These are the dashed lines shown in Figure 2. Geolinks are similar to the connections used in a threaded binary tree (Fenner and Loizou, 1984).

- **Ocean link**: Depressions high in the mountains may overflow down escarpments to depressions far below. In this case, the depressions do not overflow into each other: the relationship is one-way. There can be multiple such escarpments, so this can happen multiple times. In such cases, each group of depressions forms a proper binary tree. However, the root of one of the trees has both an *ocean link* and a geolink to a leaf node of the downstream binary tree. In Figure 2, both #11 and #12 are the root nodes of a set of nested depressions. #12 has an ocean link (heavy arrow) to #4, one of the leaf depressions of #11. #12 also has a geolink (dotted arrow) to #4. #11 itself has an ocean link and a geolink to the ocean. In many of the algorithms discussed below, ocean-linked nodes are processed similarly to children; however, information is usually not passed across ocean links. Oceanlinks are used solely for guiding traversals of the depression hierarchy whereas water is passed through geolinks.

3 The Algorithm

The Fill-Spill-Merge algorithm consists of several steps, outlined here, depicted in Figures 3 and 4, and shown in flowchart form in Figure 5. First (Section 3.1), surface water needs to move downhill, either to the ocean (i.e., a designated sink region or the map edge) or to collect in pit cells – the deepest points within leaf depressions. This operation takes place across all the cells of the DEM. Second (Section 3.2), water is redistributed across the depression hierarchy such that any depressions that have filled sufficiently must spill over into neighboring depressions and, if both depressions are full, flood their parent to merge into a single, larger body of water within a meta-depression. This operation is done without explicitly considering the cells of the DEM, which makes it very fast. Third and finally (Section 3.3), the water within the depression hierarchy is translated into an extent and depth of flooding across the topographic surface (DEM).

Computing a depression hierarchy (Barnes et al., 2020) is a necessary precursor to running Fill-Spill-Merge. The specific outputs from the depression hierarchy that are used in the Fill-Spill-Merge algorithm are:

- *DH*: the depression hierarchy itself.

- *Flowdirs*: a matrix of flow directions, indicating which of a cell's neighbors receives its flow. Because Priority-Flood (Barnes et al., 2014) is used to generate the depression hierarchy, flat areas are automatically resolved.

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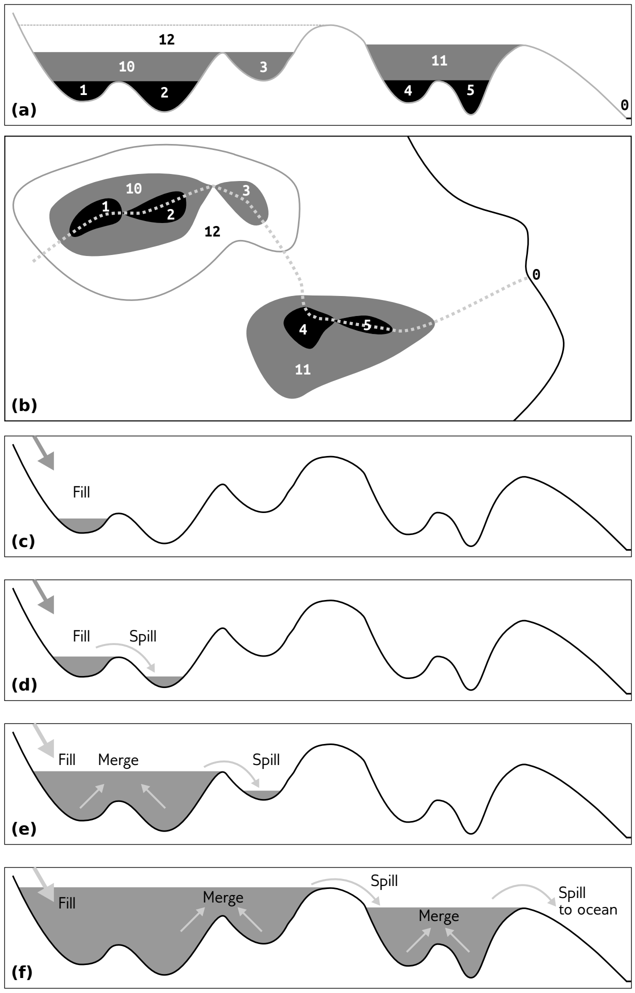

Figure 3. Fill-Spill-Merge process. Water moves through topographic depressions by filling them, spilling over sills, and merging to form meta-depressions. **(a)** Topographic cross section with labeled leaf depressions and their parents, following the left-hand side of the depression hierarchy in Figure 2. “0” represents the ocean; other numbers represent leaves and parents that together form depressions and meta-depressions. **(b)** Map showing this depression structure; the cross-section in (a) follows the dotted gray line. **(c)** A water source to the left begins to fill Depression 1. **(d)** Continued water input causes Depression 1 to overflow and spill into Depression 2. **(e)** Depression 2 fills, causing Depressions 1 and 2 to fill their parent (10) and merge to form a metadepression. This metadepression overflows into Depression 3. **(f)** Depression 3 fills and merges with Meta-Depression 10 (1 and 2 being implied members based on their position in the hierarchy) to flood their parent, 12. After Meta-Depression 12 overspills, it enters Depression 4, which then fills and spills into Depression 5. After Depression 5 floods, its waters join with those from Depression 4 to fill Meta-Depression 11, which then spills to the ocean. Figures 4 and 5 describe the algorithm in more specific detail.

