Comparison of rainfall generators with regionalisation for the estimation of rainfall erosivity at ungauged sites

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Abstract. The assessment of rainfall erosivity is one of the main inputs in determining soil erosion. To calculate mean annual rainfall erosivity (R), long-term high-resolution observed rainfall time series are required, which are often not available. To overcome the issue of limited data availability in space, four methods are employed: the direct regionalisation of R, the regionalisation of 5 minute rainfall, the disaggregation of daily rainfall into 5 minute timesteps, and the use of a regionalised stochastic rainfall model. In addition the minimum time series length necessary to adequately estimate R is investigated. The impact of station density is considered for each of the methods. The study is carried out using 159 recording and 150 non-recording (daily) rainfall stations in the federal state of Lower Saxony, Germany. Results show that the direct regionalisation of mean annual erosivity leads to the best results in terms of relative bias and relative root mean square error (RMSE). This is followed by the regionalisation of the 5 minute rainfall data, which yields better results than the rainfall generation models, namely an alternating renewal model (ARM) and a multiplicative cascade model (Disagg). However, a key advantage of using regionalised rainfall models is the generation of rainfall time series that can be used for the estimation of the erosive event characteristics, which is not possible through the direct regionalisation of R. Using the stochastic ARM it can be shown that in most cases more than 60 years of data is needed in order to obtain a stable estimate of the annual rainfall erosivity. Estimation of soil erosion based on only 5 or 10 years of data can lead to uncertain values. Such short time series are often used when regionalisation is applied. Moreover, it was also found that resolution of measuring device has a significant effect on the rainfall erosivity and coarser data resolution can lead to high relative bias.

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1 Introduction

Intense soil erosion has a significant impact on the environment, for example it can present a major threat for agricultural production or lead to increased sedimentation and pollution in rivers, which can also affect aquatic organisms. Soil erosion
35   modelling can be applied in order to detect the most critical areas and design suitable measures for soil loss reduction. One of the most frequently applied models for the soil erosion modelling is the Universal Soil Loss Equation (USLE) and in its follow-up version named Revised Universal Soil Loss Equation (RUSLE) (e.g., Renard et al., 1997). In the scope of the RUSLE model, soil erosion is described with six factors, one of which is the rainfall erosivity factor R (in MJ mm ha-1 h-1, Renard et al., 1997). Rainfall erosivity is often characterised by large spatial and temporal variability (e.g., Bezak et al., 2020, 2021a,
40   2021b; 2021c; Panagos et al., 2016, 2017; Petek et al., 2018), which means that rainfall erosivity estimations are not a straightforward task and require adequate temporal and spatial data resolution.

In order to obtain robust rainfall erosivity values, high-resolution observed rainfall estimates are needed, ideally with a 1 or 5 minute resolution (Dunkerley, 2019). However, rainfall data at this resolution is often only available for shorter periods of observation (e.g. 10 or 20 years) or not at all. A solution to overcome this shortcoming is the use of stochastic rainfall models
45   that allows the generation of long time series of arbitrary length. A few studies have investigated the possibility of applying stochastic precipitation models to generate rainfall time series that can be used to estimate rainfall erosivity (e.g., Jebari et al., 2012; Lobo et al., 2015; de Oliveira et al., 2018; Haas et al., 2018). Different methods have been developed for the generation of rainfall itself such as cluster-based models (e.g., Onof et al., 2000, Onof and Wang, 2020), cascade models (e.g., Molnar and Burlando, 2005; Müller and Haberlandt, 2018), method-of-fragments (e.g., Breinl and Di Baldassarre, 2019) or alternating
50   renewal models (e.g., Callau Poduje and Haberlandt, 2017), or as part of weather generators (Peleg et al., 2016). The parameters of these methods are estimated based on observations and the complexity of the models depends on the selected time step of data. Thus, most studies are focused on daily data. For example, CLImate GENerator (CLIGEN) was applied to obtain daily rainfall estimates and calculate rainfall erosivity using daily data (e.g., de Oliveira et al., 2018; Lobo et al., 2015; Wang et al., 2018). However, some shortcomings such as sensitivity to the selection of input parameters of the above mentioned models
55   have been reported in the literature (e.g., Meyer et al., 2008; Haas et al., 2018). On the other hand, temporal high-resolution time series (i.e. 5 minutes) are not frequently generated using stochastic rainfall models, although, in recent years advancements have been made in terms of high frequency rainfall data simulations (e.g., Haberlandt et al., 2008; Vandenberghe et al., 2011; Vernieuwe et al., 2015, Callau Poduje and Haberlandt, 2017, Müller-Thomy, 2020).

Thus, a few studies (e.g., De Oliveira et al., 2018; Haas et al., 2018) have investigated if stochastic rainfall models are able to
60   correctly predict the rainfall erosivity pattern at specific locations. In the case that a stochastic rainfall model is able to mimic the rainfall erosivity characteristics, generated long-term high-resolution rainfall time series should allow a robust estimation of annual and monthly erosivity pattern. Similarly, a limited number of studies (e.g., Angulo-Martinez et al., 2009) have investigated the issue of observed high-resolution data in space.

Therefore, the main aim of this study is to evaluate and compare different rainfall generators and regionalisation approaches
65 in order to obtain either directly or indirectly rainfall erosivity estimates for ungauged locations. Given a lack of high-resolution
rainfall time series, the research question is whether these methods can adequately reproduce observed rainfall erosivity
statistics. In addition, one further research question is posed. Given the existence of a high-resolution rainfall time series, how
long should this time series be in order to obtain stable site rainfall erosivity statistics?

All tests are performed via leave-one-out cross validation, as the premise of this study is that high-resolution time series are
70 not widely available. Additionally, the effect of station density on the regionalisation performance is assessed by performing
each test at five different station counts. To minimise the sampling uncertainty, 20 realisations of each test at each station
density are performed.

