ESD response to the report of Professor Furbish

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We would like to thank Professor Furbish for a thorough and encouraging review. His comment about riverbed collisions was an eye-opener. Below we address all the issues raised in the report.

1. **APPLICATION OF THE MODEL TO ABRASION IN RIVERS** We propose to add a new subsection, after subsection 2.4, where we discuss how our model can be explored to make predictions about fluvial environments. Here follows the suggested subsection:

Fluvial abrasion

Here we interpret the intuitive picture of fluvial abrasion in the context of our statistical model. In our model a fluvial environment may be represented by a *fluvial population*, consisting of $N+1$ particles: a very large number (N) of small particles X^i ($i = 1, 2, \dots N$) representing the pebbles carried by the river and one very large particle Y representing the riverbed. Such a scenario can not be explored directly in the context of our continuum model, however, as we will discuss in detail in Subsection 3.3, the discrete model can capture this situation even in the limit as $N \to \infty$.

To make a meaningful characterization of geologically relevant scenarios, we will regard two extreme cases which represent brackets on geological processes. In both cases we assume that the mass evolution is driven by binary collisions and we regard the limit as $N, Y \to \infty$. Since we are interested in the mass evolution of pebbles (and in the current paper we are not interested in the mass evolution of the riverbed) we will denote the relative variance of the pebble population (i.e. all X^i particles, the riverbed Y not included) by $R(t)$. Our aim is to establish the sign of $R_t(t)$ as the main qualitative feature of collective dynamics.

In the first extreme scenario we assume that particles are chosen uniformly from the full *fluvial population*: i.e., the riverbed has no special role. In this case *almost all* collisions will happen among a pair of small particles (X^i, X^j) thus the presence of the riverbed has no impact on the evolution of $R(t)$. For this extreme case all predictions of our continuum model remain valid: $r = 0.5$ will be a critical parameter value above which we see focusing ($R_t < 0$), below which we see dispersing ($R_t > 0$) behaviour. At the critical value $r = 0.5$ our model predicts neutral behaviour with $R_t = 0$.

In the second extreme scenario we assume that the small particles *only* collide with the riverbed (large particle), i.e., we only have (X^i, Y)-type collisions. This means that the evolution for each of the small particles is an identical process, controlled by the binary collision law (1). In the $Y \to \infty$ limit each individual small particle X^i will thus evolve as

$$X^i(t) = X^i(0)e^{-t} \tag{9}$$

and thus follow Sternberg's Law. It is easy to show that for any initial distribution for the masses $X^i(0)$, in this process we have $R_t = 0$. The large Y-particle (riverbed) will lose some mass as well but in this publication we are not interested in that part of the process.

Intuitively it is clear that any geologically relevant process is in-between the above two extreme cases and, although we do not deliver a rigorous proof, it appears plausible that in a geologically relevant setting R_t will be also bounded by the two evolutions predicted for the two extreme scenarios. As for the second extreme scenario we have $R_t = 0$ we expect that for any intermediate scenario the sign of R_t will agree with the sign of R_t based on the first extreme scenario. This would imply that all our qualitative predictions remain valid in fluvial environments.

2. **ESTIMATE FOR EXTREMAL r VALUE**. We propose to add a new Appendix where we explain our estimate for the parameter r in case of smooth gradient flows (laminar flows). Here follows the suggested Appendix:

Estimating physically possible values of r

In the paper we assumed that the particle collision probability depends on the volume of the particles as

$$P(X) \propto X^r. \tag{A24}$$

Here we investigate two extreme scenarios, associated with the collision probabilities $P_{\text{smooth}}(X)$ and $P_{\text{turbulent}}(X)$ where we expect r to assume its extremal values.

The first is the smooth gradient flow. In such a case the driving fluid has a strong but on a particle size scale constant velocity gradient in one of the spatial directions. Such situations may arise e.g. in shallow water layers. In such a case the relative velocity of the particles grows with the distance. So if we are at distance u from the center of the particle in the direction of the flow velocity gradient, the collision probability $P_{\text{smooth}}(X)$ can be estimated by the product of the velocity difference and the linear cross section of the particles (Note that $R \equiv X^{1/3}$ is the linear size of the particle):

$$P_{\text{smooth}}(X) \sim \frac{1}{R}\int_0^R u\sqrt{R^2-u^2}du = \frac{1}{3}R^2 = \frac{1}{3}X^{2/3} \tag{A25}$$

Based on (A24), this gives us an estimate for high $r = 2/3$.

The other case is a fully chaotic motion where the equipartition takes place [1]. Thus the kinetic energy of the particles ($\frac{1}{2}\rho X v^2$) is independent of their volume. Thus the speed of the particles must be proportional to $X^{-1/2}$. If we disregard correlations the particles have a cross section proportional to their projected area which is proportional to $X^{2/3}$. Combining the two gives us

$$P_{\text{turbulent}}(X) \sim X^{-1/2}X^{2/3} = X^{1/6} \tag{A26}$$

and based on (A24) we obtain $r = 1/6$. Thus it is possible to have physical scenarios apparent in nature where the value of r falls to either side of the critical value of $r_c = 0.5$ with large enough margin.

3. **DIFFUSION**

The referee is right, we have used the term 'diffusion' incorrectly. We modified the text, the new version is below.

Note that, contrary to the majority of Fokker-Planck models, our model contains solely the advection term, which readily follows from the deterministic nature of the kernel. Here we aim to figure out the collective behavior implied by (5). Nonetheless, a stochastic kernel would produce diffusion in the master equation, such a generalization might be essential for testing model predictions against experimental data.

References

[1] Uberoi, Mahinder S: Equipartition of energy and local isotropy in turbulent flows, *J. Applied Physics*, 28(10),1165–1170, 1957