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Interactive comment on "How Hack distributions of rill networks contribute to nonlinear slope length–soil loss relationships" *by* Tyler H. Doane et al.

Anonymous Referee #3

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Comments on:
How Hack distributions of rill networks contribute to nonlinear slope length-soil loss relationships

1 Review Recommendation

Overall, the paper is of interest and, to a significant degree, addresses the two main goals of (1) exploring the non-linearity of the slope length-sediment output relation and (2) deriving Hack's law for Scheidegger networks. The main reason that this reviewer found the results of the analysis to be convincing lies in the excellent comparisons between the theoretical results and the numerical results from the simulations, as illustrated in figures 4, 5, 6, and 7. While these figures probably reflect an appropriate theoretical background, it is possible that they reflect the robustness of the modelling approach.

The paper as it stands could be significantly improved with some rewriting and, as such, would be quite publishable. Since the primary goal is to demonstrate and understand the non-linearity of the length-sediment relation for a rilled surface, it would have been useful to have first discussed the relation for an unrilled surface. There are clearly reasonable sediment transport laws that lead to a non-linear response in such a case, especially since the existence of rilling indicates that the flow regime is characterized by erosional instabilities. It would then be important to attempt to explain any differences between the planar and rilled flows.

It would be useful to make a little section for related literature, and especially to add a comprehensive summary of all of the relevatn findings of Dodds and Rothman since they are employed/referred to so extensively.

The section on network geometry could be greatly improved for ease of understanding and readability by shortening and simplifying. It would be simpler to state Hacks law in deterministic form, state its inverse in deterministic form, and note that the goal is to derive its probabilistic representation from the theory of random walks in a Scheidegger network. The derivation of equation 7 as currently written, could then be simplified and disambiguated by

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Discussion paper

1. noting that width is the difference between two normally distributed and independent RVs and must be positive;
2. stating the ICs at $s = 0$, presumably as two rill sources separated by a distance $w = w_0$;
3. noting that the distribution of the difference of two independent normal distributions $z_1 - z_2$ with means (μ_1, μ_2) and variances (σ_1, σ_2) is itself a normal distribution with mean $\mu_1 - \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$;
4. noting that the first passage of a random walk with a normally distributed RV with diffusion coefficient D starting at $w = w_0$ is given by the Rayleigh distribution

$$f(w, s) = \frac{w}{4\pi D s^3} e^{-w^2/4Ds} \tag{1}$$

since, as written, the description is not clear, equation (5) is confusing, while the brief discussion of the diffusion (6) and Fokker-Planck equations will not be helpful to many readers.

1.1 Typos, etc.

1. Line 12 understanding Hack's
2. Line 34 geometrical (not topological)
3. Line 169
4. Line 197 the this
5. Line 286 , for ;
6. Line 330 length(en)

7. Line 343 geometrical (not topological)
8. Line 358 distribution a

Figure 12B needs some attention. Its not clear what its meant to be (as seen by this reviewer.)

2 Comments Concerning Longer-Term Improvements

The paper could be greatly improved if the authors are willing to put more work into both their numerical simulations and into their derivations, with the possibility of authoring two publishable papers. In terms of this option, a possible approach might be break the research into a subproject relating to each of the two main goals, with

1. the first subproject exploring the non-linearity of the relation in greater depth by running a large number of simulations over different experimental conditions and inferring the desired relations from the data;
2. the second subproject deriving Hack's law for Scheidegger networks with a deeper theoretical than is currently presented.

The first paper, for example, could describe the results of experiments that are run over a variety of rill geometries such as trapezoidal, triangular, and semi-circular etc. and over different erosion rules. This should led to interesting results, especially if variations in the results are related to different geomorphic variables.

The second paper could, for example, explore a somewhat more general approach to deriving a probability density function for the area-length relation by considering a a rilled slope of unbounded lateral extent (or some approximation) and considering an ensemble of rills. A problem with the current approach of deriving this function from the difference of just two neighboring ridges is that the two adjacent rills that define a ridge at some point s are not necessarily the ones that merge to close off the ridge since they might merge with their other adjacent rills. Intuitively this model would appear to lead to a different probability density function for area than the extended slope, with an increased probability of large and unrealistic widths. It is not clear, therefore, to this reviewer that this is the most appropriate model for deriving the Hack's law although its probably a good approximation. It is also possible that investigating an extended slope might lead to similar results to employing just two ridges or channels, but that would, of itself, be an interesting finding.

Another issue that could be explored in a second more theoretical paper is to work with a probablistic flow and transport theory to match the probablistic geometric theory. As currently written, the paper mixes probabilistic reasoning with deterministic reasoning in a somewhat simplistic manner. The geometrical modeling is probabilistic, while the modeling of the flow and erosion dynamics is deterministic. The problem with this approach is that the randomness in the whole system is driven entirely by the randomness in the geometry, which is not an entirely convincing assumption.