2 Data and study area

High-resolution observed rainfall data for 159 stations in and around the northern German federal state of Lower Saxony (Fig.
75 1) was acquired from the German Weather Service (DWD). The one minute source time series, from a combination of tipping
bucket and later drop counter measurements, were aggregated to 5 minutes for use within this project. All data used for the
project is restricted to the ten year period 2007-2016, as all 159 stations have at least 98% data availability for this period. The
study area is dominated by lowland terrain, with the only mountains of significance being the south east lying Harz mountain
range. The region is predominantly classified as having a temperate oceanic (Cfb) climate according to Köppen-Geiger (Kottek
80 et al., 2006). The far eastern portion of the study area is categorised as temperate continental (Dfb) and the Harz mountain
range as cool continental (Dfc). Fig. 2 displays the long-term seasonal rainfall and temperature variability averaged over the
German federal state of Lower Saxony. Annual rainfall varies between 950 mm in the Harz mountain range to under 400 mm
in lower lying areas east of the study area.

3 Methods

85 In this study, four different methods to calculate annual rainfall erosivity (R) for ungauged sites are applied (Fig. 3). The first
and simplest is the direct regionalisation of observed mean annual rainfall erosivity (method Direct-R). The other three methods
all generate rainfall time series first from which mean annual rainfall erosivity is subsequently calculated. The second is the
regionalisation of the observed 5 minute rainfall time series (method Direct-P). The final two methods are stochastic rainfall
models – one being a regionalised disaggregation model using regionalised daily rainfall as its source (method Disagg) (Müller-
90 Thomy, 2020) and the other a regionalised alternating renewal model (method ARM) (Callau Poduje and Haberlandt, 2017).
Detailed descriptions of the applied methods are provided in sections 3.2 and 3.3 while section 3.1 provides details about the
rainfall erosivity calculation.

3.1 Rainfall erosivity

Rainfall erosivity is one of the factors that has the highest impact on soil erosion rates. Rainfall erosivity is characterised by

95 multiple properties of rainfall events such as the kinetic energy of raindrops, rainfall intensity and rainfall duration. In order to calculate annual rainfall erosivity for a selected time span of M years, the following equation proposed by Renard et al. (1997) can be used:

$$R = \frac{\sum_n E \cdot I_{30}}{M} \tag{1}$$

where R is rainfall erosivity (MJ mm ha^{-1} h^{-1}), I_{30} is the maximum 30 minute rainfall intensity for a specific rainfall event (mm

100 h^{-1}), E is the kinetic energy of the rainfall event (MJ ha^{-1}) and n is the total number of erosive events. Equation 2 is used to derive the kinetic energy of the rainfall event:

$$E = e_B \cdot I \cdot \Delta t \tag{2}$$

where e_B is specific kinetic energy (MJ ha^{-1} mm^{-1}), I is rainfall intensity (mm h^{-1}) for the event and Δt is the time interval. Various equations for calculation of e_B were developed for different parts of the world, where most often high frequency

105 measurements using optical disdrometers are used in order to derive the $e_B - E$ relationship (e.g., Petan et al., 2010). One of the most frequently and globally used equations (e.g., Panagos et al., 2015, 2017) was proposed by Brown and Foster (1987):

$$e_B = 0{,}29 \cdot [1 - 0{,}72 \cdot exp \left(-0{,}05 \cdot I\right)] \tag{3}$$

This equation is also mentioned in the RUSLE Handbook (Renard et al., 1997), which means that it should be regarded as a standard method when one wants to estimate the soil erosion using the RUSLE method. Additionally, the equation has been

110 already applied several times to similar climatic conditions (e.g., Panagos et al., 2015) and yielding meaningful results. This equation was also used in the presented study in order to calculate specific kinetic energy. The equation 3 is valid for the rainfall intensity in the range between 0 and 250 mm/h.

Erosive rainfall events are defined according to the RUSLE methodology (Renard et al., 1997). A rainfall event is considered erosive if the total volume exceeds 12.7 mm of rain or if the maximum volume in 15 minutes is more than 6.35 mm. Here a 6

115 hour period without rain is used in order to separate two consecutive erosive rainfall events. The monthly and annual rainfall erosivity values are then calculated based on the rainfall erosivity of all erosive events.

3.2 Direct regionalisation methods

3.2.1 Method Direct-R: direct regionalisation of mean annual rainfall erosivity R

The direct interpolation of erosivity is carried out using geostatistical methods (see e.g. textbooks from Isaaks and Srivastava,

120 1989 or Goovaerts, 1997). The spatial dependence of R is modelled using an isotropic spherical variogram model:

$$\gamma(h) = c_0 + c \left(\frac{3}{2}\frac{h}{a} - \frac{1}{2}\frac{h^3}{a}\right) if \ h \leq a, \ otherwise \ \gamma(h) = c_0 + c \tag{4}$$

where c is 0.5, c_0 is 0.3 and *aeff* is 30000 km. For interpolation ordinary kriging (OK) and external drift kriging (EDK) are applied. While OK assumes spatial stationarity, EDK relaxes the intrinsic hypothesis regarding the constant mean and assumes a linear relationship between the mean of the target variable Z and one or more additional variables Y:

$$E[Z(u)|Y(u)] = a + \sum_{i=1}^{n} b_i\, Y_i(u) \tag{5}$$

The coefficients a and b_i need not be known explicitly, but are considered in building the kriging system. The additional variables Y_i need to be provided for all points with observed erosivity and for all unobserved points to be interpolated, usually on a raster. Here, the following additional variables are used in the analyses: daily mean rainfall (*Pmean*), maximum daily rainfall (*Pmax*), the 90 and 99 percent quantiles of daily rainfall (*Pq90*, *Pq99*) and the yearly wet fractions of days with rainfall higher than 12.7 mm (based on the RUSLE methodology; Renard et al., 1997) and 1 mm thresholds (*wfd12*, *wfd1*). These variables are extracted from regionalised daily data in a spatial resolution of 1 km² (NLWKN, 2019), and are calculated as long-term averages from the period 2007-2016. The interpolation uses a minimum of 8 neighbours and a maximum of 12 neighbours within a radius of 300 km. The R statistical software with the package "gstat" (Pebesma, 2004) is employed for the calculations.

3.2.2 Method Direct-P: regionalisation of 5 minute rainfall

A second approach for the regionalisation of the annual R, is to estimate the 5 minute rainfall time series for unobserved locations based on the information of nearby measured rainfall time series. In this study, the nearest neighbor approach (NN) is used for the regionalisation of the 5 minute rainfall time series from 2007-2016. This technique, also known as the Thiessen Polygon method (Thiessen, 1911), is a simple approach which assigns for each unobserved point the observed rainfall time series of the closest available rain gauge. Despite this being a basic interpolation method, a prior investigation of different interpolation techniques (nearest neighbor, inverse distance and ordinary kriging) led to the conclusion that this technique maintains the best temporal structure of rainfall at the 5 minute time scale and yields the lowest error for the calculation of erosivity. One reason for the superiority of NN compared to the other interpolation methods might be that NN is the only method which is not smoothing the extremes, which is important here for the estimation of erosive events.

