Below, Dr. Hergarten’s comments are in bold and our response is in normal font. Changes to the manuscript text are in italics.

Best regards,
Ian Delaney, on behalf of all authors

However, contrary to the explanation in your response (top of page 2), the physical dimensions in Eqs. 5 and 6 are still not correct. In detail:

1. "In this form, Condition ... has units \( m^2 s^{-1} \)." Right, but since \( \nabla \) is inverse length, \( \tilde{Q}_s \) would have units \( m^3 s^{-1} \), so the same units as \( Q_s \).

2. "In turn, when we approximate ... becomes \( ms^{-1} \)." Also correct, but then the unit of \( \tilde{Q}_s \) becomes \( m^4 s^{-1} \! \)!

3. "Units of ... make Equation 5 consistent." But not in itself because \( Q \) is volume per time and \( \nabla \) is \( m^{-1} \! \)!

I think you really have to fix this problem. As far as I can see, it would work if you define \( \tilde{Q}_s \) properly as sediment flux per unit width and then write Eqs. 5 and 6 directly in terms of \( \tilde{Q}_s \). Eq. 5 would be dimensionally correct then, and Eq. 6 would also be if you use \( \tilde{Q}_s \) everywhere and remove \( w \). Maybe you could also write Eq. 8 in terms of \( \tilde{Q}_{sc} \) (since this is the genuine form).

However, you really have to be careful not to mess up channel width and cell size then since rates of erosion and deposition refer to the area covered by channels only (as far as I can see). This is why I am so picky at this point. I accept that there are many assumptions and simplifications which may be questioned, but I want to be sure that you transferred your ideas into your model correctly.

I would also ask you to perform another round of checking the text, in particular the parts that were subject to your extensive revision.

We appreciate the comment about the units and have worked to make this clearer.

To improve this issue, we have removed \( \nabla \) from Equation 6 and replaced it with simply \( \tilde{Q}_s \), sediment mobilization. In each case below, the units for \( \tilde{Q}_s \) are \( m^2 s^{-1} \). By dividing by width of patch of the glacier bed \( w \), we approximate \( \nabla \cdot Q_s \) as \( \frac{\tilde{Q}_s}{w} \), which has units \( ms^{-1} \! \). We had considered re-writing Equation 8 for unit transport width. However, this would omit the channel width scaling that is needed to quantify sediment transport across the glacier bed with evolving channel geometry. We have also pointed out that \( w \) is not channel width, but rather the width of a patch of glacier bed over which water may access sediment and remains consistent over the glacier, so \( w \) and \( w_c \) are different values. For instance, Equation 1 in Wickert and Schildgen (2019) uses a variable valley width in their mass conservation relationship. Because \( w \) remains constant here, the term \( \frac{\partial w}{\partial x} \), while in 1 dimension in Wickert and Schildgen (2019) would be set equal to 0. In the implementation of the equation \( w \) would be cell width, which remains constant as we use a regular square grid here.

The text now reads:

\[
\frac{\partial H}{\partial t} \Bigg|_{\text{till evolution}} = - \nabla \cdot \tilde{Q}_s + \dot{m}_t, \tag{1}
\]

sediment transport bedrock erosion
$H$ is till thickness and $t$ is time (Table ??). The first term represents fluvial sediment transport processes, where $\nabla \cdot Q_s$ represents sediment mobilization or deposition. The second term captures bedrock erosion processes, where $\dot{m}_t$ is a bedrock erosion rate. We calculate sediment mobilization in both supply- and transport-limited conditions. Divergence of the sediment flux is evaluated by approximating $\nabla \cdot Q_s$ with $\tilde{Q}_s$ using a similar mobilization scheme as in Delaney et al. (2019)

$$\tilde{Q}_s = \begin{cases} 
\frac{Q_{sc} - Q_s}{l} & \text{if } \frac{Q_{sc} - Q_s}{l} \leq \dot{m}_t w \\
0 & \text{if } H = H_{lim} \text{ and } \frac{Q_{sc} - Q_s}{l} \leq 0 \\
\frac{Q_{sc} - Q_s}{l} \sigma(H) + \dot{m}_t w (1 - \sigma(H)) & \text{otherwise}
\end{cases} \tag{2c}$$

