I wish to thank the reviewers for their insightful comments, which have led to a significantly improved paper. I apologize for the length of time it has taken me to provide a revised manuscript; my primary focus for the past few months has been on single-parenting during the pandemic and the basics of my professional obligations such as teaching and servicing funded projects. I appreciate everyone's patience and request that if the revised paper falls short in some important way that I be given one more opportunity to strengthen the manuscript. Here I summarize the points of each reviewer, respond to each point, and state how the revised manuscript has been modified to address each comment.

Reviewer 1 (Gordon Grant):

Q: "*I remain somewhat skeptical of both the approach and the results. My skepticism is rooted in both some of the underlying theory and assumptions used to derive the mathematical relations that underpin this work, and also the wide variance and assumptions in the data used to validate the methods. I cannot put my finger on a single "smoking gun" amidst all of this, but I was quite surprised that given all the assumptions and uncertainties, the predicted relationships were almost spot-on the widely published values (starting with Leopold and Maddock) for the hydraulic geometry relationships. Herein lies the intrigue, but I'm not convinced that the right answer was obtained for the right reasons. To his credit, the author acknowledges the fragility of some of the assumptions, and calls for additional work to better understand key relationships. Thus, this paper will stimulate discussion and work and deserves to be shared with the broader community, irrespective of any misgivings.*"

A: I appreciate that the reviewer sees value in the paper and wishes to see it published despite his skepticism.

Q: "*The postulated relation between bankfull height and clay content builds on previous work, but as the author points out, the assumption in this earlier work was that co-hesion limited erosion by increasing the required near-bank shear stresses for fluvial entrainment. Here the focus is on gravitationally-driven stress failures, modeled with a very simple 1-D relationship (Eq. 2), that when parametrized, gives rise to an even sim-pler linear equation between bankfull depth and maximum gravitationally stable bank height. This relation is tested at the scale of the Mississippi River basin by comparing bankfull height derived from USGS gage station data with predicted height calculated from clay content derived for each station using gNATSGO soil survey data and esti-mated with a 10km moving window. Not surprisingly, the resulting scatter plot is. . .well, scattered with a weak positive trend (Fig. 2A). The author does synthetically consider what the error in this relationship might look like; this error analysis is not propagated hrough the rest of the analysis, however. At the end of the day, I remain skeptical that broad-scale soil survey data can be used to parametrize clay content for point data (USGS stations). To be fair, in the discussion the author does consider other sources of potential error beyond data error, including the use of stage data as a proxy for bank height, and other possible channel adjustments to varying clay content (i.e., varying bank angle, widening). But taken together, these uncertainties raise doubts about the validity of the paper's central claim that bankfull height is primarily controlled by gravi-tational slope failure.*"

A: Thank you for this comment. First, let me state that my discussion paper did not claim that gravitational failure is the dominant process occurring in channel widening and bank migration more generally. Fluvial scour and gravitational failure influence and complement one another to such an extent that it is difficult to clearly identify a dominant control for even a single alluvial channel, let alone for alluvial channels in general. Fluvial scour transports slumped material away from the bank, likely keeping bank angles higher than they would be without fluvial scour. Gravitational failure transports material from higher on the bank to the toe (often following fluvial scour, which is maximized at the toe), likely keeping bank angles lower than they would be without gravitational failure. The relative importance of these two processes may be reflected in the bank angle, i.e., a bank angle persistently much less than vertical may indicate the dominance of gravitational failure while a bank angle persistently at or above 90˚ (i.e., a cantilever or overhang) may indicate the dominance of fluvial scour. The following text has been added to the manuscript on this point:

"That said, gravitational failure and fluvial shear stresses act in concert in such a way that identifying which process is dominant may be difficult. Fluvial scour transports slumped material away from the bank, likely keeping bank angles higher than they would be without fluvial scour. Gravitational failure transports material from higher on the bank to the toe (often following fluvial scour, which is maximized at the toe), likely keeping bank angles lower than they would be without gravitational failure. More research is needed, but the relative importance of these two processes may be reflected in the bank angle, i.e., a bank angle persistently much less than vertical may indicate the dominance of gravitational failure while a bank angle persistently at or above 90˚ (i.e., a cantilever or overhang) may indicate the dominance of fluvial scour."

Given that both gravitational failure and fluvial scour play important and complementary roles, which process do data suggest is the more dominant control on alluvial channel morphology? This question can be addressed by comparing the channel geometries predicted by models based on gravitational failure and fluvial scour to data. Dunne and Jerolmack (2020) tested the fluvial scour hypothesis using the same alluvial channel morphology dataset (i.e., Dunne and Jerolmack, 2018) used in my paper. The authors explain why their model underpredicts data by approximately 2 orders of magnitude in channels with $D_{50} < 1$ cm (Fig. 2A of their 2020 paper) this way: "For fine-grained rivers with $D_{50} < 1$ cm, however, we see rivers peel off of the Shields curve; the smaller the riverbed grain size, the larger bankfull shear stress deviates from the threshold expectation… we infer that this departure… represents the point where τ_c of cohesive banks, which is rarely measured, becomes larger than τ_c of noncohesive bed sediments, on average."

The predictions of a model based on gravitational failure, while scattered, are consistent with data for all grain sizes. The scatter of the model predictions can be expected to be largely a consequence of lack of data on bank angle.

Text added on this point:

"Absent site-specific data for bank angles, the largest source of uncertainty in the proportionality coefficient in Eq. (4) as applied to specific locations is likely the bank angle, since relatively modest variations in bank angle (e.g., from 90˚ to 75˚) are associated differences in Ns of approximately a factor of 3 while other sources of uncertainty (e.g., between cohesion and clay content as quantified by Eq. (3)) are smaller. Section 4 provides discussion on how uncertainty in bank angle and other factors such as bank vegetation limit the precision of Eq. (4) to specific locations. The primary of objective of this paper, however, is to document an increase, on average, in bankfull channel depth with increasing clay content:

$$h \approx 0.35\, p_c. \tag{5}$$

assuming that bankfull depth is approximately equal to the maximum gravitationally stable bank height."

The only systematic deviation of a model based on gravitational failure occurs for shallow channels (< 2 m deep) with very low clay contents, a result that, as noted in the discussion paper, is likely a result of an upward bias in clay content estimates at very low clay contents.
Text added on this point:
"This upward bias may be associated with the difficulty of measuring very low clay contents in the field. Clay contents estimated in the field can only be constrained to be within the range from 0 to 10% clay. If the actual clay content is close to 0 (e.g., < 1%), the clay content estimate is likely to be overestimated by a much larger fraction than would be the case for a larger clay content (e.g., 5% clay is 400% greater than 1% clay but only 50% lower than 10% clay). The result may be an upward bias in clay content for clay contents less than approximately 10%, which, according to Eq. (5), may be associated with channels less than 2-3 m in bankfull depth. Channels with bankfull depths less than 2 m (Sect. 2.1) were excluded from the analysis due to this potential bias."

I believe I have done the best I can with existing data to report and discuss all the primary sources of uncertainty, even if some of the sources of uncertainty are difficult to quantify (on bank angles especially) and thus preclude the propagation of a quantified uncertainty from the start to finish of the analysis. I have also tried to emphasize to the reviewer that, while the application of Eq. (5) (the old Eq. (4)) to specific locations is inherently difficult due to lack of local constraints on bank angle, the primary of objective of this paper is to document an increase, on average, in bankfull channel depth with increasing clay content. That objective has been met in a statistically comprehensive way.

I appreciate the reviewer's skepticism that broad-scale soil survey data can be used to parameterize clay content for point data. However, I note that soil survey data are derived from point-scale measurements (SCS scientists use data from soil pits to define soil properties across areas of similar slope, geology, etc.). Moreover, rates of bank retreat at a gaging station are likely controlled not by the clay content of a single point but rather by clay content over the spatial scale of a meander bend, which for large rivers can be ~1-10 km.

Q: "*While I'm appreciative of the author's invoking the critical flow hypothesis (correct citation is Grant, 1997, not 2000 as it appears in text but not references), I'm somewhat confused by the role it plays in this story. As I understand it, the author argues that this hypothesis suggests that the range of Froude numbers for both sand and gravel-bed channels should be limited to near- or less than 1.0, and that therefore Froude number and discharge should be weakly and inversely related. The logic here is not entirely clear, and the mechanism that restricts Froude numbers is not entirely accurate. In its original form, the hypothesis argues that in steep channels (typically S>0.01), inter-actions between the free surface (particularly hydraulic jumps) and the bed result in a rough balance between forces that accelerate the flow and forces that extract energy from the flow and thus retard it, thus promoting near-critical flow conditions. This condition applies irrespective of grain size, although the author rightly points out that the actual bed features that* *set up this interaction are different for sand, gravel, and even boulder bed channels. A recent paper tests this idea in the flume and articulates the mechanism well (Piton and Recking, 2019). Most of the streams in the Dunne and Jerolmack data set have slopes much less than 0.01, and consequently much lower Fr, as the data shown in the paper point out.*"

A: I thank the reviewer for pointing out the incorrect citation, which has been corrected in the revised manuscript. The reviewer is correct that I generalized his 1997 critical flow hypothesis. The wording has been corrected in the revised paper. My reason for generalizing his hypothesis was that, even though the critical flow hypothesis was originally proposed for steep channels, the key idea (to my mind) of the critical flow hypothesis is the existence of a self-regulatory feedback in which an increase in velocity is met with an increase in drag that brings the velocity, and hence *Fr*, back down to a lower value. Similarly, a decrease in velocity tends to be met with a decrease in drag that tends to increase the velocity. It is likely that such a self-regulatory feedback is not limited to steep channels. In steep, gravel-bedded channels, faster flow tends to lead to more wave breaking and other drag-inducing processes, thereby tending to lower velocities back down to a critical value (i.e., $Fr = 1$). In less-steep, sand-bedded channels, faster flow tends to lead to the development of larger and more well-developed bedforms, which increases relative roughness and hence drag in the range of *Fr* values associated with well-developed bedforms ($Fr \sim 0.1$-1). The mechanisms for self-regulation differ between steep channels and less-steep channels, but self-regulation exists in both. In the revised manuscript, I clarify that the critical flow hypothesis applies to steep channels only and separately discuss evidence for a similar self-regulation of *Fr* in less-steep channels.

Text added on this point:

"Grant (1997) proposed a critical-flow hypothesis in which depth-averaged water velocities are self-regulated via interactions between the water flow and the channel-bed morphology. Grant (1997) argued that, in steep ($\gtrsim$ 0.01 m m-1) channels, the Froude number rarely exceeds one for extended periods of time due to interactions between the free surface and the bed that result in an approximate balance between forces that accelerate the flow and forces that extract energy from the flow. Such a balance may also extend to coarse-bedded channels, the water flow in which is prone to wave drag associated with flow around bed sediment grains that protrude above the water surface (Wohl, 2013). Wave drag can be expected to be more common in coarse-bedded channels relative to channels with finer bed sediments both because they have relatively large bed roughness elements that more readily protrude above the water surface as well as a tendency towards shallower flows as a result of their characteristically large width-to-depth ratios (Schumm, 1960).

Central to the critical-flow hypothesis is the existence of a self-regulatory feedback in which an increase in velocity is met with an increase in drag that tends to reduce the velocity and hence the Froude number. Similarly, a decrease in velocity tends to be met with a decrease in drag that tends to increase the velocity. Here it is hypothesized that such a self-regulatory feedback is not limited to steep channels. In less-steep, sand-bedded channels, faster flow tends to facilitate the development of larger and more well-developed bedforms (which tend to form at Froude numbers ~0.1-1 (e.g., Simons and Richardson, 1966)) that increase relative roughness and hence drag. The mechanisms of self-regulation and the Froude numbers at which steep and less-steep channels may achieve this self-regulation thus differ, but both are likely to have self-regulatory interactions between the flow and the bed that limit the Froude numbers of bankfull discharges. Consistent with this hypothesis, here it is documented that bankfull Froude numbers, and hence the ratio of depth-averaged water velocities to the square root of bankfull depths, tend to be within a relatively narrow range that has a weak inverse relationship to bankfull discharge."

