Revisions made to Struble and Roering based on comments by Reviewers 1 and 2. *All line number references refer to numbering in original manuscript (as referenced by reviewer).*

<u>Reviewer 1, Doane:</u>

<u>General Comments:</u>

Organization: We appreciate this point, as we tried several different organizational schemes. We prefer to keep the material in its current order, as it allows us to introduce the polynomial calculation as previous work (it is essentially the primary past work to which we compare the CWT), introduce the field site, and then discuss the new methods (CWT for C_{HT}), without getting sidetracked by the field site later.

Torrence and Compo: Perhaps we are misunderstanding, because we disagree. Torrence and Compo explicitly define the derivative of a Gaussian, but in a very slightly different functional form (see the $\frac{d^m}{d\eta^m}$ term and following in their Table 1. The leading term with the gamma function defines the amplitude; *m* refers to the order of the derivative. We use *m*=2.), but the shape of the function is the same. The primary difference is in the definition of wavelet scale. Lashermes et al. define the scale as the inverse of the function's band-pass frequency while Torrence and Compo plug a cosine function with a known frequency into the wavelet basis function (the DoG) to calculate the wavelet transform and identify the scale, *s*, at which the wavelet power spectrum reaches its maximum. Admittedly, we prefer the simplicity and more intuitive definition of Lashermes et al. (and we do not explicitly go into details on *how* Lashermes et al. and Torrence and Compo define λ to avoid introducing too much confusion). Yet, both are valid. So, for the sake of thoroughness, we will maintain usage of Torrence and Compo.

In regards to the question are "the results between the wavelet definitions for a single value of λ comparable?" We don't see why they wouldn't be. We aren't mixing the definitions and then completing a single analysis (and we aren't using the definitions to construct a wavelet with a different functional form, either). For a given λ, we use equations 7 and 8 (now eqns 9 and 10) to calculate wavelet scales, *s*, for both the Lashermes et al. and Torrence and Compo definitions, to then apply the wavelet transform (separately, once for each definition of *s*). It is only after extracting C_{HT} that we compare the two. In essence, in addition to comparing to broad methodologies (CWT to PFT), we are comparing two separate CWT methods as well.

We have opted, then, to not make any substantial adjustments regarding this comment. At first it may seem a bit of a distraction, and we struggled with the best way to present these two wavelet scale definitions. We feel, though, that it is important to acknowledge that both definitions are reasonable as they produce similar results and are both mathematically acceptable as far as we are aware.

DoG wavelets: This is a fair point. We have adjusted this section to include an equation for the Gaussian and we have also recast the equation for the Ricker wavelet such that it includes *s*. We've also taken advantage of this adjustment to discuss more carefully the property of convolutions that allows for simultaneous low pass filtering and curvature calculation.

Line comments:

Line 33: Removed phrase.

Line 36: Changed to "landscape properties."

Line 51: Modified these few sentences.

Line 56: Replaced "recorder" with "record."

Lines 73-85: We appreciate this comment, but it largely ignores much of the previous work done that demonstrates the importance of using more involved curvature calculation techniques as well as the necessity of using high-resolution lidar data, as opposed to 10-m data, which are not of sufficient quality to measure curvature (even robust statistics cannot necessarily get around the data quality issues with 10-m data compared to lidar). Sources such as Grieve et al. (2016), Passalacqua et al. (2010), and Ganti et al. (2012) are a few examples (we cite them in the paper). Our enhanced discussion of the convolution hopefully helps this discussion some.

Line 74: Adjusted to "types of curvature (i.e. tangential, planform, Laplacian, etc.)"

Line 80: We've adjusted to read "reduce the impact of topographic roughness," as it isn't always erroneous, but simply an undesired part of the signal.

Line 80: We wish to maintain the transport reference, as often much of this noise is produced through stochastic processes. We have adjusted the sentence, though, to read "sediment transport and surface perturbations," as the deviations to the topographic surface are what constitutes the noise.

Line 157: Added citations for Foufoula-Georgiou and Kumar (1994) and Lashermes et al. (2007) at the end of the next sentence. In addition, the edits we've made to this section no longer have this sentence in its original form, which more clearly necessitated discussing derivatives and convolutions; however, we have added text that discusses why the convolution is useful.

Line 157: Convention for the Ricker/Mexican Hat wavelet. It has introduced confusion for us several times, but the Mexican Hat wavelet is explicitly defined as the negative second derivative. We also discussed with the reviewer in person that the output coefficients use a different curvature convention than is typical in geomorphology (convex vs concave values).

Eqs. 7 & 8: We have adjusted the wording here (we felt that saying that s had no physical meaning to be a bit more abstract than we wished and didn't convey what we were trying to get across). Since our workflow involves selecting λ and solving for s (not the opposite as presented in the review) we considered instead having these equations written as s=…, but given we are discussing smoothing scales (corresponding to λ), we decided to maintain the equations in their current form. That said, we've adjusted more wording here to try and make clear that we are solving for s after selecting a range of λ. We also followed the suggestion of writing the Gaussian and wavelet functions with s included. As we elaborated on above, we maintain usage of Torrence and Compo.