3.3 Stochastic rainfall models

3.3.1 Method ARM: an alternating renewal rainfall model

The stochastic alternating renewal model (ARM) is based on the theory of renewal processes. Rainfall is described as a series of independent alternating wet and dry spells, described by the three variables: wet spell amount (WSA), wet spell duration (WSD) and dry spell duration (DSD) (Fig. 4).

Probability distributions are fitted to observations of these variables, with observed rainfall events being limited by a minimum WSA (1 mm) and DSD (60 min). Synthetic rainfall time series of arbitrary length can be generated by producing random variates of the distributions.

Additionally, the temporal distribution of rainfall within a wet spell is described by a double exponential function conditioned on the wet spell time to peak (WSTP - modelled using a uniform distribution) and wet spell peak intensity (WSPI - modelled using a copula, see below), and WSA (Fig. 5). Full details of the model can be found in Callau and Haberlandt (2017), with the following alterations which have been found to provide a better model performance, especially when considering regionalisation:

- an Khoudraji Gumbel copula describes the dependence between WSA and WSD
- a Tawn copula describes the dependence between WSD and the ratio WSPI:WSA
- the three parameter Weibull distribution is used instead of the four parameter Kappa distribution for the variables DSD and WSA, being more robust in a regionalisation setting.

3.3.2 Method Disagg: cascade model

Another possibility to generate high-resolution rainfall time series is to disaggregate daily time series, which exist most often for longer time periods and with higher station densities. For this study the micro-canonical cascade model after Müller-Thomy (2019, 2020) is applied due to its performance in previous studies (e.g. Müller and Haberlandt, 2015, 2018). The general cascade model scheme with disaggregating one coarse time step into "b" finer time steps is illustrated in Fig. 6. From daily rainfall amounts three time steps with 8 h duration are generated (b=3). For all further disaggregation steps b=2 is applied, resulting in temporal resolutions of Δt={4h, 2h, 1h, 30 min, 15 min, 7.5 min}. To achieve Δt=5 min, a linear transformation is applied. The rainfall amount is conserved exactly in each disaggregation step. For a detailed description of the cascade model the reader is referred to the description of the preceding cascade model in Müller-Thomy (2020).

The cascade model parameters are estimated by the aggregation of observed 5 min time series of the recording stations available in each density scenario. As daily time series the aggregated 5 min time series of all 159 stations are used, independent of the applied density scenario. To investigate how suitable is the disaggregation of daily information for unobserved locations, a regionalisation of the daily precipitation is done prior to the application of the cascade model. Here ordinary kriging approach (OK) was used for the regionalisation of daily rainfall time series from 2007-2016. The interpolation uses an isotropic exponential variogram as in Eqn. 6, a minimum of 4 neighbours and a maximum of 12 neighbours within a radius of 150 km.

$$\gamma(h) = c_0 + c \left[1 - exp(-\frac{h}{a})\right] \tag{6}$$

where c_0 is 0.4, c is 1.5 and $aeff$ is 186280 km.

3.4 Regionalisation schemes and station selection

In order to adequately test the performance of the different methods, all regionalisations have been performed in a cross validation mode at varying station densities. Five station densities were chosen – 20% (N=32), 40% (N=64), 60% (N=96), 80% (N=128) and 100% (N=159) of the total available station count. Stations were selected at random and are consistent across the different testing methods presented above. Important to note is that the cross validation is only performed on the stations chosen at the 20% level (N=32) – the higher densities only add further supplementary stations available for the

185   regionalisation, with the assumption that the regionalisation performance will improve at a higher available station count. Furthermore, 20 realisations are performed for each density to minimise the influence of station selection on the results. This scheme is illustrated graphically in Fig. 7.

3.5 Effect of time series length on the calculation or mean annual R

One further research question of interest is how long a rainfall time series must be in order to achieve a stable (i.e. robust)
190   result of mean annual rainfall erosivity. The importance of such a research question lies in the fact that rainfall series used to calculate annual erosivity are often of limited length. Therefore, using only a few years of data could perhaps lead to uncertain estimations of rainfall erosivity, which could affect the estimated soil erosion (i.e. under- or overestimation).

Using the alternating renewal rainfall model described in section 3.3.1, it is investigated how many years of data are needed in order to obtain stable estimations of the annual rainfall erosivity. For this purpose, 18 stations with the longest observation
195   time series length are selected to achieve better model fitting. For each station the ARM model described in section 3.3.1 is fitted. Afterwards, 200 years of synthetic rainfall time series are generated 50 times for all 18 stations. The effect of time series length is investigated by calculating mean annual rainfall erosivity for each station and for each realisation using separately 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 140, 160, 180 and 200 years of data. Results within ±20% of the mean annual erosivity calculated for 200 years are considered stable.

200   **3.6 Evaluation criteria**

To assess the relative performance of the four different methods (Fig. 3), three different evaluation criteria are chosen. The first is the Pearson correlation coefficient r_{xy} between the annual rainfall erosivity calculated from observed rainfall and the estimated mean annual erosivity, i.e. regionalised by the different methods, at a given station density scenario across all realisations and the 32 reference stations. It is worth remembering that this measure is an indication of linear correlation only.
205   It is given by the formula:

$$r_{xy} = cov(x, y)/\sigma(x) * \sigma(y) \tag{7}$$

where cov is the covariance and $\sigma(x)$ and $\sigma(y)$ are the standard deviations of x and y (rainfall erosivity calculate from observed and simulated data), respectively. The second is the relative root mean square error (RMSE) of the mean annual rainfall erosivity for a given station density scenario across all realisations and stations, given by equation 8:

210   $$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[\frac{y-x}{x}\right]^2} \tag{8}$$

where x refers to mean annual rainfall erosivity estimated from observations, y is the estimated mean annual rainfall erosivity estimated by the different methods proposed in this work and N is the number of stations. The third is the median relative bias rB_{med} of the mean annual erosivity. For a given station density scenario the relative bias rB is calculated over all 32 stations for one realisation, and the median relative Bias rB_{med} is provided as summarizing result (referred to as relative Bias in the
215   following):

$$rBmed = median(\frac{y-x}{x}) \tag{9}$$

Additionally, a comparison using absolute rainfall erosivity values is also provided.

