Report on the article entitled "The transport of sediment mixtures examined with a birth-death model for grain-size fractions"

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I provide a partial report on the paper submitted by Shawn M. Chartrand and David J. Furbish to *Earth Surface dynamics*. I stopped reading after Section 2 because I failed to understand the theoretical developments.

First note that when I introduced the concept of collective entrainment in my 2008 JFM paper, I was referring to a positive feedback loop in the stochastic process: the higher the number of moving particles, the higher the probability of entrainment. In other words, particle entrainment does not depend only on fluid forces, but also on the presence of other particles. This definition is different from the one used by the authors Line 21 on page 1. Among other things, I never stated that collective entrainment implies that two or more particles can be entrained at the same time (and in fact this scenario would conflict with the model's core assumptions).

I have a hard time understanding how the model has been built. Here are the main points that were unclear to me:

- On p. 6, the authors say that they took inspiration from my 2010 JGR paper. If so, Eq. (5) represents the ensemble-averaged time evolution for the number of moving particles n_j, and this means that in Eq. (5) n_j is the mean number of particles (thus a deterministic variable); otherwise, the randomly fluctuating part (colored noise) is missing. They then took the Fourier transform of this equation. I do not understand why they introduced a Fourier transform of the entrainment rate λ_j, whereas the other parameters are assumed to be constant. In my model, n was the random variable, and the parameters λ, μ, and σ were constant (for fixed hydraulic conditions). The paragraph L20-32 on p. 7 seems to indicate that the entrainment rate λ was a random variable (in my papers, λ is the constant rate of a Poisson process; and so, entrainment occurs randomly, but its rate is constant). They then interpreted Eq. (8) as a low pass filter on λ. This leads me to think that they have entirely changed the nature of the problem I addressed: in my papers, hydraulic conditions are constant, and so are the entrainment and deposition parameters λ, μ, and σ. The number of moving particles n varies randomly as a result of random events described within the framework of jump Markov Processes (birth-death immigration-emigration processes). As far as I understand the submitted paper, λ is the random variable, μ and σ are constant, and n is a deterministic variable subject to a random forcing.
- Later, on p. 10, the authors assumed that the entrainment rate λ could be described using an autoregressive process. As λ was assumed to an AR(2) process, this means that the whole process was no longer Markovian because λ depended on what occurred not only at time $t - \delta t$, but also $t - 2\delta t$. This is an important assumption that is not justified. In principle, it should be possible to justify this assumption by

plotting the partial autocorrelation function of λ (for instance, see the supporting information of my 2020 JGR paper), but as λ is not something that is easy to measure experimentally, it is in practice difficult to ensure that λ time variations can be described by an AR(2) process. Note that this assumption conflicts with the assumption that the number of moving particles is a Markov process. If n variations cannot described using a Markovian process, then Eq. (5) cannot be used, or at least the reference to Ancey (2010) is not the right one.

- On p. 11, I understand that the authors combined the Fourier transforms of n and the power spectral density function of λ. If I am not mistaken, then there is a problem because their spectral density of n is just the norm of the Fourier transform of n, whereas the power spectral density of λ is the Fourier transform of the autocorrelation of λ. Without further guidance, I cannot understand how all these elements were combined to create Figures 3 and 4.

I recommend major revision. The theoretical developments should be better explained and revised according to the points above. The authors are free to contact me for further information.