Q: "*More to the point, in my view, the primary control on Fr is channel slope, and I suspect that this is what is behind the weak inverse correlation between Fr and discharge (Fig. 3A). The simple theoretical dependency between Fr and S is shown in Grant (1997; ig. 4); a more sophisticated treatment is given in Palucis and Lamb (2017; Fig. 2). Since smaller streams tend to be steeper than larger ones, my hunch is that discharge is more of a proxy for slope than a driver as in Fig. 3A. This slope dependency is also probably lurking behind the grain-size/Fr relationship in Fig. 3B as well. None of this fundamentally invalidates the argument being made by the author but I think these relations should be acknowledged, as they have bearing on the physical mechanisms underlying the presentation.*"

A: Thank you for this comment. My work demonstrates how alluvial channel geometric variables interrelate. I used bankfull discharge as an independent variable because it has traditionally been used for this purpose. I did not and would not argue that any variable is more fundamental, significant, or a "driver" than any other variable. Bankfull Froude number depends explicitly on channel depth and velocity, the latter of which depends on slope and bed roughness. As such, bankfull Froude number certainly depends on channel depth, channel slope, and bed roughness (the latter of which depends on grain size and all of the factors that control bedform geometry (if bedforms are present)). I know of no basis for ranking the relative importance of any of these controls. This view is consistent with Palucis and Lamb (2017), who, despite relating specific channel morphologic types to specific slopes, document the importance of channel width and grain size in controlling channel type (e.g., "step-pools form in near supercritical flow or when channel width is narrow compared to bed grain size") and conclude that "certain bed slopes have unique channel morphologies because the process variables covary systematically with bed slope."

Reviewer 2 (Roberto Fernandez):

Q: "*1. The use of the data by Dafalla (2013) seems misrepresented in this manuscript (Figure S1). Cohesion values reported by Dafalla (2013) correspond to pure sand, sand and clay mixtures with 5*

In addition, Dafalla (2013) shows that for the same clay content (15

I really believe that the approach proposed in this manuscript has a lot of potential and others have included geotechnical considerations in models for stream restoration (e.g. CONCEPTS, see Langendoen et al. 2001; RVR Meander, see Motta et al. 2012). I would encourage the author to dig out some more references regarding cohesion estimates for soils that are more relevant for the Mississippi River Basin. For example, Masada (2009) presents a very extensive report on geotechnical parameters for the state of Ohio and includes different relations between sediment properties (cohesion for example) and soil composition (amount of silt, amount of clay, etc.). Their approach is specific to highway embankments but the results of their tests might be more general in terms of cohesion values in relation to clay contents."

A: I greatly appreciate the reviewer's drawing my attention to these additional sources of data, Masada (2009) in particular. I have scoured the literature for additional sources of data but have not found any. Masada (2009) is not a good source of data on the relationship between cohesion and clay content because he presents cohesion versus clay content for a very narrow range of clay contents (e.g., 27%-34% in Fig. E6). Also, I prefer to use data from direct shear tests rather than the triaxial shear tests of Masada (2009) because direct shear tests are more similar to gravitational shear failures. Dafalla (2013) is also a good source of data because he presents data for cohesion versus clay content at fixed moisture contents. Assuming that there is no systematic variation between short and tall banks in the water content at which channel banks tend to fail, it is the dependence of cohesion on clay content at similar moisture contents (or averaged across all moisture contents) that is most appropriate for use in the model.

The reviewer is correct that the sand and clay mixtures considered by Dafalla (2013) were obtained by mixing sand and clay as opposed to being natural soils. However, I don't think creating such mixtures necessarily undermines the quality of the data or the applicability of the results to natural soils. Masada (2009) considered natural soils but they were almost entirely glacial in origin, hence not necessarily a better analog for alluvial soils than the mixtures studied by Dafalla (2013).

Text added on the Dafalla (2013) data:
"Bank material cohesion varies linearly from 0 (for cohesionless sand) to approximately 90 kPa (for pure clay) for moisture contents in the range of 7 to 40% according to a least-squares linear regression of the data from Dafalla (2013) (Fig. S1):

$$C \approx (900 \pm 70)p_c. \quad (3)$$

where the units of C is Pa and pc is percent. The uncertainty in Eq. (3) is the standard error (i.e., one standard deviation) resulting from the regression."
and the caption for Figure S1 provides some additional technical information:
"Figure S1. Plot of cohesion, C, as a function of percent clay content, pc, from the experiments of Dafalla (2013) demonstrating the approximately linear nature of the relationship, i.e., the exponent of a power-law relationship between C and pc (shown as a line above) determined by a least-squares linear regression to the logarithms of the data is 0.78 ± 0.12 where 0.12 is the standard error. Note that three data points from Dafalla (2013) were not included because the clay contents were zero and hence the logarithms were undefined. However, those data points were included in the linear least-squares regression of non-log-transformed data reported in the paper that resulted in the coefficient of proportionality of 900 ± 70 in Eq. (3)."

Q: "*2. The use of equation (2) might not be appropriate for riverbanks. The use of that equation as presented by Chen (1969) and Terzaghi et al. (1996; p. 271-272) is for soil embankments located above the water table. Several authors have used it in the past as discussed by ASCE (1988) but even there, the authors suggest that critical depth approaches are not accurate when the most common bank failure mechanisms for riverbanks are due to tension cracks that cause toppling or cantilever failures. Assuming the equation is indeed an appropriate approach for riverbanks, I would en-courage the author to explore the sensitivity of its input variables to other values. Chen (1969) shows a wide range of Ns values that depend on the internal friction angle of the material (which is sensitive to moisture content) and the actual slope of the bank. The smallest stable bank height would be given by the smallest possible safety param-eter Ns so why not explore a range of Ns values. When the channel has low flow, the bank might be quite dry and its maximum stable height would be quite different from that obtained with a saturated bank (e.g. during the falling limb of a hydrograph where the river stage is getting lower but the bank remains saturated). It would be very useful to see these considerations in the analysis. The author discusses the issue* *briefly but more details regarding bank failure mechanisms and their prevalence might strengthen the manuscript.*"

A: The reviewer is correct that the analysis of Chen et al. (1969) is for unsaturated banks only. I have augmented the results of Chen et al. (1969) with other studies to provide a more accurate basis for the 0.35 proportionality coefficient between percent clay content and critical bank height. Text added on this point:

"The maximum stable height, hc, of an alluvial channel bank subject to gravitational shear failure is proportional to bank-material cohesion, C (Taylor, 1937; Terzaghi and Peck, 1967; Hunter and Schuster, 1968; Chen et al., 1969; ASCE, 1999):

$$h_\mathrm{c} = \frac{N_s}{\rho g} C, \qquad (2)$$

where ρ is the bulk density of the bank material, g is the acceleration due to gravity, and Ns is a stability parameter dependent on the geometry of the potential failure surface (e.g., planar, log-spiral, or circular), the pore pressure of the bank material (which is governed by the water table position if the pore pressure is assumed to be hydrostatic), and the angles of the bank and of internal friction (see Table 1 for a list of variables).

In order to estimate a reference Ns value appropriate for understanding how gravitational stability may influence the scaling of alluvial channel bankfull depths to discharge, a steep bank (i.e., near-vertical at the top of the bank but decreasing to approximately 45˚ near the toe) with an internal friction angle of 35˚ (typical for a loamy or clayey sand), near-saturated conditions, and a log-spiral potential failure surface were assumed. Near-saturated conditions are consistent with the fact that gravitational shear failure has been documented to occur most frequently during the falling limbs of flood discharges when pore pressures tend to be associated with near-saturated conditions (e.g., Casagli et al., 1999; Simon et al., 2000). Chen et al. (1969) derived Ns values for prescribed angles of the bank and of internal friction for unsaturated conditions. For a friction angle of 35˚, Ns values in Table 1 of Chen et al. (1969) decrease with increasing bank angle from Ns = 22 for a 60˚ bank to Ns = 12 for a 75˚ bank and Ns = 7.5 for a vertical bank. Hunter and Schuster (1968) limited their analysis to cases with no internal friction (hence their absolute Ns values are not applicable here) but documented an approximately 3-fold decrease in Ns values from unsaturated conditions (i.e., M = hwγw/hcγ′ ≈ 1, where is hw is the depth to the water table below the top of the bank, γw is the unit weight of water, and γ′ is the submerged unit weight of the bank material) to near-saturated conditions (i.e., M = 0). Combining the results of Chen et al. (1969) and Hunter and Schuster (1968) suggests that Ns values for a saturated bank with an internal angle of friction of 35˚ vary from approximately 7.3 for a 60˚ bank to Ns ≈ 4 for a 75˚ bank and 2.5 for a vertical bank.

Bank material cohesion varies linearly from 0 (for cohesionless sand) to approximately 90 kPa (for pure clay) for moisture contents in the range of 7 to 40% according to a least-squares linear regression of the data from Dafalla (2013) (Fig. S1):

$$C \approx (900 \pm 70) p_\mathrm{c}. \qquad (3)$$

where the units of C is Pa and pc is percent. The uncertainty in Eq. (3) is the standard error (i.e., one standard deviation) resulting from the regression.

Combining Eqns. (2) and (3) and assuming a bulk density of 1700 kg m-3 and a representative value of Ns ≈ 6 (corresponding to a near-saturated bank with an angle of approximately 65˚, i.e., an average angle for a bank that is near-vertical at the top and decreases to an angle of approximately 45˚ near the toe) yields

$$h_\mathrm{c} \approx 0.35 p_\mathrm{c}. \qquad (4)$$

Absent site-specific data for bank angles, the largest source of uncertainty in the proportionality coefficient in Eq. (4) as applied to specific locations is likely the bank angle, since relatively modest variations in bank angle (e.g., from 90˚ to 75˚) are associated differences in Ns of approximately a factor of 3 while other sources of uncertainty (e.g., between cohesion and clay content as quantified by Eq. (3)) are smaller. Section 4 provides discussion on how uncertainty in bank angle and other factors such as bank vegetation limit the precision of Eq. (4) to specific locations. The primary of objective of this paper, however, is to document an increase, on average, in bankfull channel depth with increasing clay content:

$$h \approx 0.35\, p_c. \qquad (5)$$

assuming that bankfull depth is approximately equal to the maximum gravitationally stable bank height."

Q: "*3. Sensitivity analysis: Figures 2b and 2c present results for bank heights based on a synthetic dataset. If the author estimated clay contents using averaging windows for a soils dataset, why not extract second order statistics from it and use them directly instead of creating a synthetic dataset?*"

A: I don't think that second-order statistics (e.g., coefficient of variation) necessarily capture all of the potential uncertainty in a dataset. For example, spatial variations in the values of a dataset that systematically over- or under-predict actual values won't correctly capture the true uncertainty. I also think creating a synthetic dataset has advantages. For example, a key goal was to demonstrate that an upward bias can exist in very low clay contents. I don't think using second-order statistics in the averaging would necessarily have the same potential to demonstrate this phenomenon.

Q: "*4. Use of the Mississippi River Basin data: The author clearly states why the MRB data are used. However, not knowing much about the many different locations along the basin, I have a few questions. (1) What percentage of the cross sections analyzed can be considered natural? (2) Did the author discard those locations where the navigable channels are maintained by the US Army Corps of Engineers? (3) Of the many stations used, how many might be influenced by river control structures (dams, wing dams, chevrons, etc.) or road infrastructure (e.g. culverts, bridges)?*"

A: Using Google Earth, I have examined the locations of the 387 stations and have found no instances where cross-sections are located close to infrastructure. This lack of overlap between the U.S.G.S. station locations and infrastructure may be partly due to the fact that my filtering criteria (lines 88-92) did an effective job at removing stations with stage-discharge relationships that are affected by infrastructure.

Q: "*5. Figures 3 and 4: It is not at all clear why the author includes regression plots of the Dunne and Jerolmack (2018) dataset. Based on the abstract and introduction, it was unexpected that a different dataset appears in the manuscript and becomes the focus of the second half. I understand the use of the dataset for Figure 5, which is new but the content of Figures 3 and 4, is not. I would encourage the author to make it clear to the reader earlier that the DJ dataset is a substantial part of the analysis and to state explicitly the novelty of including figures 3 and 4.*"

A: The opening sentence of the abstract makes clear that the problem I am tackling is the power-law scaling of bankfull depths, widths, depth-averaged water velocities, and along-channel slopes to bankfull discharge in alluvial channels. The first part of my paper deals with control on channel depth only, so it stands to reason that there must be another part of the paper that extends the work on channel depth to other aspects of channel geometry using additional principles. Figures 3 and 4 (including the reporting of best-fit exponents) have not been published elsewhere and their inclusion is important for meeting the goals of the paper.