4 Results and discussion

4.1 Mean annual erosivity R

220   The main focus of this study is to evaluate the performance of the four different tested methods in reproducing observed mean annual erosivity. For the direct regionalisation of erosivity (Direct-R), EDK with mean annual precipitation (P_{mean}) as external drift was found to provide the best performance. The direct regionalisation provides the best results overall, both in terms of accuracy and precision. The Pearson correlation (Fig. 8) by this method is larger than for the other three methods, which show more or less the same result. Its root mean square error is also lower than the other methods (Fig. 8). The rBmed is the only

225   performance criteria where this method is not clearly better (Fig. 8). In terms of rBmed the Direct-R method is followed by Direct-P. ARM and Disagg in most cases yield relative bias larger than 10 and 20%, respectively. In terms of relative RMSE and Pearson Correlation the ARM and Disagg methods perform relatively similarly. The median result over all realisations shows slightly better results for the direct regionalisation of rainfall (Direct-P) compared to stochastic rainfall models, although the range of results is greater than for Direct-R. The disaggregation model performs worse than the other methods with a >20%

230   underestimation of mean annual erosivity. With increasing station density, the median result is generally improved, in particular for the Pearson correlation (Fig. 8). This is less noticeable for rBmed, as bias likely indicates the inherent ability of the underlying method to replicate the mean annual R (Fig. 8). Moreover, it seems that for ARM and Direct-P the results change greatly between realisations (i.e. larger spread of the results), meaning that the station setting is more decisive, with Direct-P being the most affected. The Direct-R and Disagg methods seem to be more robust in terms of station setting, which

235   indicates their usage for regions with low station densities. Thus, it seems that for areas similar to that of Lower Saxony (i.e. relatively flat areas with small changes in erosivity), the Direct-R method is preferred in case that one needs to estimate the rainfall erosivity for the ungauged locations. The application of this method is also the simplest of the four methods (Fig. 3), especially when compared to the ARM and Disagg methods, since these two methods require development of the stochastic rainfall models and estimation of the model parameters. Thus, in terms of mean annual rainfall erosivity, the use of stochastic

240   rainfall models does not provide an improvement of the results in terms of performance in space (Fig. 8). In terms of the number of stations needed to provide good estimates of rainfall erosivity for the ungauged locations, it is clear that higher station counts available for the regionalisation yield better results, which could be expected (Fig. 8). Thus, for the Direct-R the performance D100>D80>D60>D40>D20 (Fig. 8) with somewhat diminishing improvements in performance above D80. Similar conclusions can be made for the Direct-P method while for the ARM and Disagg this relationship is not so evident for

245   the rBmed and RMSE (Fig. 8). Therefore, even for topographically relatively homogeneous areas (compared to for example

Alpine area) such as Lower Saxony, the accuracy of rainfall erosivity maps greatly depends on the number of stations used for the interpolation.

4.2 Erosive event characteristics

Although the main focus of the study is the calculation of the mean annual erosivity R, in some specific applications more

250 details about the rainfall erosivity at a location might be required. For example, studies that investigate changes in erosivity patterns in time (i.e. climate change studies) might be interested in changes in the number of erosive events or erosive events characteristics. In such a case the Direct-R method cannot be used since this method is only able to provide estimates of the mean annual rainfall erosivity and does not provide a time series of erosive events. Naturally a similar procedure as in the case for Direct-R (section 3.2.1) could be used for any other variable of interest (e.g. number of erosive events, mean duration of

255 the erosive events). However, the EDK setup used for mean annual R is unlikely to be optimal for other variables, which means that an optimal EDK setup would need to be investigated for each variable of interest separately and thus could be considered as relatively time consuming.

However, three of the four methods presented in this study, namely Direct-P, ARM and Disagg, are able to reproduce erosive events themselves from which the mean annual R is calculated. How well the three methods reproduce erosive event

260 characteristics is explored in this section. Fig. 9 displays the median relative bias of different erosive event variables. The mean annual event count and precipitation sum (first two box plots) is best represented by the Direct-P method. Whereas the median error is close to zero, the range of errors is well over 100% when one considers outliers. The Disagg method represents the event count well but underestimates the annual volume slightly, whereas the ARM method underestimates both the annual count and volume (Fig. 9). Performance improves slightly for all methods with increasing station density (not shown).

265 For both the mean event duration and volume (Fig. 9, last two box plots), the results again show that the Direct-P method is the most efficient. The Disagg method significantly overestimates the event duration and at the same time underestimates the event volume. However, its overall performance is achieved already for the low density scenarios (not shown here). An overestimation of the erosive events duration is to be expected as previously identified by Jebari et al. (2012) with underestimations higher than 40%. Here the overestimations are higher as the disaggregation inherits the errors caused by the

270 regionalization of the daily rainfall. Because of the unbiased estimator, OK tends to smooth the spatial structures of the rainfall, leading to overestimation of the low intensities (explaining the longer event durations) and underestimation of the extreme ones (explaining the lower event volumes). The performance of the disaggregation may improve if another regionalization method that captures better the temporal variability is employed.

The ARM method underestimates both mean event duration and mean event volume, which explains the underestimation in

275 both annual number of events and their volume. Thus, it can be seen that Direct-P outperforms the ARM and Disagg methods not only in terms of annual rainfall erosivity but also in terms of specific events characteristics. In case that one would like to obtain characteristics of the erosive events for the ungauged sites the Direct-P method should be preferred since it is able to produce acceptable results and yields better performance compared with the two tested stochastic rainfall models.