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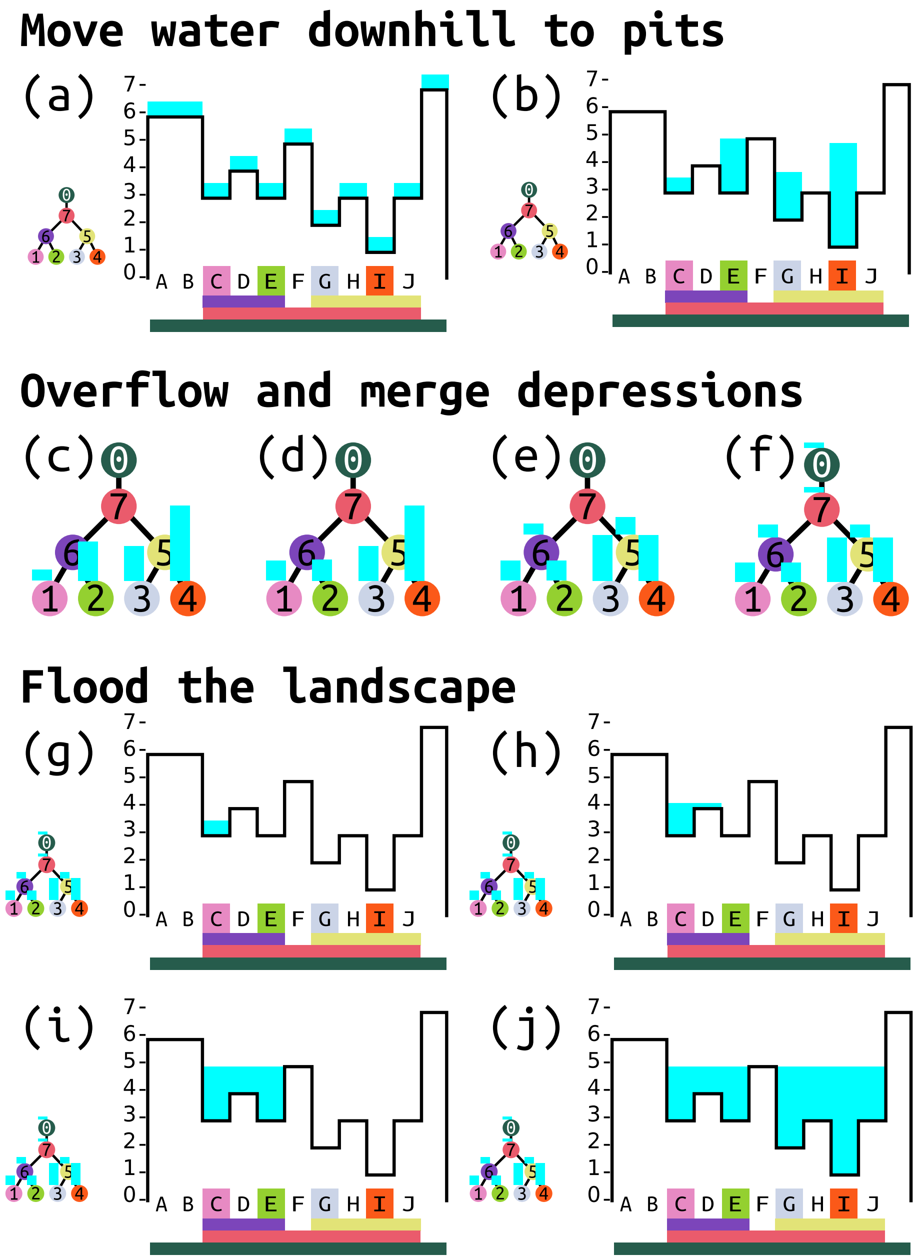

Figure 4. Visual Overview of the Algorithm. In this figure the heights of the water bars are non-additive: only the changes between panels are important. The algorithm consists of three major stages (Figure 5). From its initial distribution (A), water is moved downhill into pit cells (B, §3.1). Water is then moved within the depression hierarchy (C–F, §3.2): water in depressions with insufficient volume overflows first into their sibling depressions (D) and then – if the sibling depression becomes filled – passes to their parents (E, F). Any leftover water overflows into the ocean (F) and is forgotten. Depressions to be flooded are then identified and flooded (§3.3) starting from an arbitrarily-chosen pit cell (G–J).

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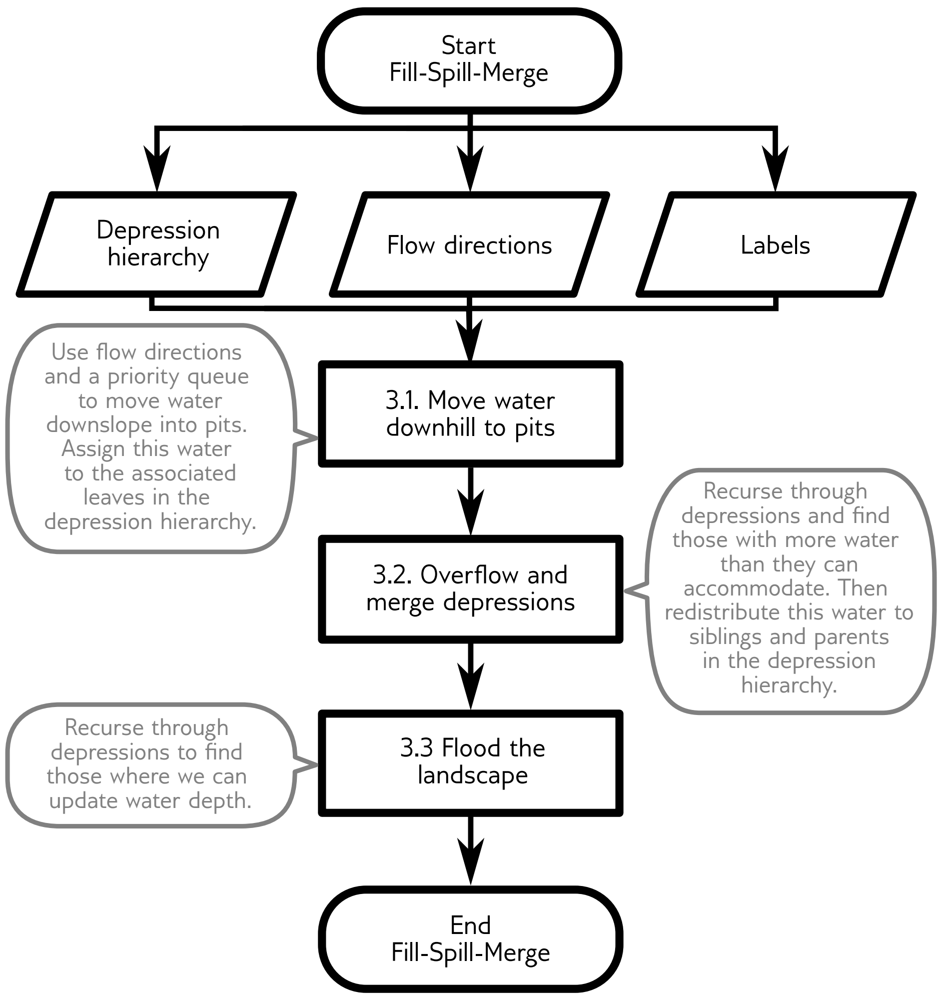

Figure 5. Flowchart showing the main steps taken by the algorithm. These steps are described in more detail in §3.1 to §3.3.

– *Labels*: a matrix indicating the leaf depression to which each cell belongs.

By routing water according to the DH, we significantly accelerate the compute speed and ensure that the full network of depressions is a topologically correct directed tree. Each of the following subsections details one of the numbered steps along the central path of the flowchart shown in Figure 5.