Q: "*6. Figure 5: I have a few specific questions about the analysis leading to Fig. 5. (1) What is the number (and percentage) of cases that report ripples/dunes over the entire Ohata (2017) dataset? (2) For those reporting ripples/dunes, what is the number and percentage of measurements obtained in the laboratory and in the field? (3) For those in the field, how many are for large rivers? Cisneros et al. (2020) show that traditional dune scaling equations overestimate the size of dunes in large rivers and propose the following relation between dune height (H) and water depth (h) – H 0.056h - 0.12h. 4) Are the only sources of roughness in the DJ data the ripples/dunes or gravel size? What about bars, meandering, vegetation?*"

A: 1) 1574 (42%) of the 3790 data points in Ohata et al. (2019) have ripples or dunes (noted in revised manuscript), 2) of that 42%, 19% are in the field and the remaining 81% are in the laboratory (noted in revised manuscript), 3) 3.2% (123 out of 3791) of the data points are from rivers with h > 5 m. 4) I am assuming that the dominant (not only) sources of roughness in the channels of the DJ data are ripples/dunes or gravel clasts. Long-wavelength topographic features such as bars and meanders are not likely to be dominant roughness/drag-inducing elements given that the presence/absence of the flow separation that tends to dominate drag depends sensitively on the maximum slope of bedforms and other obstacles to the flow, with slopes in excess of 0.2 m/m generally needed to trigger the flow separation (though surface curvature also plays an important role in addition to slope; see below). I concede that vegetation can be a dominant source of roughness on the beds of ephemeral channels, and some of the scatter in my analysis may be a result of vegetation-induced bed roughness.
Text added on this point:
"Long-wavelength topographic features such as bars and meanders are not likely to be dominant roughness/drag-inducing elements given that the presence/absence of the flow separation that tends to dominate drag depends sensitively on the maximum slope of bedforms and other obstacles to the flow, with slopes in excess of 0.2 m/m generally needed to trigger flow separation (e.g., Lefebvre et al., 2014). Vegetation can certainly be a dominant source of roughness on the beds of ephemeral channels, however, and some of the scatter in the analysis of this paper may be a result of vegetation-induced bed roughness."

Cisneros et al. (2020) demonstrates that the lee-side angle of many bedforms in large alluvial channels is lower that empirical equations predict and argues that such lower angles means that flow separation and hence drag is less significant than empirical models would suggest in large rivers. It is important to note, however, that flow separation depends sensitively on the surface curvature in the zone of the maximum adverse pressure gradient (Lamballais et al., 2010), not just on the relative height or lee-side angle of bedforms. As such, more research is needed to conclude that flow separation is rare on the lee sides of bedforms in large alluvial channels.

Q: "*As a final general comment, I was hoping to see more analysis on the Mississippi River Basin dataset and comparisons between it and the DJ dataset where possible. The manuscript seems to be split between two separate analyses but the abstract and introduction do not suggest that. I recommend the author to modify these initial sec-tions as necessary and compare the MRB data with the DJ data where possible. What kind of relation does the author obtain between bankfull depth and bankfull discharge for the MRB under the geotechnical considerations? On the other hand, could clay contents (and cohesion) be estimated with a revised version of equation (4) for other rivers in the world where soil data is not readily available?*"

A: See my response to point 5 on the apparent split in the manuscript. It should be possible to infer clay content values for some rivers *if bank angles and moisture contents were also well constrained*. We do not have such data yet for many alluvial channel cross sections across a sizable region.

Q: "*1) How do the bankfull estimates found here for the MRB compare to those of Dong et al (2019). This reference appears in the introduction but is not mentioned in the discussion.*
A: Since Dong et al. (2019) deal with the Selenga River Delta and my paper deals with the MRB, it is difficult to make a direct comparison with their results.

Q: *"2. I did not understand the fourth criteria used to keep a USGS gaging station in the analysis of the MRB."*
A: I apologize that this was not clearer. The estimation of bankfull stage requires fitting stage to discharge for the smallest and largest values. The bankfull stage is estimated to be where these two lines meet. In order to verify that the low-flow fit is reasonable, I retained only those stations for which the extrapolation of the fit of low-flow values passes "close" to the correct value: zero flow depth at zero discharge. In cases where the extrapolation of the fit does not pass close to zero, the data are likely of insufficient quality or require an adjustment based on data that are not publicly available. What represents "close" should not be based on an absolute error, e.g., 0.5 m, because such a criterion would require that low-flow fits for a deep channel be much more accurate than one for a shallow channel. So, instead, I defined "close" as within 50% of the bankfull stage from zero. That is, if the bankfull stage is 5 m, then the extrapolation of the low-flow fit to zero discharge must yield a stage within 2.5 m of zero. Similarly, if the bankfull stage is 2 m, then the extrapolation of the low-flow fit to zero discharge must be within 1 m of zero.
Text added on this point:
"The analysis presented here began by including data from all U.S.G.S. gaging stations in the MRB with available peak discharge data (U.S. Geological Survey, 2020). Only those stations for which the slope of the high-flow linear regression is at least five times smaller than the slope of the low-flow linear regression were retained. In addition, only those stations that had at least 20 years of data, have a contributing area larger than 100 km2, are not located close to major infrastructure (based on an inspection of each station location in Google Earth imagery), and have a resulting bankfull depth of greater than 2 m were retained. Channels with bankfull depths less 2 m were removed because such channels tend to be associated with low clay contents that are inherently difficult to estimate in the field (see Sect. 3.1 for more detail on the potential bias associated with estimating low clay contents). In order to further filter out stations where the low-flow linear regression is potentially unrepresentative of the hydraulic behavior of in-channel discharges, only those stations for which the extrapolation of the low-flow linear regression is close to zero flow

depth at zero discharge were retained. Stations for which the low-flow regression does not extrapolate to a flow stage close to zero at zero discharge may have gage height data that are not an accurate proxy for flow stage and/or have other data quality issues that preclude an accurate estimate of bankfull depth using the stage-discharge rating curve. What represents "close" should not be based on an absolute error, e.g., 0.5 m, because such a criterion would require that the low-flow regressions for deep channels be relatively more accurate than those for shallow channels. Here "close" was defined as being within 50% of the bankfull stage from zero. That is, if the bankfull stage is 5 m, then the extrapolation of the low-flow regression to zero discharge must yield a stage within 2.5 m of zero in order for that station to be retained in the analysis. Similarly, if the bankfull stage is 2 m, then the extrapolation of the low-flow fit to zero discharge must be within 1 m of zero. A total of 387 stations met these criteria."

Q: *"3. If the analysis discards rivers with depths smaller than 2m, why is the 0.5m to 1.5m soil depth the only section considered for the analysis. What about river sections with different bank layers? The author mentions that soil data below 1.5m is not reliable but how valid is it to assume a uniform soil profile for the entire channel depth? How sensitive is the proposed model to this assumption?"*

A: I assume that the texture of the top 1.5 m of the bank (for which data is readily available) is representative of the entire bank. I understand that this assumption may be violated in many cases. However, given that the floodplain deposits that comprise many banks are the depositional products of channels where well-sorted sediments tend to be the norm, it is reasonable to expect that the texture of the uppermost 1.5 m of the bank will correlate strongly with bank material at greater depths in many cases.

Q: *"4. Line 150(and other locations) - Ohata et al 2017 (not 2019)."*
A: That you for pointing out this typo. It has been fixed in the revision.

Q: "5. Line 151 "How did the author "cross-reference" the Ohata et al. dataset with the Dunne and Jerolmack dataset?

A: Cross-referencing refers to the development of a curve in *Fr* vs. d_{50} space that separates channels that have ripples and/or dunes from those that do not, and using that curve to infer that the vast majority of sand-bedded channels in the Dunne and Jerolmack dataset have ripples and dunes. This is done by assuming the existence of ripples and dunes in channels of the D&J dataset that have *Fr* and d_{50} values that sit above and to the left (Fig. 3B) of the envelope curve separating channels with and without ripples and dunes in the Ohata et al. (2017) dataset.
Text added on this point:
"By cross-referencing those results with the Dunne and Jerolmack (D&J) (2018) global dataset (i.e. by drawing a curve in F vs. d_{50} space that separates channels that have ripples and/or dunes from those that do not, and assuming the existence of ripples and dunes in channels of the D&J dataset that have F and d_{50} values that sit above and to the left (Fig. 3B) of the envelope curve separating channels with and without ripples and dunes in the Ohata et al. (2017) dataset), Sect. 3.3 demonstrates that 96% of sand-bedded channels in the D&J global dataset have F and d_{50} values conducive to ripple and/or dune development and therefore have a roughness that is likely dominated by bedforms rather than by bed sediment grains."

Q: "6. Line 214 "what is the equation of the curve (envelope) used to identify the conditions conducive to dune/ripple development?"

A: There is no equation. This is simply a drawn curve. Noted in revised manuscript.

Q: "7. Line 259 (and other locations) "Chen 1969 (not 1971)"

A: Typo fixed.

Q: 8. Line 286 "Vegetation. What are the predominant types of vegetation along the MRB? How deep are their roots? Root length might set slump block thickness. Vegetation might be the most relevant factor in shallow channels (up to max root length) and geotechnical considerations might be more relevant in deeper channels where roots might not stabilize the full bank.

A: The stability of any bank is primarily a function of its weakest portion or layer, i.e., failure is more likely to occur in a zone of low shear strength compared to a zone of high shear strength, all else being equal. Failure of a weaker zone underlying a stronger zone can create a cantilever, but such cantilevers cannot continue to grow indefinitely, hence rates of long-term retreat in stronger and weaker zones of the same bank will tend to be similar. This, together with the fact that vegetation is likely to strengthen just the uppermost approximately 2 m of channel banks (globally, >99% of roots are found in the top 2 m of the soil (Jackson et al., 1996)), suggests that bank material shear strength is not likely to be controlled primarily by vegetation in channels deeper than 2 m. I concede that some plant roots can exceed (even greatly exceed) 2 m in depth. However, plant roots become quite small in density at > 2 m depth based on Jackson et al. (1996). I understand that this view appears to contradict the dependence of bank retreat rates on vegetation that has been documented in many studies (e.g., Ielpi and Lapôtre, 2020). I am not stating that vegetation has no effect on bank stability. Rather, I am stating that I know of no study that has attributed bank shear strength to vegetation that has accounted for the fact that wetter climates with more vegetation also tend to have more clay-rich soils. As such, much of the apparent control of vegetation could be due to clay content.
Text added on this point:
"Bank vegetation plays a significant role in controlling bank stability, but it is unlikely that such control is responsible for the scaling relationships that are the focus of this paper, as such scaling relationships exist across climatic regions with very different vegetation characteristics. In addition, the stability of any bank is primarily a function of its weakest portion or layer, i.e., failure is more likely to occur in a zone of low shear strength compared to a zone of high shear strength, all else being equal. Failure of a weaker zone underlying a stronger zone can create a cantilever, but such cantilevers cannot continue to grow indefinitely, hence rates of long-term retreat in stronger and weaker zones of the same bank will tend to be similar. This, together with the fact that vegetation is likely to strengthen just the uppermost approximately 2 m of channel banks (globally, >99% of roots are found in the uppermost 2 m of the soil (Jackson et al., 1996)), suggests that bank material shear strength is unlikely to be controlled primarily by vegetation in channels deeper than 2 m. A dependence of bank retreat rates on vegetation has been documented in many studies (e.g., Ielpi and Lapôtre, 2020). However, such studies may not fully account for the fact that wetter climates with more vegetation also tend to have more clay-rich soils, leaving open the

question of whether it is bank vegetation or material texture that is most responsible for bank resistance to erosion."

Reviewer 3 (Christopher Hackney):

Q: "*I find the section between lines 30 and 34 quite confusing. The author states that channel incision or floodplain deposition may increase bank height (OK so far). This causes banks to collapse once a critical bank height is reached. The subsequent failure results in channel widening which tends to increase water depths back towards the stable bank height. It is this last bit that I find counter to the previous few lines. I follow the argument that a wider channel results in slower flows, but the reduction in flows is a result of the increased channel capacity due to increasing width, and so there is no need for the channel to continue to incise of raise water depths to hold the same volume of water, as the increased width accounts for this. Furthermore, it has been shown recently that over long time frames, channels tend to maintain an equilibrium width (and thus presumably bank height; see Mason and Mohrig, 2019) and that the channel adjustment due to bank collapse also often sees increased deposition on the inner bank (see van de Lageweg et al. 2014). A clearer explaination of how bank failures can result in increased water depths would be welcome.*"

A: The reviewer is correct that the sentence "Bank failure results in channel widening, which may reduce depth-averaged water velocities and therefore tend to increase water depths (to convey similar water discharges) back towards the maximum stable bank height" was confusing and did not accurately convey my conceptual model for how alluvial channels self-regulate to a maximum depth comparable to the maximum stable bank height. I have replaced it with: "The gravitational failure of channel banks may partially control bankfull depths via a self-regulatory mechanism in which channel incision and/or floodplain deposition tend to increase bank height, triggering bank failure when a critical bank height, dependent on bank material cohesion, is exceeded (Andrews, 1982), thus introducing new sediment into the channel bed that, as it is redistributed by fluvial processes, tends to reduce the channel depth back towards a critical value."