4.3 Annual rainfall erosivity – data length sensitivity

280 Despite the fact that Direct-R and Direct-P methods yield better performance than the evaluated stochastic rainfall models, the benefit of the latter is the ability to generate long time series of arbitrary length of high-resolution data for (almost) unobserved areas, where Direct-R and Direct-P are not applicable at all. The goal of this section is to investigate how long the synthetic time series should be in order to obtain a stable estimate of the mean annual rainfall erosivity, which is most frequently used as an input to the soil erosion models (Panagos et al., 2015; 2017). Using the ARM model and the methodology described in

285 section 3.5 it is investigated how many years of data is needed in order to obtain stable annual rainfall erosivity estimation. Important to remember is that the ARM model performance is better than what was shown in sections 4.1 and 4.2 as the ARM model in this section is fitted directly to observations and not regionalised. Fig. 10 shows results of this investigation. It can be seen that the variability between different realizations is quite high (Fig. 10). Moreover, investigation of the intersection between the 5 and 95% quantile values and +-20% variation of the mean annual rainfall erosivity indicates that in case of 5%

290 quantile in most cases 60 years of data are needed in order to obtain a value that is between the +-20% interval (Fig. 10). This indicates that calculations of the annual rainfall erosivity for the soil erosion modelling using USLE or RUSLE (e.g., Renard et al., 1997; Panagos et al., 2017) using only a limited sample size (e.g., less than 5 or 10 years) will likely result in a greater than +-20% difference to the long term mean. Consequently, a similar impact (i.e. over- or under-estimation) on the calculated soil erosion rates will be obtained if one applies the RUSLE equation for the prediction.

5 Conclusions

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This study evaluates four methods that can be used to estimate the annual mean erosivity (R) in space. Based on the presented results the following conclusions can be drawn:

1. Both tested direct regionalisation methods (Direct-R and Direct-P) outperform the tested stochastic rainfall models (ARM and Disagg), with slightly better results for Direct-R. Furthermore, in terms of method complexity, this can

300 be regarded as the simplest since it does not require the fitting of any model parameters. Differences among tested methods are relatively large, for example, in relative bias up to 25%.

The main drawback of the Direct-R method is that it cannot be used to estimate the number of erosive events or mean event duration without applying the model to every variable separately (e.g., number of erosive events, annual rainfall erosivity). Therefore, the Direct-P method has the advantage that it is able to generate high-resolution time series of erosive events for

305 ungauged sites. Therefore, information about the number of erosive events and the characteristics of erosive events can be determined as well.

2. In terms of the characteristics of the erosive events, the Direct-P method yields better performance than both tested stochastic rainfall models.
3. Both rainfall generators have proven their applicability in the field of soil erosion modelling since they are able to

310 produce long synthetic series of the high-resolution data.
4. Cross validation methodology using multiple density scenarios (Fig. 7) has indicated that performance of all methods slightly increases with increasing station density.

Investigation of the impact of time series length on the annual rainfall erosivity for 18 stations is carried out using the ARM model:

315   5.   More than 60 years of data is required in the case that one would like to obtain rainfall erosivity estimates within 20% of the actual long-term mean annual rainfall erosivity.

Thus, this conclusion is of critical importance for soil erosion studies where rainfall erosivity estimates are used as input, since in most cases the high-resolution data used to estimate rainfall erosivity is much shorter than 60 years. So in cases where only 5-10 years of observed rainfall data is available, the estimated mean annual rainfall erosivity can be up to ±100% in comparison

320 to the actual long-term mean annual rainfall erosivity, which can be reduced by the application of one of the here used rainfall generators.

It should be noted that the approaches presented in this paper should be applied and tested for further case studies with different rainfall and topographical characteristics than for Lower Saxony, which is mostly flat and without major orographic obstacles. Additionally, some study limitations and lessons learned can also be made based on the presented results and conclusions such

325 as that resolution of the measurement device, which has evolved in the recent decades, has a significant effect on the calculated rainfall erosivity and relative bias (Supplement; Fig. S1).

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Competing Interest

Authors declare no conflict of interest.

Author contribution

All authors developed the concepts of the manuscript. R.P. conducted most of the calculations with support of the B.S., H.M.T., U.H., N.B. N.B. drafted the first version of the manuscript, all authors contributed to writing and editing of the manuscript.

Code and data availability

Data can be requested from the German Weather Service (DWD), the codes used in this study are freely available upon request from the first author.

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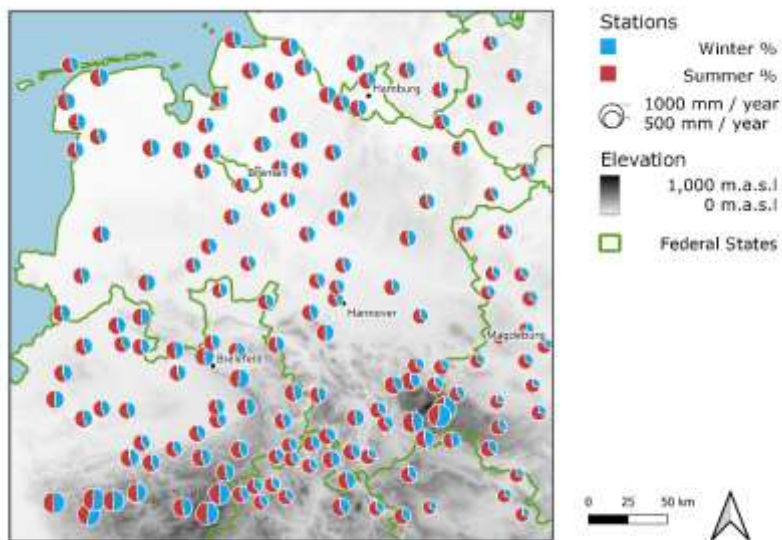
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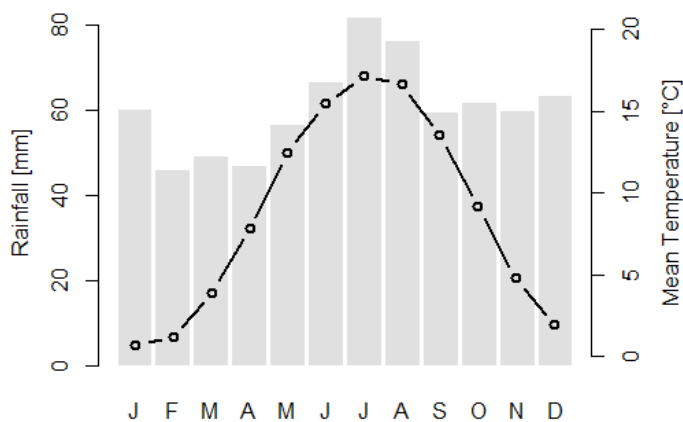
Figure 1: Location of all recording stations (N=159). Pie charts indicate relative annual volume (radius) and percentage split between summer (red) and winter (blue) precipitation.