3.1 Move Water Downhill to Pits

We route water in a similar way to standard flow-accumulation algorithms (Mark, 1988; Wallis et al., 2009; Barnes, 2017), but for completeness summarize our approach here. Flow directions for each cell have already been identified by the DH. Each cell calculates how many of its neighbors flow into it. We call this value the cell's dependency count, as it describes the number of immediate upstream cells whose flow accumulation must be resolved before flow accumulation at the given cell can be computed. Local maxima in the DEM are identified as those cells that receive no flow from any neighbor. These local maxima are placed in a queue. Cells are then popped (i.e., noted while being removed) from this queue. The cells determine how much flow they generate locally (perhaps referring to matrix of rainfall values) and add this to their flow accumulation value. They then add their flow accumulation to their downstream neighbor's and set their own flow accumulation value to zero. The neighbor's dependency count is then decremented. If the neighbor's dependency count has reached zero during this step, it is added to the end of the queue. This process of accumulating flow, passing it downstream, decrementing the dependency count, and adding cells to the queue continues until the queue is empty, at which point every cell on the map has been visited and any water has been moved downslope. Braun and Willett (2013) present an alternative formulation based on a depth-first traversal, but Barnes (2019) demonstrates that a breadth-first ordering, such as that presented here, is better suited to parallelism.

When the accumulated flow reaches the pit cell of a depression, the downhill-directed flow routing stops because there is no downhill neighbor to receive the flow. At this point, all of the flow-accumulated water in the pit cell is moved into the pit cell's associated leaf depression in the DH. That is, the water is moved out of the geographic space and into the topologic space. This then enables mass-conserving depression flooding via rapid Fill-Spill-Merge calculations, as detailed below.

3.2 Overflow and Merge Depressions

At this point, the Fill-Spill-Merge algorithm has routed all of the surface water into either the ocean or into the leaf nodes of the DH. The next step is to redistribute this water through the DH to nodes with enough volume to contain the water, and to send any excess water to the ocean. This set of operations can be performed entirely in the depression hierarchy without reference to the digital elevation model.

Intuitively, the process of filling, spilling, and merging can be visualized as occurring from leaf nodes to their parents (Figure 3). Water must be redistributed such that leaf depressions containing more water than they can hold spill over into their neighboring depression. If this neighboring depression is already full, then the excess water must pass to the parent of both the depression and its neighbor. This process continues recursively until either the supplied water is exhausted or this water reaches the ultimate parent, the ocean. In this latter case, all excess water is dropped from the model and the ocean is unaffected.

To efficiently redistribute water, the Fill-Spill-Merge algorithm performs nested depth-first traversals of the DH. The outer traversal is post-order and considers each meta-depression in turn, from the most deeply nested to the least. For each meta-depression, an inner traversal handles its overflows by moving water to its sibling (starting by filling the sibling's descendants) and, if there's any left, passing it to the depression's parent. In this way, the outer traversal maintains an invariant: any meta-depression it has processed does not contain an overflow.

The outer traversal of the DH (which is, after all, a forest of binary trees) begins with the ocean. For each depression, the algorithm first recurses into the depression's left child and then into its right child. If any oceanlinks are found, the algorithm also recurses into them. In the post-order portion of the traversal (which starts from the leaves and moves back up through the depression hierarchy), the algorithm identifies any depressions containing more water than they can accommodate. This

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process continues until the recursion returns to the ocean, at which point any additional water is assumed to be added to the ocean without impacting sea level.

When an overfilled depression is located, the inner traversal redistributes this water. Let us call this overfilled depression A and note that it contains some amount of excess water — that is, water beyond its depression capacity. Our goal is to distribute this fixed amount of excess into neighbouring depressions. At each step below, the amount of this excess water remaining to be distributed will either remain the same or decrease. When we pass water into a depression, it can go to one of three places: the depression itself, its sibling, or its parent. Distributing the water to any of these places may itself cause an overflow. Therefore, the inner (pre-order) traversal comprises the following steps:

1. Call the depression that we are currently considering B. This may be the depression we originally considered, depression A, or it may be some other depression reached during the steps detailed below. We add water to B until either it fills or all of the water is used. At this point, this part of the algorithm can terminate if: (i) there is no water left, (ii) B is the parent of A, (iii) B acts as a parent of A by receiving its overflow via an oceanlink–geolink pair while not being a sibling or descendant, or (iv) B is the ocean.

2. Otherwise, if B has a sibling and the sibling's water volume is less than its depression volume, then start from Step 1 with the new B set as the depression pointed to by the current B's geolink.

3. Otherwise, if B has no sibling or the sibling's water volume is equal to its depression volume, then start from Step 1 with the new B set as the parent of the current B. (Note that the parent may be the ocean or a node reached via an oceanlink).

During each such pass through the inner traversal, water moves at most one step in the DH towards the ocean.

The next step of the outer traversal, which begins one level in the DH closer to the ocean, identifies a less nested metadepression for which the inner traversal might need to be run. If this step were not supplied with information about prior water redistribution, it could cause multiple traversals of a subtree of the DH, which would be computationally wasteful. To prevent this, the inner traversal returns the ID of the final node in which it placed water: this node is the only node in the traversal with spare capacity so future traversals can begin there. Therefore, on subsequent overflows, if such a cached value is available, then the recursion skips directly to that node. This ensures that all the calls to this part of the algorithm take no more than $O(N)$ time collectively.

The following examples uses the geometry from Figure 2 to describe a set of inner traversals, starting with an overflowing Depression #12. Step numbers mirror those above; numbers in parentheses indicate the number of recursions – that is, the number of times that the inner-traversal algorithm has returned to Step 1:

1 Depression #12 fills and overflows.

2 Depression #12's water overflows into Depression #4, which is not full, following its geolink.

1(r1) Depression #4 acts as Depression #12's parent via a geolink–oceanlink pair. The inner traversal terminates.

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At this point, the outer traversal moves one level closer to the ocean, and the inner traversal repeats, this time starting at Depression #4.

1 Depression #4 fills and overflows.

2 Depression #4's water overflows into its sibling, Depression #5, which is not full and is a leaf depression. If Depression #5 had descendants, water overflowing from Depression #4 would have followed a geolink to one of these.

1(r1) Depression #5s fills and overflows.

2(r1) Depression #4 is full.

3(r1) Depression #5 overflows into its parent, Depression #11.

1(r2) Depression #11 overflows into the ocean; the inner traversal terminates.

Now the outer traversal moves yet another level closer to the ocean, and the new inner traversal starts at Depression #11.

1 Depression #11 fills and overflows.

2 Depression #11 has no sibling.

3 Depression #11 overflows into its parent, the ocean; all remaining excess water is absorbed into an infinite sink.

1(r1) The now-selected node is the ocean; the inner traversal terminates.