Q: "*The author also notes a few potential limitations to the data exploration in the discus-sion, but does so briefly and in passing. I would like to see a more developed discus-sion around the role of vegetation induced cohesion, and also the role of failed material (particularly in clay rich soils which are more likely to fail and persist in blocks at the base of the river bank), as these are likely to be key local controls on any variation in the relationships the author has presented. Another potential source of variaiton that isn't raised but may also be important is the role of floodplain topography in defining bankful depths and bank heights. If a bank is eroding through a scroll-bar then it is likely that following a failure the local bank height may decrease as a result of variable local topography (i.e. on a floodplain where elevation is sloping away from the bank). Following the model presented here, will that new bank remain stable until its critical height either is reached through channel incision or build up from floodplain deposition? Following on from this, on Ln 272 the author states that an increase in bank height caused by floodplain deposition may trigger bank failure. Presumably, to deposit ma-terial on the bank the flow needs to be over-bank. Therefore, is it the deposition of the material on the bank during these flows, for the increased water velocity and bank shear stresses that will induce this erosion?*"

A: Discussion items are, by nature, relatively brief (in that they are limited to subsections of the Discussion section only). To the extent that my discussion items are relatively brief, I think this is appropriate given that there is no indication that vegetation-induced cohesion causes the scaling of alluvial channel geometry that is the subject of this paper. Also please see my response to reviewer 2's comment 8 on the issue of vegetation.

The revised manuscript mentions the role of failed bank material in the context of the interplay between fluvial scour and gravitational failure in contributing to channel width adjustment.

Regarding the final question, either incision or floodplain deposition can trigger bank failure: it is not one or the other. Whether a channel incises depends on changes in base level and sediment supply that depend on tectonic and climatic processes that operate at scales much larger than a channel reach. A channel that is relatively shallow relative to its bankfull depth will tend to flood overbank more often than a deeper channel, thereby promoting deposition until the channel deepens to the point where channel banks fail. The model of this paper invokes a long-term dynamic equilibrium between channel deepening processes (channel incision and/or floodplain deposition) and channel shallowing processes (bank retreat, which introduces sediment into that channel that, once redistributed laterally, causes channel aggradation). My response to the reviewer's first concern clarifies (I hope) this aspect of the conceptual model in the revised manuscript.

Q: "*Overall, I think a lot of the issues raised above come down to the temporal scale being examoined here and there is a need for some discussion the manuscript around the ime-scales over which these processes may become the dominant factor and how time-averaging of the other processes involved in river bank erosion occurs. Could the author examine any historical rates of bank erosion for sites analyses in the manuscript to see whether the theoretical model holds for different time periods?*"

A: I appreciate this suggestion. However, while rates of bank erosion are available over multiple time scales, what matters to this paper is channel widening. Channel widening likely occurs most abruptly during channel incision periods (following climatic changes, for example), after which bank retreat continues but channel widening does not because cut bank migration is approximately balanced by point-bar progradation (e.g., Mason and Mohrig, 2019). I am not aware of any available data on channel widening data over multiple time scales. I also wish to emphasize, as I did in my responses to reviewer 1, that I am not arguing that gravitational failure is the dominant processes setting channel depth and width. Fluvial scour and gravitational failure influence and complement one another so intimately that it is difficult to clearly identify a dominant control.

Thank you,

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Controls on the hydraulic geometry of alluvial channels: bank stability to gravitational failure, the critical-flow hypothesis, and conservation of mass and energy

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Abstract. The bankfull depths, widths, depth-averaged water velocities, and along-channel slopes of alluvial channels are approximately power-law functions of bankfull discharge across many orders of magnitude. What mechanisms give rise to these patterns is one of the central questions of fluvial geomorphology. Here it is proposed that the bankfull depths of alluvial channels are partially controlled by the maximum heights of gravitationally stable channel banks, which depend on bank material cohesion and hence on clay content. The bankfull depths predicted by a bank-stability model correlate with observed bankfull depths estimated from the bends in the stage-discharge rating curves of 387 U.S. Geological Survey gaging stations in the Mississippi River Basin. It is further proposed that depth-averaged water velocities scale with bankfull depths as a result of a self-regulatory feedback among water flow, relative roughness, and channel-bed morphology that limits depth-averaged water velocities to be within a relatively narrow range associated with Froude numbers that have a weak inverse relationship to bankfull discharge. Given these constraints on channel depths and water velocities, bankfull widths and along-channel slopes consistent with observations follow by conservation of mass and energy of water flow.

1 Introduction

The bankfull depths, h, widths, w, depth-averaged water velocities, v, and along-channel slopes, S, of alluvial channels exhibit power-law relationships with bankfull discharge, Q:

$$h \propto Q^k, w \propto Q^b, v \propto Q^m, S \propto Q^z \tag{1}$$

where $k \approx 0.4$, $b \approx 0.5$, $m \approx 0.1$, and $z \approx -0.4$ (Leopold and Maddock, 1953). Many studies have proposed a process-based understanding of these patterns (see reviews by Ferguson (1986) and Singh (2003)), but none has achieved widespread acceptance.

Schumm (1960) documented that alluvial channels with sand-rich bed and bank material tend to be wide and shallow, while alluvial channels with silt-and-clay-rich bed and bank material tend to be narrow and deep. Schumm's findings have led nearly all subsequent researchers to consider the resistance of bank material to fluvial erosion to be the key factor controlling alluvial channel width (e.g., Parker, 1979; Eaton and Millar, 2004; Dunne and Jerolmack, ~~2019~~2020). Bank retreat, however, is also driven by gravitational failure (ASCE, 1999), a process that limits bank heights to values that depend on bank material cohesion and hence on clay content. The gravitational failure of channel banks may partially control bankfull depths via a self-regulatory mechanism in which channel incision and/or floodplain deposition ~~cause an~~tend to increase~~ in~~ bank height, triggering bank failure when a critical bank height, dependent on bank material cohesion, is exceeded (Andrews, 1982~~). Bank failure results in ~~), thus introducing new sediment into the channel ~~widening, which may~~ bed that, as it is redistributed by fluvial processes, tends to reduce the channel depth~~-averaged water velocities and therefore tend to increase water depths (to convey similar water discharges)~~ back towards ~~the maximum stable bank height~~a critical value. This paper demonstrates that bankfull depths predicted by a bank-stability model correlate with observed bankfull depths estimated from the bends in the stage-discharge rating curves of 387 U.S. Geological Survey (U.S.G.S.) gaging stations in the Mississippi River Basin (MRB). This analysis supports the hypothesis that the gravitational failure of channel banks partially controls bankfull depths and complements the recent work of Dong et al. (2019) that documented a relationship between bank-material texture and the hydraulic geometry of alluvial channels in the Selenga River Delta.

~~G~~rant (~~2000~~1997) proposed a critical-flow hypothesis in which depth-averaged water velocities are self-regulated via interactions ~~among~~between the water flow~~, relative roughness,~~ and the channel-bed morphology. Grant (~~2000~~1997) argued that, ~~for gravel-bedded~~in steep ($\gtrsim$ 0.01 m m^{-1}) channels~~ above a threshold~~, the Froude number ~~close to 1, much~~rarely exceeds one for extended periods of ~~the gravitational potential energy~~ time due to interactions between the free surface and the bed that ~~would otherwise~~ result in ~~increased~~ an approximate balance between forces that accelerate the flow and forces that extract energy from the flow. Such a balance may also extend to coarse-bedded channels, the water ~~velocities~~flow in which is ~~lost~~prone to wave drag~~, especially that~~ associated with flow around bed sediment grains ~~protruding~~that protrude above the water surface (Wohl, 2013). ~~For~~Wave drag can be expected to be more common in coarse-bedded channels relative to channels with finer bed sediments both because they have relatively large bed roughness elements that more readily protrude above the water surface as well as a tendency towards shallower flows as a result of their characteristically large width-to-depth ratios (Schumm, 1960).

Central to the critical-flow hypothesis is the existence of a self-regulatory feedback in which an increase in velocity is met with an increase in drag that tends to reduce the velocity and hence the Froude number. Similarly, a decrease in velocity tends to be met with a decrease in drag that tends to increase the velocity. Here it is hypothesized that such a self-regulatory feedback is not limited to steep channels. In less-steep, sand-bedded channels, faster flow tends to facilitate the development of larger and more well-developed bedforms (which tend to form at Froude numbers ~~typically in the range of~~ ~0.1-1~~, a similar increase in drag occurs due to the formation of ripples and/or dunes that tend to develop in that range of Froude numbers (~~ (e.g., Simons and Richardson, 1966~~). These additional energy-loss mechanisms associated with critical or near-critical flow conditions, over and above the drag associated with subcritical flow over a smooth bed, can be expected to result in reduced depth-averaged water velocities and hence an increase in flow depths, resulting in lower~~)) that increase relative roughness and hence drag. The mechanisms of self-regulation and the Froude numbers at which steep and less-steep channels may achieve this self-regulation thus differ, but both are likely to have self-regulatory interactions between the flow and the bed that limit the Froude numbers ~~and reduced relative roughness that, in turn, tend to increase in water velocities back towards a critical or near-critical state. Here it is proposed that the critical-flow hypothesis constrains~~ of bankfull discharges. Consistent with this hypothesis, here it is documented that bankfull Froude numbers, and hence the ratio of depth-averaged water velocities to the square root of bankfull depths, tend to be within a relatively narrow range that has a weak inverse relationship to bankfull discharge.

The bankfull widths of alluvial channels are set by the requirement that channels convey geomorphically effective water discharges. Conservation of mass, together with the clay-content control of bankfull depths and the Froude-number control of water velocities, thus constrains bankfull widths.

Conservation of energy constrains along-channel slopes. The conversion of gravitational potential energy into the kinetic energy of water leads to a relationship among along-channel slopes, bankfull Froude numbers, bankfull depths, and the size of the largest bed roughness elements, which in gravel-bedded channels tend to be bed sediment grains and in sand-bedded channels tend to be ripples and/or dunes.

2 Methods

2.1 Controls on bankfull depths

The maximum stable height, h_c, of an alluvial channel bank subject to gravitational shear failure is ~~(Chen et al., 1971~~proportional to bank-material cohesion, C (Taylor, 1937; Terzaghi and Peck, 1967; Hunter and Schuster, 1968; Chen et al., 1969; ASCE, 1999):

$$h_c = \frac{N_s}{\rho g} C, \qquad (2)$$

w~~here ~~where ρ is the bulk density of the bank material, g is the acceleration due to gravity, and N_s is a stability parameter dependent on the geometry of the potential failure surface (e.g., planar, log-spiral, or circular), the pore pressure of the bank material (which is governed by the water table position if the pore pressure is assumed to be hydrostatic), and the angles of the bank and of internal friction~~, ρ is the bulk density of the bank material, g is the acceleration due to gravity, and C is the bank material cohesion~~ (see Table 1 for a list of variables).

In order to estimate a reference N_s value appropriate for understanding how gravitational stability may influence the scaling of alluvial channel bankfull depths to discharge, a steep bank (i.e., near-vertical at the top of the bank but decreasing to approximately 45˚ near the toe) with an internal friction angle of 35˚ (typical for a loamy or clayey sand), near-saturated conditions, and a log-spiral potential failure surface were assumed. Near-saturated conditions are consistent with the fact that gravitational shear failure has been documented to occur most frequently during the falling limbs of flood discharges when pore pressures tend to be associated with near-saturated conditions (e.g., Casagli et al., 1999; Simon et al., 2000). Chen et al. (1969) derived N_s values for prescribed angles of the bank and of internal friction for unsaturated conditions. For a friction angle of 35˚, N_s values in Table 1 of Chen et al. (1969) decrease with increasing bank angle from $N_s = 22$ for a 60˚ bank to $N_s = 12$ for a 75˚ bank and $N_s = 7.5$ for a vertical bank. Hunter and Schuster (1968) limited their analysis to cases with no internal friction (hence their absolute N_s values are not applicable here) but documented an approximately 3-fold decrease in N_s values from unsaturated conditions (i.e., $M = h_w\gamma_w/h_c\gamma' \approx 1$, where is h_w is the depth to the water table below the top of the bank, γ_w is the unit weight of water, and γ' is the submerged unit weight of the bank material) to near-saturated conditions (i.e., $M = 0$). Combining the results of Chen et al. (1969) and Hunter and Schuster (1968) suggests that N_s values for a saturated bank with an internal angle of friction of 35˚ vary from approximately 7.3 for a 60˚ bank to $N_s \approx 4$ for a 75˚ bank and 2.5 for a vertical bank.

