

Supplementary material

Text S1: Concave-up knickpoint migration (decreasing U and $n < 1$)

Figure S1a shows the evolution of a channel profile under $n < 1$ and a step-decrease in the tectonic uplift rate from U_0 to U_1 , leading to the migration of a concave-up knickpoint (Figure S1b). Similar to the model shown in Figure 1, channel segments below and above the knickpoint are in equilibrium with respect to U_1 and U_0 , respectively. Figure S1b shows the change in knickpoint elevation when it migrates from point A to D, during time interval dt . The elevation difference, segment DG, can be expressed as:

$$DG = z_{t+dt}(x + dx) - z_{t+dt}(x) = \left(\frac{\partial z}{\partial x}\right)_1 \cdot v_H \cdot d, \quad (S1)$$

where v_H is the knickpoint celerity. Using the geometric relationship:

$$10 \quad DG = BG - BD, \quad (S2)$$

$$BD = -(U_1 - U_0) \cdot d, \quad (S3)$$

$$BG = \left(\frac{\partial z}{\partial x}\right)_0 \cdot v_H \cdot d, \quad (S4)$$

Equations (S1-S4) can be combined to derive v_H :

$$15 \quad v_H = \frac{(U_1 - U_0)}{\left(\frac{\partial z}{\partial x}\right)_1 - \left(\frac{\partial z}{\partial x}\right)_0}, \quad (S5)$$

Equations (13) and (S5) are identical, indicating that the celerity of concave-up knickpoints formed in response to decreasing U and $n < 1$ and convex-up knickpoints formed in response to increasing U and $n > 1$ is the same.

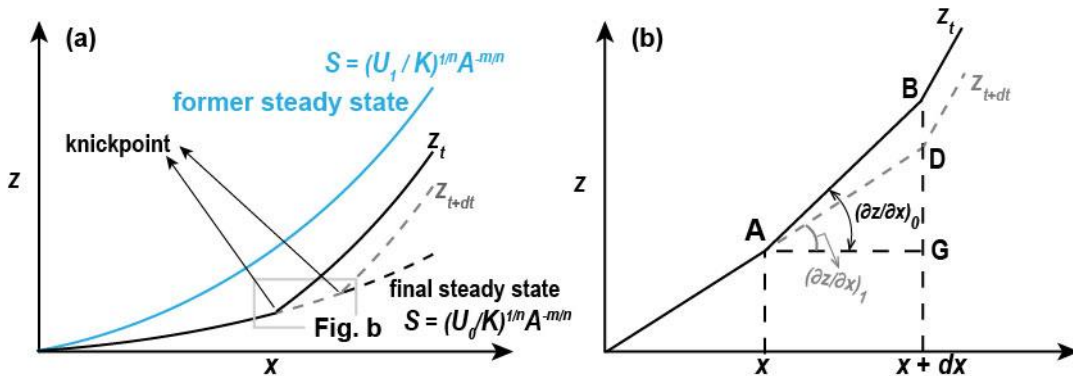


Figure S1: (a) The adjustment of channel profile to a decrease in the tectonic uplift rate from U_0 to U_1 . (b) Knickpoint (concave-up) migration under decreasing U and $n < 1$. Knickpoint retreats from point A to D during time interval dt , with a transition of channel profile from z_t to z_{t+dt} .

Text S2: Theoretical comparison between the model of kinematic wave speed and slope-break knickpoint retreat

S2.1 Difference in estimating knickpoint age

We compare and contrast the slope break migration model derived here to the kinematic wave speed model (e.g. Rosenbloom and Anderson, 1994; Weissel and Seidl, 1998; and Oskin and Burbank, 2007). In the wave speed model, the horizontal speed for knickpoint migration ($v_{H_{Ro}}$) is:

$$v_{H_{Ro}} = K \cdot A^m \cdot S^{n-1}, \quad (S6)$$

The linear form ($n = 1$) has been widely used to calculate knickpoint age and infer regional incision/uplift history (Goren et al., 2021).

