Hilltop curvature as a proxy for erosion rate: Wavelets enable rapid computation and reveal systematic underestimation

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Abstract. Estimation of erosion rate is an important component of landscape evolution studies, particularly in settings where transience or spatial variability in uplift or erosion generates diverse landform morphologies. While bedrock rivers are often used to constrain the timing and magnitude of changes in baselevel lowering, hilltop curvature (or convexity), C_{HT}, provides

10 an additional opportunity to map variations in erosion rate given that average slope angle becomes insensitive to erosion rate owing to threshold slope processes. C_{HT} measurement techniques applied in prior studies (e.g. polynomial functions), however, tend to be computationally expensive when they rely on high resolution topographic data such as lidar, limiting the spatial extent of hillslope geomorphic studies to small study regions. Alternative techniques such as spectral tools like continuous wavelet transforms present an opportunity to rapidly document trends in hilltop convexity across expansive areas.

15 Here, we demonstrate how continuous wavelet transforms (CWTs) can be used to calculate the Laplacian of elevation, which we utilize to estimate erosion rate in three catchments of the Oregon Coast Range that exhibit varying slope angle, slope length, and hilltop convexity, implying differential erosion. We observe that C_{HT} values calculated with the CWT are similar to those obtained from 2D polynomial functions. Consistent with recent studies, we find that erosion rates estimated with C_{HT} from both CWTs and 2D polynomial functions are consistent with erosion rates constrained with cosmogenic

20 radionuclides from stream sediments. Importantly, our CWT approach calculates curvature 10^2-10^3 times more quickly than 2D polynomials. As such, continuous wavelet transforms provide a compelling approach to rapidly quantify regional variations in erosion rate as well as lithology, structure, and hillslope sediment transport processes, which are encoded in hillslope morphology. Finally, we test the accuracy of CWT and 2D polynomial techniques by constructing a series of synthetic hillslopes generated by a theoretical nonlinear transport model that exhibit a range of erosion rates and topographic

25 noise characteristics. Notably, we find that neither CWTs nor 2D polynomials reproduce the theoretically prescribed C_{HT}

value for hillslopes experiencing moderate to fast erosion rates, even when no topographic noise is added. Rather, C_{HT} is systematically underestimated, producing a power law relationship between erosion rate and C_{HT} that can be attributed to artifacts from the increasing prominence of planar hillslopes that narrow the zone of hilltop convexity as erosion rate increases. As such, we recommend careful consideration of measurement length scale when applying C_{HT} to estimate erosion
30   rate in moderate to fast-eroding landscapes, where curvature measurement techniques may be prone to systematic underestimation.

1 Introduction

The morphology of landscapes, including river channels and hillslopes, adjusts to conform to exogenic perturbations such as uplift and climate as well as spatial variations in lithology, geologic structure, and biology. As such,
35   numerous studies have taken advantage of landscape morphology to estimate rates and timing of perturbations to these boundary conditions. In bedrock rivers, for instance, geomorphic transport laws have been formulated to allow for linkages between landscape form and process, including from measurements such as channel steepness and χ, a metric that integrates drainage area along a channel profile (Kirby and Whipple, 2001; Perron and Royden, 2013; Royden and Perron, 2013). These tools have been effectively utilized to estimate and map spatial variations in uplift, quantify the timing and rates of
40   landscape transience and uplift history, and predict drainage basin reorganization (e.g. Barnhart et al., 2020; Dietrich et al., 2003; Fox, 2019; Kirby and Whipple, 2001, 2012; Roberts and White, 2010; Willett et al., 2014; Wobus et al., 2006).

Similarly, hillslope geomorphic transport laws formulated for soil mantled landscapes allow for estimation of uplift and erosion rates as well as prediction of the migration of hillcrests in response to landscape transience (Forte and Whipple, 2018; Mohren et al., 2020; Mudd, 2017; Mudd and Furbish, 2007, 2005; Roering, 2008; Roering et al., 2007, 2001, 1999).
45   Over 100 years ago, it was proposed that hillslope form, specifically slope and curvature, may be an effective predictor of erosion rate, as hillslopes steepen and lengthen to accommodate increases in baselevel lowering (Gilbert, 1909, 1877). However, hillslopes do not continue to steepen as baselevel lowering progressively increases to faster and faster rates (e.g. Howard, 1994; Penck, 1953; Schumm, 1967; Strahler, 1950). Rather, hillslope gradients approach a threshold value as erosion rate increases, such that gradient becomes invariant and insensitive to further increases in baselevel lowering
50   (Andrews and Bucknam, 1987; Burbank et al., 1996; DiBiase et al., 2012; Larsen and Montgomery, 2012; Montgomery, 2001; Roering et al., 1999). This 'nonlinear' formulation, such that sediment flux varies nonlinearly with slope, implies that slope angle becomes insensitive to baselevel lowering due to threshold-dependent processes such as landsliding as well as granular creep (BenDror and Goren, 2018; DiBiase et al., 2012; Ferdowsi et al., 2018; Gabet, 2000; Larsen and Montgomery, 2012; Montgomery, 2001; Ouimet et al., 2009; Roering et al., 2001).

55   Despite the insensitivity of hillslope gradient in rapidly eroding landscapes, soil mantled hillslopes remain an effective recorder of landscape transience and uplift. Specifically, hilltop curvature continues to respond to baselevel lowering when uplift and erosion rates are high, even as slope becomes insensitive to ever-increasing erosion rate (Hurst et al., 2012; Mohren et al., 2020; Roering et al., 2007). For a one-dimensional hillslope at steady state, erosion rate, E, can be estimated as

60    $E = -\frac{\rho_s}{\rho_r} D C_{HT}$,   (1)

where ρ_s and ρ_r are the density of soil and bedrock, respectively, D is the soil transport coefficient or diffusivity, and C_{HT} is curvature at the hilltop (Roering et al., 2007). Using this formulation, Hurst et al. (2012) demonstrated in the Sierra Nevada, California, that C_{HT} records erosion rate in both low-relief, low-slope headwater catchments of the Feather River as well as in high-relief catchments that have already adjusted to a faster baselevel lowering rate where hillslopes approach a threshold

65   angle. Similarly, Hurst et al. (2013) observed that hillslopes that are translating through an uplift gradient along the San Andreas Fault actively steepen and become sharper (C_{HT} becomes more negative) as they traverse the zone of high uplift and hillslope gradients become invariant. The hillslopes then decay, that is slopes become gentler and curvatures become less sharp, as they reenter the region of low background uplift (Hurst et al., 2013). Similarly, Clubb et al. (2020) observed that steep channels and sharp hilltops record uplift along the Mendocino Triple Junction in northern California, and they note that

70   the lag in hillslope response time relative to the bedrock channels records the northward migration of the Mendocino Triple Junction.

    Past studies that couple geomorphic transport laws and hilltop curvature have typically relied on curvature calculated from 2D polynomial functions fit to the topographic surface (PFTs; e.g. Roering et al., 1999). While a variety of polynomial forms and curvature definitions have been utilized (e.g. Minár et al., 2020; Moore et al., 1991), Hurst et al.

75   (2012) found that 6 term functions were sufficient for measuring curvature to estimate erosion rate. Specifically, Hurst et al. (2012) used least squared regression to fit a surface, z, to topography, such that,

$z = ax^2 + by^2 + cxy + dx + ey + f$,   (2)

where curvature, or more specifically the Laplacian of elevation, $\nabla^2 z$, is denoted as

$\nabla^2 z = 2a + 2b.$ (3)

80   To reduce topographic roughness due to stochastic sediment perturbations such as tree throw pits as well as noise in the digital topographic data, they applied the PFT over a scale, λ (L in Hurst et al., (2012)), which defines the size of the polynomial kernel that is fit to the surface. The value of λ can be obtained by analysis of the scale dependency of roughness metrics (e.g. Hurst et al., 2012; Roering et al., 2010). As elaborated in the methodology proposed by Hurst et al. (2012), the PFT is not required to pass through each digital elevation model (DEM) node; hence, λ can be understood as a smoothing

85   scale, thus measuring the background C_{HT} and removing topographic noise.

While the application of PFTs has proven useful for calculating curvature to estimate erosion rate and predict spatial and temporal variations in uplift (e.g. Clubb et al., 2020; Godard et al., 2020; Hurst et al., 2019, 2013, 2012; Mohren et al., 2020; Roering et al., 2007), PFTs are computationally cumbersome, hindering large-scale exploitation of high-resolution topographic datasets that have become increasingly available. Here, we demonstrate that 2D continuous wavelet transforms

90 (CWTs) provide an alternative and computationally efficient approach to calculating hilltop curvature, operating 10^2 to $>10^3$ times faster than PFTs. We establish the similarity of the output CWT C_{HT} values to those produced by PFTs, and we compare estimated erosion rates calculated from C_{HT} values to erosion rates measured with cosmogenic radionuclides (CRN) in catchments in the Oregon Coast Range. In addition, we test the relative accuracy of the CWT and PFT approaches by applying them to synthetic hillslopes with known erosion rates generated by a nonlinear transport model and superimposed

95 topographic noise. We find that both techniques systematically underestimate C_{HT} at moderate to high erosion rates and appear to approximate a square root relationship between C_{HT} and erosion rate as erosion rate increases, consistent with a recent study (Gabet et al., 2021).

2 Study Site: Oregon Coast Range

We selected the Oregon Coast Range (OCR) to compare CWTs and PFTs as hilltop curvature measurement

100 techniques, as it is a region that has been extensively studied in the geomorphic literature, exhibits relatively uniform topography over intra-catchment scales while exhibiting diversity in hillslope form and erosion rate across the axis of the range, and has negligible spatial variability in climate. The OCR is an unglaciated humid landscape that parallels the Cascadia Subduction Zone and is characterized by cool, wet winters when the majority of the annual 1-2 m of precipitation falls, and warm dry summers (PRISM Climate Group, 2016). The dominant tree populations are composed of Douglas-fir

105 (*Pseudotsuga menziesii*) and western hemlock (*Tsuga heterophylla*) that reside on hillslopes that are soil-mantled throughout the range. Soils are thickest in colluvial hollows and unchannelized valleys (~1-2 m), thinnest (~0.5 m) on planar hillslopes and hilltops, and are primarily produced stochastically through tree throw and bioturbation (Dietrich and Dunne, 1978). Colluvial hollows are periodically evacuated by shallow landslides that mobilize into debris flows (Benda and Dunne, 1997; Dietrich and Dunne, 1978; Penserini et al., 2017; Stock and Dietrich, 2003). Erosion rate, measured using techniques

110 including CRNs, ^{14}C dating, and fluvial and colluvial sediment flux, usually cluster at approximately 0.1 mm yr^{-1} (Balco et al., 2013; Bierman et al., 2001; Heimsath et al., 2001; Penserini et al., 2017; Reneau and Dietrich, 1991), though these rates can temporally and spatially vary dramatically (Almond et al., 2007; Marshall et al., 2015; Sweeney et al., 2012). Average OCR erosion rates approximately correspond with uplift rates calculated from abandoned marine terraces, ranging from <0.05 to >0.4 mm yr^{-1} (Kelsey et al., 1996), as well as from fluvial strath terraces which range from 0.1 to 0.3 mm yr^{-1}

115 (Personius, 1995), which has led to suggestions that the OCR may approximate steady state. Nonetheless, deviations from uniform erosion have been noted based on morphologic trends as well as soil properties (Almond et al., 2007; Sweeney, et al., 2012).