445   **Figure 2: Long-term climate (1881-2019) averaged across the German federal state of Lower Saxony that is investigated in the scope of this study. Source: German Weather Service (DWD) Climate Data Center (CDC) (CDC, 2020).**

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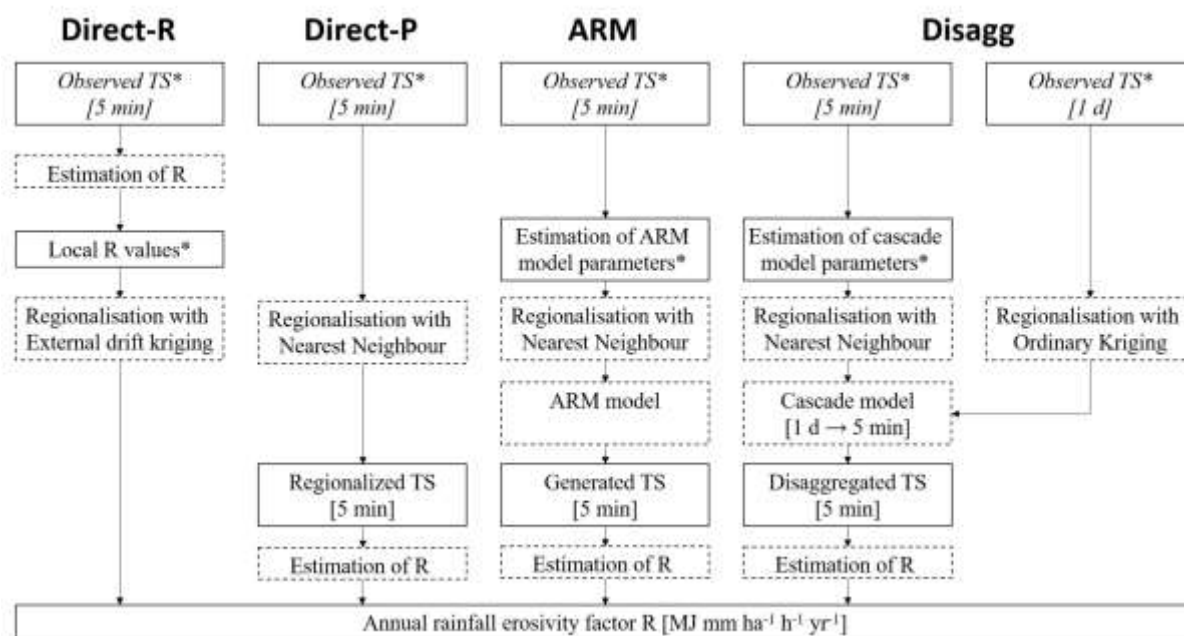
Figure 3: Overview of the estimation of the annual rainfall erosivity (R) including all applied methods (dashed boxes) and data sets
450 **(solid boxes, italic written represent input data sets) ['TS' are rainfall time series; '*' indicates data sets affected by the station density scenarios].**
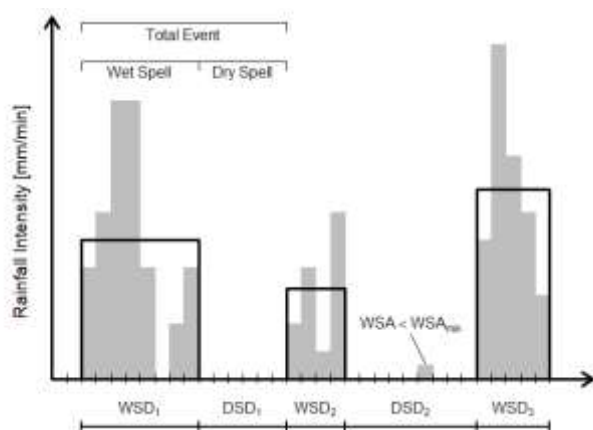
Figure 4: Schematic of the external structure of the ARM model. The black boxes describe rainfall events derived from observations.

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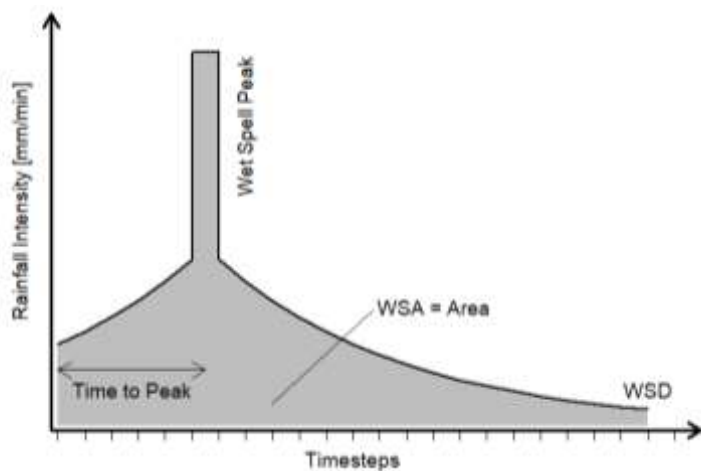
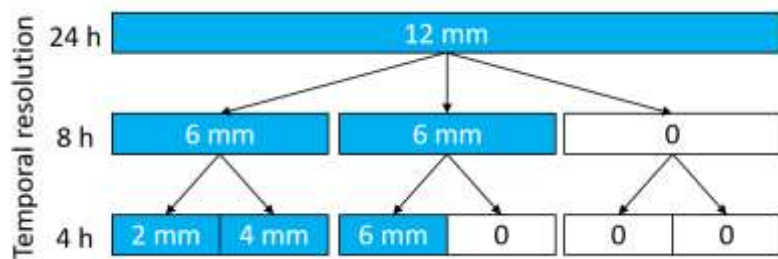


Figure 5: Internal structure of ARM model according to the Callau and Haberlandt (2017).