Section 3.3: The calculation was elaborated on in section 3.1 (i.e. the convolution). We can see how this title introduces confusion, so we've adjusted the section name to "Hilltop extraction."

Figure 3: We agree. We had gone back and forth on this, since, as you say, a and b are dependent on our setup. We've replaced a and b with c and d.

Figure 4: Perhaps not, but we think some readers will be interested where curvature is measurements are most likely to deviate from each other (i.e. high magnitudes, which partially plays into our synthetic analysis later in the paper). We've opted to keep this here.

Section 4.1: This is a great question! We have added an additional subplot to Figure 3 (after removing the extraneous subplots per a previous comment) that shows that the CWT increases in relative speed as DEM size grows. For scaling to large areas, this is a key advantage of the CWT, so thank you again for suggesting this! We have modified language throughout the manuscript that highlights this updated result.

Line 290: We refer back to our earlier explanations. They are indeed "different," but it is due to a different way of defining the scale. Rather than thinking about a single value *s* producing different λ, we are picking a single λ, which produces two different *s* (as we mentioned above, we have adjusted text to try and make this clearer). That does, in a manner of speaking, produce two different wavelets, but it is simply the size that differs (the functional form is the same; they are just stretched differently).

Figure 5C: These curvature values originally caused some concern, as the hillslopes are so broad and grade so gradually into gentle valleys, that the landscape is largely semi-planar. The mode values near-zero are due to clipping off the positive curvatures from the dataset. While not numerous in number (i.e. number of nodes), the positive curvatures, combined with very low magnitude negative curvatures, results in a mode near 0. Applying C_{HT} to these hilltops kind of pushes C_{HT} to its limit, as these hillslopes exhibit such little relief. Fortunately, the mean and median curvatures are negative and produce erosion rates consistent with the CRN erosion rate.

Figure 6: Thank you!

Line 328: True, though even Roth et al.'s rough site was still comparatively smooth relative to other locations in the OCR. We've added some additional citations. The goal here, regardless, is not to fully mimic a specific landscape, but pick a physically reasonable roughness value. Figure 9 additionally includes the hillslopes with no noise, so the choice of highlighting the 50 cm noisy surfaces seems reasonable.

Figure 7: Suggested change made.

Line 410: We haven't checked this specifically with the wavelets. But given the consistency of results between the wavelet and polynomial fits, the results from Grieve et al. (2016) are relevant here. We also refer back to our comment for Lines 73-85. Curvature will be systematically underestimated when using 10-m data. In addition, using a 10-m DEM does not simply constitute taking a high-resolution lidar DEM and using those data spaced every 10 meters. The decrease in data quality and sources with the 10-m DEM is nontrivial.

Line 427: We've re-worded the sentence following the suggestion. We've maintained the reference to linear diffusion as a parenthetical comment at the end of the sentence, as we like the reference to linear diffusion in the next sentence.

Line 530: We aren't fully sure what the question is. Is it that we used plural "relationships?" (The relationships will depend on diffusivity (i.e. producing multiple different slopes), so we wish to maintain the plural form). Is it about why we used power law and square root? (We, and Gabet et al., do not always observe a perfect square root form, so we want to maintain the flexibility for a more generic power law, while highlighting the potential square root relationship that Gabet et al. note.).

Reviewer 2:

General Comments:

One or two figures of DEMs:

We have added a figure that has a 3D perspective of our three catchments, with a general focus on each representative hilltop. For each of these 3D-viewed DEMs, we have included a hillslope profile for the representative hilltop.

Hilltop identification and cutoff parameters: Please see our response to the comment at line 199).

Other types of curvature: Wavelets have been used in geomorphology for other uses, including mapping channel heads and drainage networks. We had alluded to this in the text, but we've added a bit more in the second paragraph of section 3.1. We've also added some more supporting text and some other useful citations in the Discussion. (The Ricker wavelet is specific to the Laplacian, but that is not to say that other wavelets may have the capacity to calculate other types of curvature. That is beyond the scope of this work, however.).

Figure quality: We apologize for the rasterization effects for some of the figures. We think this may have been an issue when converting the manuscript file to a pdf. We will try to ensure that the revised manuscript pdf does not have the same problem (as the original figures do not have this issue).

Guidelines for C_{HT} bias: We aren't completely sure what is being suggested in regards to making changes to Figure 10b. That figure is already showing the ratio of measured/actual C_{HT} as a function of erosion rate (for a given λ). Perhaps you are suggesting we include a hillshade of sorts that demonstrates this? Unfortunately, generalization in that way is a challenge for figure 10B, since any hillslope form will depend on the diffusivity (indeed a key point of Figure 10B). But we have added profiles of three example synthetic hillslopes in figure 10A, since diffusivity is limited to a single value.

Specific comments:

Line 73: We struggled with finding an appropriate acronym for "polynomial functions fit to the topographic surface." We feel that PFT is a reasonably concise acronym (Polynomial Fit to Topography). We have added an additional parenthetical to make this hopefully more obvious.