Bank material cohesion varies ~~approximately~~ linearly from 0 (for cohesionless sand) to ~~100~~approximately 90 kPa (for pure clay) for moisture contents in the range of 7 to 40% according to a least-squares linear regression of the data from Dafalla (2013) (Fig. S1~~). Adopting~~):

$$C \approx (900 \pm 70) p_c. \qquad (3)$$

where the units of C is Pa and p_c is percent. The uncertainty in Eq. (3) is the standard error (i.e., one standard deviation) resulting from the regression.

~~Combining Eqns. (~~2) and (3) and assuming a bulk density of 1700 kg m^{-3} and a representative value of $N_s \approx 6$ (corresponding to a ~~range of angles of the bank and of internal friction, including a vertical bank with an angle of internal friction of 25° (Table 1 of Chen et al., 1971)) results in an approximately linear relationship between the maximum stable bank height and the percent clay content,~~ ~~p_c, of the bank material (i.e., the floodplain deposits adjacent to the channel) with a~~near-saturated bank with an angle of approximately 65°, i.e., an average angle for a bank that is near-vertical at the top and decreases to an angle of approximately 45° near the toe) yields

$$h_c \approx 0.35 p_c. \qquad (4)$$

Absent site-specific data for bank angles, the largest source of uncertainty in the proportionality coefficient ~~of 0.35 (e.g.,~~ ~~$h_c \approx 35$ m for pure clay bank material):~~in Eq. (4) as applied to specific locations is likely the bank angle, since relatively modest variations in bank angle (e.g., from 90° to 75°) are associated differences in N_s of approximately a factor of 3 while other sources of uncertainty (e.g., between cohesion and clay content as quantified by Eq. (3)) are smaller. Section 4 provides discussion on how uncertainty in bank angle and other factors such as bank vegetation limit the precision of Eq. (4) to specific locations. The primary of objective of this paper, however, is to document an increase, on average, in bankfull channel depth with increasing clay content:

$$\cancel{h_c \approx} h \approx 0.35 \cancel{p_c.}$$

$$\cancel{(3)}\ p_c. \qquad (5)$$

~~The model of this paper posits~~assuming that bankfull depth is approximately equal to the maximum gravitationally stable bank height. ~~With that assumption, Eq. (3) becomes~~

$$h \approx 0.35 \cancel{p_c.} \qquad \cancel{(4)}$$

To test ~~Eq. (4~~the tendency for channel depth to increase, on average, with increasing bank material clay content as predicted by Eq. (5), the bankfull depths for 387 U.S.G.S. gaging stations in the MRB were estimated using the bends in the stage-discharge rating curves (Fig. S2~~).~~) for each station. Predictions of bankfull depth using Eq. (~~4~~5) were then compared to the observed bankfull depths derived from the rating curves. The gNATSGO soil database (Soil Survey Staff, 2019) was used to estimate the percent clay content of the floodplain deposits adjacent to each station. This analysis focuses on the MRB because there is no readily available soils database for any other continental-scale river basin that resolves floodplains in comparable detail and is based on a similar richness of field-based soil texture measurements.

The bankfull depth for each U.S.G.S. gaging station was estimated using the intersection of the linear regressions of peak annual gage height (used as a proxy for stage) to peak annual discharge obtained using the five smallest and five largest discharges in each record (shown as gray circles in the example of Fig. S2). This intersection, or bend, in the stage-discharge rating curve can identify the stage and discharge above which overbank flow occurs (Copeland et al., 2000), provided that the slope of the high-flow linear regression is much smaller than the slope of the low-flow linear regression~~.~~, which is a signature of the abrupt widening of flows as they transition from in-channel to overbank.

The analysis ~~of this paper~~presented here began by including data from all U.S.G.S. gaging stations in the MRB with available peak discharge data (U.S. Geological Survey, 2020). ~~In order to maximize the accuracy of the analysis, however~~Only those stations for which the slope of the high-flow linear regression is at least five times smaller than the slope of the low-flow linear regression were retained. In addition, only those stations that ~~met the following criteria were retained: 1)~~had at least 20 years of data, ~~2)~~have a contributing area larger than 100 km^2, ~~3) at least a factor of five decrease in the slope of the stage-discharge rating curve from low flows to high flows, 4) a low-flow linear regression that extrapolates to a flow stage that has an absolute value within 50% of the bankfull~~ are not located close to major infrastructure (based on an inspection of each station location in Google Earth imagery), and have a resulting bankfull depth of greater than 2 m were retained. Channels with bankfull depths less 2 m were removed because such channels tend to be associated with low clay contents that are inherently difficult to estimate in the field (see Sect. 3.1 for more detail on the potential bias associated with estimating low clay contents). In order to further filter out stations where the low-flow linear regression is potentially unrepresentative of the hydraulic behavior of in-channel discharges, only those stations for which the extrapolation of the low-flow linear regression is close to zero flow depth at zero discharge~~, and 5) a resulting bankfull depth greater than 2 m. A total of 387 stations met these criteria.~~ were retained. Stations for which the low-flow regression does not extrapolate to a flow stage close~~, i.e., within 50% of the bankfull depth,~~ to zero at zero discharge may ~~indicate that the~~ have gage height ~~is~~data that are not an accurate proxy for flow stage and/or ~~that~~have other data quality issues ~~or river management practices~~that preclude an accurate estimate of bankfull depth using the stage-discharge rating curve. What represents "close" should not be based on an absolute error, e.g., 0.5 m, because such a criterion would require that the low-flow regressions for deep channels be relatively more accurate than those for shallow channels. Here "close" was defined as being within 50% of the bankfull stage from zero. That is, if the bankfull stage is 5 m, then the extrapolation of the low-flow regression to zero discharge must yield a stage within 2.5 m of zero in order for that station to be retained in the analysis. Similarly, if the bankfull stage is 2 m, then the extrapolation of the low-flow fit to zero discharge must be within 1 m of zero. A total of 387 stations met these criteria.

To estimate the floodplain clay content for each of the 387 U.S.G.S. gaging stations, the depth-averaged percent clay content from soil depths of 5 to 150 cm was computed for every pixel within the MRB using gNATSGO. These depths were chosen to avoid the soil horizon close the surface (typically the O and/or A horizon, which ~~can~~may have clay contents reflective of surficial biological processes that are not representative of the rest of the profile) and because soil properties at depths greater than 150 cm can be inconsistently encoded in U.S. soil databases (Miller and White, 1998). A moving geometric mean (averaging distance of 10 km) of percent clay content was computed within floodplains mapped by Nardi et al. (2019). Because some U.S.G.S. gaging stations are located in channels with narrow floodplains that are not resolved in Nardi et al. (2019), the Nardi et al. (2019) floodplain map was augmented with single-pixel-width valleys defined by pixels with contributing areas larger than 100 km^2 following a steepest-descent routing of contributing area within the National Map Digital Elevation Model (DEM) (Archuleta et al., 2017). Bankfull depths predicted by Eq. (~~4~~5) were then compared to observed bankfull depths using a Pearson correlation coefficient, a root-mean-squared error (RMSE), the percentage of values correctly predicted to within a factor of 2, and a *p*-value that quantifies the likelihood of the null hypothesis that the predicted and observed bankfull depths may be correlated by chance.

To assess the potential impact of errors in measurements of percent clay content on predictions of the maximum stable bank height and therefore of bankfull depth using Eq. (~~4~~5), synthetic predictions for bankfull depths, $h_{\text{pred,syn}}$, were generated equal to 0.35 times samples of synthetic percent clay content, $p_{\text{c,syn}}$, drawn from a lognormal distribution designed to mimic the distribution of bankfull depths of U.S.G.S. gaging stations in the MRB, plus a normally distributed random error with a mean of zero and standard deviation of σ:

$$h_{\text{pred,syn}} = 0.35\left(p_{\text{c,syn}} + \sigma\eta\right)$$ (~~5~~6)

where η is a normally distributed random variable with a mean of zero and a standard deviation of 1. For σ = 0, Eq. (~~5~~6) produces synthetic data precisely equal to Eq. (~~4~~5). With finite values of σ, Eq. (~~5~~6) produces synthetic data with scatter that can be used to assess how errors in percent clay content may affect the relationships between observed and predicted bankfull depths.

2.2 Controls on depth-averaged water velocities and bankfull widths

The critical-flow hypothesis implies that bankfull Froude numbers, *F*, are limited to a relatively narrow range of values with an upper limit close to 1 for gravel-bedded channels and a similarly narrow but somewhat lower range of values for sand-bedded channels. In Sect. 3.2 it is demonstrated that this variation in *F* can be quantified using a power-law relationship between *F* and *Q*:

$$F \propto Q^n.$$ (~~6~~7)

The power-law exponents reported in this paper were determined via a least-squares linear regression of the logarithms of the data. The definition of Froude number provides a linkage among depth-averaged bankfull water velocities, bankfull Froude numbers, and bankfull depths:

$$v = F\sqrt{gh}.$$ (~~7~~8)

Equations (~~6~~7) and (~~7~~8) thus constrain the value of *m* to be

$$m = \frac{k}{2} + n.$$ (~~8~~9)

The exponent *b* in the relationship between bankfull width and discharge can be constrained by conservation of mass of water assuming steady, uniform flow, consistent with many previous models for the downstream hydraulic geometry of alluvial channels (e.g., Lindley, 1919; Smith, 1974; Ferguson, 1986; Huang et al., 2004; Julien, 2014):

$$b = 1 - k - m.$$ (~~9~~10)

2.3 Controls on along-channel slopes

The Darcy-Weisbach equation

$$v = \sqrt{\frac{8ghS}{f}},$$ (~~10~~11)

is based on conservation of energy for steady, uniform flow, i.e., that the gravitational potential energy per unit mass, ghS, produces a depth-averaged water velocity associated with a drag force per unit mass exerted by the channel bed on the water equal to $(f/8)v^2$, where f is the friction factor (e.g., Ferguson, 1986). Equation (~~10~~11) can be rewritten as

$$S = \frac{F^2}{8} f. \qquad (\cancel{11}12)$$

Here the Variable Power Equation (VPE) of Ferguson (2007) is used to quantify f:

$$(8/f)^{1/2} = a_1 a_2 \beta^{-1} / \left(a_1^2 + a_2^2 \beta^{-5/3}\right)^{1/2} \qquad (\cancel{12}13)$$

where a_1 and a_2 are coefficients (equal to 6.1 and 2.4 based on the least-squares minimum error for velocity in the calibration performed by Ferguson (2007)), and β is the relative roughness. The VPE equation was chosen because it provides a transition between the Manning-Strickler 1/3rd power scaling of friction factor to relative roughness for $\beta \gg 1$ (nearly all sand-bedded channels and some gravel-bedded channels) to a quadratic scaling when β is $\lesssim 1$ (channels with very coarse beds) that accords well with available data (Ferguson, 2007).

~~R~~elative roughness depends on whether or not bedforms are present. Table S2 of Ohata et al. (~~2019~~2017) identifies the range of F and d_{50} values conducive to ripple and/or dune development in alluvial channels. ~~By cross-referencing those results with the Dunne and Jerolmack (D&J) (2018) global dataset, Sect.~~Of the 3790 data points in the Ohata et al. (2017) dataset, 1574 (42%) have ripples or dunes, of which 19% are in the field and the remaining 81% are in the laboratory. By cross-referencing those results with the Dunne and Jerolmack (D&J) (2018) global dataset (i.e. by drawing a curve in F vs. d_{50} space that separates channels that have ripples and/or dunes from those that do not, and assuming the existence of ripples and dunes in channels of the D&J dataset that have F and d_{50} values that sit above and to the left (Fig. 3B) of the envelope curve separating channels with and without ripples and dunes in the Ohata et al. (2017) dataset), Sect. 3.3 demonstrates that 96% of sand-bedded channels in the D&J global dataset have F and d_{50} values conducive to ripple and/or dune development and therefore have a roughness that is likely dominated by bedforms rather than by bed sediment grains. The D&J global dataset includes 789 observations of d_{50}, S, and, h, and 711 observations of d_{50}, S, w, h, and Q drawn from the literature.

