We greatly appreciate the constructive and thoughtful review, and largely agree with the recommendations made. Following each of the comments, we provide our amendment to the manuscript or the justification in deviating from the comment, in normal font.

Thank you for your time and efforts to comment on our work.

Best regards,

Ian Delaney, on behalf of all authors.

- **For the synthetic ice sheet case, this study finds that sediment discharge in the model run decreases after the climate warms and reaches a stable regime. The decrease occurs due to sediment exhaustion from increased water discharge, and associated sediment transport, which removed till that was unable to be transported in a cooler climate with less available meltwater able to transport sediment.**

  I find this model behavior problematic to generalize: it would be an interesting finding, depletion of sediment flux, yet the outcome is entirely controlled by the model assumptions on initial till height and bedrock sediment production. So, it is hard to draw any conclusions on this process being of relevance?

  I think the paper would be strengthened by omitting this case study, it oversimplifies the ice sheet system by too much.

  Results on the effect of spatially-distributed fluxes seem of more of importance, and this effect is more pronounced in the alpine glacier case. The alpine glacier case recreates high sediment concentrations in early season, and his is explained by spatial variability in the till distribution and sediment transport access to these sediment patches. This effect has been observed in proglacial streams.

  We appreciate these comments and can understand why this section could be cumbersome. After consideration, we agree with Dr. Overeem in the conclusion that the paper will be enhanced by omitting this section.

- **It seems that there is a large process parametrization discrepancy between the general bedrock sliding law applied (with no grainsize dependence and validated over glacial-deglacial scales) and the heavily grainsize dependent sediment transport law (Engelund-Hansen). A note of caution should probably already be added into the results section, and refer to the later discussion section on this topic.**

  This is a very good point and one of the assumptions of the sliding-erosion relationship is the presence of basal debris to provide a means of abrading the bedrock (Hallet, 1979). In the introduction, we note that this contributes to sediment transport. Additionally, comments regarding the role of debris concentration have been added to the new **Model Limitations** Section.

On the structure of the paper:

- **Title: I recommend simplifying the title.**

  Suggestions: "Bedrock erosion and sediment transport variations across a glacier bed controlled by glacier behavior and hydrology" or "Modeling of the spatially distributed nature of subglacial sediment erosion and transport dynamics"?

  We agree with this comment and find the current title a bit long and convoluted. We have changed it to *Modeling of the spatially distributed nature of subglacial sediment erosion and transport dynamics.*

- **Introduction: can be tightened. Please read carefully through and omit some of the sections that are repetitive or wander.**

The introduction has been shortened and some content has been omitted from the current draft.

- **Model description: I do appreciate the review of the hydraulic model, even if it previously been well described in Delaney, 2019, but is needed here to have this paper be a stand-alone contribution.**

  We have updated this section to include a complete description of the hydraulic model. The primary addition contains the explanation of the shape factors, such s, that impact the shape of the subglacial channel and channel width, w_c, which is required for sediment transport relationships.

  The section now reads:

  SUGSET_2D requires a hydraulics model as a means to route sediment and water through the subglacial environment and to evaluate the sediment transport capacity of this water, based upon the hydraulic gradient, channel size and water flux (Table 1, Section 2; e.g. Walder and Fowler, 1994; Alley et al., 1997). The hydraulic model here is based on the premise that subglacial water flows along the hydraulic potential gradient and glacier ice pressurizes water at its bed (Shreve, 1972). We aim simulate key characteristics of an Röthlisberger-channel without explicitly describing properties such as creep closure and pressure melt of channel walls.