$\tilde{Q}_s$ is sediment mobilization across a width of the glacier bed $w$ perpendicular to the water’s flow direction. Note that $w$ is not necessarily the channel width, but rather a representative width across the glacier bed over which sediment can be accessed by water flowing through the subglacial channel (Figure 2). $Q_{sc}$ is the sediment transport capacity or the maximum amount of sediment that could be transported under the given hydraulic conditions. $l$ is a characteristic length-scale for sediment mobilization, over which sediment mobilization adjusts to sediment transport conditions. $\sigma$ is a sigmoidal function of $H$

$$\sigma(H) = \left(1 + \exp \left( \frac{2 - \Delta \sigma H}{5} \right) \right)^{-1}, \tag{1}$$

which enables a smooth transition from transport- to supply- limited transport in Equation (2c). If $H$, the till thickness, is greater than $3 \Delta \sigma$, then the impact on sediment mobilization is negligible and the system is in a transport-limited regime. When $H = \Delta \sigma$, then $\sigma(H)$ is close to 0 and sediment transport is in a supply-limited regime; no significant sediment mobilization takes place.

References


We thank the reviewer for their detailed review, which has greatly improved the manuscript. Generally, we have adapted the text to the reviewer's comments. A figure describing the grid and the locations of different parameters has been included in the text, additionally, we have modified the section describing the routing of the sediment and water, with the aim of improving the clarity. Additionally, we have modified or omitted comments that the reviewer points to that may not be supported in the text.

Below, Reviewer 4's comments are in bold and our response is in normal font. Changes to the manuscript text are in italics.

Best regards,
Ian Delaney, on behalf of all authors

1 General Comments

1. For Section 2.1, the main goal of this section is to provide a way to “estimate” the Hydraulic diameter Dh. The main assumption is that Dh, a characteristic size of subglacial conduits, does not change too much over a certain period. With such an assumption, the Qw in Equation 1 can be estimated using a time-averaged value of discharge during the period. And the pressure gradient can be estimated from the Shreve potential gradient. With these two pieces of information, the Dh in Equation 1 can be approximately estimated and kept unchanged in other conditions. Though the paper finally achieves this goal in lines 109-111, I suggest the authors add the key motivations and assumptions of this Section at the very beginning of the Section, which will improve the readability of section 2.1.

We have modified the beginning of this section slightly to read:

SUGSET_2D requires a hydraulic model as a means to route sediment and water through the subglacial environment. The hydraulic model determines the sediment transport capacity of the subglacial water, based upon the gradient of the hydraulic potential, channel size, and water flux (Table 1, Section 2.2; e.g., [Walder and Fowler 1994; Alley et al., 1997]). The hydraulic model is based on the assumption that subglacial water flows along the hydraulic potential gradient, the weight of ice pressurizes water at the bed [Shreve, 1972], and the channel size varies over a substantially longer time scale compared to water discharge. This model includes characteristics of a Röthlisberger-channel without explicitly describing properties such as creep closure and pressure melt of channel walls [Röthlisberger, 1972].

The gradient of the hydraulic potential of a subglacial channel $\Psi$ (at a certain location and time) can be determined with a known hydraulic diameter $D_h$, (a function of channel size and shape) and water discharge $Q_w$... 

We believe that this adequately and clearly describes the key components and intent of the section, in terms of 1) the need for the hydraulics model, 2) the link between the hydraulic diameter in Equation 1 and the size and shape of the glacial channel.

2. Section 2.3.1 need improvements in two aspects: better descriptions for the grid and its sizes/area etc, donor, receiver, and the final linear combinations used for equation 18, 19, and 21; better descriptions for the routing schemes for equations 18-22. First, Section 2.3.1 is trying to obtain distributed values for hydraulic potential, water discharge, sediment discharge, and till height in a 2D domain. In a 2D domain, any grid (with grid id i) has multiple neighboring grids that can exchange potential/discharge/height to its donor grid. And how much exchanged is further affected by the grid size and the approach to calculating gradients between neighbor-
ing grids. In short, such a process is similar to the numerical discretization process and usually very complex. In this paper, lines 174-178, 184-185, 190-191, 192-193, 196, and 197 aim to describe these processes, however, are not successful, in my perspective, to explain how the grid size, weights, and equations (18.19,21) are organized together. To help clarify the equations in this section, I suggest the authors sketch to carefully describe mesh, donor, receiver, and variables defined at each donor grid in a 2D grid domain and then use this sketch to improve the description for equations 16-22. To draw such a sketch, the authors can check figure 3.1.1.1 or 3.1.1.2 at this link: [http://www.thevisualroom.com/finite_volume_method_3.html](http://www.thevisualroom.com/finite_volume_method_3.html). In this sketch, it is necessary to clarify the following definitions: lambda, nr, wr,j, phi,i, phi,j in Equation 18; delta, nd, wd,j, Qwi, Qwj in equation 19, lambda and nd, Qsi, Qsj in equation 21, and Hi, mti in equation 22. With the clarification of these terms in the sketch, section 2.3.1 will become clear.

