This review focuses on the implementation description and presentation of the model. The reviewer identifies points of confusion in the description of the model. We take these comments seriously and have addressed them below.

The reviewer's comments are in bold, while our response is in normal font, and excerpts from the manuscript are in italics.

Best regards,

Ian Delaney, on behalf of all authors.

- **(1) Sect. 2.2**

  The main part where I am not immediately convinced that is is correct is Sect. 2.2 about sediment transport, starting from the balance (Exner) equation (Eq. 4). In its genuine form, such an equation would be written in 2D in terms of a sediment flux per unit width (m^2/s) instead of the sediment flux (m^3/s). In the 1D version (Delaney et al., JGR, 2019, Eq. 9), the sediment flux is used in combination with a channel width w. However, the version introduced here uses a length scale l that describes a "characteristic length-scale for sediment mobilization, over which sediment mobilization adjusts to sediment transport conditions." This length scale is a longitudinal length scale (in flow direction), while the formulation of the balance equation in terms of the flux requires a length scale perpendicular to the flow direction (such as the channel width). So a am quite sure that Eq. 4 is not correct in the form it is written, but I cannot assess whether it is correct in the implementation and whether it affects the results in case it is not correct.

  Many thanks for this comment and given the fundamental nature of this equation, any misunderstandings must be clarified. We have altered the equation so that now it reads:

  The model simulates the evolution of a subglacial till height, which we define as transportable sediment below the glacier due to glacier erosion and fluvial sediment transport. Fluvial sediment transport in supply- and transport-limited regimes mobilize and deposit sediment, adding or removing material from the till layer (Brinkerhoff et al., 2017; Delaney et al., 2019). Conversely, erosive processes such as abrasion and quarrying add material to the layer. To quantify these processes, we implement the Exner Equation (Figure 2; Exner, 1920a,b; Paola and Voller, 2005), a mass conservation relationship, to solve for the till layer height given the erosive and fluvial conditions.

  $$\underbrace{\frac{\partial H}{\partial t}}_{\text{till evolution}} = - \underbrace{\nabla \cdot Q_s}_{\text{sediment transport}} + \underbrace{\dot{m}_t}_{\text{bedrock erosion}} \tag{1}$$

  H is till thickness and t is time (Table 1). The first term represents fluvial sediment transport processes, where $\nabla \cdot Q_s$ represents sediment mobilization in either supply- or transport-limited regimes. The second term captures bedrock erosion processes, where $\dot{m}_t$ is a bedrock erosion rate.

  There also seems to be a problem with the physical dimensions in Eq. 5 (beyond that the divergence of the sediment flux at the left-hand size should not be called "sediment discharge"). If $\dot{m}_t$ is indeed an erosion rate (m/s) as defined earlier, it is not consistent with the other properties, which are fluxes per length (m^2 s^{-1}).

  We thank the reviewer for the careful examination of this text and the accompanying equation as several inconsistencies and mistakes were identified. The relevant equation and text now read:

Divergence of the sediment flux is calculated by approximating $\nabla \cdot Q_s$ with $\frac{\nabla \cdot \widetilde{Q_s}}{w}$ and using the mobilization scheme from Delaney et al. (2019)

$$\frac{\nabla \cdot \widetilde{Q_s}}{w} = \begin{cases} \dfrac{Q_{sc} - Q_s}{l} & \text{if } \frac{Q_{sc}-Q_s}{l} \leq \dot{m}_t w \quad \text{(2a)} \\ 0 & \text{if } H = H_{lim} \quad \& \quad \frac{Q_{sc}-Q_s}{l} \leq 0 \quad \text{(2b)} \\ \dfrac{Q_{sc} - Q_s}{l} \sigma(H) + \dot{m}_t w \, (1 - \sigma(H)) & \text{otherwise} \quad \text{(2c)} \end{cases}$$

w is the width of a patch of glacier bed, prependicular to the direction of water flow. Q_{sc} is sediment transport capacity, or the amount of sediment that could be transported given hydraulic conditions. l is a characteristic length-scale for sediment mobilization, over which sediment mobilization adjusts to sediment transport conditions. σ is a sigmoidal function of H

$$\sigma(H) = \left(1 + \exp\left(\frac{2 - \Delta\sigma H}{5}\right)\right)^{-1}, \tag{3}$$

that enables smooth transition over the range: $H = 2\Delta\sigma^{-1} \pm \Delta\sigma^{-1}$ in Equation 2c. $\Delta\sigma$ is a value below which σ substantially deviates from 1, and reduces sediment mobilization.