  The hydraulic gradient for a section of channel at a location below the glacier at a certain time Ψ can be determined with a known hydraulic radius D_h and water discharge Q_w of that channel, given the Darcy-Weissbach equation for fluid flow through a pipe

$$\Psi = s\, f_r\, \rho_w\, \frac{Q_w^2}{D_h^5}. \qquad (1)$$

  the density of water is given by ρ_w, f_r represents the Darcy-Weisbach friction factor, and s accounts for channel geometry Hooke et al. (1990)

$$s = \frac{2\,(\beta - \sin\beta)^2}{(\frac{\beta}{2} + \sin\frac{\beta}{2})^4}, \qquad (2)$$

  where β is the central angle of the circular segment representing the channel edge. Smaller values of β result in broad channels and $\beta = \pi$ corresponds to a semicircular channel. The channel width w_c is given by

$$w_c = 2\sin\frac{\beta}{2}\sqrt{\frac{2\,S}{\beta - \sin\beta}}, \qquad (3)$$

  where S is the cross-sectional area of the channel given by

$$S = \frac{D_h^2}{2}\,\frac{\left(\frac{\beta}{2} + \sin\frac{\beta}{2}\right)^2}{\beta - \sin\beta}. \qquad (4)$$

  To approximate hydraulic diameter D_h, we assign a representative water discharge Q_w^* to Q_w, by taking a quantile of water discharge over a certain prior time period (days), over which the subglacial conduit responds to hydraulic conditions (source quantile; c.f. de Fleurian et al., 2018; Delaney et al., 2019; Nanni et al., 2020). The source quantile is assumed to be the water discharge over which the hydraulic radius responds to changes in water discharge. For instance, we would not expect a short-lived increase in water discharge due to a short precipitation event to greatly impact the hydraulic radius, where as the hydraulic radius would respond to a prolonged increase in water discharge from the increased melt.

  This time represents the time period, assumed to be days, over-which hydraulic radius D_h evolves in response to Q_w (e.g. Nanni et al., 2020). Q_w and Q_w comprise the amount of melt produce upglacier, such that no storage exists in the model.

  We then evaluate D_h, the hydraulic radius given

$$D_h = \left(s\, f_r\, \rho_w\, \frac{Q_w^{*\,2}}{\Psi^*}\right)^{\frac{1}{5}}. \qquad (5)$$

Ψ^* is a representative hydraulic gradient at overburden pressure, evaluated using the Shreve potential gradient

$$\Psi^* = \nabla(\rho_i\, g\, (z_s - z_b) + \rho_w\, g\, z_b), \tag{6}$$

where z_s and z_b are surface and bed elevations, respectively, ρ_i is the density of ice and g is the gravitational acceleration constant.

With knowledge of D_h, we insert the instantaneous value of Q_w into Equation 1 to evaluate the instantaneous hydraulic gradient Ψ. We note that to prevent unreasonable water pressures when Q_w^* rapidly increases and D_h is small, the model limits the minimal cross-sectional area S to $0.5\,\mathrm{m}^2$.

- **I found the order in Section 2.2 non-intuitive. Indeed, it is important to detail the Exner equation approach first, but then begin with bedrock erosion parametrization, as that is the term that produces sediment, and then describe sediment transport.**

  We appreciate the issue with the ordering of the equation. However, we choose to present sediment transport first, as this depends largely on the hydraulics, which are presented directly above. However, we have modified the Equation so that it's ordering now aligns with the text.

  The model simulates the evolution of a subglacial till height, which we define as transportable sediment below the glacier due to glacier erosion and fluvial sediment transport. Fluvial sediment transport in supply- and transport-limited regimes mobilize and deposit sediment, adding or removing material from the till layer (Brinkerhoff et al., 2017; Delaney et al., 2019). Conversely, erosive processes such as abrasion and quarrying add material to the layer. To quantify these processes, we implement the Exner Equation (Figure 2; Exner, 1920a,b; Paola and Voller, 2005), a mass conservation relationship, to solve for the till layer height given the erosive and fluvial conditions.

$$\underbrace{\frac{\partial H}{\partial t}}_{\text{till evolution}} = -\underbrace{\nabla \cdot Q_s}_{\text{sediment transport}} + \underbrace{\dot{m}_t}_{\text{bedrock erosion}} \tag{7}$$

  H is till thickness and t is time (Table 1). The first term represents fluvial sediment transport processes, where $\nabla \cdot Q_s$ represents sediment mobilization in either supply- or transport-limited regimes. The second term captures bedrock erosion processes, where $\dot{m}_t$ is a bedrock erosion rate.