We have added the following figure to the text.

![Routing scheme on the grid](image)

**Figure 1:** Routing scheme on the grid. Solid lines represent cell boundaries, blue squares are cell centers, and red squares are cell edges. $\phi$, the hydraulic potential, decreases in the direction of arrows so that water and sediment generally flow left to right and top to bottom. Edge length ($\lambda$) and cell area ($\delta$) are shown. Cell numbers refer to identification in the stack ($s_i$). Select cells denote the weight of donors $w_{d,i,j}$, number of donors $n_d$, donor cells $d$, number of receivers $n_r$, and receiver cells $r_s$. Variables and their respective locations on the grid are shown. Some red and blue squares have been removed in some cells for clarity.

Additionally, we have modified Section 2.3.1 to improve its clarity. The specifics are addressed in the comments below.

3. For the result analyses: in Section 3.1.2, the texts in the paper are not consistent with the Figures shown in Figure 4. In Figure 4 caption, it says "a) Seasonally varying water discharge (Qw) increases from year 10 to 20, while till height (H) decreases."
b) Annual sediment discharge (green) increases over with increasing melt”. However, Figure 4b sediment discharge does not show an increasing trend, but shows a “decrease, increase, and then decrease” trend. A similar problem occurs for figures 5b,d,f at lines 257-258. In the author’s response, the author mentioned this may be caused by using an old version of the manuscript, the authors are suggested to carefully re-exam all results in Section 3.1.2 to make sure the texts in the current version agree with the figures.

We thank the reviewer for the comments and have carefully reviewed the text to examine these inconsistencies. With regard to the issue in Figure 4 (now Figure 5), the intended meaning was that the increase in $Q_S$ is small compared to the relatively small decreases in $Q_s$. The remainder of the text has been examined and modified to address these specific variations.

4. For the Conclusion section, the main problem is the Conclusion is not supported by quantitative evidence. The Conclusion in the current version has 4 paragraphs: the first one introduces the 2D model; the second one discusses the limitation, the third one discusses the future work, and the last one has two sentences commenting on the results and two sentences discussing further work. From my understanding, a Conclusion should include what you did and what you have discovered, and provide quantitative evidence to support your discovery. The first paragraph did describe the 2D model, which is good. But no quantitative evidence to support your discovery makes the Conclusion very weak. Also, limitation and future work should NOT be the main texts in the Conclusion because they are not the ‘Novel contribution’ of the paper. Therefore, I believe the Conclusion section requires significant improvement, which needs to provide concrete QUANTITATIVE evidence to support the main Conclusions of the paper.

We agree that the conclusion can be strengthened and have made changes to the section. While the quantitative evidence that is presented in the conclusion is quite limited, given the nature of the paper, we have provided a summary of the model, and our opinion that the model provides the basis for modeling subglacial sediment dynamics, the model’s limitations, and the model’s significance.

A two-dimensional subglacial sediment transport model, SUGSET_2D, evolves a till layer in response to changing subglacial hydraulic conditions. The model represents sediment transport in supply- and transport-limited regimes, and sediment and water are routed across the bed in response to changing hydraulic conditions in two horizontal dimensions. The till layer is supplied with sediment either from bedrock erosion or by existing sediment, represented by the initial condition. Model cases utilize geometries and hydrological forcings from a synthetic case and Griesgletscher, an alpine glacier in the Swiss Alps.

The interdependence of a large number of parameters and their interaction with one another, for instance, sliding and erosion (Equations 12 to 15), point to the complexity of sediment transport in the subglacial system. Furthermore, the model’s limited representation of the magnitude of interannual variability in the Griesgletscher simulation, from 2011 to 2017, points to processes not completely represented in this application of the model. This misfit could come from poorly constrained parameters and external factors, such as model inputs that may limit the model’s accurate representation of sediment discharge observations. These include interannual variability of glacier velocity and, thus, bedrock erosion, changing glacier topography that routes water to different patches of the glacier bed over time, and routing of water to the glacier bed.