460 **Figure 6: General scheme of the cascade model for the first two disaggregation steps with exemplary rainfall amounts for a daily total of 12 mm (blue boxes = wet time steps).**

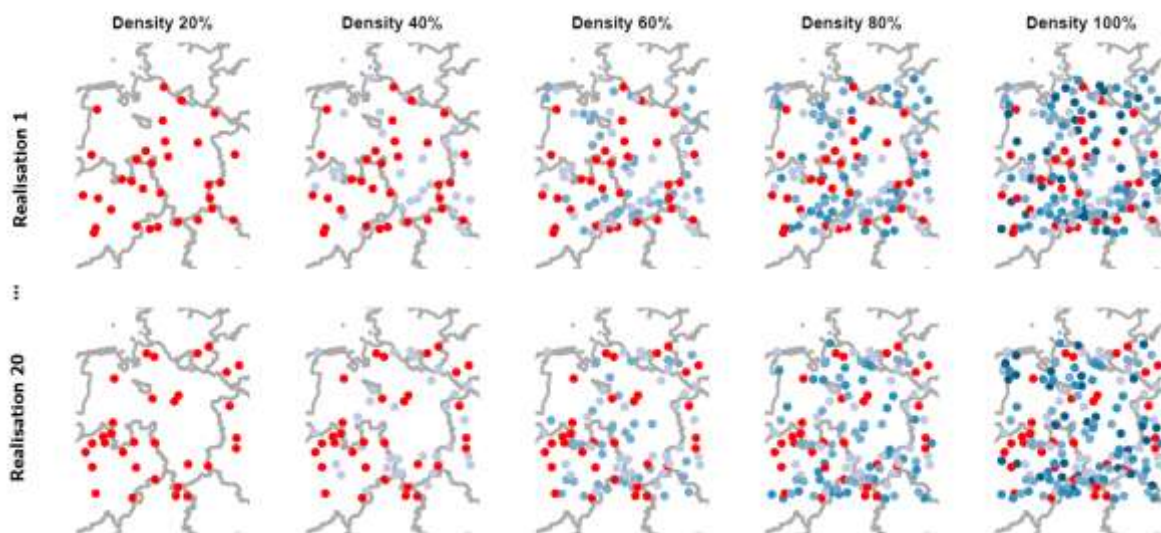



Figure 7: Five different station density scenarios shown for two realisations. The red stations are used for the cross validation for all station densities. The additional stations (blue) provide supplementary information for the regionalisation.

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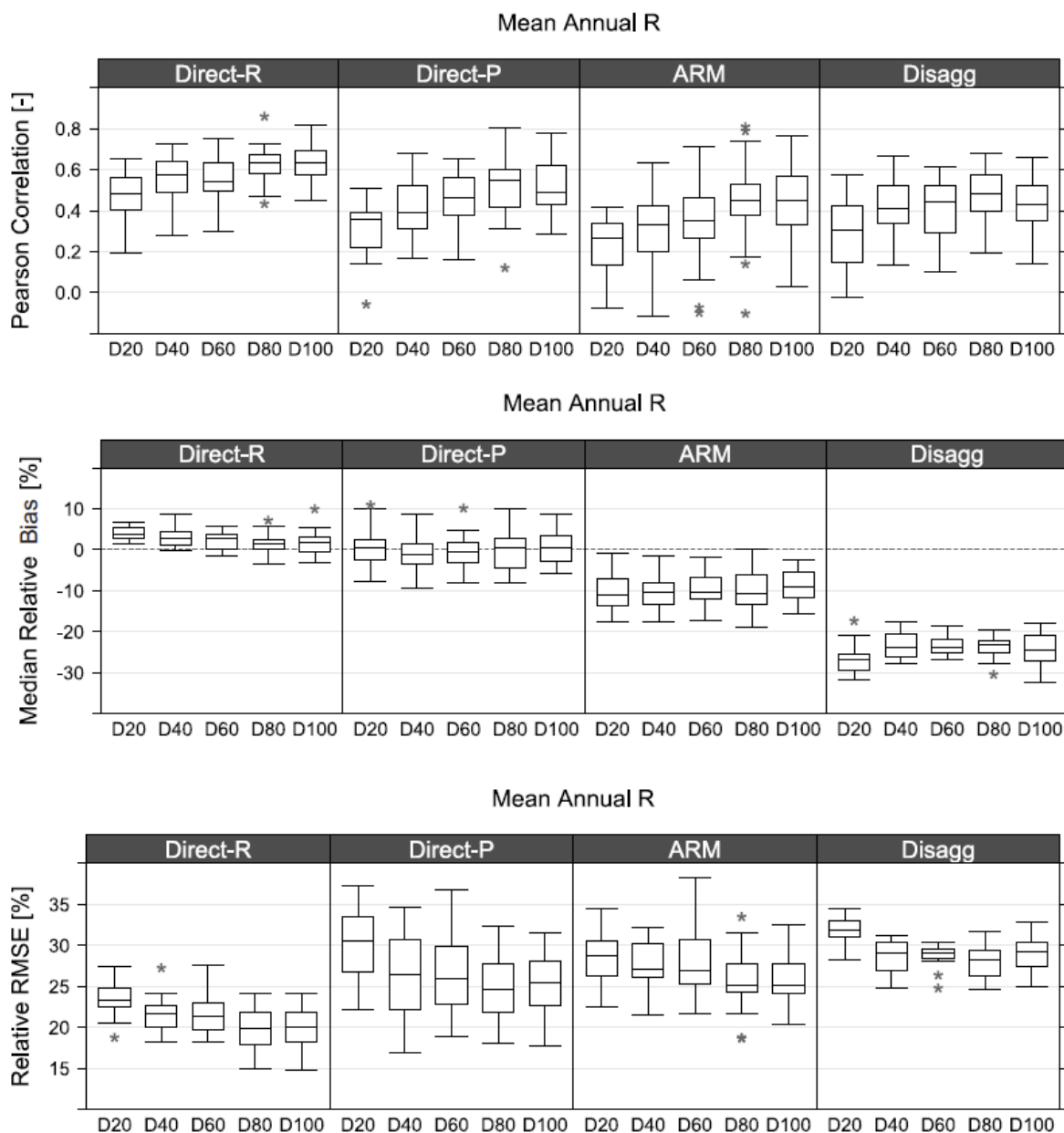
Figure 8: Pearson's Correlation Coefficient, Relative Bias and Relative RMSE results for the four tested methods. Box plots show the relevant statistic for the 20 realisations (N = 20) at each density scenario (eg. D20 corresponds to the 20% station density scenario).