- **One suggestion to make each of the source terms and in-out fluxes clear is to add a diagram of a mass fluxes in the Exner equation, and label the processes. Like Fig 1 in Paola and Voller, but then made specific for this special implementation.**

  We appreciate this comment and agree that such a figure would make interpretation of the relevant processes easier. In turn, we have made the following figure.

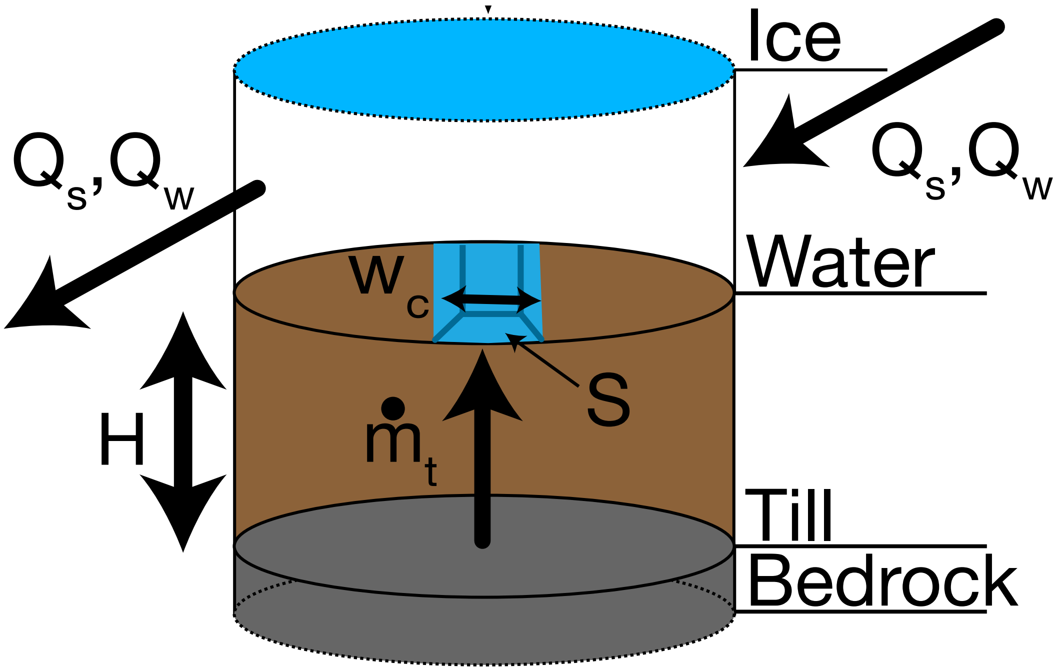

Figure 1: Illustration of terms in Equation 7, detailing the layers of bedrock, till, water and ice.

Line-by-line minor comments:

- **line 6: the concept of 'sediment connectivity' needs explanation or definition before being invoked as an important control. Either explain within the abstract, or omit.** The text now reads:

  However, sediment discharge depends heavily on sediment connectivity, the movement of sediment between its detachment in source areas and its deposition in sinks. In turn, the geoscience community needs modeling frameworks that describe subglacial sediment discharge in two spatial dimensions over time.

- **Line 14: this sentence is unclear."We find that sediment grainsize plays an important role. Smaller sediment sizes....**

  The text now reads:

  Additionally, the model's capacity to represent the data depends greatly on the sediment grain size parameter in the model, impacting the sediment transport capacity of the subglacial water.

- **Line 24: possibly add this paper by Dongfeng Li 2021, although the increasing sediment loads are not just attributed to glacier melt but also due to permafrost thaw and rainfall change. Li, D., Lu, X, Overeem, I., Walling, D., Syvitski, J., Kettner, A.J., Bookhagen B., Zhou, Y., Zhang, T., 2021). Exceptional increases in fluvial sediment fluxes in a warmer and wetter High Mountain Asia. Science, 10.1126/science.abi9649**

  Many thanks for the recommendation. Citation fits well here and is added.

- **Line 30: replace with: 'are limiting nutrients in the oceanic ecosystem'**

  Done.

- **Line 45: add flow after . . . water**

  Done.