Additional insights into subglacial erosion and sediment transport processes over decadal timescales can be gained from more sophisticated parameterizations of bedrock erosion and subglacial hydrology. Even so, the foundational processes of the model presented
here should be considered when examining subglacial sediment transport processes at seasonal to decadal scales. These processes include: 1) fluvial transport of subglacial sediment across a glacier’s bed in two dimensions in supply- and transport-limited regimes, 2) spatially-distributed bedrock erosion and sediment production, and 3) variable water routing in response to changing melt and hydraulic conditions. It is our hope that the model will be applied in the context of field observations to evaluate and isolate subglacial processes controlling sediment discharge from glaciers as they change.

2 Specific Comments

1. **Line 88: change hydraulic gradient to hydraulic pressure gradient**
   We have changed the text to read: *gradient of the hydraulic potential*.

2. **Equation 1: maybe change \( \Psi \) to \( \Psi(x, y, t) \), which helps to clarify the pressure gradient is a time-dependent and 2D distribution variable.**
   We have chosen not to present \( \Psi \) as \( \Psi(x, y, t) \) as we intend for this section to establish the theory for the hydraulics model. However, we have added the following text to the paragraph preceding Equation 1:
   
   *The gradient of the hydraulic potential of a subglacial channel \( \Psi \) (at a certain location and time) can be determined with a known hydraulic diameter \( D_h \) (a function of channel size and shape) and water discharge \( Q_w \).*

3. **Equation 2: can you a sketch the angle \( \beta \) in Figure 1 or 2?**
   Figure 2 has been modified to the following:

   ![Figure 2](image_url)

   Figure 2: Illustration of model cell (a), detailing the layers of bedrock, till, water, and ice. Characteristics of the subglacial channel are noted as a polygon but shown in one dimension for clarity in (b) with Hooke angle parameterization with two different channel shapes for different values of \( \beta \), Equations 2, 9 and 10.

4. **Line 96: change “establish” to “estimate” or “approximate”?**
   We have changed the text to “establish.”
5. Line 98: change “we call the source percentile” to “which is called the source percentile”.
Done.

6. Line 97-98: How the surface melt and resulting discharge control the subglacial conduit diameter is a complicated problem. Could you add a few citations to support this assumption, if any?
We appreciate this comment. Upon re-reading the text, we have moved the citations in the next sentence to this one.

7. Line 103: I think $Q_w$ is the instantaneous discharge, while $Q'_w$ is a representative scale of the instantaneous discharge. What is the “total instantaneous amount”?
The text has been clarified to read: *We sum the prescribed melt rate $\dot{m}_w$ up the glacier to define $Q_w$, not considering englacial water storage...* The intent of this sentence is to make clear that the variations in water discharge occur as a result of melt, thus processes such as englacial storage that account for the transit time of water through the glacier are not accounted for.

8. Line 96-104: some of the texts here are confusing. $Q_w$ is the water inside the subglacial conduits. Without considering ice melting inside the conduit, such water should be discharged from surface melting, therefore, I would say water in the conduit equals the surface melting into the conduit, based on mass conservation, therefore, $Q_w = m_w$. In general, the surface melting varies with time, so here you assume $m_w$ is estimated by $Q'_w$ which could be a time-averaged value over a certain period (hours to days as mentioned). In thinking in this way, it is logical converting equation 1 to equation 3. However, lines 96, 99-100, and 103-104 cause confusion. I would suggest revising the texts between 96-104 to better reflect the logic as I suggested above.
We appreciate the reviewer’s comment on this matter. We have modified the text to the following to address the comments.

