Notes on Miller-Jerolmack Particle Attrition

This is not a formal review of this paper. Rather, please view the comments below as notes to myself, aimed at my trying to understand elements of the analysis.

The experiments and analyses reported in this paper are clever.

1) Because in physics and mathematics a "double pendulum" has very specific meaning and significance,

https://en.wikipedia.org/wiki/Double_pendulum ,

the authors might consider using "dual pendulum" or "paired pendulum."

2) The presentation by Kok (2011) is a bit misleading. A value of -2 is not claimed by Gilvarry and Bergstrom (1961), and Astrom (2006) actually focuses on fracture size, not fragment size. Eq. (1) in Kok (2011) is a power-law distribution. His Eq. (2) is sort of like a Weibull distribution, but is not. Moreover, it is incorrect for Kok (2011) to say that N_f represents a number of fragments. It is in fact a number density.

Here is my take on where Eq. (2) in Kok (2011) comes from. To simplify notation, let $N_f(D_f) \to n_x(x)$ with $D_f \to x$. Now let $f_x(x)$ denote the probability density function of the fragment sizes x, and let N denote the total number of fragments. Then

$$n_x(x) = N f_x(x) \tag{1}$$

is the number density of fragment sizes. Now

$$\mathrm{d}n_x(x) = N f_x(x)\,\mathrm{d}x \tag{2}$$

is interpreted as the number of fragments within the small interval x to $x + \mathrm{d}x$. (Note that this wording is consistent with that of Gilvarry (1961).)

We now write

$$y = \ln x \tag{3}$$

In turn,

$$\mathrm{d}y = \mathrm{d}(\ln x) = \frac{\mathrm{d}x}{x} \tag{4}$$

Note that the parentheses are implied in writing $\mathrm{d}\ln x = \mathrm{d}(\ln x)$, as in Kok (2011). This means that $\mathrm{d}x = x\mathrm{d}\ln x$. Whereas $\mathrm{d}x$ denotes a small increment of x in arithmetic space, $\mathrm{d}\ln x$ denotes a small increment of log space. Now,

$$\mathrm{d}n_x(x) = N x f_x(x)\,\mathrm{d}\ln x \tag{5}$$

That is,

$$\frac{\mathrm{d}n_x(x)}{\mathrm{d}\ln x} = Nxf_x(x) \tag{6}$$

The power law distribution in Eq. (1) of Kok (2011) in effect neglects the exponential part of his Eq. (2). Moreover, if his Eq. (1) is correct, then it implies that $f_x(x) \sim x^{-3}$ such that the cumulative distribution goes as x^{-2}. This is a rather heavy-tailed distribution whose mean may be undefined. Gilvarry and Bergstrom (1961) find that the cumulative distribution of fragment volume v (rather than linear size x) goes as $v^{-2/3}$.

By writing the distribution as in Eq. (6) above (like Eq. (2) in Kok (2011)), the data can be fit using bins of specified log increment — as with a histogram — rather than the usual manner of constructing an exceedance probability plot. This is rather clever, as it avoids the issue of censorship, which, if not addressed in constructing empirical exceedance probabilities, can change the slope of the fitted line. The caveat is that N must be relatively large to support the binning.

That said, I hope I am not misinterpreting how Miller and Jerolmack constructed the plot in Figure 9.

djf