- **Line 51: although if there is little sediment embedded there is little abrasion!**

  This is an important point and the text now reads:

  Bedrock erosion and fluvial sediment transport vary depending on the timescales and characteristics of individual glaciers. For instance, erosive processes likely dominate over sediment transport processes on glaciers with minimal sediment storage, large concentrations of subglacial debris and steep gradients (Hallet, 1979; Humphrey and Raymond, 1994; Herman et al., 2015; Ugelvig and Egholm, 2018; Herman et al., 2021).

- **Line 90: please clarify this approach: is there a single hydraulic diameter and single associated water discharge for the entire distributed drainage system? Or dothese properties Q* and Dh get assigned/calculated for individual drainage channels?**

  We appreciate this comment and understand the need to clarify this matter.

  They text at line 90 now reads: *The hydraulic gradient for a section of channel at a location below the glacier at a certain time* Ψ *can be determined with a known hydraulic radius* D_h *and water discharge* Q_w *of that channel, given the Darcy-Weissbach equation for fluid flow through a pipe. . .*

- **Line 94: this selection of representative discharge seems really difficult and perhaps arbitrary? how do you decide ahead of time what quantile (or did you mean quartile?).**

  Selection of a representative discharge does require the assumption of time periods of the response of channel's size. Both numerical modeling (e.g. de Fleurian et al., 2018) and observations of subglacial hydrology (Nanni et al., 2020) suggest that this time period is on the order of days. With regard to selecting a representative quantile, we select this value with the assumption that certain water discharges influence the size of the conduit.

  The text now reads:

  To approximate hydraulic diameter D_h*, we assign a representative water discharge* Q_w^* *to* Q_w*, by taking a quantile of water discharge over a certain prior time period (days), over which the subglacial conduit responds to hydraulic conditions (source quantile; c.f. de Fleurian et al., 2018; Delaney et al., 2019; Nanni et al., 2020). The source quantile is assumed to be the water discharge over which the hydraulic radius responds to changes in water discharge. For instance, we would not expect a short-lived increase in water discharge due to a short precipitation event to greatly impact the hydraulic radius, where as the hydraulic radius would respond to a prolonged increase in water discharge from the increased melt.*

  We have also discussed the limitations and implications of our subglacial hydrology parameterization in the Section **Model limitations**.

- **Line 103: please spell out R-channel at its first occurrence.**

  Done. This sentence is also moved to the top of this Section.

- **Line 124: I am not sure about the limit, H-lim, at 10 cm, this seems really arbitrary. Sediment transport in a pressurized pipe flow can probably easily scour bedforms to 4-5 times that depth almost instantaneously? I understand that perhaps there is little data to constrain this parameter, but it may be prudent to explore whether the model is sensitive to this setting.**

  We have discussed this parameter to the **Model Limitations** section. While bedforms can be scoured to 4-5 times that depth, we suggest that this may only happen at a very specific event, where water is routed to a new patch of the glacier bed, or if sediment is thick during the model initialization. Over the time periods that we examine in this model, we believe that it is not as imperative to capture these peak events. Rather, we choose this small value in order to reduce the spin-up period and response time of the model, as larger values of H_{lim} would mean that longer time periods are needed for the till to accumulate and then be transported from the glacier in regions such as over deepenings.

- **line 165: this switch in describing code is a bit out of style, perhaps better to describe this in the code documentation as opposed to command lines in the paper.**

  This line has been omitted.

- **line 177 Reference is oddly formatted, please make conform journal requirements**

  This line has been omitted as well.

- **line 184 First explain why this would be a non-stiff problem, and then state that this solver is apprporiate for non-stiff problems.**

  The text now reads:

  In turn, to discretize the problem in time, the model implements the VCABM solver (Hairer et al., 1992; Radhakrishnan and Hindmarsh, 1993) from the package DifferentialEquations.jl (Rackauckas and Nie, 2017) to evolve till layer height H. This solver implements an adaptive time step and uses a linear multistep method (Adams-Moulton) that is well-suited for non-stiff problems, which is optimal here due to the rapid fluctuations in sediment transport that can occur in response to variable hydraulic conditions.