We pinpointed catchments in the OCR that exhibit a range of hilltop curvatures for analysis. Specifically, we focus on Hadsall Creek (43.983° N, -123.823° W), the North Fork Smith River (NFSR; 43.963° N, -123.811° W), and Bear Creek

120 (44.181° N, -123.371° W). Hadsall Creek and the NFSR are catchments in the central OCR that share a drainage divide (Fig. 1A). Hadsall Creek is characterised by steep channels and hillslopes with evenly spaced ridges and valleys where incision is dominated by debris flows (Penserini et al., 2017). Contrastingly, the NFSR, which is erosionally isolated from baselevel by an Oligocene-age gabbroic dike that has pinned the fluvial channel, exhibits comparatively gentle channel and hillslope angles as well as longer soil residence times (Sweeney et al., 2012). CRN measurements have recorded catchment-averaged

125 erosion rates at Hadsall Creek and the NFSR of 0.113±0.018 mm yr^{-1} and 0.058±0.0054 mm yr^{-1}, respectively (recalculated from Penserini et al., 2017; Table 1). We also utilize hillslopes within three small sub-catchments that drain to Bear Creek (Fig. 1B), a tributary to the Long Tom River on the eastern margin of the OCR in the southwestern Willamette Valley (WV). Hillslopes within Bear Creek and the western margin of the WV exhibit gentle slopes, weathered soils with long residence times >150 kyr (Almond et al., 2007), and are bounded by broad alluviated valleys. We additionally report a newly collected

130 CRN-derived catchment-averaged erosion rate for the northern Bear Creek subcatchment that we study here (Fig. 1B).

The spatial proximity of Bear Creek, Hadsall Creek, and the NFSR make them well-suited to compare C_{HT} measurement techniques, as other factors that may influence morphology, such as climate and lithology, remain relatively invariant. All three catchments are within the Tyee Formation, a ~3 km thick sequence of gently dipping Eocene turbidite deposits characterized by a sequence of sandstone and siltstone interbeds (Baldwin, 1956; Heller and Dickinson, 1985;

135 Lovell, 1969). While variability in sandstone-siltstone ratios in the Tyee Formation result in latitudinal north-south variations in deep-seated landsliding (Roering et al., 2005), our three study sites are within sufficient proximity to each other such that lithologic variability in setting hillslope morphology should be limited. In addition, while common elsewhere in the OCR (Franczyk et al., 2019; LaHusen et al., 2020; Roering et al., 2005), the sites we have selected for analysis do not exhibit pronounced evidence of deep-seated landslides, which may bias C_{HT} values, complicating comparison to known erosion rates

140 from CRN analysis. As such, Hadsall Creek, the NFSR, and Bear Creek provide an ideal spectrum of hillslopes that allows for assessment of C_{HT} measurement techniques.

3 Methods

3.1 Curvature calculation: polynomial fit and continuous wavelet transform

We used PFTs to calculate curvature of the Hadsall and Bear Creeks and NFSR lidar DEMs (grid spacing of 0.9144

145 m), as enumerated in Equations 2 and 3. In order to identify and remove the topographic impact of stochastic sediment transport processes such as tree throw, we calculated PFT curvature rasters using variable kernel sizes, corresponding to a range of smoothing scales, specifically for λ=5-141 m (the diameter of the polynomial kernel requires odd dimensions).

In contrast to PFTs, CWTs are computationally efficient and can provide a variety of outputs depending on the analysis and type of wavelet used (e.g. Foufoula-Georgiou and Kumar, 1994 and references therein). Here, we applied a 2D CWT using the Ricker wavelet (often known as the Mexican Hat wavelet). The Ricker wavelet has been used in geomorphology to map and estimate landslide ages based on surface roughness (Booth et al., 2009; LaHusen et al., 2020), identify dominant landforms at particular wavelengths (Struble et al., 2021), extract channel networks (Lashermes et al., 2007; Passalacqua et al., 2010), and other topographic spectral analyses including mapping faults and predicting lithospheric thickness (e.g. Audet, 2014; Jordan and Schott, 2005; Malamud and Turcotte, 2001). The Ricker wavelet, ψ, is the negative, second derivative of a 2D Gaussian function (Derivative of a Gaussian (DoG) wavelets constitute a wavelet family), and is defined as

$$\psi(x,y) = (2 - x^2 - y^2) \exp\left[-\frac{1}{2}(x^2 + y^2)\right]. \tag{4}$$

As the negative second DoG, the output wavelet coefficients of the Ricker wavelet provide a measure of the Laplacian over the input scale of interest. The generalized 2D CWT of topography, z, at location (u,v), then is given as

$$C(s,u,v) = \frac{1}{s}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z(x,y)\psi\left(\frac{x-u}{s}, \frac{y-v}{s}\right) dx dy, \tag{5}$$

where s (σ in Lashermes et al., 2007) is a parameter that sets the size of the wavelet kernel, specifically the standard deviation of the DoG. Equation 5, notably, is a convolution of z and ψ, expressed as

$$C(s,u,v) = z(x,y) * \psi\left(\frac{x-u}{s}, \frac{y-v}{s}\right), \tag{6}$$

where $*$ represents the convolution. The output scaled wavelet coefficients, C, are the Laplacian values that we use to extract C_{HT}, which we elaborate on below.

Similar to the application of PFTs to estimate erosion rate, it is necessary to select a measurement scale that effectively smooths over stochastic sediment transport perturbations and noise that is inherent to topographic datasets and DEMs and does not represent long-term morphology reflective of baselevel lowering (Hurst et al., 2012; Roering et al., 2010). Thus, it is important to utilize an appropriately scaled wavelet (akin to a kernel size) to generate curvature values that are appropriate to represent C_{HT}. Several definitions for the smoothing scale of a DoG wavelet exist. Torrence and Compo (1998) define the smoothing scale, λ, for an mth DoG as

$$\lambda = \frac{2\pi s}{\sqrt{m+\frac{1}{2}}}. \tag{7}$$

For the Ricker wavelet, m=2. Alternatively, Lashermes et al. (2007) define the Ricker wavelet smoothing scale as

$$\lambda = \sqrt{2}\pi s . \tag{8}$$

175 To clarify, while λ represents the physical scale at which topography is smoothed, *s* specifically defines the scale of the wavelet function and does not have a specific physical interpretation beyond its relationship with λ in Equations 7 and 8 and is not interchangeable with λ. While the Torrence and Compo (1998; TC98) and Lashermes et al. (2007; L07) λ definitions generate similar smoothing scales, the output Laplacian values may be sufficiently diverse to produce significantly different erosion rate estimates depending on the choice. Thus, we utilize both definitions for comparison to the curvature values

180 produced from the PFT.

We applied the CWT and PFT for λ values that correspond to the scales at which topographic noise manifests in topographic data. The CWT can only be applied for $s>1$, which for DEMs with a grid spacing of ~1 m with the odd-dimensions constraint of the PFT, places a lower λ limit of 5 m. We additionally tested larger λ (up to 141 m) to isolate the consistency between the CWT and PFT. For each smoothing scale, λ, for which we calculated curvature, we solved for s in

185 Equations 7 and 8 to construct the appropriately sized Ricker wavelet (Equation 4). We then applied the CWT to the OCR lidar DEMs for smoothing scales of 5-141 m (same as PFT) and produced C_{HT} values for the CWT and PFT methods, denoted as C_{HT-W} and C_{HT-P}, respectively.

3.2 Computational efficiency of curvature values

190 We compared the efficiency of calculating curvature with a PFT to the CWT, including both definitions of wavelet smoothing scale, λ (TC98 and L07; Equations 7, 8). We measured curvature for $\lambda=5-197$ m in MATLAB on a personal laptop with 16 GB of RAM. To account for potential variations in calculation time that may result from variable landscape morphology, we utilized sample regions of the Hadsall Creek and Bear Creek DEMs, as they represent the high and low erosion rate end members of our test sites. Each DEM was a 513x513 single precision grid with a cell size of 0.9144 m.

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3.3 Hilltop curvature calculation: extraction of hilltops

We calculated curvature at every pixel of our DEMs, but C_{HT} requires limiting curvature values to hilltop pixels. Therefore, we extracted hilltop masks in MATLAB with TopoToolbox (Scherler and Schwanghart, 2020; Schwanghart and Scherler, 2014), and we refined the hilltop masks by only considering locations where C_{HT} is negative (convex) and where

200 local hillslope gradient is less than 0.4, above which a greater proportion of hillslope sediment transport can be classified as nonlinear. We manually removed drainage divides mapped in low-relief valley bottoms and where flow routing is interrupted by roads, which are common in the OCR and introduce noisy high-magnitude curvatures. While the signature of deep-seated landslides is generally absent from our study catchments, if it appeared in the DEM that there has been a history of bedrock slope instability, we filtered hilltops proximal to mapped landslides. We also did not consider hilltops that exhibit

205 prominent asymmetry. Thus, at Hadsall Creek and NFSR, we neglected all hilltops at the main drainage divide (Fig. 1A). At the Bear Creek catchments, we similarly removed all hilltops at the main drainage divide (the northeast divide in Fig. 1B) except for those that border adjacent catchments that are likely experiencing the same baselevel imposed by Bear Creek (southwest divide in Fig. 1B). Finally, to visualize the scale-dependency of C_{HT} (Hurst et al., 2012; Roering et al., 2010), we

selected a single representative hilltop in each catchment (~100-200 m long; Fig. 1), chosen such that it approximates the
210 average curvature for the catchment when compared to curvature measurements taken for all hilltops (Fig. 2).

3.4.1 Erosion rates estimated from hilltop curvature

We applied the CWT and PFT to the Hadsall Creek, NFSR, and Bear Creek lidar DEMs, and estimated erosion rates. In the OCR, Roering et al. (2010) observed a scaling break in curvature at 15 m, corresponding to the length scale at
215 which pit and mound topography from tree throw pits is removed. We observe similar scaling breaks in hilltop curvature for selected hilltops at $\lambda \approx 15$-20 m (Fig. 2), though we note that the clarity of this scaling break depends on the size of the study area and consistency or lack thereof of small pit and mound topography in a landscape. Thus, while the scaling break that distinguishes the effective scale at which topographic noise is filtered out may differ between the DEMs and catchments we analyse here, we find that the scaling breaks do not clearly or systematically differ from those observed by Hurst et al.,
220 (2012) and Roering et al. (2010; Fig. 2). Thus, we used a smoothing scale of $\lambda = 15$ m for the PFT and CWT in each OCR catchment to estimate erosion rate as enumerated in Equation 1. We assumed that $\frac{\rho_s}{\rho_r} = 0.5$ and D=0.003 m^2 yr^{-1} (Roering et al., 2007). We compared the mean and variance of these estimated erosion rates to CRN-derived erosion rates in each OCR study catchment.

3.4.2 Erosion rates from cosmogenic radionuclides

To test the efficacy of C_{HT} as a proxy for erosion rate, we compare erosion rates estimated from C_{HT} to those estimated from CRNs in stream sediments. We collected stream sediments from the western tributary to Bear Creek that we study here (Fig. 1B; 44.186 °N, -123.375° W) to estimate erosion rate with cosmogenic ^{10}Be (Balco et al., 2013; Heimsath et al., 2001). We used the online calculator CRONUS (Balco et al., 2008) to calculate erosion rate for the sample, which
230 incorporates the material from the upstream drainage area and assumes steady erosion over the CRN integration timescale (Table 3). We additionally recalculate the erosion rates for Hadsall Creek and NFSR from the CRN data previously reported by Penserini et al. (2017; Table 2, 3).

3.5 Construction of synthetic hillslopes to test C_{HT} measurements

235 We utilized synthetic hillslopes generated from a theoretical model to compare the accuracy of hilltop curvature calculated using the PFT and CWT as well as test how well these approaches can predict erosion rate. We used the functional form for a 1D hillslope experiencing nonlinear diffusion given as

$$z = \frac{DS_c^2}{2(\rho_r/\rho_s)E}\left[\ln\left(\frac{1}{2}\left(\sqrt{1+\left(\frac{2\left(\frac{\rho_r}{\rho_s}\right)Ex}{DS_c}\right)^2}+1\right)\right) - \sqrt{1+\left(\frac{2\left(\frac{\rho_r}{\rho_s}\right)Ex}{DS_c}\right)^2}+1\right],$$ (9)

8

where E is the erosion rate calculated using Equation 1, S_c is the threshold, or critical, slope angle, and x is distance along the

240   hillslope profile (Roering et al., 2007). We extended the hillslope profile solution perpendicular to the x-axis to construct a

2D synthetic hillslope on a 201x201 m grid (Fig. 7, 8, S2). Odd hillslope dimensions ensure the existence of a hilltop pixel in

the middle of the domain. We utilized the PFT and CWT, including both CWT definitions for the wavelet scale λ (Equations

7, 8; TC98, L07), to calculate C_{HT-W} and C_{HT-P} of the synthetic hillslopes for several different scenarios. Specifically, we

considered various dimensionless erosion rates, E*, given by:

245    $$E^* = \frac{2E\left(\frac{\rho_r}{\rho_s}\right)L_H}{DS_c} = \frac{2C_{HT}L_H}{S_c},$$ (10)

where L_H is hillslope length (Roering et al., 2007). In testing the ability of the CWT and PFT to predict hilltop curvature, we

generate hillslopes with a range of E* values that can account for variations in E, C_{HT}, L_H, and S_c. For instance, low (high)

E* values may correspond to low (high) E, C_{HT}, or L_H as well as high (low) S_c, or some combination thereof. We specifically

tested E* values of 1, 10, 30, and 100. While E*=100 is an extreme case and may only be rarely observed in natural

250   landscapes that are eroding rapidly and also manage to maintain a soil mantle, such as badlands, E* values of 1, 10, and 30

have been readily observed in multiple landscapes (Clubb et al., 2020; Hurst et al., 2019, 2013; Marshall and Roering, 2014;

Roering et al., 2007).