Line 82: Rectangular window; no. We have modified language to hopefully clarify this.

Line 88: This is true. That said, our results demonstrate that the difference in computation speed for the wavelet and polynomial is substantial enough such that if we were to also just apply the wavelets to the hilltops, and not the whole DEM, we would see a similar reduction in processing time. We have chosen not to change the text in this location, as the PFT is slower than the CWT; see our response to the comment on line 406, however, where we did make an adjustment to address this point.

Line 144: The data source is listed in the Code and Data Availability Section. We have added these other details to the text.

Lines 164-165: Adjusted text to clarify the distinction.

Line 166: We are not sure what the confusion is referring to here. At this point of the paper, we are discussing the smoothing scale needed to remove topographic noise. Is the confusion surrounding "estimating" erosion rate? We go into how erosion rate is calculated in equation 1. We use the word estimate as it is dependent on D, which we point out later in the paper (and which Gabet et al. emphasize) may exhibit uncertainty. Furthermore, we "estimate" erosion rate, since "measuring" erosion rate is preferably done with cosmogenic nuclides. Unfortunately, broad application of CRNs in expansive regions is often cost prohibitive. Note that for the comment at section 3.4.1 we adjusted language to more explicitly show that we calculated curvature and used hilltop masks to extract C_{HT} (We think this is the primary source of confusion). Later in the section, we go into details on erosion rate.

Line 192: Added 2.60 GHz CPU

Line 194: Yes. Text adjusted.

Line 199: Added statement about restriction of first-order divide lengths (yes, we used the DIVIDEobj function introduced in Schwanghart and Sherler, 2020).

Line 205: We've adjusted this sentence to clarify that we removed drainage divides that share a divide with neighboring drainage basins, where disequilibrium (and thus asymmetry) may be an issue.

Line 210: Added these details.

Section 3.4.1. We adjusted the text to clarify that we measure curvature, then use hilltop masks to extract C_{HT}. We now wait until later in the section to explicitly mention estimating erosion rate.

The identification of scaling breaks is a specific step to estimate the erosion rate. We've maintained the language we had in this case.

Line 220: The D we used here is from previously published work. We added a brief statement to make clear it was estimated in the OCR and added a citation for Roering et al. (1999), in addition to the citation we already had for Roering et al. (2007). In other words, we did not use our hilltop curvature measurements with CRN erosion rates to estimate D, and then use that D to recalculate erosion rate (there's no circularity here).

Line 265: That is simply the non-scaled noise. We added a reference to the ±1 m in the sentence that specifies that we are testing amplitudes by scaling those original noisy surfaces.

Line 320: Yes it is, thank you for bringing this to our attention! We have added a sentence at the end of this paragraph pointing that out.

Line 406: At the end of these few sentences, we added a brief statement clarifying this. We stress, though, that the computation advantages of the wavelet would allow for curvature calculation of entire DEMs, with no extra cost! This is of use to not just hilltop enthusiasts, but to those who may wish to calculate curvature elsewhere (i.e. Lashermes et al., 2007; Passalacqua et al., 2010). In addition, while addressing comments from the other reviewer, we made an addition to Figure 3 that shows that the CWT becomes even more efficient as DEM size increase.

Line 412: Indeed. While rivers have the advantage that you point out (you don't necessarily need high resolution data for large regional studies), hillslope analyses do not have that luxury. We have opted to keep this sentence in its current form.

Line 421: We think that discussing the introduction of planar side slopes into the measurement as hilltops narrow is a fairly conceptual explanation. As you rightly point out, this is fairly easy to visualize for the PFT. For the CWT, however, it is much less intuitive, as the actual calculation takes places in the Fourier domain. Spectral techniques struggle with sharp edges and abrupt transitions (such as sharp hilltops!). Hence, we think that the explanation we've included where "the CWT and PFT kernels have become sufficiently large to incorporate planar side slopes" is a straightforward explanation for why both methods begin to underestimate curvature.

Line 436: Thinking about other filtering schemes is a good point and something we've considered quite a bit. Unfortunately, due to a particular property of convolutions (i.e. taking the derivative of the smoothing function and then convolving it with topography is identical to smoothing to topography and then taking the derivative), your suggestion is what we did here. In other words, the wavelet simultaneously applies a low pass filter (since it's the second derivative of a Gaussian) while calculating curvature. That's what makes this such an intriguing problem; where topographic noise and hilltops have similar characteristic scales, there isn't a clear way to adequately remove the noise while maintaining the underlying hillslope signal. Definitely worthy of future work! We have made some additions to the methods section that makes this property of convolutions clear.

Figure 9: We have adjusted the y-axes in the middle two rows, as this was where the most avoidable inconsistencies were. Unfortunately, to make sure the figures are readable, we had to leave the first row ($E^*=1$) figures unchanged. We have added text to the caption, though, that points out that the y-axes between figures in this row differ.

Apologies again on the rasterization. Will make sure the pdf conversion doesn't produce similar issues this time.

Table 3: We moved the new sample to its own table and added such details.