For gravel-(and-coarser)-bedded channels, relative roughness is defined in the calibration of Ferguson (2007) as the ratio of 84th percentile of bed grain diameter to the hydraulic radius. Since the analysis of this paper relates Eq. (~~12~~13) to data from the D&J global dataset that uses d_{50} instead than d_{84}, it is assumed that $d_{50} \approx d_{84}/2$ and, because $w > 10h$ for all points in the D&J global dataset, that the bankfull hydraulic radius is approximately equal to the bankfull depth. The relative roughness for gravel-bedded channels, β_g, in the D&J global dataset can, therefore, be approximated as

$$\beta_g \approx \frac{2d_{50}}{h}. \qquad (\cancel{13}14)$$

For sand-bedded channels, relative roughness can by estimated as (Bathurst, 1993):

$$\beta_s \approx \frac{1.1H(1-e^{-25\alpha})}{3h} . \qquad (\cancel{14}15)$$

where H is the bedform height and α is the ratio of bedform height to length, L.

Combining Eqs. (~~11)-(14~~12)-(15) gives an equation for the along-channel slopes of gravel-bedded channels consistent with conservation of energy:

$$S_\mathrm{g} = \frac{F^2}{a_1^2 a_2^2}\left(\frac{2d_{50}}{h}\right)^2\left(a_1^2 + a_2^2\left(\frac{h}{2d_{50}}\right)^{5/3}\right),$$ (~~15~~16)

and an analogous equation for sand-bedded channels:

$$S_\mathrm{s} = \frac{F^2}{a_1^2 a_2^2}\left(\frac{1.1H(1-e^{-25\alpha})}{3h}\right)^2\left(a_1^2 + a_2^2\left(\frac{3h}{1.1H(1-e^{-25\alpha})}\right)^{5/3}\right).$$ (~~16~~17)

The heights and lengths of ripples and dunes can be estimated as (Yalin, 1964):

$$H \approx \frac{h}{6}\left(1 - \frac{\tau_\mathrm{c}}{\tau_0}\right),$$ (~~17~~18)

and

$$L \approx 1000 d_{50}.$$ (~~18~~19)

Equation (~~16~~17), therefore, can be rewritten in terms of d_{50}/h for a prescribed value of α as

$$S_\mathrm{s} \approx \frac{F^2}{a_1^2 a_2^2}\left(\frac{18000\alpha(1-e^{-25\alpha})}{1.1\left(1-\frac{\tau_\mathrm{c}}{\tau_0}\right)^2}\frac{d_{50}}{h}\right)^2\left(a_1^2 + a_2^2\left(\frac{1.1\left(1-\frac{\tau_\mathrm{c}}{\tau_0}\right)^2}{18000\alpha(1-e^{-25\alpha})}\frac{h}{d_{50}}\right)^{5/3}\right).$$ (~~19~~20)

3 Results

3.1 Controls on bankfull depths

Figure 1 illustrates the tendency for the clay contents of floodplain deposits adjacent to many smaller channels in the MRB to be lower than those of larger channels. For example, the North and South Platte Rivers have typical floodplain clay contents <10%, while the Platte River has typical floodplain clay contents of ≈10-20%, and the Missouri and Lower Mississippi Rivers have typical clay contents of ≈20-30%. There are many relatively small channels, however, that have clay contents > 30% due to clay-rich local bedrock. As such, there isn't a precise, one-to-one correlation between clay content and contributing area or discharge, but rather a general tendency for channels conveying larger discharges to have more clay-rich floodplain deposits.

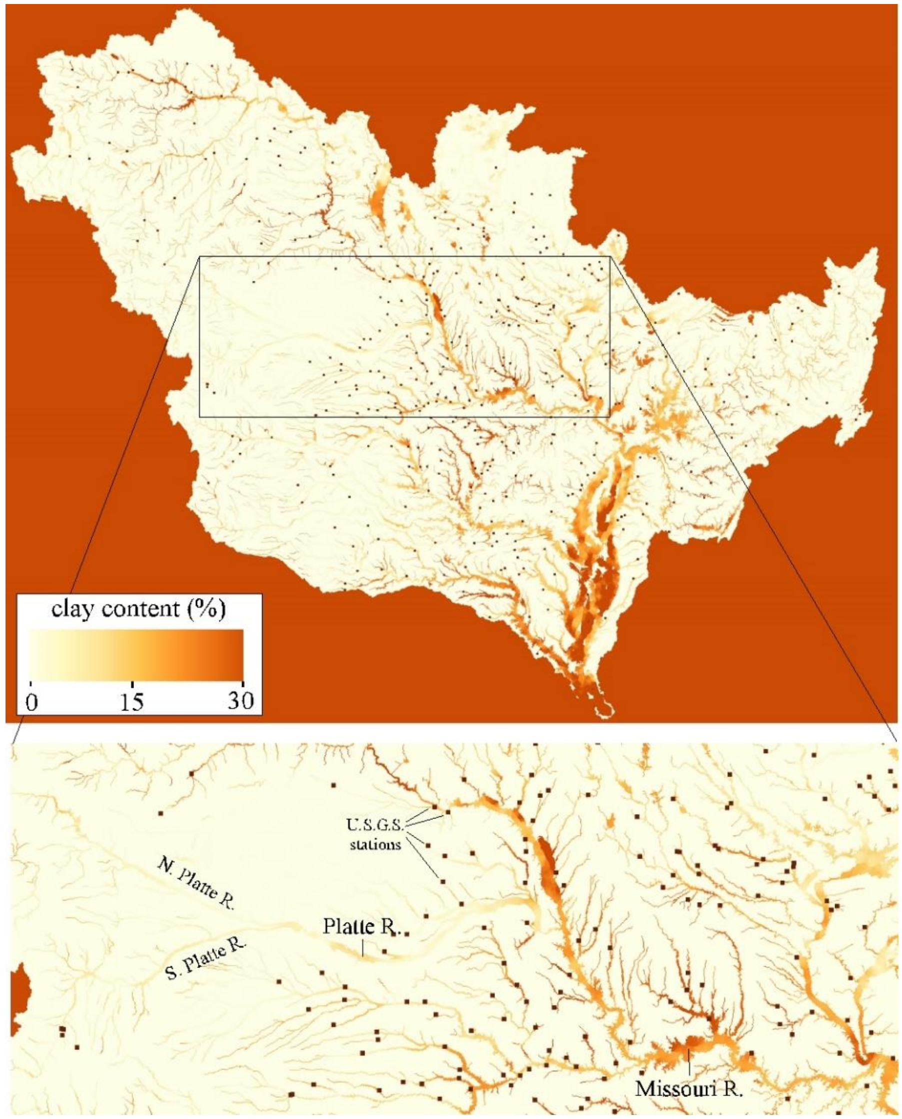
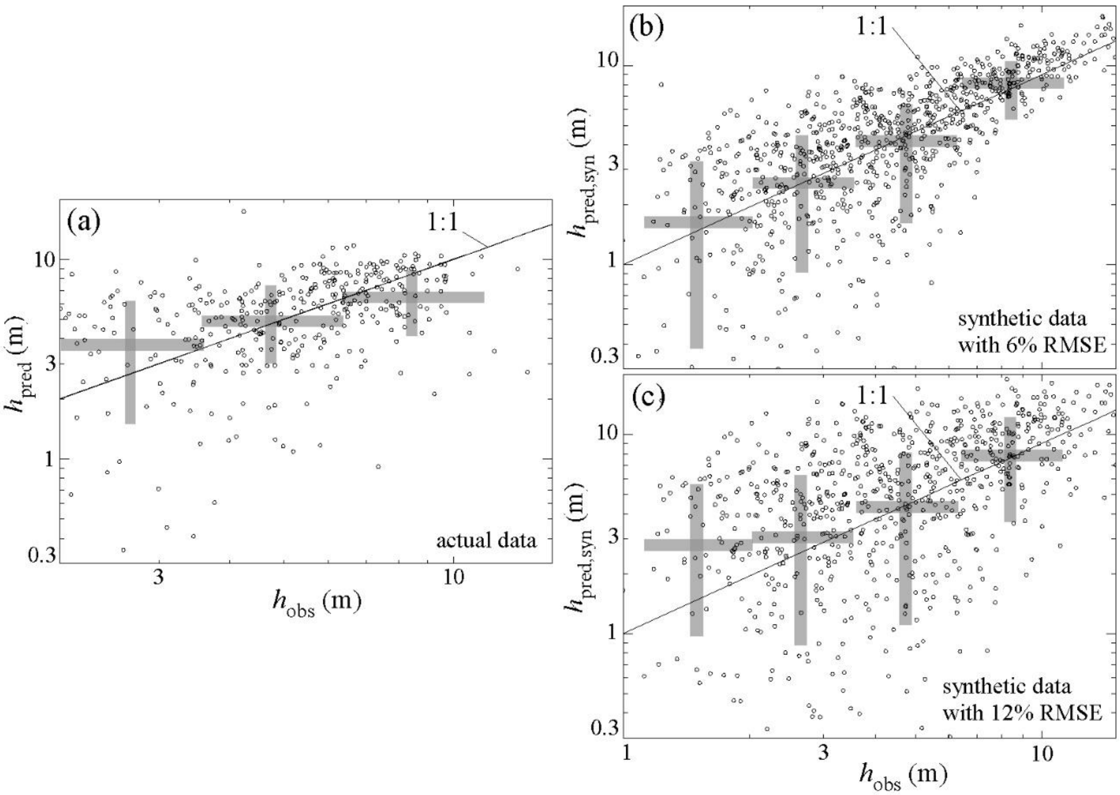


Figure 1: Color map of the average floodplain deposit clay content in the Mississippi River Basin. Also shown are the locations of the 387 U.S.G.S. gaging stations where predicted bankfull depths were computed using Eq. (45) and the observed bankfull depths were estimated based on the bends in the stage-discharge rating curves.

Figure 2(a) compares the bankfull depths predicted by Eq. (45) to observed bankfull depths estimated from the bends in the stage-discharge rating curves. Predicted bankfull depths have a Pearson correlation coefficient of 0.42 with observed bankfull depths, a RMSE of 1.7 m, and 84% of the data points are within a factor of 2 of the observed bankfull depth. The p value, i.e., the chance that the correlation between h_{pred} and h_{obs} could have occurred by mere chance, is $\sim 10^{-17}$.

Figures 2(b) and 2(c) illustrate the potential impact of errors in percent clay contents on predicted bankfull depths using Eq. (5) with $\sigma = 0.06$ and 0.12 (6 and 12%), respectively. These σ values were chosen because, while gNATSGO does not provide an error estimate, a recent soil property dataset created using machine learning algorithms has an estimated RMSE of 12% (Ramcharan et al., 2018) and a value half that size allows for the effect of error size to be assessed. For a relatively small error ($\sigma = 6\%$), there is a spread of values around the 1:1 line, with a larger relative spread for smaller clay contents, i.e., a 6% error results in a 100% relative error for a percent clay content of 6% (i.e.,

$h_{pred,syn} \approx 2$ m) but a 50% relative error for a percent clay content of 12% (i.e., $h_{pred,syn} \approx 4$ m). As the σ value increases to 12%, the spread of values around the 1:1 line increases as expected but $h_{pred,syn}$ values less than ~~a few~~approximately 2 meters also appear to be biased upward (i.e., the geometric mean of $h_{pred,syn}$ values deviate from the 1:1 line). ~~This bias may be a result of percent clay content being bounded at zero, thus retaining the spread of values on the high side of the expected value only. Systematic deviations of h_{pred} from h_{obs} at low clay contents (i.e., percent clay contents ≲ 10% and predicted bankfull depths ≲ 3 m) may, therefore, be due to biases associated with the difficulty of measuring low clay contents with high relative precision. It is for this reason that stations with observed bankfull depths less than 2 m were not retained in the analysis (Sect. 2.1).~~



This upward bias may be associated with the difficulty of measuring very low clay contents in the field. Clay contents estimated in the field can only be constrained to be within the range from 0 to 10% clay. If the actual clay content is close to 0 (e.g., < 1%), the clay content estimate is likely to be overestimated by a much larger fraction than would be the case for a larger clay content (e.g., 5% clay is 400% greater than 1% clay but only 50% lower than 10% clay). The result may be an upward bias in clay content for clay contents less than approximately 10%, which, according to Eq. (5), may be associated with channels less than 2-3 m in bankfull depth. Channels with bankfull depths less than 2 m (Sect. 2.1) were excluded from the analysis due to this potential bias.