In addition, to account for natural topographic roughness that the CWT and PFT smooth over to estimate C_{HT}, we

introduce noise to the synthetic hillslopes in the form of white (β=0), pink (β=-1), and red, or Brownian, (β=-2) noise, where

255   β is spectral slope. White noise denotes a random surface where all wavenumbers (frequencies) have equal amplitude, or

spectral power. Conversely, spectral power density varies inversely (β=-1) with wavenumber for pink noise, such that low

wavenumbers have higher intensity. Similarly, red noise exhibits higher spectral power at low wavenumbers, but more

dramatically than for pink noise. While hillslope spectra will vary between landscapes and likely exhibit a combination of

different spectral slopes depending on the scale of analysis, red noise surfaces generally best describe topographic noise in

260   natural landscapes while white noise surfaces are comparatively the least likely (e.g. Booth et al., 2009; García-Serrana et al.,

2018; Marshall and Roering, 2014; Pelletier and Field, 2016; Perron et al., 2008). We generated each noisy surface of values

normally distributed about 0 with the standard deviation ranging from -1 m (pits) to 1 m (mounds; Konowalczyk, 2021). For

each type of noise, we tested how the amplitude of the noise affects calculated C_{HT} by scaling the noise distributions by

0.1%, 0.5%, and 5% of hillslope length (L_H=100 m). In other words, we test cases where the standard deviation of the noise,

265   σ, is σ=0.001L_H, σ=0.005L_H, and σ=0.05L_H, corresponding to 1σ values of 10 cm, 50 cm, and 5 m, respectively. While

topographic noise with a distribution of amplitudes with a standard deviation of 5 m is likely unphysical for soil mantled

landscapes, this extreme case allows us to clearly test how different topographic parameters affect calculated values of E*

and how well each measurement technique can filter out noise.

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4 Results

4.1 Computational efficiency of CWT and PFT curvature calculation

We find that the CWT is dramatically more efficient at calculating hilltop curvature than the PFT. Curvature

275 calculation time depends on smoothing scale, λ, with large kernel sizes taking longest for both the PFT and CWT. Specifically, we compared curvature calculation times for the PFT and CWT in selected portions of the Hadsall Creek and Bear Creek catchments for $\lambda=5$-197 m. We find that for the 513x513 single precision grid, the PFT takes ~4-4.5 seconds to calculate curvature at the smallest scales and ~30 seconds to calculate curvature at larger scales (Fig. 3A, B). Measurement time does not vary greatly between the fast and slowly eroding landscape DEMs. By comparison, for $\lambda=5$-200m, both the

280 CWT L07 and TC98 definitions for λ calculate curvature at the smallest scales in ~0.0039-0.004 seconds while at larger scales they calculate curvature in ~2.2-2.3 seconds (Fig. 3A, B). Comparing the two techniques, we find that at the smallest smoothing scales ($\lambda=5$m) the CWT operates >10^3 times faster than the PFT, while at larger scales where λ approaches 200 m, the CWT still outpaces the PFT by over an order of magnitude (Fig. 3C, D).

285 ### 4.2 Similarity of C_{HT-P} and C_{HT-W}

We utilized 2D CWTs and PFTs to calculate C_{HT-W} and C_{HT-P} for a range of λ in the OCR catchments of Hadsall Creek, NFSR, and Bear Creek. We find that C_{HT-W} and C_{HT-P} are similar when using λ values of 5-30 m. Specifically, Fig. 2 compares output C_{HT-W} using both λ length scale definitions (Equations 7, 8) and C_{HT-P} for the representative hilltop in each catchment. Mean measured C_{HT-W} and C_{HT-P} values differ the most at small smoothing scales, where signal to noise ratio

290 (topographic noise to underlying C_{HT}) is highest (Fig. 2A, D, G). At these small smoothing scales, the standard deviation of C_{HT-P} is larger than that of C_{HT-W} (Fig. 2B, E, H). Mean C_{HT-W} for both L07 and TC98 λ definitions are similar, as are the output standard deviations (Fig. 2). However, we observe that mean C_{HT-W} calculated using the TC98 definition of λ is lower in magnitude than that of L07 (Fig. 2; Table 1). This is not unexpected, however, since λ, as defined by TC98 in Equation 7, is effectively smaller than that of L07 defined in Equation 8, for a given wavelet scale, s. Figure 4 compares the output C_{HT}

295 measurements from each technique by plotting C_{HT} for individual DEM nodes for $\lambda=15$ m. If measurements from each technique are in agreement, their output C_{HT} values should plot as a 1:1 line. Indeed, C_{HT-W} for TC98 λ is lower than that of L07 for both the representative hilltop and all mapped hilltops, with the largest deviation occurring on the sharpest hilltops (Fig. 4C, F, I). Similarly, mean C_{HT-W} (TC98 and L07) is lower than C_{HT-P}, particularly for high magnitude curvatures. Nevertheless, the output values from each definition do not vary dramatically, particularly when considering the C_{HT} for

300 DEM nodes corresponding to representative hilltops (Fig. 4).

We additionally plot probability density functions (PDF) of measured C_{HT-W} and C_{HT-P} for each catchment (Fig. 5, S1). Notably, the shape of each PDF is similar between measurement techniques but is shifted along the x-axis due to the variable definitions of λ. This shift is further illustration of the deviation from a 1:1 relationship between each measurement technique as observed in Fig. 4. Similar to the greater deviation between calculated C_{HT} at curvature extrema in Fig. 4, we

305 observe greater offset between PDFs in the distribution tails, while the peaks remain similar. We observe this consistency

between PDF peaks reflected in the mean C_{HT} of the PDFs, which are similar regardless of measurement technique (Fig. 5; Table 1, 2).

4.3 Erosion rate calculated with C_{HT} and cosmogenic radionuclides

310 We utilized C_{HT} for $\lambda=15$ m to estimate erosion rate. Erosion rates calculated from C_{HT-P} and C_{HT-W} for all mapped hilltops and the representative hilltop in each catchment can be found in Table 2. We observe that C_{HT-P} and C_{HT-W} produce expected relative pattern of erosion rate in our OCR catchments. That is, calculated erosion rate from C_{HT} is fastest at Hadsall Creek and slowest at Bear Creek, as revealed by our cosmogenic erosion rate data (Fig. 5, Table 2). Notably, we observe that C_{HT}-generated erosion rates (mean ± standard deviation) fall within or near the measurement uncertainty of the CRN erosion rate for both the representative hilltop and all hilltops (Table 2). For instance, CRN-measured erosion rates are

315 0.113 ± 0.018 mm yr^{-1} at Hadsall Creek, 0.058 ± 0.0054 mm yr^{-1} for the NFSR, and 0.008 ± 0.0007 mm yr^{-1} at Bear Creek (Table 3). Similarly, for the case of the representative hilltop and using the TC98 λ definition, we find C_{HT}-calculated erosion rates of 0.178 ± 0.030 mm yr^{-1} at Hadsall Creek, 0.088 ± 0.025 mm yr^{-1} for the NFSR, and 0.007 ± 0.005 mm yr^{-1} at Bear Creek. The rates calculated with the PFT and L07 λ definition are similar, whether considering the representative hilltop or all mapped hilltops in each catchment (Table 2; Fig. 5, 6, S1). Finally, we observe linear correlation between C_{HT}-calculated and

320 CRN-measured erosion rates at our OCR catchments, consistent with the relationship between C_{HT} and E expected in Equation 1 (Fig. 6).

4.4 Testing of C_{HT} extraction with synthetic hillslopes

We calculated C_{HT-P} and C_{HT-W} for a series of synthetic hillslopes with a range of dimensionless erosion rates, E*,

325 and topographic noise (Fig. 7, 8). We observe that the ability of the CWT and PFT to reproduce the defined curvature at particular λ depends on the dimensionless erosion rate, E*, though the type and magnitude of added noise contributes to uncertainty in appropriate λ values to be used to calculate erosion rate. We focus on synthetic hillslopes where no noise has been added as well as where noise amplitude $\sigma=0.5\%$ L_H, as the magnitude of noise in this case ($\sigma=50$ cm) is a reasonable physical approximation of noise and surface roughness in natural landscapes (e.g. Roth et al., 2020). The cases where noise

330 amplitude is defined by $\sigma=0.1\%$ L_H ($\sigma=10$ cm) and $\sigma=5\%$ L_H ($\sigma=5$ m) can be found in the Supplemental Information (Fig. S5-S10).

4.4.1 Slowly eroding synthetic hillslopes, E*=1

We observe that for E*=1, both the PFT and CWT reasonably predict the model-defined C_{HT} (and thus E) at

335 moderate smoothing scales. Specifically, when $\sigma=0.5\%$ L_H, C_{HT-W} and C_{HT-P} converge on the defined C_{HT} when $\lambda>\sim9-11$ m for white noise, $\lambda>\sim15-19$ m for pink noise, and $\lambda>\sim13$ m for red noise (Fig. 9B-D). At smaller λ, the signal to noise ratio is too high for noise be adequately filtered by either the PFT or CWT. This mirrors past results in natural landscapes, where a sufficiently large smoothing scale must be selected to smooth over topographic noise and recover an accurate C_{HT} (Hurst et al., 2012; Roering et al., 2007). Notably, when E*=1, the hillslopes are not sufficiently steep to approach Sc (Fig. 7, 8). Thus,

340 even at the largest smoothing scales, the CWT and PFT accurately record curvature (Fig. 9A-D). In natural landscapes, however, valley bottoms will introduce positive curvatures, which will cause an increase in curvature (i.e. become less negative), at smoothing scales that approach the hillslope length, which has also been utilized to constrain an optimal smoothing scale (Hurst et al., 2012), and which we observe in OCR catchments (Fig. 2A, D, G).

We observe that the uncertainty in C_{HT}, which we define as the standard deviation of C_{HT} along the hilltop, is
345 highest at the smallest smoothing scales (Fig. S3, S4). Notably, we observe for all noise types that at small smoothing scales of λ=5-~13m, C_{HT-P} exhibits higher uncertainty than C_{HT-W}. As λ increases, the uncertainty in C_{HT-P} and C_{HT-W} diminishes as topographic noise is progressively filtered. Because red noise includes higher spectral power at long wavelengths, we observe that the decrease in C_{HT} uncertainty occurs at larger smoothing scales, converging towards 0 at scales of >17 m (Fig. S4).

350 We find that when no noise is added to the synthetic hillslopes, C_{HT-P} and C_{HT-W} accurately predict C_{HT} at all scales (Fig. 9A). While there is some deviation between measured and defined C_{HT} at larger scales, this deviation is exceptionally small (<0.5%) and is primarily a result of edge effects that may not be fully clipped for both the CWT and PFT at the edge of the synthetic hillslope domain. Uncertainty in C_{HT-W} and C_{HT-P} is near 0 when no surface noise is added, with deviations again primarily due to the presence of edge effects that are not fully clipped off at the hillslope tips (Fig. S4). Finally, we
355 observe that for a given style and amplitude of added topographic noise, the uncertainty in C_{HT} does not vary with changes in E* (Fig. S4). We do not vary topographic noise as a function of E*, so equal uncertainty over a range of E* values indicates that variable hillslope form as defined by E* does not affect the uncertainty in C_{HT} *along* the hilltop. Given the convolutional form of the CWT in Equation 6 and the distributive property of convolutions (i.e $f * (g + h) = (f * g) + (f * h)$, where f is the wavelet, g is the synthetic hillslope, and h is surface noise), the standard deviation of C_{HT} remaining constant as a
360 function of E* is not unexpected.