- **Line 202-208 this section has several 'disclaimers' on your assumptions that would be better suited for the discussion of model limitations later in the paper.**

  We have removed the topics from the here and moved them to a new section titled **Model limitations**.

- **Line 245: I am not intimately familiar with SHMIP, but does the rate of temperature offset originate from those setups? Do I understand it right that the total warming scenario is an added 15 degrees C to diurnal amplitude? Over 30 years? That warming rate seems really abrupt, and unprecedented even under the most catastrophic warming scenarios?**

  This warming is abrupt and rapid. The intent is not to mimic real warming scenarios, but rather to demonstrate the behavior of the model as climate warms. The results suggest that additional meltwater accessing sediment below a greater portion of the glacier bed will result in great amounts of sediment transport. We also note that while the glacier morphology and hydrology parameterizations are from SHMIP, SHIMP does not implement any warming in its hydrological forcing.

- **Line 255: line 255 replace till height by thickness?**

  We appreciate this comment. However, we have used till height throughout the paper and think it more appropriate.

- **Line 372. Repeat for careless readers, what are the last three time spans?**

  The text now reads:

  The model captures the last three measured yearly sums of sediment discharge well...

- **Line 374. Isn't one of the important changes of this approach that you do have a way to access different patches of the glacier bed? Could it be model underestimation of till thickness and erosion instead?**

  While the mismatch between the data and the model could result from underestimated till thickness, we find this unlikely given the weak dependence of the outputs on the model's initial condition or the production of sediment. Other runs in the ensemble experienced greater bedrock erosion rates and till heights as initial conditions. However, these model runs overestimated the sediment discharge for the last three years of the study. As a result, we believe that the mismatch is due to processes not considered in the model, rather than a limited exploration of the parameter space.

Figures

- **Figure 2: This figure is really helpful in providing a feel for the dimensions of the subglacial drainage network , flow velocity magnitude (small), and spatial distribution of discharge.**

  Thank you.

- **Figure 3. Recommend this figure to go to an appendix. It is not too well explained in the text and seems important for a manual of the model, not for the scientific findings.**

  This figure has been removed from the text.

- **Figure 5: Perhaps improve this figure by changing the aspect ratio of these figures, at present they are hard to read. May help to add 2 initial panels with icesheet topography map and ice flow velocity? And then have the 'maps' of the basal conditions.**

  Inline with the recommendations by the reviewer to remove the ice sheet case, this figure has also been removed.

- **Figure 7**

  - **May help to add 2 initial panels with icesheet topography map and ice flow velocity? And then have the 'maps' of the basal conditions.**

    We value this comment and have added two panels with ice thickness and bedrock erosion rate. We included the bedrock erosion rate given that it is a function of basal sliding velocity, so the figure also yields information about glacier sliding.

  - **Perhaps improve this figure by changing the aspect ratio of these figures, at present they are hard to read.**

    We have only adjusted the aspect ratio with respect to the additional two panels for the comment above. However, the individual panels are constrained in order to keep the x and y axes scaled equally.

  - **Great to see this as animations.** Thank you.

- **Figure 9. Remove 'likely ' from the caption.**

  Done.

- **Figure 10. Add to the figure caption what parameters are modeled vs observed.**

  The text now reads:

  Water discharge, an input modeled for Griesgletscher in (Delaney et al., 2018), and sediment discharge, output of the model, from Griesgletscher (a). Below is modeled outputs of sediment transport capacity and average till height (b). Note that sediment discharge capacity is roughly one order of magnitude larger than sediment transport discharge. Additionally, the reduction in till-height H through this model run shows that sediment is transported from the glacier bed at a greater rate than it is produced.

References

Alley, R. B., Cuffey, K. M., Evenson, E. B., Strasser, J. C., Lawson, D. E., and Larson, G. J.: How glaciers entrain and transport basal sediment: physical constraints, Quaternary Science Reviews, 16, 1017–1038, doi:10.1016/S0277-3791(97)00034-6, 1997.