At this point, the outer traversal moves one level closer to the ocean, and arrives at the ocean. The outer traversal also terminates.

3.3 Flood the landscape

After water moves through the DH (Section 3.2, above), each node in the DH exists in one of the three following states:

1. **Empty:** The depression's water volume is equal to zero. In this case, nothing needs to be done. The depression's descendants might contain water, but the water never propagates to this level of the DH.

2. **Full:** The depression's water volume is equal to the volume of the depression itself. In this case, the depression is entirely full. If the depression's parent contains water, then the calculation of water depth is dealt with at a higher stage in the DH. If the depression's parent is empty, or if the depression's parent is the ocean, then the calculation is performed at this level as described below.

3. **Partially filled:** The depression's water volume is less than its depression volume. In this case, the depth of water across the depression and all its descendants' cells must be calculated at this level so that the depression fills to an appropriate level. This is described below and indicated as the *marginal volume* on Figure 2e.

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The next step is to distribute this water across the DEM, appropriately flooding geographic depressions.

Given the three states described above, the algorithm locates the highest-level node within each binary tree that contains water. It does so by first traversing from the ocean to each leaf depression by recursively traveling to each node's children in turn. Each time it reaches a leaf, the algorithm notes its label and pit cell. After identifying each of these, the algorithm reverses direction, moving from child to parent so long as the parent node contains water. Therefore, this traversal towards the ocean ends at the highest-level node whose parent does not contain water. Call this node L. The water volume contained within the depression will only very rarely be exactly enough to perfectly flood it; therefore, we must spread water across the depression to create a flat water surface.

To calculate water level within a depression, the algorithm begins by picking an arbitrary pit cell within it, and then uses this as a seed from which to start building a priority queue through the depression. The priority queue returns cells ordered from lowest to highest elevation. At each step through the priority queue, the algorithm checks whether a depression whose outlet is at this elevation would have enough volume to hold the water. If so, the algorithm exits, having successfully defined the flooded area. If not, it continues to build the priority queue.

To expand this brief conceptual discussion into a more formal set of steps, let us begin by calling the active cell – that is, the one that is currently being considered by the algorithm – c_p. This cell is initially the arbitrary pit mentioned above, and is added to the priority queue. The algorithm marks c_p, which stands for "**c**ell of current highest **p**riority", as *visited*; all other cells remain unvisited. The algorithm then follows these steps:

1. Pop c_p from the priority queue and use its elevation to calculate the volume of water that can be accommodated in the set of cells processed so far (Equation 3, below). If this volume is enough to accommodate the volume of water available, exit the loop and compute the final water level (Equation 4, below). Otherwise, proceed to Step 2.

2. Add the former c_p (which was popped in Step 1) to a plain queue, which records all of the cells scanned so far; these cells will later be inundated.

3. Add the cells neighboring the former c_p that are not marked as *visited* to one of two lists. If an unvisited neighboring cell shares a label with L or any of its descendants, then this neighboring cell is added to the priority queue. Each of these neighboring cells is then marked as *visited*.

4. Choose the lowest-elevation cell in the priority queue and label it as the new c_p and return to Step 1. If the priority queue is empty, then all cells in the same meta-depression as c_p or its descendants have been visited and we are now guaranteed to have sufficient depression volume to hold all of the water.

Step 1 in this approach requires an efficient way to determine the volume of a depression below any given elevation. To do so, we imagine a hypothetical outlet that drains the depression. If the depression is full enough that of its all cells receive water, then the elevation of this hypothetical outlet is simply that of the topographic outlet from the depression. If the depression is not yet completely filled, it can be visualized as a pipe in the side of the depression that is an infinite sink for any water entering it, thereby acting analogously to an overflow drain below the edge of a sink or bathtub. If we call the elevation of this hypothetical

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outlet is o and a depression contains cells of elevations $\{a,b,c,d,\ldots\}$, then the capacity of the depression is

$$(o-a)+(o-b)+(o-c)+(o-d)+\ldots = No-a-b-c-d-\ldots \tag{1}$$

$$= No - \sum^{N}(\text{elevations}) \tag{2}$$

Now, consider cells $c_i = c_1,\ldots,c_N$ in the plain queue (i.e., those that have been visited and popped from the priority queue), we can calculate the volume of the depression below that of the last cell popped from the priority queue, the sill z_s, as:

$$V_{dep,z_s} = z_\mathrm{s} N - \sum_{i=1}^{N} z_i \tag{3}$$

Here, $V_{\mathrm{dep,z_s}}$ is the volume of the depression below z_s, and z_i is the elevation of cell c_i. Thus, if we keep track of the number of cells in a depression and their total elevation, it is possible to calculate the volume of a depression at any hypothetical outlet level.

Once $V_{\mathrm{dep,z_s}}$ is greater than or equal to the volume of water in the depression, V_w, the plain queue contains all cells to be flooded. At this point, the algorithm updates z_w, which is the water level within this depression. If $V_w = V_{\mathrm{dep,z_s}}$, the algorithm sets $z_w = z_N$. If instead $V_w < V_{\mathrm{dep,z_s}}$, the available volume is greater than the water volume, and the algorithm calculates z_w in the depression as follows:

$$z_w = \frac{1}{N}\left(V_w + \sum_{i=1}^{N} z_i\right). \tag{4}$$

We call this the Lake-Level Equation (LLE). The conditional usage of the LLE described above is purely for computational efficiency: if $V_w = V_{\mathrm{dep,z_s}}$, its solution is that $z_w = z_N$.

After solving for the water-surface elevation, the algorithm pops each cell in the plain queue ($c_i = c_1,\ldots,c_N$), corresponding to the flooded region, and sets its water elevation to the computed z_w. This is the final step of the Fill-Spill-Merge algorithm. At this point, it outputs a file representing the topography plus water thickness across the domain (i.e., topography with depressions filled or partially filled with water).

4 Theoretical Analysis

Here we use computational complexity as a means of contrasting the expected run-time of our algorithm against previous algorithms such as FlowFill. To do so, we describe a simple iterative solver similar to FlowFill whose goal is to determine an appropriate water level for a depression. The solver operates on a one-dimensional domain of cells bounded by high cliffs on either side in which each cell may have a column of water. At each step, if the solver finds a discontinuity in water levels between two cells, it responds by averaging the heights of the cells' water columns. (The solver we describe is known as Jacobi's method.) The challenge we present to this solver is a direct analogue of routing flow along a stretch of river with negligible gradient and is very similar to routing flow across the surface of a lake or ocean.