4.4.2 Moderate to fast eroding synthetic hillslopes, E*≥10

We observe that both the CWT and PFT produce biased C_{HT} as E* increases. The deviation between the model-defined and measured C_{HT} progressively grows for larger E*. Specifically, for the case of λ=15 m, when E*=10, we find that
365 C_{HT-W} and C_{HT-P} are within ~10% of the defined C_{HT}, with modest dependencies on the type of topographic noise (Fig. 9F, G, H). However, C_{HT-P} and C_{HT-W} are underestimated by >20% for E*=30 hillslopes and by 60% for E*=100 slopes when λ=15 m. This deviation occurs for hillslopes constructed with topographic noise of all types as well as the synthetic hillslopes without added noise (Fig. 9I-L). Even for small λ, we observe that C_{HT} is systematically underestimated. For the case of E*=30, we observe that C_{HT-W} and C_{HT-P} deviate by 10-25% for λ<15 m, with the smallest λ (~5-7 m) exhibiting the least
370 deviation, with C_{HT-P} and C_{HT-W} falling within ~10% of the known C_{HT}. C_{HT} is reasonably recovered at λ=5 m for the red noise E*=30 hillslope, despite the noise dominating C_{HT-W} and C_{HT-P} when λ=5 m for the E*=1, 10 hillslopes. Given the added noise is constant between E* values, this accurate recovery of C_{HT} for E*=30 when λ=5 m may indicate that planar hillslopes introduce curvature values sufficiently near-zero to cancel out the positive (concave) noise. For λ>15 m, we

observe that C_{HT} is underestimated by at least 25% for all E*=30 hillslopes and >60% for E*=100 hillslopes. As λ increases,
375 this deviation systematically grows such that when λ=35, C_{HT} is underestimated by half for E*=30 hillslopes and ~80% for
exceptionally narrow hillslopes where E*=100 (Fig. 9I-P). Importantly, we observe these major deviations for the hillslopes
with no added noise as well, indicating that topographic noise is not solely responsible for biased C_{HT}.

5 Discussion

380   Application of CWTs and PFTs to measure C_{HT} and estimate erosion rate in soil mantled landscapes such as the
OCR produces erosion rate values that are in agreement with those collected from CRNs in stream sediments, though with
dramatically disparate efficiencies. Yet, we also observe that while both techniques accurately reproduce hillslope
morphology in synthetic landscapes experiencing modest dimensionless erosion rates, both techniques exhibit systematic
bias where dimensionless erosion rate is moderate to high, calling into question the accuracy of past estimates of erosion rate
385 in landscapes that are experiencing moderate to rapid erosion rates. Nevertheless, CWTs are an exciting tool to be added to
hillslope geomorphometric analyses, particularly as high-resolution topographic datasets continue to grow and classification
of topographic roughness, particularly on the hillslope scale, continues to improve.

5.1 C_{HT} measurement and erosion rate estimation in natural landscapes: Oregon Coast Range

390   We utilized CWTs and PFTs to estimate erosion rate in a landscape that has been thoroughly studied in past
geomorphology studies. Encouragingly, C_{HT}-calculated erosion rate in Hadsall Creek, NFSR, and Bear Creek reproduce
CRN-measured erosion rates from each site. We also observe, however, that some variability in measured C_{HT} reinforces the
need to use caution when selecting hilltops at which curvature will be extracted, especially in landscapes where topographic
noise, including from anthropogenic sources such as roads, as well as landslides and variable lithology, may introduce
395 inaccurate measurements of curvature. Indeed, despite careful selection of hilltops, calculated C_{HT} exhibit a wide range of
values (Fig. 4, 5, S1). Fortunately, the catchments we have sampled here exhibit few to no deep-seated landslides and are
mapped entirely within the Tyee Formation, which exhibits little variability over small spatial scales. Also, while there are
numerous forest and logging roads throughout the OCR, they are easily identifiable in lidar data and are limited to a small
portion of hilltops. Hence, while haphazard selection of hilltops without a predefined methodology for trimming hilltops
400 should be avoided, our observed agreement between estimated erosion rates for all selected hilltops in a catchment and
representative hilltops emphasizes that mild to moderate trimming of hilltop masks is sufficient for estimating an accurate
erosion rate (Table 2, Fig. 5, S2). Finally, agreement between TC98 and L07 λ definitions and CRN erosion rates suggests
that either definition is reasonable for calculating C_{HT}. However, careful and informed selection of λ when calculating
erosion rate remains paramount.

5.2 Rapid calculation of C$_{HT}$

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We have demonstrated that CWTs calculate C$_{HT}$ >10^3 times faster than PFTs at smoothing scales of λ=5 m. At smoothing scales often utilized to estimate C$_{HT}$ (~10-30 m), the CWT operates >10^2 times faster. Even at the largest smoothing scales we test (up to 197 m), the CWT operates ~14-15 times faster than the PFT. This dramatic difference in curvature calculation time opens many doors for utilizing hilltop curvature in topographic analyses of landscapes that require

410 consideration of large spatial scales. What's more, the ability of the CWT to operate so efficiently on high-resolution lidar data does not necessitate that coarse data be used to analyse large regions, as has generally been the case for past geomorphic analyses of regional and continental-scale bedrock rivers. Rather, the ability of the CWT to calculate hilltop curvature over large spatial scales with such speed means that the limiting factor for large landscape analyses where lidar data is available is no longer the operating time of the measurement technique, but rather the ability of existing systems to store vast quantities

415 of high-resolution topographic data and curvature-related products! In addition to C$_{HT}$ measurement, the rapidity of the CWT will allow for large-scale analyses of other landscape characteristics well-suited to spectral analyses including mapping landslides (Booth et al., 2009; LaHusen et al., 2020), quantifying surface roughness (Doane et al., 2019; Roth et al., 2020), and mapping landforms (Black et al., 2017; Perron et al., 2008; Struble et al., 2021).

420 **5.3 C$_{HT}$ underestimated in moderate to fast-eroding landscapes**

We find that both the CWT and PFT are unable to reproduce accurate C$_{HT}$ at moderate to fast dimensionless erosion rates. Disagreement between measured and defined C$_{HT}$ for a given E* can be conceptualized primarily as a biasing of curvature measurement as hilltops progressively narrow and steepen in response to faster erosion rates. Specifically, since we utilize a nonlinear diffusion framework to construct the synthetic hillslopes (Equation 9), planar side slopes begin to

425 develop and advance towards the hilltop as the hillslope gradient approaches the critical slope angle, S$_c$, at moderate to fast E*. The formation of planar hillslopes means, by definition, that curvature does not accurately reflect C$_{HT}$ along the entire hillslope length, as would be the case for a hillslope experiencing linear diffusion. The E*=1 synthetic hillslope, while also constructed with Equation 9, can be approximated as experiencing linear diffusion, as slopes are not sufficiently steep to approach S$_C$ and develop planarity. In this case, even as λ increases, C$_{HT-W}$ and C$_{HT-P}$ accurately recover the actual C$_{HT}$. We

430 observe that at these slow erosion rates (E*=1-10), the main obstacle to recovering an accurate C$_{HT}$ is topographic noise (Fig. 9A-H). As we have applied here, and has been previously demonstrated (Hurst et al., 2012; Roering et al., 2007), careful selection of a λ sufficiently large to remove such noise, but not so large such that concave valley bottoms introduce positive curvatures, still allows for accurate calculation of C$_{HT}$, particularly for E*=1. By contrast, in cases where E* is sufficiently high to develop planar side slopes, once λ reaches a sufficiently high value to remove topographic noise, the CWT and PFT

435 kernels have become sufficiently large to incorporate planar slopes into the curvature measurements, thus underpredicting the actual value of C$_{HT}$. In these cases, topographic noise is a secondary impediment to accurate C$_{HT}$ measurement, preventing utilization of a sufficiently small λ that avoids planar hillslopes. Furthermore, if E* is sufficiently large, planar

side slopes may appear close enough to the hilltop to disqualify almost any smoothing scale, which is clear from our synthetic hillslopes with no added noise (Fig. 9E, I, M). Importantly, even at modest E*=10, planar slopes begin to bias C_{HT}

440   (Fig. 9E).

The grid resolution of digital topographic data has been recognized to affect measurements of topographic curvature and hillslope sediment flux (e.g. Ganti et al., 2012; Grieve et al., 2016b). However, the deviation between known and measured C_{HT} we note here is intrinsic to the form of hillslopes that are described by the nonlinear diffusion model. As E* increases and the hilltop undergoes a concomitant increase in C_{HT}, a smaller λ is ideally required to avoid the planer side

445   slopes and accurately calculate C_{HT}. Unfortunately, however, λ can only be decreased so much before topographic noise and stochastic and disturbance-driven processes begin to overwhelm the calculated curvature values (Hurst et al., 2012, 2013; Roering et al., 2007; Fig. 2, 9). As such, increasing the resolution of topographic data, while desirable for characterizing hillslope sediment transport processes, will not by itself alleviate the systematic deviation between measured and model-specified C_{HT}, as such high-resolution data will also be recording the stochastic signals that deviate from the underlying

450   hillslope form (Roth et al., 2020). However, improved characterization of the distribution of roughness and microtopography in landscapes and how they may vary with erosion rate may provide a remedy for estimating erosion rate from topography and defining a better-informed λ, particularly in landscapes where hilltops are conspicuously sharp and where topographic resolution continues to improve.

Importantly, we stress that neither CWTs nor PFTs are, at this time, capable of accurately estimating hilltop

455   curvature at moderate to high E*, even when λ is small (Fig. 9). We observe that the CWT and PFT systematically underpredict E* when E*=100 (Fig. 9M-P). However, we acknowledge that E* values of 100 are perhaps unreasonably high for most natural landscapes, with perhaps a few notable exceptions (e.g. Taiwan, Himalaya, New Zealand). More so, soil production limits (e.g. DiBiase et al., 2012; Heimsath et al., 1997; Neely et al., 2019) imply that these settings may exhibit processes that are not well represented with the soil creep model employed here. Regardless, the CWT and PFT clearly

460   underpredict C_{HT} when E*=30 and exhibit underpredicted C_{HT} when E*=10, *even in the most ideal case when synthetic hillslopes have no added noise*. Similar values of E* have been recorded in numerous natural landscapes (Clubb et al., 2020; Grieve et al., 2016a; Hurst et al., 2019, 2013, 2012; Marshall and Roering, 2014; Roering et al., 2007).

5.4 Does hilltop curvature vary linearly with erosion rate?

465   The systematic underestimation of C_{HT} that we observe here has important implications for interpreting erosion rates and hillslope surface processes in natural soil-mantled landscapes that are not eroding slowly. Specifically, our results here urge caution when applying hilltop curvature measurement techniques to natural soil mantled landscapes eroding at moderate to rapid rates and where hilltops are correspondingly sharp. While C_{HT} has been found to generally agree with independently calculated erosion rates following Equation 1, the measurement artifact we have observed here calls into

470   question the accuracy of calculated erosion rates from C_{HT} in natural landscapes in past studies. Recent observations put forward by Gabet et al. (2021), show that hilltop curvature varies with the square root of erosion rate, which implies a square

root relationship between hillslope diffusivity, D, and erosion rate, representing a deviation from the long-held view that C_{HT} varies linearly with erosion rate (Equation 1). Here we use a synthetic hillslope simulation to explore whether these findings may be influenced by a systematic bias in the estimation of C_{HT} as described above (Fig. 9). Specifically, we followed the

475   methodology laid out by Gabet et al. (2021) and reproduced a C_{HT}-E relationship from synthetic hillslopes with *no added noise*. While Gabet et al. (2021) constructed synthetic hillslope profiles to account for the effect of grid spacing on calculated C_{HT}, we additionally consider the role of smoothing scale, λ, on estimation of C_{HT}. In order to facilitate comparison, we initially selected λ=14 m, the same scale that Gabet et al. (2021) applied at each of their field sites, to calculate C_{HT}. We constructed a series of synthetic hillslopes described by Equation 9 for E*=1-100, which corresponds to erosion rates of

480   ~0.01-1 mm yr^{-1} (assuming D=0.003 m^2 yr^{-1} and $\frac{\rho_s}{\rho_r}$=0.5). We used the CWT to calculate curvature; a PFT could be used as well, which would be consistent with the Gabet et al. (2021) methodology. However, as we have demonstrated, C_{HT-P} and C_{HT-W} are similar for both natural hillslopes and synthetic hillslopes with no added noise (Fig. 2, 9).