- **(2) Sect. 2.3**

  While I am confident that the numerical implementation is sound, I am not happy about the way it is described in this section. To be honest, I found it even more confusing than enlightening. The cited work by Bovy et al. (2016) used a standard finite-volume discretization of the fluxes, while the algorithm proposed by Braun and Willett (2013) used a single-flow-direction (D8) scheme (if I am not wrong). I read that you use a multi-flow-direction scheme, but it is not clear what the difference toward typical continuum schemes (finite volume) is or whether it is the same. Instead of (or in addition to) mentioning functions and software packages, I would ask you to state the equations that are finally solved.

  - **(a) How exactly is the flux (water/sediment) distributed among the available directions? It looks as if this changes trough time, but I did not get it completely.**

    The text in **Section 2.3.1 Routing algorithm and implementation** now reads as:

    Sediment and water are routed down the hydraulic gradient using a multi-cell routing scheme (Quinn et al., 1991), implemented in a similar way as Bovy et al. (2016), but on a regular grid. Sediments and water move from one cell to another using a steepest-descent algorithm, based upon the hydraulic potential. This routing scheme returns a stack, from which contains information about the order of cells to perform the calculations. The model evaluates the hydraulic potential at every time step, first, the flotation fraction for a cell at a given time is calculated by $f_f = \frac{\phi_o}{\phi^*}$, where hydraulic potential ϕ_o comes from

    $$\phi_o = \sum_{j=1}^{n_r} \phi_{0_j} \cdot w_{r_j} \; + \; \Psi_j \; \cdot \delta^{\frac{1}{2}} \cdot w_{r_j}. \tag{4}$$

    Here, where Ψ_n comes from Equation 1, δ is the area of a cell on a regular grid yielding cell length, n_r is the number of receivers that the cell has, and w_r is the proportion of water flowing from one cell to the other. The operation executed on a cell by cell basis using the routing scheme above, beginning at the glacier terminus and moving up glacier.

We distribute the mean value of f_f across the glacier, then implement the routing scheme for the hydraulic potential determined from the Shreve potential as

$$\phi = f_f \, \rho_i \, g \, (z_s - z_b) + \rho_w \, g \, z_b. \tag{5}$$

The node ordering algorithm is executed every time step in response to diurnal variations in water pressure and thus variable routing of subglacial water in response to changing hydraulic conditions (e.g. Iken and Bindschadler, 1986; Chu et al., 2016). However, to improve stability during periods of rapidly changing sediment transport conditions, we reorder the stack, based upon the hydraulic conditions to the nearest 6 *mn. Smaller solving tolerances increase the computational time due to 1) increased accuracy of the solution and 2) the reassessment of flow fractions between the adjacent cells, which results in different routing configurations as the model converges. We fill closed basins or over-deepenings in the hydraulic potential to maintain continuous sediment transport through the domain. The model uses an external algorithm, that contains routes flow and fills basins based upon rasterised values of the hydraulic potential.*

Using this routing scheme, we are able to evaluate the water discharge in a cell from melt upstream as

$$Q_w = \dot{m}_w \cdot \delta \, + \sum_{j=1}^{n_d} Q_{w_j} \cdot w_{r_j}, \tag{6}$$

where $\dot{m}_w$ is a melt water source term and n_d is the number of donors of that cell. The sediment discharge Q_s into a cell is like-wise computed as

$$Q_s = \sum_{j=1}^{n_d} Q_s \cdot w_{r_j}. \tag{7}$$

Q_s is then used to evaluated the $\nabla \cdot Q_s$ given Equation 2 and subsequently, the change in till height using Equation 1. In both Equation 6 and 6, the operations are conducted given the node ordering information in the stack such that the flux in to a cell depends on the flux through the catchment above it.