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For our analysis, we imagine that the system is initialized with a high column of water on the left and no water anywhere else. We call the cell with the water Cell 1. We call the cells to its right 2, 3, 4, and so on. During the solver's first step, Cell 1 is initialized. On its second step, Cell 1 averages its height with Cell 2. On the third step, Cell 2 averages with Cell 3 and Cell 1 then averages with Cell 2. On the fourth step, Cell 3 averages to 4, 2 averages to 3, and 1 averages with 2. Thus, the number of cells affected at each step are: 1, 2, 3, 4, and so on. Since there must be *at least* as many steps as there are cells, we can say that there are N steps. The total time, t_{compute}, is then

$$t_{\mathrm{compute}} = \sum_{i=1}^{N} i = \frac{N(N+1)}{2} \tag{5}$$

Thus, for any model (Callaghan and Wickert, 2019; Fan et al., 2013) that uses a scheme similar to our simple solver, the time required to solve the model is in $O(N^2)$.

In contrast, the new algorithm runs in $O(N \log N)$ time in the worst case. Moving water downhill (Section 3.1) is a flow-accumulation algorithm. This is known to take $O(N)$ time (Mark, 1988) and efficient variants exist for performing flow accumulation in parallel on large datasets (Barnes, 2017) and on GPUs (Barnes, 2019), though for simplicity we do not use these techniques here. Moving water within the depression hierarchy (Section 3.2) requires a depth-first post-order traversal of the entire hierarchy. This type of traversal is a foundational algorithm in computer science and takes $O(N)$ time. Each node in this traversal has the potential to overflow, which also results in a depth-first traversal, thereby requiring up to $O(N)$ time. However, by using a jump table that persists between calls to the overflow function, we ensure that it is able to identify the target of the overflow in amortized constant time; that is, the function is able to skip over fully-filled depressions. Finally, the algorithm floods the digital elevation model from the pit cells up. This requires a depth-first post-order traversal, which calls a flooding function (Section 3.3) on select subtrees of the DH. The depth-first traversal takes $O(N)$ time, as described above. The priority queue used for flooding nominally takes $O(N \log N)$ time in the worst case for floating-point data and $O(N)$ time in the worst case for integer data (Barnes et al., 2014). However, with specialized data structures the time can be reduced to $O(N)$ for both floating-point and integer data (Barnes et al., 2014). Most real datasets consist of many small depressions whose cell counts $N_{\mathrm{cells-in-dep}}$ are much smaller than the total number of cells in the digital elevation model. Therefore, the actual time is for this step is $O(N_{\mathrm{dep}} N_{\mathrm{cells-in-dep}})$, where N_{dep} is the total number of depressions and $N_{\mathrm{dep}} N_{\mathrm{cells-in-dep}}$ can be much less than N. Because the worst-case time complexity of any operation is $O(N)$, this bounds the time of the algorithm as a whole. However, to reduce the potential for bugs, we use the C++ standard library's $O(N \log N)$ priority queue in our implementation, at the cost of reduced performance.

To put this in more concrete terms, consider a long stretch of low-gradient river. Such a feature poses a lower bound on the time of our simple solver. North America's Red River of the North runs for 885 km with a gradient that is often on the order of 0.03 m km^{-1}. On a 90 m grid of floating-point data, the river would be 9,833 cells long. Our simple (Jacobi) solver would then take an estimated 97 million time units to reach a solution, whereas the new solver that we describe in this paper would take 9,833 time units, a 10,000× speed-up. Our empirical results, below, support both the theory and this expected value.

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Dataset	Dimensions	Cells	FSM Time (s)	Total Time (s)
Madagascar	2000×1000	$2.0 \cdot 10^6$	0.1	0.4
U.S. Great Basin	1920×2400	$4.6 \cdot 10^6$	0.2	8.7
Australia	5640×4200	$2.3 \cdot 10^7$	9.1	15.6
Africa	9480×9000	$8.5 \cdot 10^7$	65.3	118.0
N&S America	18720×17400	$3.2 \cdot 10^8$	53.2	231.6
Minnesota 30m topobathy	34742×23831	$8.2 \cdot 10^8$	307.8	792.6

Table 1. Datasets used, their dimensions, and algorithm wall-times. Tests were performed on the Comet cluster run by XSEDE (see main text for full specifications). Times for Fill-Spill-Merge ("FSM Time") alone and this time plus the depression hierarchy construction time ("Total Time") are shown. Topographic data for Madagascar, the U.S. Great Basin, Australia, Africa, and North & South America, were clipped from the global GEBCO_08 30-arcsecond global combined topographic and bathymetric elevation data set (GEBCO, 2010). The Minnesota 30m topobathy data is the merged result of two data sources. The topography is resampled from the Minnesota Geospatial Information Office's 1m LiDAR Elevation Dataset (MNGEO - Minnesota Geospatial Information Office, 2019). Bathymetric data were provided by the Minnesota Department of Natural Resources (MNDNR - Minnesota Department of Natural Resources, 2014). Richard Lively of the Minnesota Geological Survey merged and combined these data sets.

5 Empirical Tests

We have implemented the algorithm described above in `C++11` using the Geospatial Data Abstraction Library (GDAL) (GDAL Development Team, 2016) to read and write data. There are 981 lines of code of which 50% are or contain comments. The code can be acquired from https://github.com/r-barnes/Barnes2020-FillSpillMerge and Zenodo (Barnes and Callaghan, 2020).

Tests were run on the Comet machine of the Extreme Science and Engineering Discovery Environment (XSEDE) (Towns et al., 2014). Each node of the machine has 2.5 GHz Intel Xeon E5-2680v3 processors with 24 cores per node and 128 GB of DDR4 DRAM. Code was compiled using GNU g++ 7.2.0 with full optimizations enabled. Scaling tests on datasets spanning three orders of magnitude in terms of their number of cells are shown in Table 1. The GuessCompx package written in the R programming language by Agenis-Nevers et al. (2019) shows that an $O(N \log N)$ scaling relationship gives the best fit to the data, which agrees with the theory. Further tests are described in our Applications section (§6), below.

6 Applications

Given a depression hierarchy, Fill-Spill-Merge provides an efficient method to route water across any surface while taking depressions into account. Furthermore, Fill-Spill-Merge can be used to assess which depressions are most important in day-to-day or seasonal changes to the hydrologic system. For example, small depressions will become flooded and spill over even with relatively small amounts of water reaching them, while larger depressions may not be completely filled. These depressions impact the hydrologic connectivity of the landscape (Callaghan and Wickert, 2019).