We observe that erosion rate and C_{HT} do not vary linearly as expected from Equation 1 for all erosion rates (Fig. 10). While the relationship between erosion rate and measured hilltop curvature (we plot the absolute value, $|C_{HT}|$, to allow

485   visualization of positive values) is linear as expected from Equation 1 for erosion rates of 0.01-0.08 mm yr^{-1}, the measured and actual synthetic values of $|C_{HT}|$ begin to clearly diverge for erosion rates >0.08 mm yr^{-1} (Fig. 10A), though some deviation exists at erosion rates as low as ~0.03 mm yr^{-1} (Fig. 10B; blue squares). As this deviation increases with E*, it approximates a square root relationship between erosion rate and hilltop curvature. Importantly, the erosion rate at which this deviation occurs is heavily dependent on smoothing scale and diffusivity. We tested a range of diffusivities (D=0.001-0.005

490   m^2 yr^{-1}) for λ=14 m and λ=20 m and plotted the ratio of measured C_{HT} to the actual C_{HT} (Fig. 10B). We find that for smaller D, the deviation between measured and model-defined C_{HT} occurs at slower erosion rates, while λ dictates the magnitude of deviation (Fig. 10B). Thus, while the erosion rates at which we observe significant deviation between measured and model-defined C_{HT} tend to be higher than those found in many landscapes (for D=0.003 m^2 yr^{-1} and λ=14 m), including those tested by Gabet et al. (2021), the strong dependency of this deviation on diffusivity and smoothing scale warrants caution in

495   interpretations of nonlinear relationships between hilltop curvature and erosion rate. We encourage future work to investigate climatic and other factors that dictate hillslope diffusivity and the potential coupling between diffusivity and erosion rate (e.g. Richardson et al., 2019), although care must be taken to ensure that observed relationships do not result from measurement artifacts that result from the underlying hillslope form.

Current hilltop curvature measurement techniques do not have a well-defined capability to filter topographic noise

500   that is inherent to all landscapes and topographic datasets while establishing an unbiased value of C_{HT} at elevated E*. As such, estimates of erosion rates using Equation 1 should be considered minimum erosion rates, particularly in landscapes with conspicuously sharp hilltops. These results strongly motivate future investigation of the structure of topographic noise in landscapes due to underlying processes such as trees throw and other sources of bioturbation, as well as noise inherent to

digital topographic data. Improved understanding of the structure of topographic surface roughness may facilitate future accurate morphologic estimates of erosion rate in fast-eroding landscapes.

6. Conclusions

We utilized 2D continuous wavelet transforms to calculate hilltop curvature in three catchments in the Oregon Coast Range that exhibit a diversity of hillslopes. We found that the measured hilltop curvature values are comparable to those calculated from fitting 2D polynomial functions to topography to calculate curvature, a method that has been commonly applied elsewhere. Both techniques produce estimates of erosion rate that are consistent with those independently constrained from cosmogenic radionuclides in stream sediments. Specifically, we find that erosion rate calculated with the CWT is ~0.156±0.055 mm yr^{-1} in Hadsall Creek, 0.1±0.05 mm yr^{-1} in the North Fork Smith River, and 0.01±0.008 mm yr^{-1} in three small catchments that drain to Bear Creek. We further we find that the 2D continuous wavelet transform operates 10^2 to $>10^3$ times faster than the 2D polynomial when applied at smoothing scales that are commonly used for calculating hilltop curvature (~5-30 m). This dramatic disparity in operation time opens numerous doors for widespread topographic analysis as high-resolution topographic data becomes increasingly available.

We additionally test the accuracy of both the wavelet transform and polynomial by constructing synthetic hillslopes following a nonlinear diffusive hillslope geomorphic transport law. Synthetic hillslopes were constructed with and without added surface noise of various types (white, pink, red/Brownian) and exhibited various forms corresponding to a range of dimensionless erosion rates. We find that both the wavelet transform and polynomial are able to reproduce hilltop curvature for slow dimensionless erosion rates (E*=1-10). However, we also observe that both techniques produce underestimated values of C_{HT} when E*≥10, as planar hillslopes begin to systematically bias the calculated curvature at the hilltop. While this is in part due to the required smoothing of topography to remove added noise, which in natural landscapes is due to stochastic transport processes as well as noise inherent in digital topographic data, we also find that curvature is underestimated on synthetic hillslopes where there is no added noise. At moderate to high dimensionless erosion rates (E*=30-100), we find that hilltop curvature is systematically underestimated as hillslopes become progressively narrower near the hilltop. This systematic deviation from the defined and measured hilltop curvature has key implications for predicting erosion rates in soil mantled landscapes. In landscapes eroding at moderate to rapid rates, erosion rates calculated with hilltop curvature should be considered a minimum. Finally, we demonstrate that underestimation of synthetic hilltop curvature at moderate to fast erosion rates results in apparent power law and square root relationships between erosion rate and hilltop curvature. This previously observed relationship from natural hillslopes has led to suggestions that hillslope diffusivity may also vary as the square root of erosion rate. Our results here, however, demonstrate that this is likely a measurement artifact introduced by planar hillslopes biasing hilltop curvature measurements as hilltops progressively narrow and steepen. Future hillslope geomorphic work must more clearly characterize the roughness characteristics of soil mantled hillslopes and develop methods that smooth and remove topographic noise while maintaining an unbiased hilltop curvature measurement, if hilltop curvature is to be applied in rapidly eroding landscapes.

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Code and Data Availability

545 We utilized TopoToolbox (https://topotoolbox.wordpress.com/download; Schwanghart and Scherler, 2014) code in this paper, which is freely available. We additionally used wavelet codes from the Automated Landslide Mapping toolkit (ALMtools) by Adam Booth (http://web.pdx.edu/~boothad/tools.html; Booth et al., 2009). Additional MATLAB scripts, including for synthetic hillslope construction, are available at https://github.com/wtstruble. All utilized lidar DEMs are publicly available from the Oregon Department of Geology and Mineral Industries (https://www.oregongeology.org/lidar/).

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Author Contributions

WS and JR conceived of and designed the study. WS developed the study and completed the analysis. WS prepared the manuscript with contributions from JR.

555 **Competing Interests**

The authors declare that they have no conflict of interest.

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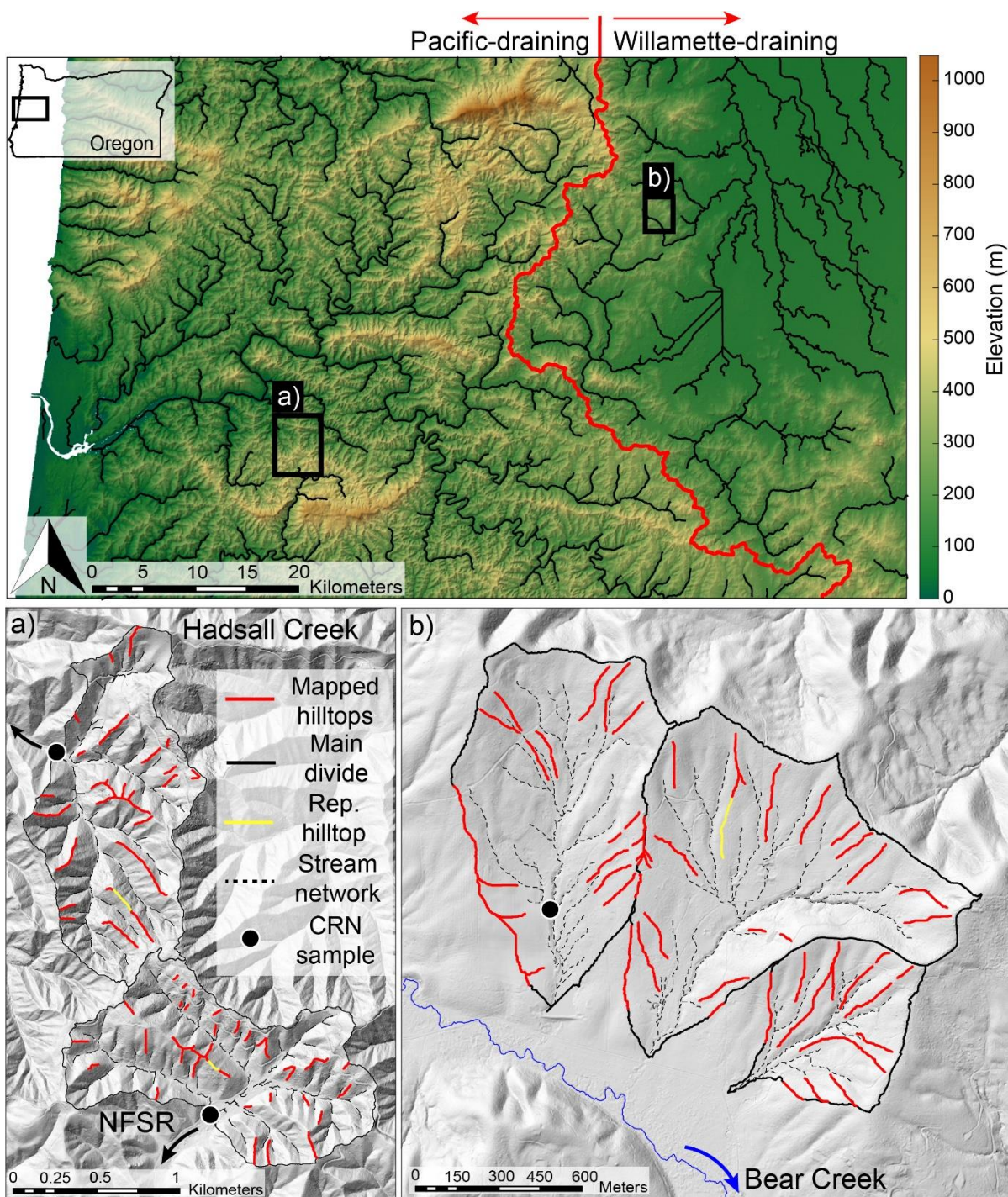
Figure 1: Oregon Coast Range study sites. Note the drainage divide (red) between catchments that flow directly to the Pacific Ocean and those that flow east into the Willamette River, which then flows northward to the Columbia River. A) Hadsall Creek and the North Fork Smith River (NFSR). B) The three catchments that flow to Bear Creek. Arrows in A) and B) denote river flow direction.
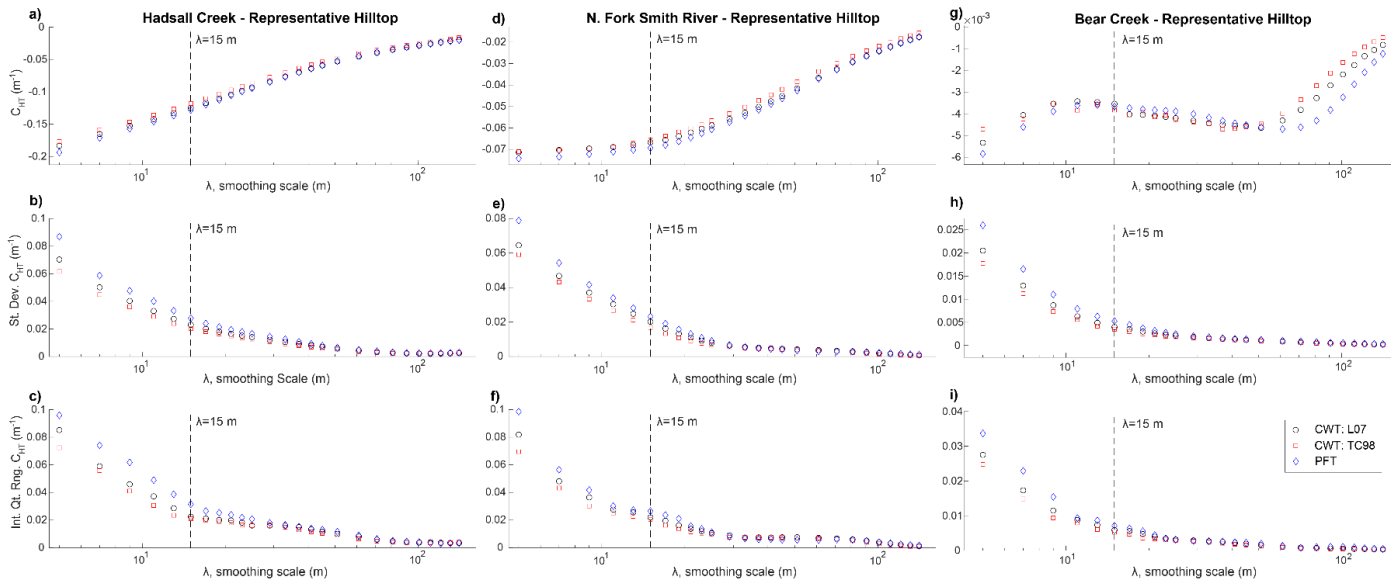
Figure 2: Curvature extracted from representative hilltop at Hadsall Creek, NFSR, and Bear Creek for a range of λ. Upper row is C_{HT} measurements, second row is the standard deviation of C_{HT}, and the bottom row is the interquartile range of C_{HT}. Note that the scaling break that identifies where tree throw pits are filtered out depends on the size of the considered hillslope and consistency of pit-mound topographic in a landscape. Here, though, a break exists at λ≈15 m for Hadsall Creek (especially apparent in standard deviation and interquartile range) and the NFSR and Bear Creek at λ≈11-20 m (note that second break at ~60 m in Bear Creek corresponds to the introduction of concave valleys). These scaling breaks are generally consistent with those observed for the OCR by Roering et al. (2010) and are visible for CWT and PFT λ definitions.