We sum the prescribed melt rate $\dot{m}_w$ up the glacier to define $Q_w$, not considering englacial water storage. Percentile $s_p$ over a response time period prior to the timestep $s_a$ is applied to $Q_w$ to evaluate a characteristic water discharge $Q'_w$ that represents the size of the conduit (hours to days; c.f. [Gimbert et al., 2016] [de Fleurian et al., 2018] [Delaney et al., 2019] [Nanni et al., 2020]). The timescales, $s_a$, and characteristic water discharges ($s_p$ and $Q'_w$), responsible for changes in subglacial conduit size are poorly constrained, yet their impact can be intuited. For instance, short-lived increases in water discharge due to an hour of precipitation will not greatly impact the hydraulic diameter of the subglacial channel, whereas prolonged melt would increase the hydraulic diameter. *We assume that the hydraulic diameter $D_h$ of the channel results from a characteristic water discharge $Q'_w$ which is evaluated by the source percentile of water discharge over a certain time period $s_p$ and a response time of the channel size $s_a$, that remains consistent throughout the model run (Table 1; [Delaney et al., 2019]).* We sum the prescribed melt rate $\dot{m}_w$ up the glacier to define $Q_w$, not considering englacial water storage. Percentile $s_p$ over a response time period prior to the timestep $s_a$ is applied to $Q_w$ to evaluate a characteristic water discharge $Q'_w$ that represents the size of the conduit (hours to days; c.f. [Gimbert et al., 2016] [de Fleurian et al., 2018] [Delaney et al., 2019] [Nanni et al., 2020]). The timescales, $s_a$, and characteristic water discharges ($s_p$ and $Q'_w$), responsible for changes in subglacial conduit size are poorly constrained, yet their impact can be intuited. For instance, short-lived increases in water discharge due to an
hour of precipitation will not greatly impact the hydraulic diameter of the subglacial channel, whereas prolonged melt would increase the hydraulic diameter.

While we understand the reviewer’s recommendation to convert Equation 1 to Equation 3, we have chosen not to. Equation 1 represents the fundamental form of the Darcy-Weisbach, while Equation 3 represents our implementation and application of it. As a result, we believe it best to present Equation 1 first, as to clarify the basis for our application of the equation.

9. **Line 99-102:** Two variables, sa and sp are introduced. And further explanations are added. However, these two variables are not used in Equations 3 and 4. They may be not useful to help us understand why equation 3 could be derived from equation 1, but may confuse readers. I suggest removing this information from lines 96-104.

We have moved the description of these variables and their influence on the hydraulic diameter down the text. The text is presented in the comment above.

10. **Line 109:** is Dh a function of x and y?

In the model implementation, $D_h$ varies in $x$ and $y$ directions. However, in this section we present the theoretical underpinnings of the model, as opposed to the model implementation, so we have not represented $D_h$ as a function of $x$ and $y$. As mentioned above, we have added text to clarify that both $\Psi$ and $D_h$ vary in $x$ and $y$ directions.

11. **Line 109:** Add one sentence at the beginning of this paragraph: With the data of representative surface melting rate $Q_{w^*}$ and the static hydraulic pressure gradient, a representative hydraulic diameter $D_h$ can be estimated. For a given short period, such a $D_h$ is assumed time-independent and used in Equation 1.

We thank the reviewer for the input and appreciate the clarification offered. It has been implemented. The text reads: With knowledge of $D_h$, we insert the instantaneous value of $Q_w$ into Equation 1 to evaluate the instantaneous gradient of the hydraulic potential $\Psi$.

12. **Line 124:** How does this conversion happen? Do you mean $Q_s = Q_s /w$?

See the response to the next question.

13. **Equation 6:** What is the necessity to use $Q_s$ rather than $Q_s$? Trying to use variables as less as possible, if existing variables are sufficient to tell the story. Revise equation 6a to a formula of $Q_s$ but not $Q_s$.

The other reviewer pointed to concerns with this equation and it has been modified somewhat. The term $\tilde{Q}_s$ is sediment mobilization across a width of the glacier bed perpendicular to the flow of water, thus it has units $m^2 s^{-1}$. This is different than sediment flux (units: $m^3 s^{-1}$) or $\lambda \cdot Q_s$ (units: $m s^{-1}$) in Equation 5. We scale the equation with the width of the glacier bed to implement the divergence of the flux in Equation 5. To clarify this matter we have adjusted the text to read:

We calculate sediment mobilization in both supply- and transport-limited conditions. Divergence of the sediment flux is evaluated by approximating $\nabla \cdot Q_s$ with $\tilde{Q}_s$ using a similar mobilization scheme as in Delaney et al. (2019)

$$\tilde{Q}_s = \begin{cases} 
\frac{Q_{sc} - Q_s}{l} & \text{if } \frac{Q_{sc} - Q_s}{l} \leq \dot{m}_t w \\
0 & \text{if } H = H_{lim} \text{ and } \frac{Q_{sc} - Q_s}{l} \leq 0 \\
\frac{Q_{sc} - Q_s}{l} \sigma(H) + \dot{m}_t w (1 - \sigma(H)) & \text{otherwise} 
\end{cases} \quad (1a)$$