- **(b) What does "integrating Equation 1" (line 166) exactly mean here? Typically, "integrating" a differential equation is somehow one-dimensional or upstream in a tree, like the algorithm made popular by Braun and Willett (2013). However, if there are multiple flow directions, upstream paths may meet again, so that you would end up at different values of phi by integrating over different paths. I see that you are solving a system of equations, but I cannot see how.** The text now reads:

  We implement a regular grid for discretization. Spatial discretization must be substantially smaller than characteristic length-scale, l, in Equation 2. We then solve Equation 1 for till height H for given initial and boundary conditions in response to till production $\dot{m}_t$ and divergence of the sediment discharge Q_s using an explicit time integration scheme.

  We have omitted the citation Braun and Willett (2013) from the text, as this could be taken as misleading given the method we use.

- **(c) It would be good to state the balance equation for the water and for the sediment for each grid cell explicitly and in such a way that it becomes clear which system of equations finally has to be solved.** Dr Overeem, the other reviewer, brought up a similar point. A figure of the mass balance of sediment and water for a cell has been added to the text. Additionally, as shown above, we have included equations for the mass balance of sediment in and out of the cells.

- **(d) Also about the discretization: I guess you are assuming one single conduit of a given hydraulic radius for each grid cell. This may be questionable, but of course not necessarily bad. A similar assumption is typically made in karst evolution models. It should be discussed at least briefly. And is this conduit directed, or will it change its direction if phi changes?**

  Indeed, the model operates by assuming a single conduit with a given hydraulic radius for each cell. Because the hydraulic potential in each cell and time step may be evaluated, given the methods described in **Section 2.3.1 Routing algorithm and implementation**. By evaluating the hydraulic gradients in x and y directions, the water leaving the cell will be directed accordingly.

- **(3) Lines 104-105: $Q{\cdot}_w$ or Q_w? And I did not understand how the water pressures can increase to unreasonable values if the flux rapidly increases. I can imagine that this happens if D$_h$ is small for a single$-$flow$-$direction model, but why do the alternative flow directions not help here?**

  This happens in the model if D$_h$, and thus cross-section S, is small and water flux rapidly increases. While the 2D routing scheme does accommodate high water pressures, alternative flow directions does not necessarily accommodate the changes if D$_h$ remains small in the surrounding locations as well.

- **(4) Line 137: Here you use "capital S" (the cross section area), which is definitely different flow "lowercase s" used in Eq. 1 (probably a nondimensional factor).**

  A more thorough explanation of the hydraulics model has been added, including a complete explanation of the relationship between hydraulic diameter D_h and channel cross-section S. The text is included in the response to Dr. Overeem's review.

- **(5) Line 144: As far as I know, the lower bound I$_{er}$ $= 2/3$ mentioned here was obtained from a worldwide comparison of glaciers under different conditions. So I am not sure whether this low value is relevant for Eq. 10.**

  We understand the reviewer's concern with the low values of I$_{er}$, however, we choose to test the model against the range of values available in the literature.

- **(6) Lines 144-149: Some other studies use the relation from Eq. 11 for the deformation velocity and a similar relation with an exponent n-1 instead of n+1 for the sliding velocity u_b. This relation predicts that deformation becomes more and more relevant if the thickness of the ice layer $z_s - z_b$ increases. Using this relation in my own work, I was even told by reviewers that the sliding velocity cannot be predicted from the deformation velocity at all. I do not share this point of view, but I think it should be discussed why even a weaker assumption than the relation typically used is employed here.**

  We thank the reviewer for this comment. Given the uncertainties in glacier sliding behavior, the relationship between glacier dynamics, sliding and erosion is yet more complex. The sliding relationship has been modified to a relationship between driving stress and sliding velocity, given the Weertman sliding relationship. The text and model have been changed in the following relationship with glacier sliding is used.

  u_b is assumed to be relates to basal shear stress (τ_b Weertman, 1957) given the following relationship

  $$u_b = B_s \tau_b^m, \tag{8}$$

  B_s is a constant and we assume the exponent m is equal to 1. We assume that τ_b is equal to driving stress (Cuffey and Paterson, 2010)

  $$\tau_b = \rho_i \, g \, h \, (\sin \alpha), \tag{9}$$

where ρ_i is the density of ice, h is the glacier thickness and α is the surface slope of the glacier.