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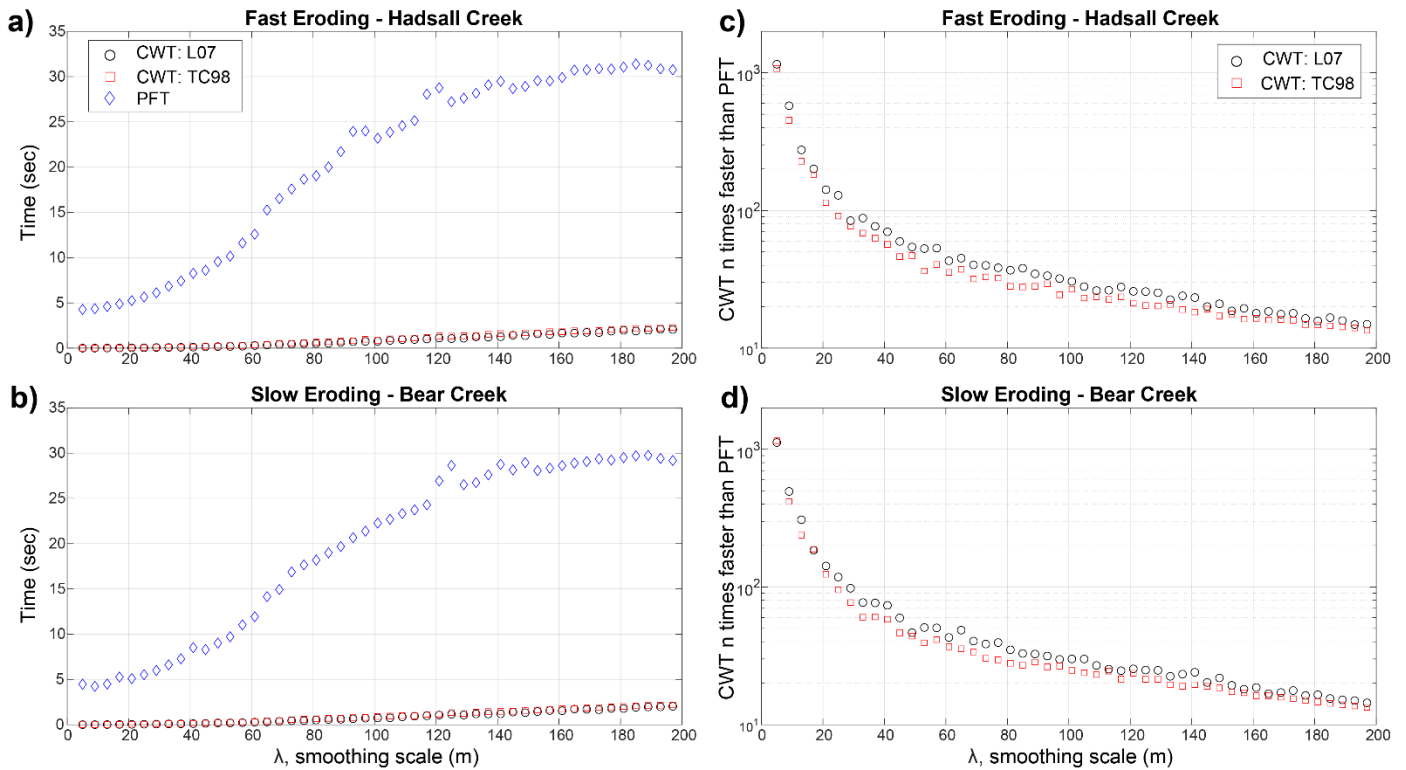
Figure 3: Speed of the CWT and PFT for different smoothing scales, λ. A, B: CWT and PFT processing time for small portions (513x513 single precision grid, cell size of 0.9144 m) of the Hadsall and Bear Creek catchments. Note the speed and consistency of the CWT. C, D: Relative speed of the CWT to the PFT, quantified as the ratio of CWT/PFT. Thus, for each smoothing scale, each point can be interpreted as the CWT being *n* times faster than the PFT. At small λ, the CWT is >1000 times faster than the PFT. The CWT remains >100 times faster than the PFT until λ≈30 m, a scale that is usually larger than most smoothing scales utilized in C_{HT} calculation.

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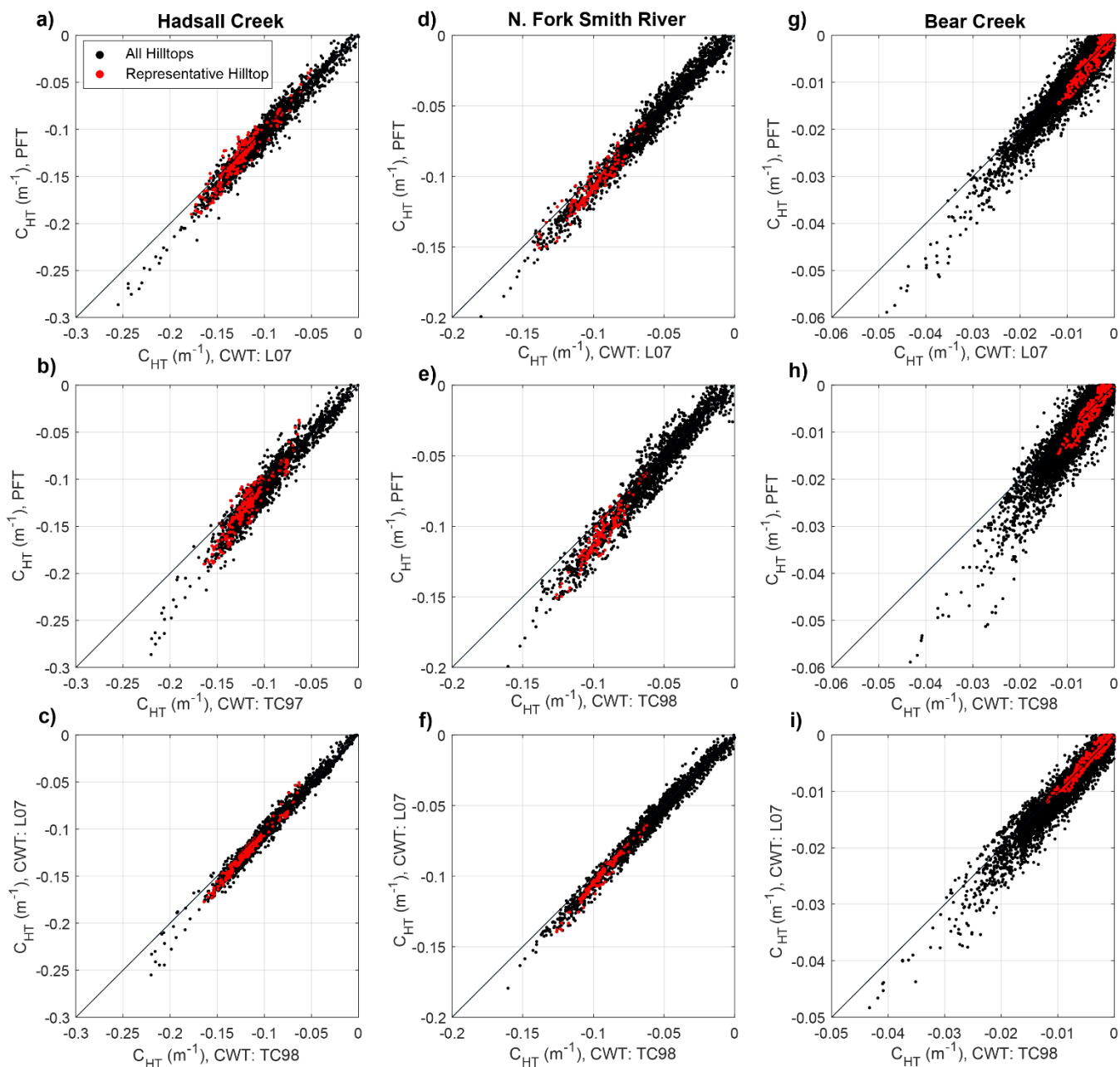
Figure 4: Comparison of C_{HT} calculation methods for λ=15 m. Black dots correspond with curvature measured at nodes for all mapped hilltops (roads, landslides, valley bottoms, etc. *removed*). Red points correspond with the representative hilltop nodes (Fig. 1). Perfect agreement between measurement techniques would plot as 1:1 line (black line). Recall that more positive (lower magnitude) C_{HT} corresponds with more gentle hillslopes (upper-right corner). See text for details. L07: Lashermes et al. (2007); TC98: Torrence and Compo (1998).

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Site	Lat. (° N)	Long. (° W)		Mean C_{HT} (m^{-1})			Median C_{HT} (m^{-1})			Standard Deviation C_{HT} (m^{-1})		
				CWT (L07)	CWT (TC98)	PFT	CWT (L07)	CWT (TC98)	PFT	CWT (L07)	CWT (TC98)	PFT
Hadsall Creek	43.983	-123.823	All Hilltops	-0.104	-0.099	-0.110	-0.111	-0.106	-0.116	0.037	0.034	0.040
			Rep. Hilltop	-0.125	-0.119	-0.129	-0.126	-0.120	-0.128	0.023	0.020	0.028
NFSR	43.963	-123.811	All Hilltops	-0.061	-0.059	-0.065	-0.059	-0.057	-0.061	0.033	0.037	0.037
			Rep. Hilltop	-0.067	-0.066	-0.069	-0.063	-0.063	-0.068	0.020	0.017	0.023
Bear Creek	44.181	-123.371	All Hilltops	-0.007	-0.006	-0.008	-0.006	-0.005	-0.006	0.006	0.005	0.007
			Rep. Hilltop	-0.005	-0.005	-0.006	-0.004	-0.004	-0.005	0.003	0.003	0.004

Table 1: C_{HT} measured at OCR study sites by the CWT and PFT for λ=15 m. L07: λ definition of Lashermes et al.
805 **(2007); TC98: λ definition of Torrence and Compo (1998). All values rounded to nearest 10-thousandth.**

Site	Lat. (° N)	Long. (° W)		Mean E (mm yr^{-1})			Median E (mm yr^{-1})			Standard Deviation E (mm yr^{-1})			CRN E (mm yr^{-1})
				CWT (L07)	CWT (TC98)	PFT	CWT (L07)	CWT (TC98)	PFT	CWT (L07)	CWT (TC98)	PFT	
Hadsall Creek	43.983	-123.823	All Hilltops	0.156	0.149	0.164	0.167	0.159	0.174	0.055	0.051	0.060	0.113±0.018*
			Rep. Hilltop	0.188	0.178	0.193	0.189	0.179	0.193	0.034	0.030	0.042	
NFSR	43.963	-123.811	All Hilltops	0.100	0.099	0.104	0.088	0.086	0.092	0.050	0.046	0.055	0.058±0.0054*
			Rep. Hilltop	0.092	0.088	0.097	0.095	0.095	0.101	0.030	0.025	0.035	
Bear Creek	44.181	-123.371	All Hilltops	0.010	0.009	0.012	0.008	0.008	0.009	0.008	0.007	0.010	0.008±0.0007
			Rep. Hilltop	0.007	0.007	0.009	0.006	0.006	0.008	0.005	0.005	0.006	

810 **Table 2: Erosion rate at OCR study sites calculated with Equation 1, assuming D=0.003 m^2 yr^{-1} and $\frac{\rho_s}{\rho_r}$=0.5. L07: λ**
definition of Lashermes et al. (2007); TC98: λ definition of Torrence and Compo (1998). All values rounded to nearest
10-thousandth.
*Recalculated erosion rates from Penserini et al. (2017); see Table 3.