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6.1 Field applications

We have compared Fill-Spill-Merge with a prior algorithm, FlowFill, at the same two sites used by Callaghan and Wickert (2019): a reach of the Sangamon River in Illinois (Figure 6) and the Río Toro basin in Argentina (Figure 7). Like Fill-Spill-Merge, FlowFill can be used to route water across a landscape while preserving real depressions, but the algorithm is significantly slower (Table 2). The two selected study sites provide very different landscapes for testing the performance of the algorithm. The Sangamon River site is located at 39.97°N, 88.72°W, in Illinois, USA. It is a low-relief, post-glacial landscape containing many closed depressions, which may impact hydrologic connectivity as a function of runoff (Lai and Anders, 2018). It furthermore contains a grid of roads and associated embankments whose elevations are significant when compared to regional relief and impact water flow paths and storage. Callaghan and Wickert (2019) resampled the 2.5 ft (0.76 m) resolution LiDAR DEM Illinois Geospatial Data Clearinghouse (2020) to 15 m resolution for analysis and manually removed several road bridges using GRASS GIS to prevent artificial pooling behind these; here we use the same modified DEM to enable a direct comparison between the algorithms. The Río Toro site is located mainly in Salta Province, Argentina, around 24.5°S, 65.8°W. This site exhibits more rugged fluvially sculpted topography (Hilley and Strecker, 2005). Callaghan and Wickert (2019) resampled the 12-m TanDEM-X DEM of this region (Krieger et al., 2013; Rizzoli et al., 2017) to 120 m resolution. Here we use this same resampled DEM for comparison.

As shown in Table 2, wall-times using Fill-Spill-Merge ranged from 0.227–0.243 s for the Sangamon River site and 0.300–0.319 s for the Río Toro site. This compares with times ranging from 20–643 s and 31-155 s, respectively, for FlowFill. These times for both sites correspond to a 86–2,645× reduction in wall-time. Since FlowFill was run with 24 processors, this translates to a 2,064–63,480× reduction in compute time. Considering that each of these example DEMs is quite small relative to modern full-resolution LiDAR-derived elevation data sets or continental-scale 30-meter DEMs (Table 1), this speed-up and its associated $O(N \log N)$ scaling provides a significant advantage for topographic analysis and solving associated problems in hydrology and geomorphology.

Although both FlowFill and Fill-Spill-Merge route water downslope, flooding depressions based on the quantity of available water, our results differ in some ways from those from those of FlowFill (Callaghan and Wickert, 2019). In both Figures 6 and 7, Fill-Spill-Merge flooded some depressions more deeply than FlowFill did, and, to a lesser extent, flooded a few depressions with less water. One possible cause for this discrepancy is FlowFill's asymptotic approach to an equilibrium water level, which may prevent small volumes of water from reaching the depression to which they belong. On the other hand, depressions with a narrow outlet could be especially prone to being overfilled by FlowFill because its cell-by-cell algorithm could dynamically dam this outlet, routing additional water into the depression. Both of these possibilities are further linked to the fact that FlowFill dynamically evolves a land-plus-water flow-routing surface, whereas Fill-Spill-Merge routes flow just over the land surface. These differences make FlowFill more useful for understanding temporal changes in surface water distribution, while Fill-Spill-Merge provides a more accurate snapshot of surface hydrology under equilibrium conditions.

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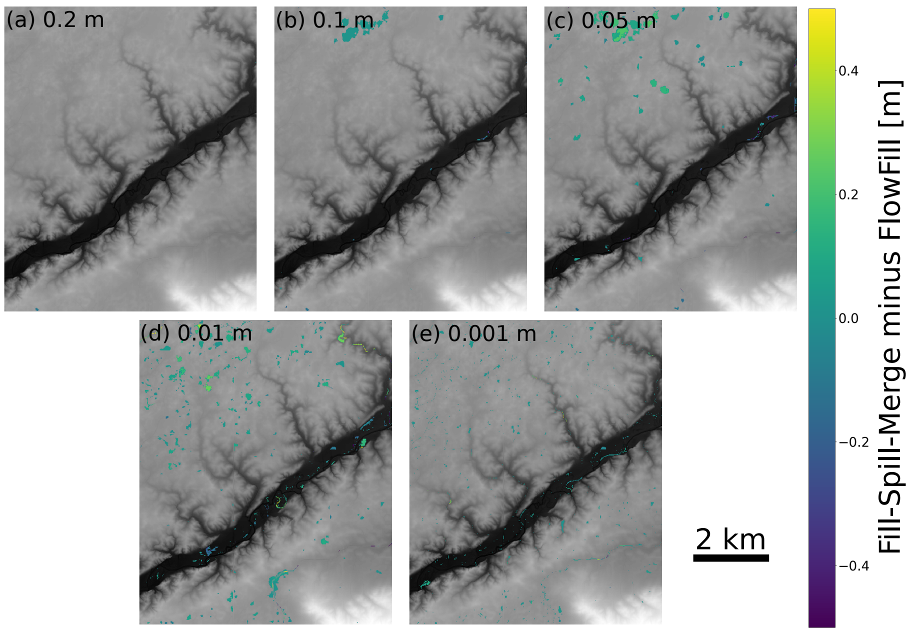

Figure 6. The difference between results of Fill-Spill-Merge and FlowFill at the Sangamon River site. The values for panels (a) to (e) indicate the depth of uniform runoff applied across the landscape for both algorithms. For example, in (a), each cell across the domain starts with 0.2 m of surface water. Green to yellow colors indicate locations where Fill-Spill-Merge had more water, and blue to purple colors indicate locations where FlowFill had more water. Differences of less than 3 mm have been masked out. Commonly, Fill-Spill-Merge retains slightly more water in depressions than FlowFill does. This could be due to the iterative nature of the FlowFill algorithm, which causes it to asymptotically approach the correct values. In some locations, FlowFill has retained more water. One possible reason for this is that some depressions have a narrow outlet, through which Fill-Spill-Merge is able to move all water as appropriate but the cell-by-cell movement of water with FlowFill can produce transient dams that reroute additional water towards these subcatchments. This DEM was prepared by Lai and Anders (2018) and Callaghan and Wickert (2019) from LiDAR topographic data provided by the Illinois State Geological Survey (Illinois Geospatial Data Clearinghouse, 2020).

