the squared in eq. (4)

$$A(t, f) = A'^2(t, f) = \frac{AS^2(f_s, t_s))}{2\pi \, h r'(t)} \, e^{-2\alpha(f)r'(t)} \, ,$$

$$A(t, f) = A'^2(t, f) = \frac{AS^2(f_s, t_s)}{2\pi \, h \, (R - |r'(t)|)} \, e^{-2\alpha(f)R} \, e^{2\alpha(f)r(t)}$$

$$A(t, f) = K'(t, f) \, e^{\beta' t} \, ,$$

identifying the terms between theoretical and real spectrograms

$$K'(t, f) = \frac{AS^2(f_s, t_s) e^{-2\alpha(f)R}}{2\pi \, h \, (R - |r'(t)|)}$$

$$\beta' = \beta = 2\,\alpha(f)\,v_m(t)$$

$$A(t, f) = K'(t, f) \, e^{\beta' t} \, ,$$

$$f(t) = b + at$$

$$F'(t, f) = \ln K'(t, f) + \beta'(t, f)\, t$$

$$F'(t, f) = \ln K + \beta\, t$$