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Catchment	Location	Concentration (atoms g^{-1} quartz)	Error (atoms g^{-1} quartz)	Erosion Rate (mm/yr)	Error (mm/yr)	Notes
Hadsall Creek	43.985°N, -123.824°W	33766.10 ([10]Be)	4666.26 ([10]Be)	0.113	0.018	Recalculated from Penserini et al. (2017)
NFSR	43.964°N, -123.811°W	70902.91 ([10]Be)	3408.59 ([10]Be)	0.058	0.0054	Recalculated from Penserini et al. (2017)
Bear Creek	44.186°N, -123.375°W	400833 ([10]Be)	8011.37 ([10]Be)	0.008	0.0007	-

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Table 3: CRN erosion rates
We used the CRONUS online calculator (Balco et al., 2008) to determine catchment-averaged erosion rates from [10]Be in stream sediment. The samples from Hadsall Creek and NFSR are recalculated from the [10]Be data presented by Penserini et al. (2017). The sample from Bear Creek is previously unpublished. Reported CRN error is from external
825 **uncertainty.**

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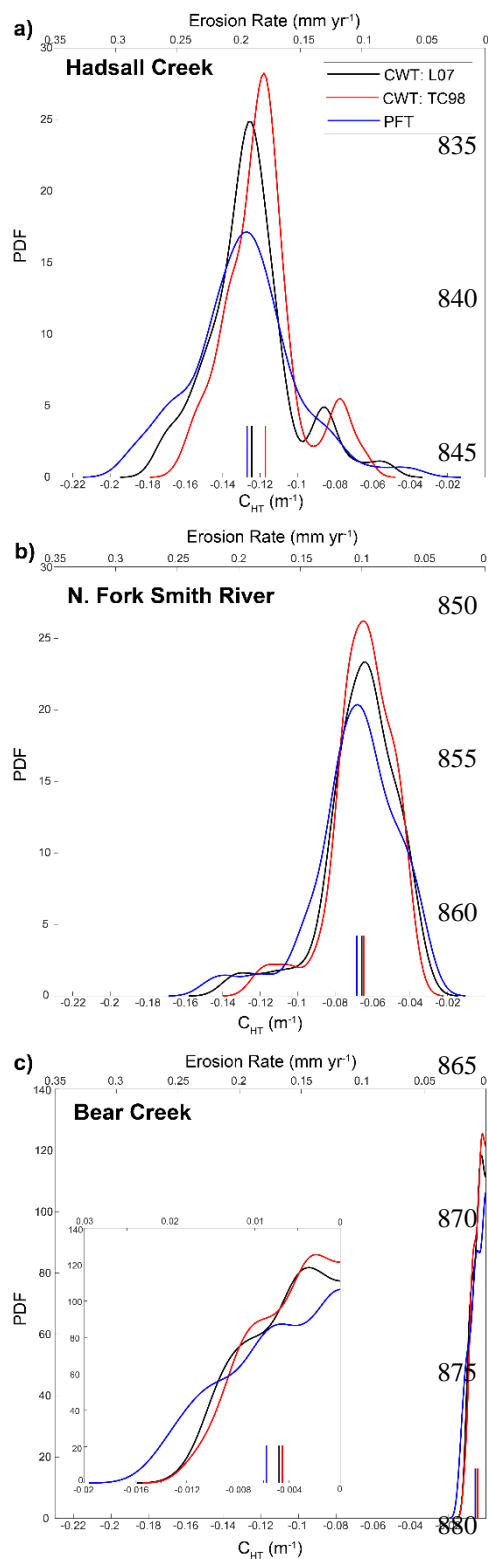
Figure 5: Probability density functions of C_{HT} (bottom x-axis) and erosion rate calculated using Equation 1 (top x-axis) for the representative hilltop at each OCR field site. See Fig. S1 for all mapped hilltops version. Note agreement between each C_{HT} calculation method. Further, note dramatic variability in C_{HT} between sites (all panels use same x-axis; inset in panel C more clearly displays distribution of C_{HT} at Bear Creek). Small vertical lines at bottom of each panel represent the mean of the plotted distribution (Table 2). Note that positive C_{HT} values are not permitted in the output PDF (C).
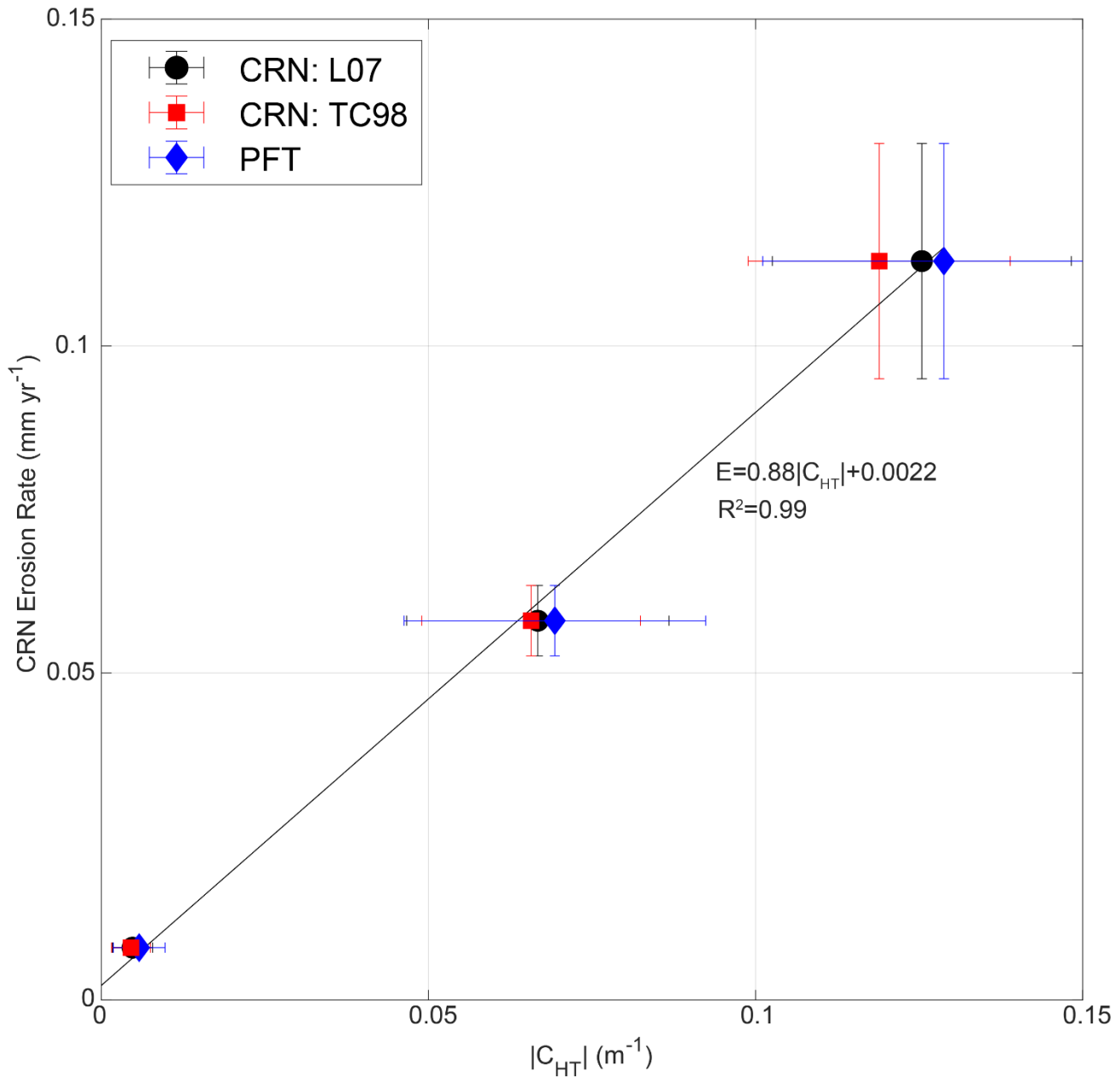
Figure 6: CRN erosion rate vs C$_{HT}$. CRN erosion rates for Bear Creek (slow E), NFSR (moderate E), and Hadsall Creek (fast E) against the absolute value of C$_{HT}$ for the representative hilltop in each catchment. Filled symbols are mean E and C$_{HT}$ values and errorbars correspond to the standard deviation of C$_{HT}$ and external uncertainty in CRN erosion rate measurements (Table 1, 3). Note that errorbars may be smaller than the size of the mean symbol for Bear Creek samples.
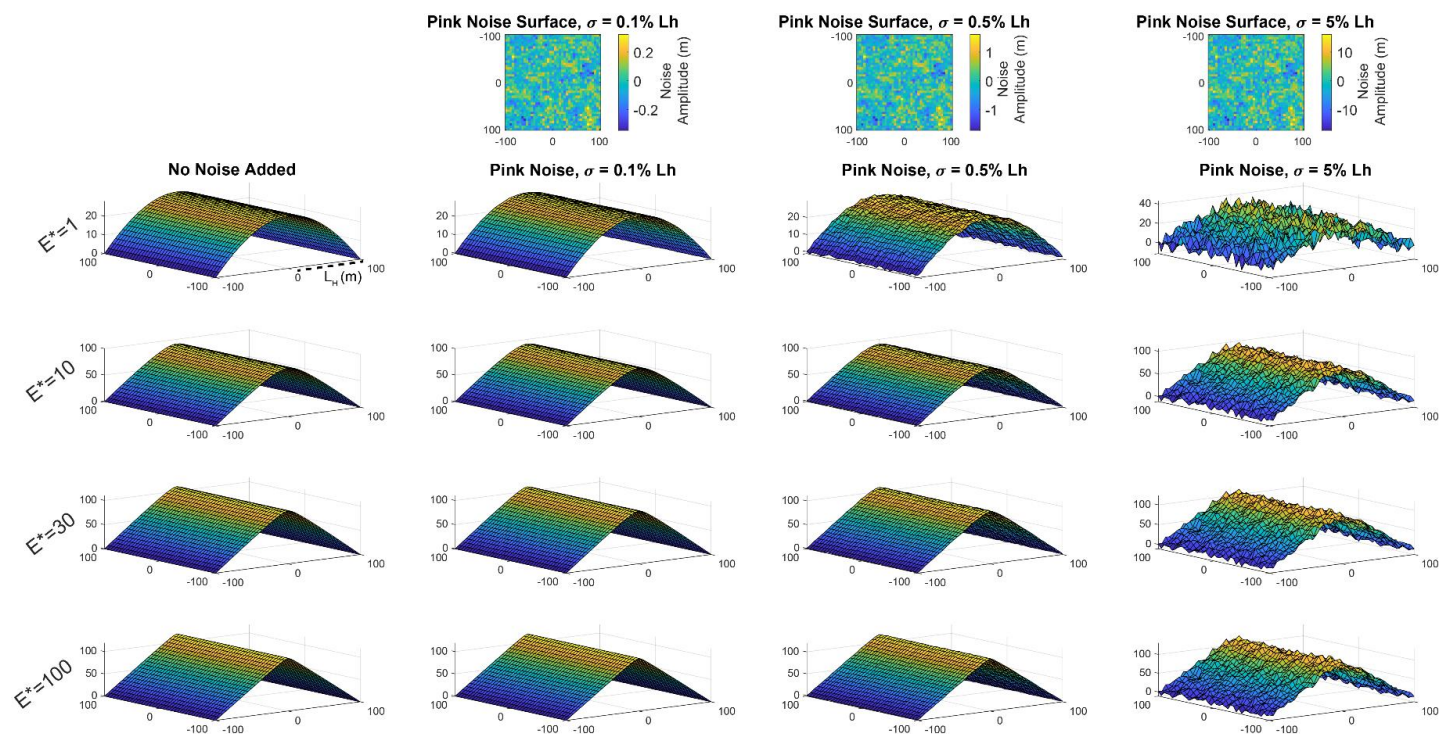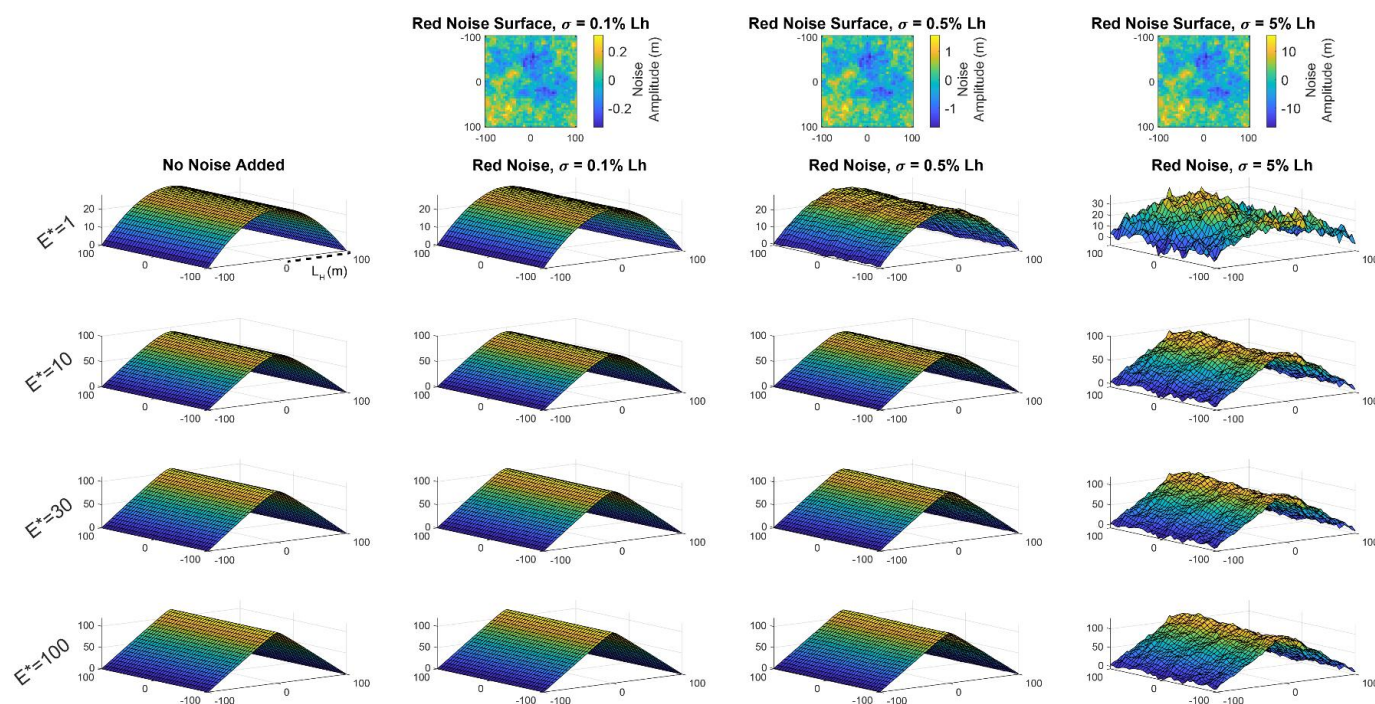
Figure 7: Synthetic hillslopes constructed using Equation 9. Upper row shows pink noise surfaces that are added to the original hillslope form (left column); yellow colors correspond with positive deviations from the hillslope (convex noise) and blue with negative deviations (concave noise). Each row of hillslopes corresponds with range of dimensionless erosion rates, from E*=1-100. Note the increased prominence of planar hillslopes as E* increases. Noise does not vary with E*; thus the magnitude of noise relative to hillslope relief is more visually apparent at lower E* (See σ=5% L$_H$ column for clear example). Note that all results in Fig. 9C, G, K, and O correspond with the third column here (σ=0.5% L$_H$). See supplemental for corresponding figures for σ=0.1%L$_H$ and σ=5%L$_H$ cases.