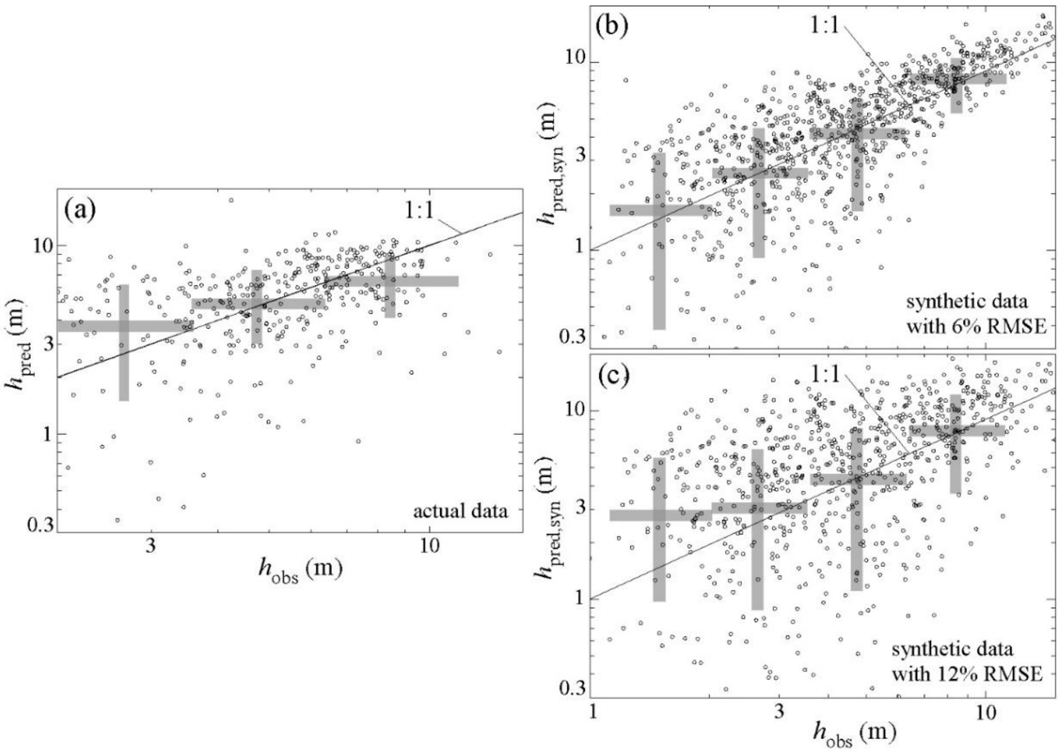


Figure 2: Plots of predicted bankfull depths, h_{pred}, calculated using Eq. (~~4~~5) versus observed bankfull depths, h_{obs}, estimated based on the bends in the stage-discharge rating curves. (a) Actual data. (b)&(c) Synthetic data (Eq. (~~5~~6)) with (b) σ = 6% and (c) σ = 12%, respectively. The small open circles are individual data points and the gray rectangles illustrate the geometric means and standard deviations within each logarithmically spaced bin. The gray rectangles are for visualization purposes only – no analyses are performed on these values.

3.2 Controls on depth-averaged water velocities and bankfull widths

Ripples and dunes tend to form at lower Froude numbers in channels with finer bed sediments (Fig. S3). The curve in Fig. S3 used to identify the range of F and d_{50} conditions conducive to ripple and/or dune development is reproduced in Fig. 3, where 96% of sand-bedded channels in the D&J global dataset are below the upper limits of F and d_{50} conducive to ripple and/or dune development identified using the Ohata et al. (~~2019~~2017) dataset. As such, it is assumed for the purposes of this analysis that sand-bedded channels in the D&J global dataset are dominated by bedform roughness while gravel-bedded channels in the D&J global dataset are dominated by grain roughness.

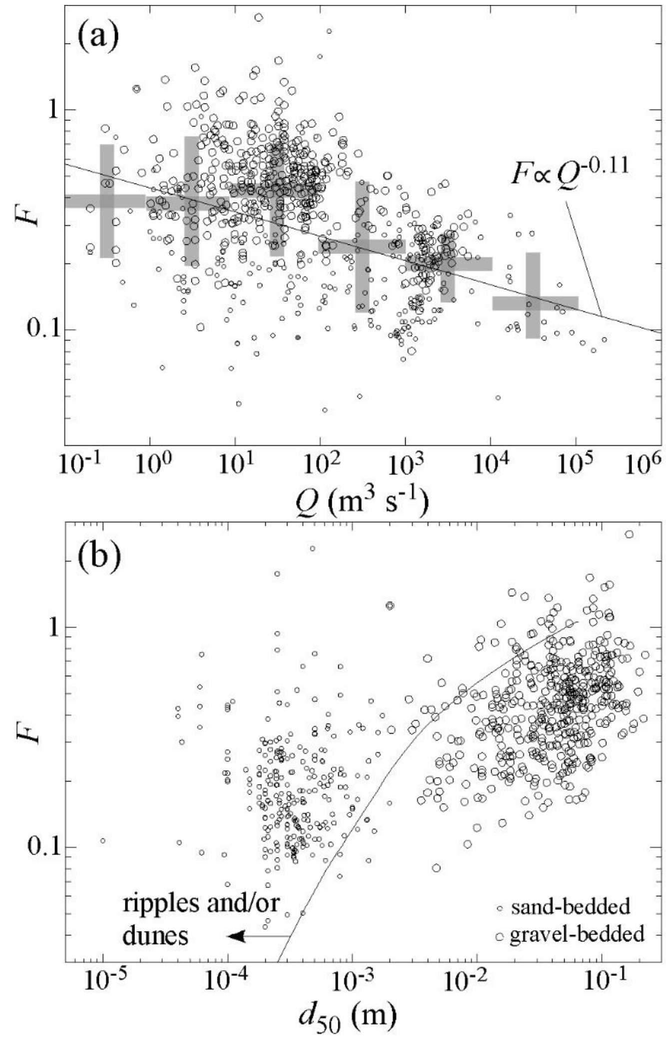


Figure 3: Bankfull Froude number relationships with bankfull discharge and median bed grain diameter. Plot of bankfull Froude number, *F*, as a function of (a) bankfull discharge, *Q*, and (b) median grain size, d_{50}, for the Dunne and Jerolmack (2018) global dataset. Small circles correspond to sand-bedded channels (d_{50} < 2 mm), large circles correspond to gravel-bedded channels (d_{50} > 2 mm). The curve in (b) defining channels that likely have ripples and/or dunes is based on the subset of 3791 field studies and experiments compiled by Ohata et al. (~~2019~~2017) and graphed in Fig. S3.

Using the D&J global dataset, alluvial channel depths and widths scale with bankfull discharges to the 0.402 ± 0.006 and 0.512 ± 0.007 powers, respectively (R^2 = 0.86 and 0.89) (Fig. 4). If we take the -0.116 ± 0.009 Froude-number-discharge scaling exponent obtained from least-squares regression to the logarithms of the data (Fig. 3(a)) and the 0.402 ± 0.006 depth-discharge scaling exponent as a starting point, Eqs. (~~8~~9) and (~~9~~10) predict a width-discharge exponent of 0.51 ± 0.01, i.e., precisely equal to that observed in the D&J global dataset.

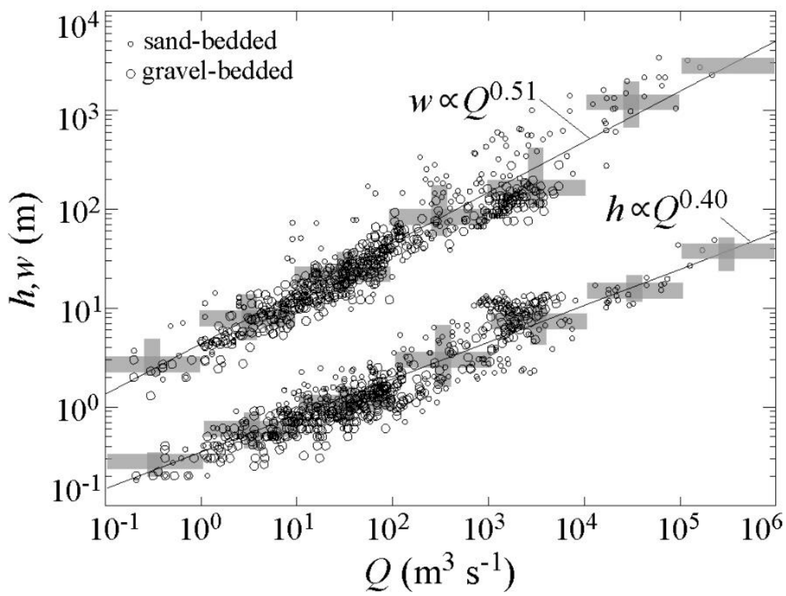


Figure 4: Plots of bankfull depth, *h*, and width, *w*, as a function of bankfull discharge, *Q*, from the Dunne and Jerolmack (2018) global dataset, along with least-squared linear regressions of the logarithms of the data, indicating *b* = 0.512 ± 0.007 and *k* = 0.402 ± 0.006 for this dataset.

3.3 Controls on along-channel slopes

Along-channel slopes predicted by Eqs. (~~15~~16) and (~~19~~20) are consistent with observed values in the D&J global dataset (Pearson correlation coefficient of 0.77) (Fig. 5). For sand-bedded channels, in which ripples and/or dunes are likely to be the dominant roughness elements, the predicted values plotted in Fig. 5 assume $\tau_c/\tau_0 \approx 0$ (consistent with the suspended-load-dominated conditions common in sand-bedded channels (Dade and Friend, 2000)) and a representative α value of 0.05 (based on the range 0.05-0.1 reported by Guy et al. (1966)). Figure S4 illustrates the sensitivity of the predictions of the along-channel slopes of sand-bedded channels to the presence/absence of bedforms and the assumed value of α. For alternative scenarios in which a) an unrealistically large value α = 0.25 is assumed, and b) bed grains are assumed to be the dominant roughness elements (i.e., no bedforms are present), Eqs. (~~15~~16) and (~~19~~20) predict along-channel slopes that are approximately an order of magnitude above and below observed values, respectively.

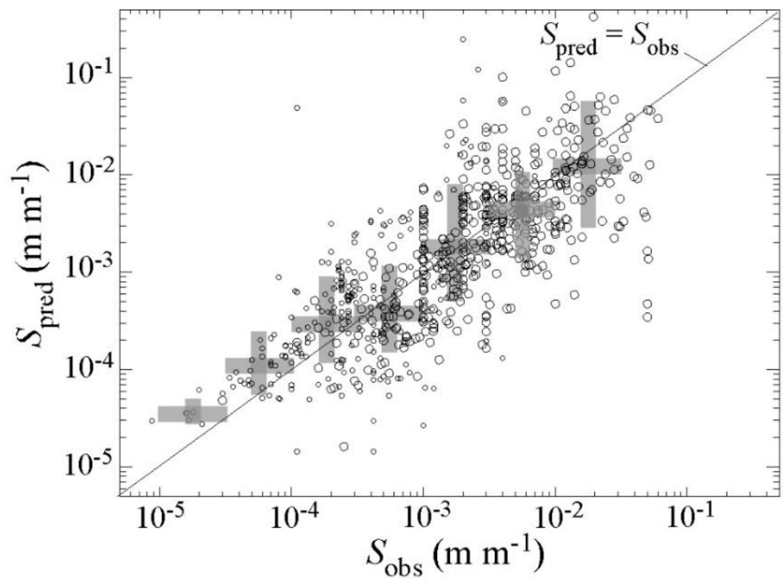


Figure 5: Plot of predicted along-channel slope, S_{pred}, using Eqs. (~~15~~16) and (~~19~~20) as a function of observed along-channel slope, S_{obs}, in the Dunne and Jerolmack (2018) global dataset. Predicted values for sand-bedded channels assume $\tau_c/\tau_0 \approx 0$ (consistent with the suspended-load-dominated conditions common in sand-bedded channels (Dade and Friend, 2000)) and $\alpha \approx 0.05$ (Guy et al., 1966).