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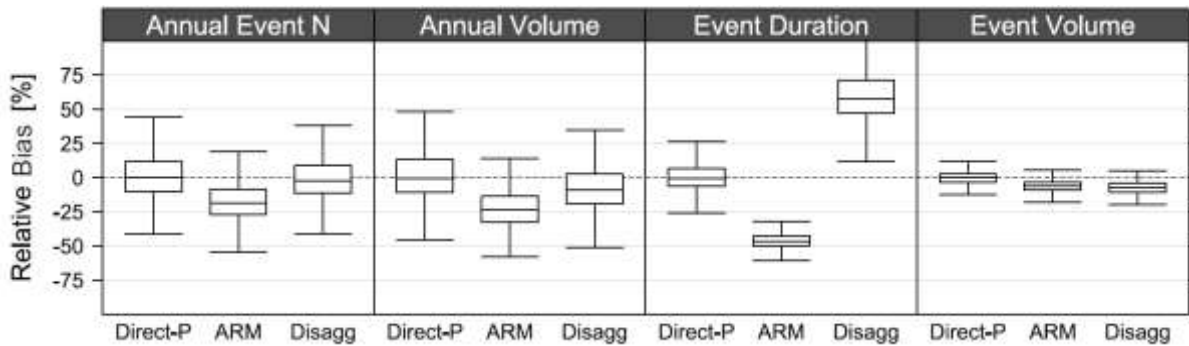
Figure 9: Relative bias of all stations (N=32) for all realisations (N=20) and density scenarios (N=5) for different erosive event variables (for each box plot, N = 3200). Annual number of erosive events, annual number of all erosive events volume, erosive event duration and erosive event volume are shown. Outliers are excluded for clarity.
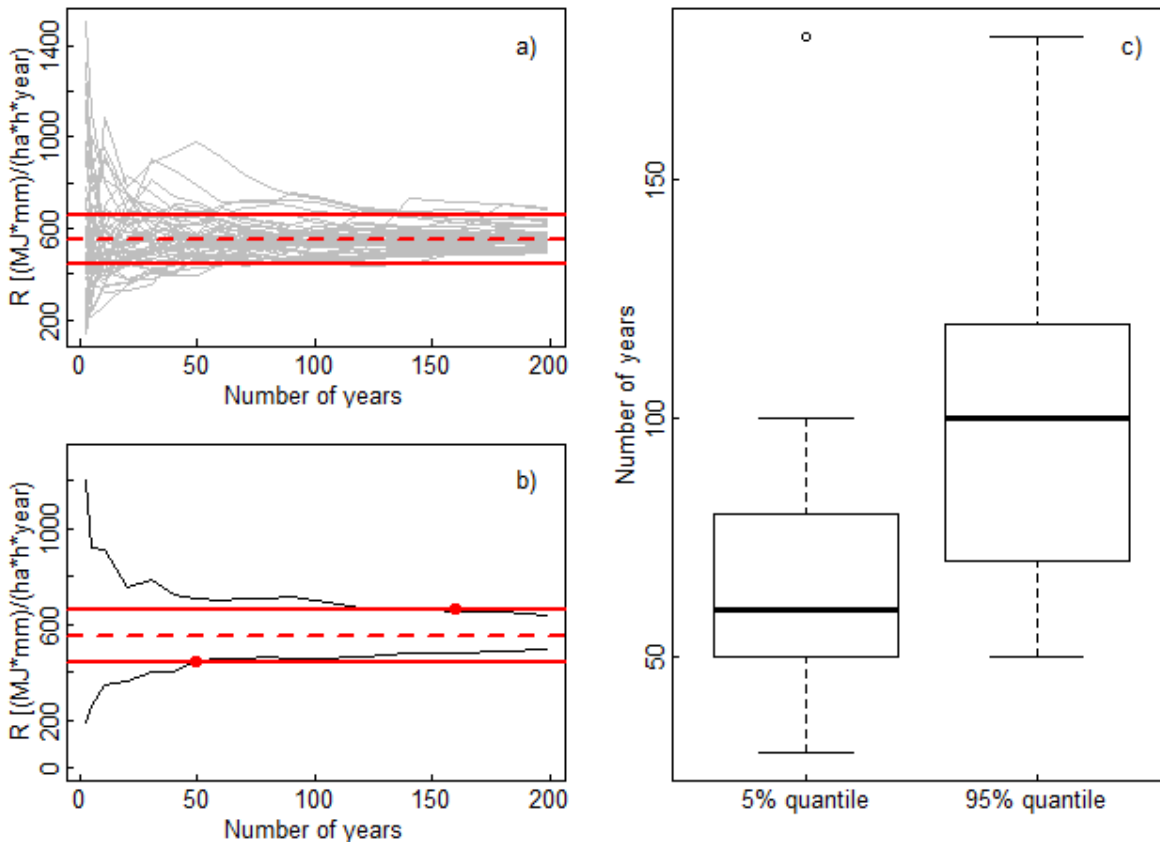
Figure 10: Relationship between mean annual rainfall erosivity and number of years used in the rainfall erosivity calculation. a) shows 50 different realizations (grey lines) together with mean annual value that is calculated using all 50 realisations with 200 years of data (solid red line) and +-20% of this mean value (dotted red line). b) shows 5% and 95% quantile values (black line) calculated based on 50 realizations shown in a) and intersections between these quantile values and +-20% interval. c) shows boxplots of intersection points for 17 stations for the 5% and 95% quantile. a) and b) are shown for station ID = 10113.

Table 1: Median values over all reference stations (N=32) and realisations (N=20). Simulated values are given for the 100% density scenario.

	Annual Erosive Events [-]	Annual Erosivity R [MJ.mm.ha^{-1}.h^{-1}]	Annual Erosive Rainfall Volume [mm]	Mean Event Duration [hrs]	Mean Event Volume [mm]
Observed	14.5	642.7	303.2	22.5	21.1
Direct-R	-	653.8	-	-	-
Direct-P	14.6	647.7	303.6	22.6	21.0
ARM	11.6	580.4	227.1	12.0	16.8
Disagg	14.2	475.5	277.8	35.9	19.6

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