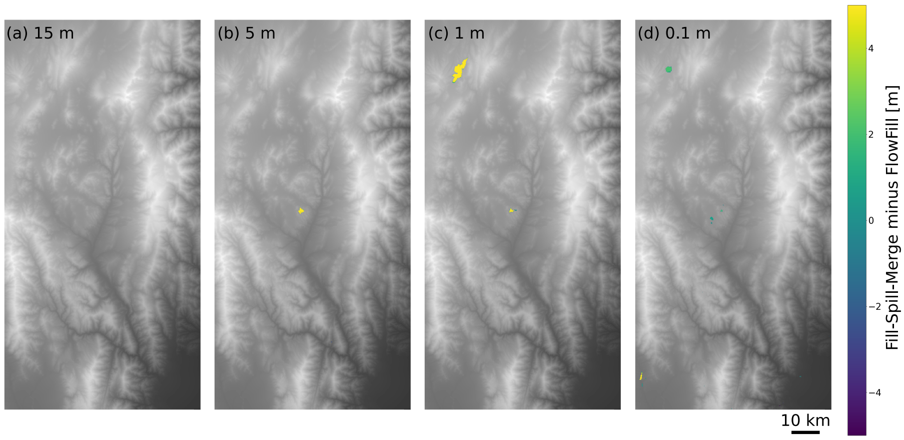

Figure 7. The difference between results of Fill-Spill-Merge and FlowFill at the Río Toro site. The values for panels (a) to (d) indicate the depth of uniform runoff applied across the landscape for both algorithms. For example, in (a), each cell across the domain starts with 15 m of surface water. Green to yellow colors indicate locations where Fill-Spill-Merge had more water, and blue to purple colors indicate locations where FlowFill had more water. Differences of less than 3 mm have been masked out. In panel (a), 15 m of water was enough to fill all depressions with both algorithms, so there are no differences between the two. The most significant difference is seen in panel (c), where Fill-Spill-Merge retained additional water in a large depression. This is likely due to transient damming of its narrow inlet in FlowFill's cell-by-cell method of moving water, which may have prevented the full volume of water from flowing into the depression. This DEM was generated with data acquired from the TanDEM-X mission (Krieger et al., 2013; Rizzoli et al., 2017).

	Sangamon			Río Toro		
Runoff depth [m]	FlowFill	FSM	Speed-up	FlowFill	FSM	Speed-Up
15	642.65	0.243	2645	154.70	0.317	488
10	626.59	0.241	2600	124.37	0.309	402
5	570.02	0.241	2365	93.56	0.300	312
1	472.33	0.241	1960	53.09	0.316	168
0.2	508.87	0.235	2165	38.30	0.316	121
0.1	464.15	0.230	2018	35.75	0.301	119
0.05	418.71	0.243	1723	33.62	0.316	106
0.01	200.81	0.227	885	32.06	0.315	102
0.001	20.12	0.235	86	30.99	0.319	97

Table 2. Time comparison of Fill-Spill-Merge vs FlowFill. Wall-times are in seconds comparing FlowFill (Callaghan and Wickert, 2019) parallelized across 24 cores versus Fill-Spill-Merge on a single core. Using FlowFill, wall-times increased with the depth of applied runoff and on flatter landscapes. Using FSM, wall-time is independent of depth of applied runoff and ruggedness of landscape, but increases for larger domains. FSM's wall-times were 86–2,645 times faster than FlowFill for these examples; compute times were 2,064–63,480 times faster.

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7 Conclusions

Here we leverage the depression hierarchy data structure (Barnes et al., 2020) to route flow through surface depressions in a realistic, yet efficient, manner. In comparison to previous approaches, such as Jacobi iteration, the new algorithm runs in log-linear time in the input size and is accompanied by extensively-commented source code. This computationally efficient algorithm may help us to better understand hydrologic connectivity and water storage across hummocky land surfaces, and is an important step forwards in recognising the importance of depressions as real-world features in digital elevation models.

Code availability. Complete, well-commented source code, an associated makefile, and correctness tests are available from https://github.com/r-barnes/Barnes2020-FillSpillMerge and Zenodo (Barnes and Callaghan, 2020).

Author contributions. KC and AW conceived the problem. RB conceived the algorithm and developed initial implementations. KC and RB completed, debugged and tested the algorithm. All authors contributed to the preparation of the manuscript.

Competing interests. The authors declare that they have no conflict of interest.

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References

Agenis-Nevers, M., Bokde, N. D., Yaseen, Z. M., and Shende, M.: GuessCompx: An empirical complexity estimation in R, arXiv:1911.01420v1, 2019.

Barnes, R.: Parallel non-divergent flow accumulation for trillion cell digital elevation models on desktops or clusters, Environmental Modelling & Software, 92, 202–212, https://doi.org/10.1016/j.envsoft.2017.02.022, 2017.

Barnes, R.: Accelerating a fluvial incision and landscape evolution model with parallelism, Geomorphology, 330, 28–39, https://doi.org/10.1016/j.geomorph.2019.01.002, 2019.

Barnes, R. and Callaghan, K.: Fill-Spill-Merge Source Code, https://doi.org/10.5281/zenodo.3755142, 2020.

Barnes, R., Lehman, C., and Mulla, D.: Priority-flood: An optimal depression-filling and watershed-labeling algorithm for digital elevation models, Computers & Geosciences, 62, 117 – 127, https://doi.org/10.1016/j.cageo.2013.04.024, 2014.

Barnes, R., Callaghan, K., and Wickert, A.: Computing water flow through complex landscapes, Part 2: Finding hierarchies in depressions and morphological segmentations, Earth Surface Dynamics, https://doi.org/10.5194/esurf-2019-34, 2020.

Braun, J. and Willett, S. D.: A very efficient O(n), implicit and parallel method to solve the stream power equation governing fluvial incision and landscape evolution, Geomorphology, 180-181, 170–179, https://doi.org/10.1016/j.geomorph.2012.10.008, 2013.

Breckenridge, A. and Johnson, T. C.: Paleohydrology of the upper Laurentian Great Lakes from the late glacial to early Holocene, Quaternary Research, 71, 397–408, https://doi.org/10.1016/j.yqres.2009.01.003, 2009.

Cabrol, N. A. and Grin, E. A.: Distribution, classification, and ages of Martian impact crater lakes, Icarus, 142, 160–172, 1999.

Callaghan, K. L. and Wickert, A. D.: Computing water flow through complex landscapes – Part 1: Incorporating depressions in flow routing using FlowFill, Earth Surface Dynamics, 7, 737–753, https://doi.org/10.5194/esurf-7-737-2019, 2019.

Fan, Y., Li, H., and Miguez-Macho, G.: Global Patterns of Groundwater Table Depth, Science, 339, 940–943, https://doi.org/10.1126/science.1229881, 2013.

Fenner, T. I. and Loizou, G.: Loop-free Algorithms for Traversing Binary Trees, BIT, 24, 33–44, https://doi.org/10.1007/BF01934513, 1984.

GDAL Development Team: GDAL - Geospatial Data Abstraction Library, Open Source Geospatial Foundation, available at http://www.gdal.org., 2016.

GEBCO: General Bathymetric Chart of the Oceans (GEBCO), GEBCO_08 grid, version 20100927, http://www.gebco.net, 2010.

Hilley, G. E. and Strecker, M. R.: Processes of oscillatory basin filling and excavation in a tectonically active orogen: Quebrada del Toro Basin, NW Argentina, GSA Bulletin, 117, 887–901, https://doi.org/10.1130/B25602.1, 2005.