(transport-limited)

$$\tilde{Q}_s = \begin{cases} 
0 & \text{if } H = H_{lim} \text{ and } \frac{Q_{sc} - Q_s}{l} \leq 0 \\
\frac{Q_{sc} - Q_s}{l} \sigma(H) + \dot{m}_t w (1 - \sigma(H)) & \text{otherwise} 
\end{cases} \quad (1b)$$

(supply-limited)
\( \tilde{Q}_s \) is sediment mobilization across a width of the glacier bed \( w \) perpendicular to the water’s flow direction. Note that \( w \) is not necessarily the channel width, but rather a representative width across the glacier bed over which sediment can be accessed by water flowing through the subglacial channel (Figure 2). \( Q_{sc} \) is the sediment transport capacity or the maximum amount of sediment that could be transported under the given hydraulic conditions. \( l \) is a characteristic length-scale for sediment mobilization, over which sediment mobilization adjusts to sediment transport conditions.

14. **Equation 15:** what is the difference between \( \tau \) in Equation 11 and \( \tau_b \) in equation 15? Do they equal to each other?

We appreciate the need for clarification on this comment. \( \tau \) is the shear stress between flowing water and the channel wall (i.e. bedrock, ice, or sediment). \( \tau_b \) is the basal shear stress in the sliding relationship. To better clarify these differences, the text now reads: We also determine the shear stress between water flowing through the channel and the sediment below . . .

15. **Equation 15:** does \( \sin \alpha = \lambda z \)?

No. This is simply the sine of the surface slope.

16. **Line 172:** What is the purpose of flow routing? Is it used to distribute water potential at different cells?

We have added the following comments to the text: A routing scheme is implemented to (a) evaluate the hydraulic potential and thus the direction of the water flow and (b) transport sediment and water across the glacier bed, to where it is expelled or deposited.

17. **Line 172:** change “multicell routing scheme” to “2-D distributed routing scheme”?

Done.

18. **Line 174:** what is a “regular grid”? square grid or rectangular grid? Also, the result in Figure 3 shows an irregular distribution, please clarify this.

Excellent point and we have modified the text to read: on a regular grid with square cells, extending in \( x \) and \( y \) directions. In the figure, a regular grid is implemented, however, values outside of the glacier have been omitted. Thus the domain has an irregular shape, but the grid is regular.

19. **Line 174:** What do “fluxes” mean? Is water flux driven by pressure gradients?

We have modified the text to read: water and sediment fluxes can pass to the four surrounding cells sharing an edge, in response to the hydraulic pressure gradient.

20. **Line 175:** Not sure what is a “stack” in Table 3. It is better to draw a 2D sketch to visualize \( st, nd, \) and \( nr \) for a single cell. See major comments 1b.

In the text we state: This routing algorithm returns a stack \( (s_1; \) Table 3), which is a vector that contains information about the order of cells to perform the calculations . . .

Additionally, we are grateful for the reviewer’s recommendation to make a figure about the grid (see above).

21. **Line 175-178:** These sentences describe the process of numerical discretization. The value of the cell, i.e., a stack defined here, is a linear combination of the values of certain variables of neighboring cells. There are different types of numerical discretizations and thus result in different linear combinations. As different combination means different discretization schemes have different accuracy, so it is generally required to describe what discretization schemes are used. The current
sentences are very general descriptions of the discretization principle. Please refer to the major comment 1b to better describe this part.

This as been addressed this above.

22. Equation 18: In lines 176-177, you defined donors and receivers, why did you only use the information of potential from receives, i.e., summation from 1 to nr in Equation 18. Here the wr,j is determined by discretization schemes. How do you calculate this?

We have slightly modified the text to read: the weight or the percentage of hydraulic potential and water or sediment discharge directed from one cell to another (w_d or w_r) . . .

The summation over 1 to nr represents the cells that are directed at a cell. For instance, if 3 cells donate to a cell, then nr = 3.