4 Discussion

The model of this paper posits that floodplain deposit clay contents partially control the maximum stable heights of channel banks~~, but~~. This control is ~~not likely~~unlikely to result in a precise correlation between clay contents and bankfull channel depths for at least two reasons besides data errors. First, the incised depth of a channel that flows through a section of higher clay content to a section of lower clay content may be more strongly controlled by the lower clay content because the downstream reach may act as the local base level of erosion for the upstream reach. Second, alluvial channels can adjust to spatial variations in bank material cohesion by varying the bank angle in addition to the bank height (Knighton, 1974). For a 35˚ angle of internal friction, for example, a bank angle of ~~45˚~~ 60˚ has a stability factor, N_s, that is approximately ~~ten~~three times larger than a vertical bank with the same cohesion (e.g., Fig. 3 of Chen et al~~., 1971).~~. (1969)). As such, an alluvial channel that flows through a section with less cohesive bank material compared to neighboring ~~reaches~~sections may adopt a less steep bank (thus increasing the stability factor, N_s) instead of, or in addition to, becoming wider and shallower in order to minimize variations in channel depth that might otherwise drive large spatial variations in rates of aggradation/incision. Further tests of the model of this paper may require a better understanding of how channel depths and/or bank angles adjust to spatial variations in bank material cohesion through a channel network.

It is also important to consider how potential errors in the data may contribute to the observed scatter in Fig. 2. The estimates of bankfull depths presented here are not exact because gage height (the height of the water above a reference point) is used as a proxy for flow stage. Also, uncertainties in clay content of just 5-10% percent are capable of creating scatter comparable to that in Fig. 2. Despite such large potential errors and the bias they may introduce into predictions of bankfull depths, the analysis presented here rules out the possibility that floodplain deposit clay contents and bankfull depths are related by chance (i.e., $p \sim 10^{-17}$) and it demonstrates that Eq. (~~4~~5) predicts bankfull depths to within a factor of 2 of the observed bankfull depths for 84% of the 387 stations included in the analysis.

The model of this paper posits that bankfull depth may be self-regulated via a tendency for an increase in bank height caused by channel incision and/or floodplain deposition to trigger bank failure when a critical bank height, dependent on bank material cohesion, is exceeded. ~~This proposed self-regulatory mechanism does not require that~~An important role for gravitational ~~shear~~ failure ~~be the dominant mechanism of bank retreat at all times (e.g., such failure may be more prevalent during periods of active incision following major climatic changes), but it does require that gravitational shear failure play an important role~~ in ~~bank retreat. Such a role~~controlling alluvial channel geometry is consistent with process-based studies of bank retreat, in which ~~bank retreat by~~ gravitational shear failure has been documented to occur frequently during the falling limbs of flood discharges when pore pressures tend to be highest (e.g., Casagli et al., 1999; Simon et al., 2000). An important role for shear failure in bank retreat is also consistent with Li et al.'s (2015) finding that bankfull depth correlates with fluid viscosity because viscosity controls the permeability of bank material and hence pore pressures and therefore susceptibility to gravitational shear failure. Fluvial erosion of the bank toe is still necessary to remove material slumped from the bank into the channel and likely plays an important role in driving bank retreat in channels with cantilevered banks (Pizzuto, 1994). However, if the bank is more than twice the height of the zone of scour, gravitational shear failure nevertheless will be the process by which the majority of material is removed from the bank (Tao et al., 2019). That said, gravitational failure and fluvial shear stresses act in concert in such a way that identifying which process is dominant may be difficult. Fluvial scour transports slumped material away from the bank, likely keeping bank angles higher than they would be without fluvial scour. Gravitational failure transports material from higher on the bank to the toe (often following fluvial scour, which is maximized at the toe), likely keeping bank angles lower than they would be without gravitational failure. More research is needed, but the relative importance of these two processes may be reflected in the bank angle, i.e., a bank angle persistently much less than vertical may indicate the dominance of gravitational failure while a bank angle persistently at or above 90° (i.e., a cantilever or overhang) may indicate the dominance of fluvial scour.

The model of this paper is simplified in at least two specific ways that bear mentioning: it does not account for tension cracks that, if present, can lower the maximum stable height below that predicted by Eq. (2) (Darby and Thorne, 1994), nor does it account for the role of vegetation in bank stability, which can increase bank heights by at least a factor of two over values predicted using bank material texture alone (Huang and Nanson, 1998).

T~~wo~~Bank vegetation plays a significant role in controlling bank stability, but it is unlikely that such control is responsible for the scaling relationships that are the focus of this paper, as such scaling relationships exist across climatic regions with very different vegetation characteristics. In addition, the stability of any bank is primarily a function of its weakest portion or layer, i.e., failure is more likely to occur in a zone of low shear strength compared to a zone of high shear strength, all else being equal. Failure of a weaker zone underlying a stronger zone can create a cantilever, but such cantilevers cannot continue to grow indefinitely, hence rates of long-term retreat in stronger and weaker zones of the same bank will tend to be similar. This, together with the fact that vegetation is likely to strengthen just the uppermost approximately 2 m of channel banks (globally, >99% of roots are found in the uppermost 2 m of the soil (Jackson et al., 1996)), suggests that bank material shear strength is unlikely to be controlled primarily by vegetation in channels deeper than 2 m. A dependence of bank retreat rates on vegetation has been documented in many studies (e.g., Ielpi and Lapôtre, 2020). However, such studies may not fully account for the fact that wetter climates with more vegetation also tend to have more clay-rich soils, leaving open the question of whether it is bank vegetation or material texture that is most responsible for bank resistance to erosion.

Four additional assumptions of the model should be emphasized. First, the model does not account for tension cracks that, if present, can lower the maximum stable height below that predicted by Eq. (2) (Darby and Thorne, 1994). Second, the model assumes a maximally incised channel, i.e., a channel that has incised to the point of reaching the threshold of bank stability quantified by Eq. (2). Quaternary climatic changes have driven cycles in which channel aggradation has been followed by a positive feedback of incision and channel narrowing (e.g., Bull, 1991) that have likely made many alluvial channels prone to an incised state. Such considerations, however, do not apply to some types of alluvial channels, e.g., small-scale channels formed in the laboratory and those in which sediment supply is not heavily influenced by climatic changes. ~~Second~~Third, the model of this paper involves no explicit constraint on the width-to-depth ratio of alluvial channels despite the fact that such a constraint may play an important role in some cases. An important concept in rill erosion is that unit stream power is maximized for a width-to-depth ratio of $\approx$ 0.5-3, with the specific value dependent on the cross-sectional functional form (e.g., Moore and Burch, 1986). A similar concept may limit incision in channels with small width-to-depth ratios (Huang and Nanson, 2000), reducing the likelihood of channels with $w/h \lesssim 1$ and hence potentially limiting h to values smaller than that set by Eq. (2) for channels with small discharges. Such a control does not seem likely in the channels of the D&J global dataset (since $w/h \gtrsim 10$), but it may play an important role in some types of channels and should be explicitly considered in future research. Fourth, the analysis assumes that the dominant sources of roughness in the channels of the D&J data are ripples/dunes or gravel clasts. Long-wavelength topographic features such as bars and meanders are not likely to be dominant roughness/drag-inducing elements given that the presence/absence of the flow separation that tends to dominate drag depends sensitively on the maximum slope of bedforms and other obstacles to the flow, with slopes in excess of 0.2 m/m generally needed to trigger flow separation (e.g., Lefebvre et al., 2014). Vegetation can certainly be a dominant source of roughness on the beds of ephemeral channels, however, and some of the scatter in the analysis of this paper may be a result of vegetation-induced bed roughness.

More data on the relationship between bank cohesion and bank material texture is needed. This study used clay content as a predictive variable for cohesion in part because a transfer function is available between clay content and cohesion based on the work of Dafalla (2013~~).~~) used in Figure S1. However, silt content also affects cohesion (Huang et al., 2006) and the findings of Schumm (1960) suggest that both the silt and clay content of bank material are relevant to understanding alluvial channel geometry. The specific surface area of bank material may be a more accurate predictive variable for cohesion than either clay content or silt and clay content (weighted equally), as specific surface area includes both the silt and clay contents but weighs the presence of clays more heavily (Huang et al., 2006).

The analysis presented here used a power-law relationship to quantify the relationship between F and Q (Eq. (~~6~~7)). However, a scaling break appears to exist in the D&J global dataset (Fig. 3(a)), with F values approximately constant for Q values less than ~10^2 m^3 s^{-1} (i.e., discharges dominated by gravel-bedded channels). This break may be consistent with the critical-flow hypothesis, i.e., steep, predominantly gravel-bedded channels in the D&J global dataset may have F values predominantly in the range of 0.3-1, independent of channel size/discharge, because the general lack of ripples and dunes in gravel-bedded channels requires that the increase in drag near critical flow conditions be caused primarily by wave drag, while in sand-bedded channels the increase in drag near critical-flow conditions is likely to be caused by ripples and/or dunes that form at a range of Froude numbers weakly dependent on bankfull discharge. It would be useful for future research to investigate the relationships among F, Q, bed texture, and the presence/absence of ripples and/or dunes to more fully test this hypothesis and its implications for potential breaks in scaling within Eq. (1).

The model of this paper implicitly includes the sediment-supply control on along-channel slope documented by Li et al. (2005), Pfeiffer et al. (2017) and Blom et al (2017). Drainage basins with higher rates of sediment supply erode faster, resulting in coarser sediments being delivered to channels, all else being equal (Attal et al., 2015). Coarser sediments increase along-channel slopes because steeper slopes are necessary to achieve critical or near-critical water velocities in alluvial channels with coarser bed sediments (Eqs. (~~15)&(19~~16)&(20)).

5 Conclusions

This paper proposed that the bankfull depths of alluvial channels may be partially controlled by the maximum heights of gravitationally stable channel banks, which depend on bank material cohesion and hence on clay content. Bankfull depths predicted by a bank-stability model correlate with the observed bankfull depths estimated using the bends in the stage-discharge rating curves for 387 U.S.G.S. gaging stations in the Mississippi River Basin. It was proposed, ~~following~~inspired by the critical-flow hypothesis of Grant (~~2000~~1997), that depth-averaged water velocities scale with bankfull depths as a result of a self-regulatory feedback among water flow, relative roughness, and channel-bed morphology that limits velocities to be within a relatively narrow range associated with Froude numbers that have a weak inverse relationship to bankfull discharge. Given these constraints on channel depths and depth-averaged water velocities, bankfull widths and along-channel slopes consistent with observations follow from conservation of mass and energy of water flow. The model of this paper provides a novel process-based understanding of the hydraulic geometry of alluvial channel networks. Section 4 identified research needs that would enable a more comprehensive test of the hypotheses and a better understanding of the bank-textural controls on the hydraulic geometry of alluvial channels more generally.

Data Availability The Supplementary Material contains all of the data used in the paper not published elsewhere.

Competing interests The author declares that he has no competing interest.

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Table 1: List of variables

Symbol	Variable Name/Description	Units	Reference value(s)
a_1	coefficient in VPE model		6.1
a_2	coefficient in VPE model		2.3
α	ratio of ripple or dune height to length		0.05
b	exponent in power-law scaling of bankfull width and discharge		0.51
β	roughness ratio		
β_g	roughness ratio for gravel-bedded channels		
β_s	roughness ratio for sand-bedded channels		
C	bank material cohesion	kPa	0-100
d_{50}	median bed grain diameter	mm	
η	normally distributed random variable		
f	friction factor		
F	bankfull Froude number		
g	acceleration due to gravity	$m\ s^{-2}$	9.81
γ_w	unit weight of water	$kN\ m^{-3}$	9.81
γ'	submerged unit weight of soil	$kN\ m^{-3}$	10
h	bankfull channel depth	m	
h_{pred}	bankfull channel depth predicted by Eq. (4)	m	
$h_{pred,syn}$	synthetic bankfull channel depth predicted by Eq. (5)	m	
h_{obs}	observed bankfull channel depth	m	
h_c	maximum stable bank height	m	
h_w	depth of water table below top of bank	m	
H	bedform height		
k	exponent in the power-law relationship between bankfull depth and discharge		0.40
L	bedform spacing		
m	exponent in the power-law relationship between velocity and bankfull discharge		0.1
M	saturation parameter in Hunter and Schuster (1968)		0-1
n	exponent in the power-law relationship between bankfull Froude number and bankfull discharge		-0.11
N_s	stability parameter in bank mass failure equation		6
p_c	percent clay content in bank material		
$p_{c,syn}$	synthetic percent clay content in bank material		
Q	bankfull discharge	$m^3\ s^{-1}$	
ρ	bank material bulk density	$kg\ m^{-3}$	1700
S	along-channel slope		
σ	standard deviation of synthetic percent clay content		0.06, 0.12
τ_c	boundary shear stress threshold for entrainment	Pa	
τ_0	boundary shear stress	Pa	
v	depth-averaged water velocity	$m\ s^{-1}$	
w	bankfull channel width	m	
z	exponent in the power-law relationship between along-channel slope and bankfull discharge		-0.4