Illinois Geospatial Data Clearinghouse: Illinois Height Modernization (ILHMP), https://clearinghouse.isgs.illinois.edu/data/elevation/illinois-height-modernization-ilhmp-lidar-data, 2020.

Jenson, S. and Domingue, J.: Extracting Topographic Structure from Digital Elevation Data for Geographic Information System Analysis, Photogrammetric Engineering and Remote Sensing, 54, 1–5, https://doi.org/0099-1112/88/5411-1593$02.25/0, 1988.

Krieger, G., Zink, M., Bachmann, M., Bräutigam, B., Schulze, D., Martone, M., Rizzoli, P., Steinbrecher, U., Antony, J. W., De Zan, F., et al.: TanDEM-X: A radar interferometer with two formation-flying satellites, Acta Astronautica, 89, 83–98, 2013.

Lai, J. and Anders, A. M.: Modeled Postglacial Landscape Evolution at the Southern Margin of the Laurentide Ice Sheet: Hydrological Connection of Uplands Controls the Pace and Style of Fluvial Network Expansion, Journal of Geophysical Research: Earth Surface, 123, 967–984, https://doi.org/10.1029/2017JF004509, 2018.

https://doi.org/10.5194/esurf-2020-31
Preprint. Discussion started: 5 May 2020

Earth Surface Dynamics
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Open Access

Li, S., MacMillan, R., Lobb, D. A., McConkey, B. G., Moulin, A., and Fraser, W. R.: Lidar DEM error analyses and topographic depression identification in a hummocky landscape in the prairie region of Canada, Geomorphology, 129, 263–275, https://doi.org/10.1016/j.geomorph.2011.02.020, 2011.

Lindsay, J. and Creed, I.: Removal of artifact depressions from digital elevation models: towards a minimum impact approach, Hydrological processes, 19, 3113–3126, https://doi.org/10.1002/hyp.5835, 2005a.

Lindsay, J. B.: Efficient hybrid breaching-filling sink removal methods for flow path enforcement in digital elevation models: Efficient Hybrid Sink Removal Methods for Flow Path Enforcement, Hydrological Processes, https://doi.org/10.1002/hyp.10648, 2015.

Lindsay, J. B.: Efficient hybrid breaching-filling sink removal methods for flow path enforcement in digital elevation models: Efficient Hybrid Sink Removal Methods for Flow Path Enforcement, Hydrological Processes, 30, 846–857, https://doi.org/10.1002/hyp.10648, http://doi.wiley.com/10.1002/hyp.10648, 2016.

Lindsay, J. B. and Creed, I. F.: Removal of artifact depressions from digital elevation models: Towards a minimum impact approach, Hydrological Processes, 19, 3113–3126, https://doi.org/10.1002/hyp.5835, 2005b.

Mark, D.: Modelling in Geomorphological Systems, chap. Network models in geomorphology, pp. 73–97, John Wiley & Sons, 1988.

Martz, L. W. and Garbrecht, J.: The treatment of flat areas and depressions in automated drainage analysis of raster digital elevation models, Hydrological Processes, 12, 843–855, https://doi.org/10.1002/(SICI)1099-1085(199805)12:6<843::AID-HYP658>3.0.CO;2-R, 1998.

Martz, L. W. and Jong, E. d.: CATCH: A FORTRAN program for measuring catchment area from digital elevation models, Computers and Geosciences, 14, 627–640, https://doi.org/10.1016/0098-3004(88)90018-0, 1988.

MNDNR - Minnesota Department of Natural Resources: Lake Bathymetric Outlines, Contours, Vegetation, and DEM, https://gisdata.mn.gov/dataset/water-lake-bathymetry, 2014.

MNGEO - Minnesota Geospatial Information Office: LiDAR Elevation Data for Minnesota, http://www.mngeo.state.mn.us/chouse/elevation/lidar.html, 2019.

O'Callaghan, J. and Mark, D.: The extraction of drainage networks from digital elevation data, Computer Vision, Graphics, and Image Processing, 28, 323–344, https://doi.org/10.1016/S0734-189X(84)80011-0, 1984.

Reheis, M.: Highest Pluvial-Lake Shorelines and Pleistocene Climate of the Western Great Basin, Quaternary Research, 52, 196–205, https://doi.org/10.1006/qres.1999.2064, 1999.

Riddick, T., Brovkin, V., Hagemann, S., and Mikolajewicz, U.: Dynamic hydrological discharge modelling for coupled climate model simulations of the last glacial cycle: the MPI-DynamicHD model version 3.0, Geoscientific Model Development, 11, 4291–4316, https://doi.org/10.5194/gmd-11-4291-2018, 2018.

Rizzoli, P., Martone, M., Gonzalez, C., Wecklich, C., Tridon, D. B., Bräutigam, B., Bachmann, M., Schulze, D., Fritz, T., Huber, M., et al.: Generation and performance assessment of the global TanDEM-X digital elevation model, ISPRS Journal of Photogrammetry and Remote Sensing, 132, 119–139, 2017.

Schwanghart, W. and Scherler, D.: Bumps in river profiles: uncertainty assessment and smoothing using quantile regression techniques, Earth Surface Dynamics, 5, 821–839, https://doi.org/10.5194/esurf-5-821-2017, 2017.

Soille, P.: Optimal removal of spurious pits in grid digital elevation models, Water Resources Research, 40, 1–9, https://doi.org/10.1029/2004WR003060, 2004.

Soille, P., Vogt, J., and Colombo, R.: Carving and adaptive drainage enforcement of grid digital elevation models, Water Resources Research, 39, 1366, https://doi.org/10.1029/2002WR001879, 2003.

https://doi.org/10.5194/esurf-2020-31
Preprint. Discussion started: 5 May 2020

Towns, J., Cockerill, T., Dahan, M., Foster, I., Gaither, K., Grimshaw, A., Hazlewood, V., Lathrop, S., Lifka, D., Peterson, G. D., et al.: XSEDE: accelerating scientific discovery, Computing in Science & Engineering, 16, 62–74, 2014.

Wallis, C., Watson, D., Tarboton, D., and Wallace, R.: Parallel flow-direction and contributing area calculation for hydrology analysis in digital elevation models, in: Preceedings of the 2009 International Conference on Parallel and Distributed Processing Techniques and Applications, Las Vegas, Nevada, http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.158.2864, 2009.

Wickert, A. D.: Reconstruction of North American drainage basins and river discharge since the Last Glacial Maximum, Earth Surface Dynamics, 4, 831–869, https://doi.org/10.5194/esurf-4-831-2016, 2016.