23. Line 186-187: It is very hard to understand this sentence by reading just words. You have to provide a figure to describe what is donor, receiver, edge length, etc. This is a typical practice for papers that involve numerical discretization algorithms. Also, for 1D grid, it only has one edge length. But for 2D grid, it has two edges and thus two edge lengths. For the regular grid, if the grid is square, then we can say it has only one edge, but if it is rectangular, it has two edges. As your paper is a 2D model, please make sure if you use a square grid or not.

We thank the reviewer for this comment and have worked to make this section clearer. A figure with the grid has been added. Additionally, in both the figure, text, and Table 3 we have clarified that the grid is square, or put differently there is a single value of edge length λ.

24. Line 188: you mentioned “beginning at the based and moving up to the glacier”, does this sentence mean the calculation is performed along a line from the base to somewhere? What does “flow paths evaluated in the routing scheme mean? You need to provide a sketch to carefully describe these details.

We have tried to address this comment by modifying the text to the following sentence: The operation is executed on a cell-by-cell basis, beginning with cells that have no receivers, such as those near the glacier terminus, and moving up the glacier using the inverted stack in st (Figure 4).

25. Equation 19: So you are applying the routing scheme to calculate a distributed discharge for water? Again, what is δ, w_d,j ? And why you only calculate summation over donor cells? These really require a detailed figure to describe this. Combining equations 19 and 18, there have too many variables, it is impossible to accurately understand the meaning and how you calculate without a detailed sketch. By the way, as you give m_w,i different value for different cell, so do you mean you prescribed a distributed meltwater source m_w,i term along a certain line?

We have modified the text slightly to read: Using the routing scheme above, we evaluate the water discharge from cell i, Q_w,i, from melt upstream as

\[ Q_w,i = \sum_{j=1}^{n_d} Q_{w,j} \cdot w_{d,i,j} + \dot{m}_{w,i} \cdot \delta \quad (2) \]

where n_d is the number of donor cells for cell i, w_{d,i,j} is the percentage of water flow from cell j to cell i, and \( \dot{m}_{w,i} \) is a prescribed meltwater source term in cell i.

26. Line 192: What is Q_s,i ? Where does this term come from and why you need to calculate this term? Most importantly, how are equations 20a-c derived? The term "like-wise" does not explain how this term is derived.
This equation is the implementation of 6 in the code. In response to this comment, we have reorganized the text slightly to read:

The amount of sediment leaving a cell \( i \), \( Q_{s,i} \), is the flux into the cell plus the sediment mobilized in the cell, which is defined as

\[
Q_{s,i} = \sum_{j=1}^{n_d} Q_{s,j} \cdot w_{d,i,j} + \tilde{Q}_{s,i} \cdot \lambda.
\]

The first term is the flux of sediment into the cell \( i \) from donor cells \( j \). The second term is sediment mobilization, \( \tilde{Q}_{s,i} \) in cell \( i \), which is computed by implementing Equation 6 as

\[
\tilde{Q}_{s,i} = \begin{cases} 
\sum_{j=1}^{n_d} \left( \frac{Q_{sc,j} - Q_{s,j}}{l} \cdot w_{d,i,j} \right) & \text{if } \sum_{j=1}^{n_d} \left( \frac{Q_{sc,j} - Q_{s,j}}{l} \right) \cdot w_{d,i,j} \leq \frac{m_{d,i}}{n_d} \\
0 & \text{if } H_j = H_{lim} \text{ and } \frac{Q_{sc,i,j} - Q_{s,i,j}}{l} \leq 0 \\
\frac{m_{t,i} \lambda}{n_d} \left(1 - \sigma(H)\right) + \sum_{j=1}^{n_d} \left( \frac{Q_{sc,j} - Q_{s,j}}{l} \right) \cdot \sigma(H) \cdot w_{d,i,j} & \text{otherwise}
\end{cases}
\]

where \( Q_{sc,j} \) is the sediment transport capacity from cell \( j \) flowing to \( i \), \( Q_{s,j} \) is sediment discharge entering from cell \( j \) to cell \( i \), \( l \) is a response length scale, and \( \lambda \) is edge length.

27. **Line 195**: Need a figure to describe what is \( \lambda \)? What is a response length scale? And where is this come from?
   Done. Response length scale \( l \) is defined above, in Equation 6.

28. **Line 196**: how is equation 21 derived? Why \( Q_{si} = \ldots \)?
   See the response to Line 192, above.