For the non-linear case, when the slope exponent, $n \neq 1$, the kinematic wave speed depends on the channel gradients, with either positive ($n > 1$) or negative ($n < 1$) correlation. Local variations in channel gradients may cause changes to the shape of the kinematic wave and potentially alter knickpoint recession rate (Whitham, 1974; Berlin and Anderson, 2007). Numerical implementation of equation (S6) is not trivial, because the slope, S , at the slope-break is ill defined. Indeed, Whipple and Tucker (1999) pointed out that the kinematic wave speed, equation (S6), and its derivations are valid for infinitesimal, step perturbations (e.g., coseismic, eustatic, or stream capture), rather than for sustained changes in uplift rates.

Assuming a small perturbation in the uplift rate, which causes no change in the channel steepness index, the channel profile below the knickpoint is in equilibrium with the tectonic forcing (Whipple and Tucker, 1999), and equation (3) could be assigned in equation (S6):

$$v_{H_{Ro}} = KA^m (k_{s-1} A^{-m/n})^{n-1} = K k_{s-1}^{n-1} A^{m/n}, \quad (S7)$$

The fluvial response time of the kinematic wave knickpoint could be expressed as (Whipple and Tucker, 1999):

$$\tau_{Ro} = \int_0^{x_p} \frac{1}{k_{s-1}^{n-1}} \cdot \frac{1}{KA(x)^{m/n}} dx = \frac{1}{KA_0^{m/n}} \frac{1}{k_{s-1}^{n-1}} \chi(x_p), \quad (S8)$$

Comparing equations (14) and (S8), the ratio in the response time of the two models, the slope-break model developed here and kinematic wave recession model, can be expressed as:

$$\frac{\tau}{\tau_{Ro}} = \frac{\frac{1}{KA_0^{m/n}} \frac{k_{s-1}^{1-\gamma_{0-1}}}{k_{s-1}^{n-1} (1-\gamma_{0-1}^n)} \chi(x_p)}{\frac{1}{KA_0^{m/n}} \frac{1}{k_{s-1}^{n-1}} \chi(x_p)} = \frac{1-\gamma_{0-1}}{1-\gamma_{0-1}^n}, \quad (S9)$$

Hence, we agree with Whipple and Tucker (1999) that, in the general case, the kinematic wave recession model is an inaccurate description of knickpoint formed following a sustained change in the rate of tectonic rock uplift. Interestingly, the two models become identical for $n = 1$.

S2.2 Similarity between the two models

50 The integral in equation (S8) defines an enclosed area below the function $A(x)^{-m/n}$ and within the interval of $(0, x_p)$. By the mean value theorem of integrals, a rectangle with an equal area can be defined, that has a length of x_p and a width equal to $A(x_1)^{-m/n}$ for $0 < x_1 \leq x_p$. This allows the following transformation of equation (S8):

$$x_p = K \tau_{\text{Ro}} k_{s_1}^{n-1} A(x_1)^{m/n}, \quad (\text{S10})$$

Equation (S10) predicts a power-law scaling between knickpoint retreat distance, x_p , and the drainage area at x_1 . Estimate x_1 55 can be done by assuming Hack's law (Hack, 1957):

$$A(x) = k_a (L - x)^h, \quad (\text{S11})$$

where k_a and h are positive constants, L is total length of the river, and x measures the distance from the outlet. Assigning equation (S11) into the term, $x_p/A_0^{m/n}$, we can derive:

$$\frac{1}{A_0^{m/n}} x_p = \int_0^{x_p} \frac{1}{k_a^{m/n} (L-x)^{hm/n}} dx = \frac{1}{k_a^{m/n} (L-x_1)^{hm/n}} x_p, \quad (\text{S12})$$

60 Thus, the position of x_1 can be derived from:

$$\frac{A_0^{m/n}}{k_a^{m/n}} \cdot \frac{x_p}{x_p} = (L - x_1)^{hm/n}, \quad (\text{S13})$$

Since Hack's law is an approximation to the drainage area distribution, we do not aim to calculate the accurate position of x_1 . More significantly, equation (S13) indicates a positive control of river total length, L , over knickpoint retreat distance, x_p . Thus, we show that (1) the models of kinematic wave and slope-break knickpoint retreat share the same mathematical form 65 only when $n = 1$ and (2) catchment drainage area, determined by channel total length (equation S11), is key in determining how far the knickpoint (both a sudden base level fall and a sustained increase in tectonic uplift rate) has moved upstream. Similar results are attained in statistic models such as in Castillo et al. (2017).