Brinkerhoff, D., Truffer, M., and Aschwanden, A.: Sediment transport drives tidewater glacier periodicity, Nature Communications, 8, 90, doi:10.1038/s41467-017-00095-5, 2017.

de Fleurian, B., Werder, M. A., Beyer, S., Brinkerhoff, D., Delaney, I., Dow, C., Downs, J., Hoffman, M., Hooke, R., Seguinot, J., and Sommers, A.: SHMIP The Subglacial Hydrology Model Intercomparison Project, Journal of Glaciology, 64, 897–916, doi:10.1017/jog.2018.78, 2018.

Delaney, I., Bauder, A., Werder, M. A., and Farinotti, D.: Regional and annual variability in subglacial sediment transport by water for two glaciers in the Swiss Alps, Frontiers in Earth Science, doi:10.3389/feart.2018.00175, 2018.

Delaney, I., Werder, M., and Farinotti, D.: A Numerical Model for Fluvial Transport of Subglacial Sediment, Journal of Geophysical Research: Earth Surface, 124, 2197–2223, doi:10.1029/2019JF005004, 2019.

Exner, F. M.: Über die Wechselwirkung zwischen Wasser und Geschiebe in flüssen, Abhandlungen der Akadamie der Wissenschaften, Wien, 134, 165–204, 1920a.

Exner, F. M.: Zur Physik der Dünen, Abhandlungen der Akadamie der Wissenschaften, Wien, 129, 929–952, 1920b.

Hairer, E., Nørsett, S. P., and Wanner, G.: Solving ordinary differential equations I: nonstiff problems, vol. 1, Springer Science & Business, URL http://link.springer.com/book/10.1007/978-3-540-78862-1, 1992.

Hallet, B.: A theoretical model of glacial abrasion, Journal of Glaciology, 23, 39–50, 1979.

Herman, F., Beyssac, O., Brughelli, M., Lane, S. N., Leprince, S., Adatte, T., Lin, J. Y. Y., Avouac, J. P., and Cox, S. C.: Erosion by an alpine glacier, Science, 350, 193–195, doi:10.1126/science.aab2386, 2015.

Herman, F., De Doncker, F., Delaney, I., Prasicek, G., and Koppes, M.: The impact of glaciers on mountain erosion, Nature Reviews Earth & Environment, 2, 422–435, doi:10.1038/s43017-021-00165-9, 2021.

Hooke, R. L., Laumann, T., and Kohler, J.: Subglacial Water Pressures and the Shape of Subglacial Conduits, Journal of Glaciology, 36, 67–71, doi:10.3189/S0022143000005566, 1990.

Humphrey, N. and Raymond, C.: Hydrology, erosion and sediment production in a surging glacier: Variegated Glacier, Alaska, 1982–83, Journal of Glaciology, 40, 539–552, 1994.

Nanni, U., Gimbert, F., Vincent, C., Gräff, D., Walter, F., Piard, L., and Moreau, L.: Quantification of seasonal and diurnal dynamics of subglacial channels using seismic observations on an Alpine glacier, The Cryosphere, 14, 1475–1496, doi:10.5194/tc-14-1475-2020, 2020.

Paola, C. and Voller, V. R.: A generalized Exner equation for sediment mass balance, Journal of Geophysical Research: Earth Surface, 110, doi:10.1029/2004JF000274, URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2004JF000274, 2005.

Rackauckas, C. and Nie, Q.: DifferentialEquations. jl–A Performant and Feature-Rich Ecosystem for Solving Differential Equations in Julia, Journal of Open Research Software, 5, 15, doi:10.5334/jors.151, 2017.

Radhakrishnan, K. and Hindmarsh, A. C.: Description and use of LSODE, the Livermore solver for ordinary differential equations, Reference Publication 1327, NASA, 1993.

Shreve, R. L.: Movement of water in glaciers, Journal of Glaciology, 11, 205–214, 1972.

Ugelvig, S. and Egholm, D.: The influence of basal-ice debris on patterns and rates of glacial erosion, Earth and Planetary Science Letters, 490, 110–121, doi:10.1016/j.epsl.2018.03.022, 2018.

Walder, J. S. and Fowler, A.: Channelized subglacial drainage over a deformable bed, Journal of Glaciology, 40, 3–15, doi:10.3189/S0022143000003750, 1994.