29. **Line 198**: Need to define cell area?
   Done.

30. **Line 215**: Need a figure to show where the edge cells are.
   Done.

31. **Section 3.1 title**: You only have one synthetic case. Use a singular form
   Done.

32. **Line 235**: It is better to reproduce and visualize the synthetic glacier geometry in your paper.
   We have referenced the figure with appropriate geometry.

33. **Line 238**: what is “laterally”? You haven’t defined any coordinates here, so no way to understand “laterally”.
   The text now reads: variable ice thickness mean that variable hydrologic gradients will occur perpendicular to the flow, thus water and sediment are routed across multiple cells.

34. **Equation 23**: What is the unit of \( T \)? For 0, is the unit K or C? Line 249: \( o \) is not the same as Celsius (oC).
   The units have been switched to C.

35. **Line 255**: Figure 4 appears earlier than Figure 3.
   This has been resolved.
36. **Line 256-257:** In Figures 5b,d,f, each figure has two lines. One is purple and the other is yellow. So which line is the "Daily-averaged sediment discharge"? However, both curves in 5b,d,f shows complex behaviors. For purple lines b and f, they show decreasing, increasing, and decreasing trends. For d, it shows an increasing, constant, and decreasing trend. The yellow lines show more complex behaviors. In short, figures 5b,d,f show different behaviors compared to what you say at line 257 (decreases until...). Please carefully analyze the figures and make your text descriptions consistent with your figure.

Maximum and average quantities of daily sediment discharge decrease until the very end of the melt season, when sediment discharge increases very slightly again.

The following text has been added to the figure caption Data are plotted at a 6 hr interval so that daily maximums and minimums are visible.

37. **Line 264:** How do you define mean till height?

The text now reads: the mean till height across the glacier.

38. **Line 295:** Can you elaborate on the 4 time periods?

We modified the text very slightly. Subglacial sediment discharge from the glacier is determined over four different time periods (2011 - 2013, 2013 - 2014, 2014 - 2015, 2015 - 2016) by differencing the bathymetry maps collected through this period and considering proglacial erosion quantities [Delaney et al.] (2018, 2019).

However, the citation for [Delaney et al.] (2018) is given, and we believe that at present the information is adequate for the model application here.

39. **Line 314:** add (see red stars in Figure 7a-c) after “... till height H0 of 2.5 cm”.

Done.

40. **Line 319:** Why is the data a constant within each year? For example, the data is constant between late 2011 - late 2013?

We have debated about how best to present the data and thought that the uneven time periods and long time spans made it so that the sum over a time period would be most logical. We also considered presenting the data as a table, however, in this case, it would be far more difficult to present the ensemble of model runs together. We have altered the plot to represent an average flux of sediment over the time period m$^3$ a$^{-1}$. This figure has been modified to:
Figure 3: Results of the parameter search (a, b, c), the frequency of parameter values that produced a rank correlation of 1 (d, e, f) and average sediment flux from model run amongst the parameter combinations over the time periods (g) in the synthetic alpine glacier. Red stars represent the optimum parameter combination with an absolute error of roughly 62,600 m³. Blue lines represent all model outputs, while the gray line represents the optimum parameter combination.

41. **Line 319:** From Figure 7a,b,c, What is the absolute error for the optimal run (red star case)? Is the value 62,600? I can not see it clearly from the y-axis in Figure 6a.  
   The text has been modified to read:  
   Red stars represent the optimum parameter combination with an absolute error of roughly 62,600 m³.

42. **Line 325:** change “short-lived” to “short-lived period”.  
   The text now reads: short periodic increases in water discharge.

43. **Line 325:** This claim needs supporting evidence. From Figure 8a, you can identify the peak values for the sediment discharge. Meanwhile, you can obtain the corresponding water discharge. You can calculate a correlation between the two values. If the correlation coefficient is high, then this claim is supported.  
   We have modified the text slightly to the following. Additionally, we have referenced Figure 10 in the text. While this figure does not have information on a correlation coefficient, it is evident that the highest water discharge values do not necessarily result in the greatest sediment discharge values.  
   Some of the peaks in sediment discharge occur during the short periodic increases in water discharge. Yet the greatest sediment discharge values do not necessarily occur at the highest water discharge values (Figure 9a and Figure 10a).

44. **Line 328:** what is “high on the glacier”?
References