Figure 8: Synthetic hillslopes constructed using Equation 9. Same as Fig. 7, but with red noise added (see
905 **supplemental for white noise example). Upper row shows red noise surfaces added to the original hillslope form (left**
column); yellow colors correspond with positive deviations from the hillslope (convex noise) and blue with negative
deviations (concave noise). Each row of hillslopes corresponds with dimensionless erosion rates from E*=1-100. Note
the increased prominence of planar hillslopes as E* increases. Noise does not vary with E*; thus the magnitude of
noise relative to hillslope relief is more visually apparent at lower E* (See σ=5% L_H column for clear example). Note
910 **that compared to Fig. 7, the surface noise exhibits longer wavelength noise, made apparent by larger concave and**
convex regions. Note that all results in Fig. 9D, H, I, and P correspond with the third column here (σ=0.5% L_H). See
**supplemental for corresponding figures for other σ=0.1%L_H and σ=5%L_H cases.**
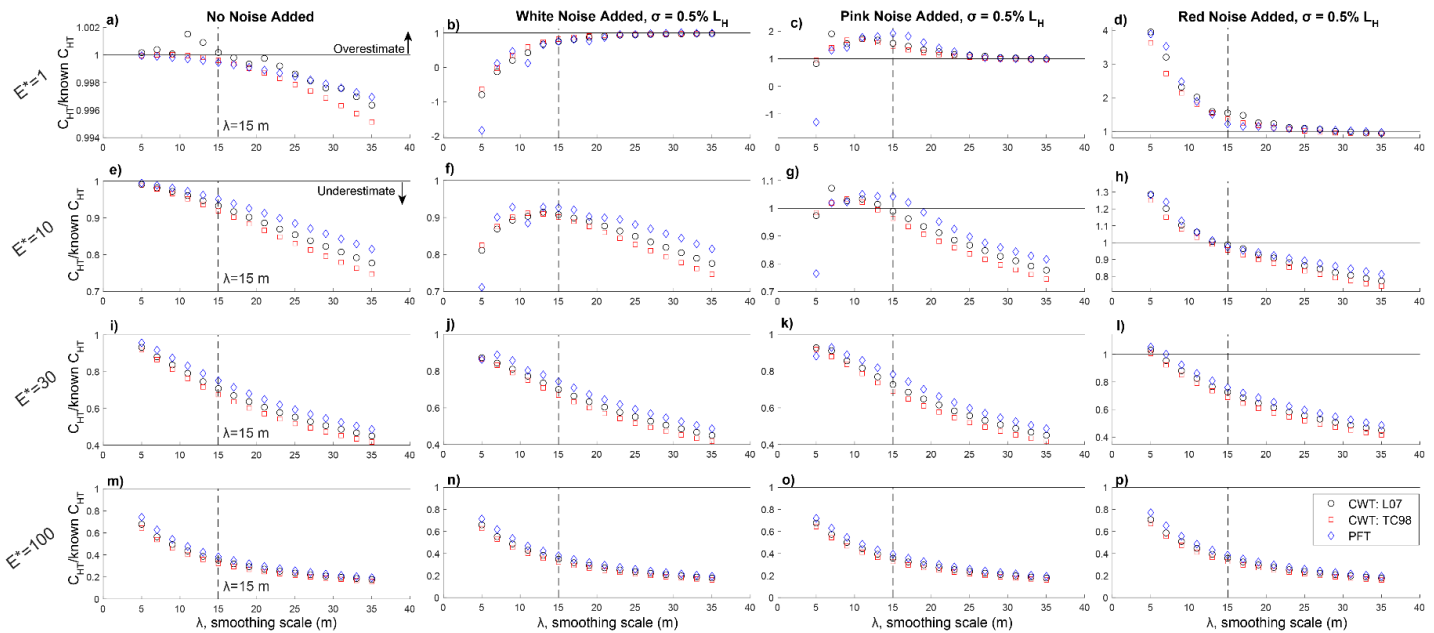
Figure 9: Ratio of C_{HT} of synthetic hillslopes where E*=1, 10, 30, and 100 measured at various smoothing scales, λ, with: no noise added (first column), σ=0.5% L_H white noise (second column), pink noise (third column), and red (Brownian) noise (fourth column). Ratio of C_{HT} is quantified as the quotient of the C_{HT-W} or C_{HT-P} and the model-specified C_{HT}. Black horizontal line in each panel corresponds with where the measured C_{HT} equals the actual synthetic C_{HT} (i.e. ratio=1). Points that plot above the line correspond with locations where C_{HT} is overestimated (sharper hilltops than expected); points that plot below are underestimations (broader hilltops that expected). See text for details but note systematic underestimation of C_{HT} as E* increases, even for the surface with no added noise. Dashed vertical line indicates λ=15 m.
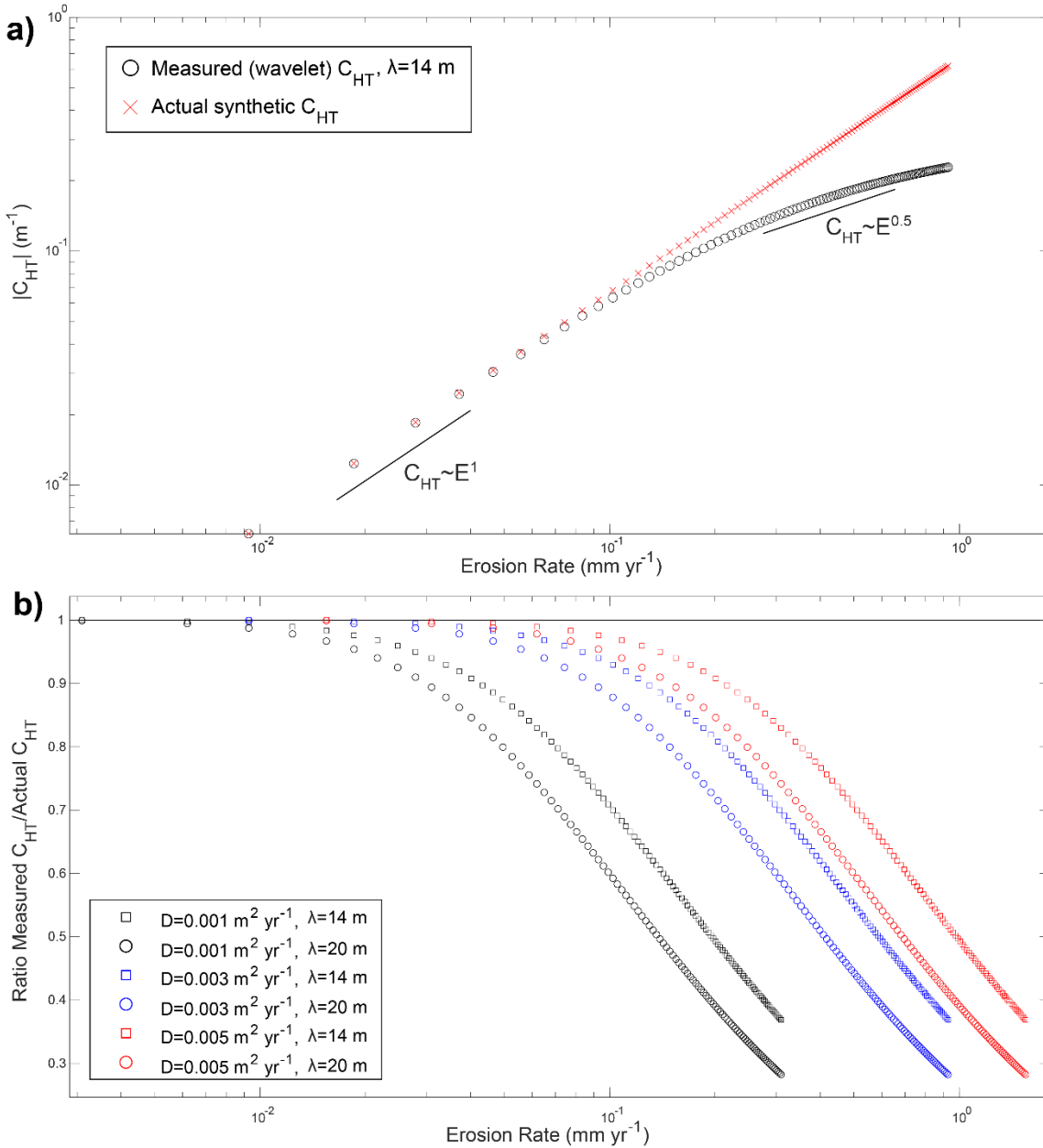
Figure 10: A) The absolute value of C_{HT} plotted against erosion rate for synthetic hillslopes constructed for E*=1-100, corresponding to erosion rates of ~0.01-1 mm yr^{-1} (assuming D=0.003 m^2 yr^{-1}, $\frac{\rho_S}{\rho_r}$=0.5). Red crosses correspond with the actual C_{HT} for each synthetic hillslope constructed for a given E*. Black circles are C_{HT-W}, using the L07 λ definition. In this case λ = 14 m. Note the linear relationship between C_{HT} and erosion rate at small erosion rates, in agreement with Equation 1. At E>~0.08 mm yr^{-1}, the relationship between measured C_{HT} and E is no longer linear but could be potentially expressed as a power law. An example square root relationship is plotted at these erosion

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935   rates for reference (Gabet et al., 2021). B) Ratio of measured C_{HT} and the actual model-defined C_{HT} for synthetic hillslopes constructed for $E^*=1$-100 using a range of diffusivities (D=0.001-0.005 m^2 yr^{-1}) and measured with λ=14 m (same as panel A) and λ=20 m. For each case, the measured C_{HT} deviates from the known value as erosion rate increases. The erosion rate at which this deviation occurs depends on diffusivity and smoothing scale, λ.