- **(7) Line 239: Why is there a need to increase DeltaT, and what is the value of DeltaT used here?**

  This test case has been removed following comments in Dr. Overeem's review, and the comment no longer applies. However, the large value of Δ T owed to the fact that a large value of Δ T was needed so that an adequate amount of water would flow through the moulins, given the location of the moulin within a single grid cell, as opposed to distributed across the glacier surface.

- **(8) Line 246: I did not get the point why H_{max} grows.**

  As in the previous comment, this test case has been removed and the comment no longer applies.

- **(9) Line 285: What is the meaning of the -5 in Eq. 16, given that DeltaT is an (adjustable) temperature offset?**

  The -5 is included to be consistent with Equation 16 in de Fleurian et al. (2018), where the original glacier set up and forcing is presented. While this value can be subtracted from Δ T, we believe it is best to leave the factor in place so that readers may compare model characteristics and values with those in de Fleurian et al. (2018).

- **(10) Lines 339-342: Nice numbers, but you somehow let the chance to analyze your model more thoroughly pass by. From Eqs. 7 and 8, we see that the transport capacity Q_{sc} is proportional to v^5 and thus also to Q_w^5. this is what you put into your model. So you could integrate Q_w^5 over, say 1 year periods, and look how well your sediment output correlates to this integral. If it correlated perfectly, then your sediment output was just what you put into your model equations, and everything else would be unimportant. I think it will not correlate perfectly, so that you can discuss which of the components of your model is important. However, this is just an idea how you could sharpen the discussion.**

  We thank the reviewer for this recommendation. The following text is included:

  Lastly, we note that the model demonstrates the limits of using hydrological relationships to evaluate sediment discharge from glaciers. Equivalent values of water discharge inputs result in model outputs of sediment discharge that varies over orders of magnitude (Figure 10, a). Sediment transport capacity could be tuned through hydraulic parameters and sediment size to improve its performance. Yet, sediment discharge is substantially lower than the sediment transport capacity substantially (Figures 4 and 8). Additionally, model outputs show that sediment discharge consistently varies over an order of magnitude for a given sediment transport capacity in the test cases (Figure 10, b). In turn, using solely the water discharge or sediment transport capacity (e.g. Equation 10) fails to consider the changes to sediment availability caused by sediment transport, especially when changes to sediment storage can take place over seasons to decades.

  Along with the following figure:

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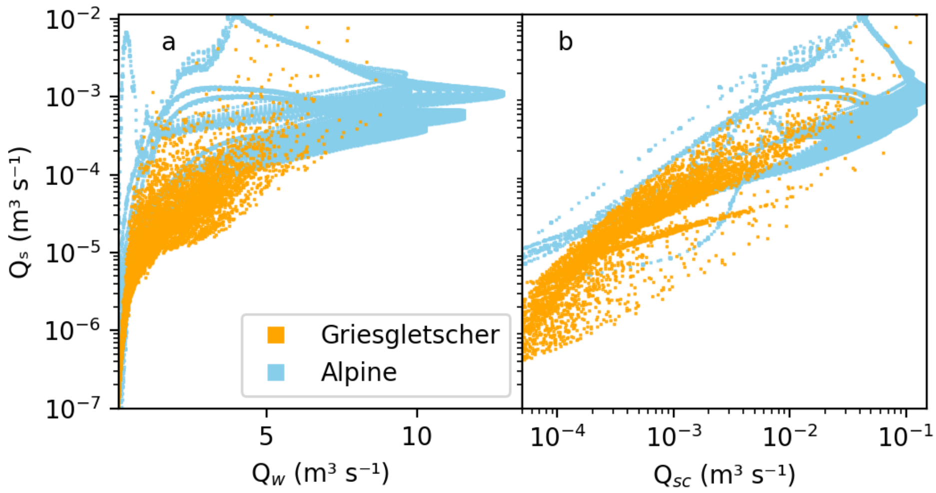

Figure 1: Model outputs of sediment discharge from the glacier compared to water discharge (a) and sediment transport capacity (b).

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