Response to Reviewer #2: MS eSurf-2022-31

Reviewer's comments are denoted by C and our responses are denoted by R, respectively. In the marked-up manuscript, the removed texts are in red and the edited/added texts are in blue color. I hope that the revised manuscript will be suitable for publication in eSurf.

General comments:

C: This article presents a synthetic physical model describing the propagation of a landslide over a slope and details analytical solutions of the equations in various cases.

R: I very much appreciate the Reviewer's time and interest in my work on the physical-mathematical model and its general exact analytical solutions. My sincere thanks to the Reviewer for the detailed comments and explicit suggestions that resulted in the substantially improved manuscript in which I appropriately addressed all the concerns raised as far as possible which are relevant to the scope of the present work.

C: My main concern regarding this work it that, though many results are said to be useful to practitioners, overall, they are presented in a very abstract way, which makes it particularly difficult for the reader to see their relevance and potential applications. The solutions that are exhibited all derive from initial configurations that appear as very arbitrary, and the results are not related to any concrete examples. Nor is any comparison made to simple benchmarks (either numerical, experimental or field-based) that might be available in the literature. I understand that the given examples are useful to demonstrate the possibilities of the model, but they do not tell about its relevance or validity.

R: The results presented in this ms are relevant to describe the earth surface process. This is evident as this paper is the direct extension of the very recently published paper in eSurf (https://doi.org/10.5194/esurf-10-165-2022, Pudasaini & Krautblatter, 2022) in constructing the general exact analytical solutions for the motion of landslide down the entire slope including the accelerating and decelerating sections. Often the exact analytical solutions may contain abstractions as such solutions are constructed following the abstract and rigorous mathematical procedures. This is natural and also applies to the analytical solutions presented in this paper. However, I have exclusively discussed on the physical and possible application aspects (Line 81-84 in the marked up ms, and in the following text). The model equation (1) and its many analytical solutions are written in very convenient forms which are the generalization of the often widely used Voellmy and Burger's solutions in landslide and fluid motions. Solutions are presented in the simplest form with necessary explanation of their relevance and potential applications. From the beginning of the model development to the construction of the new solutions to discussion of the results, I explained the relevance and applicability of the solutions, as I paid particular attention on these important features. So, in my opinion, the practitioners can follow these models and solutions. The aim of this ms is to formulate a general model and its many general exact analytical solutions in the most general and arbitrary form such that any scientists, engineers and practitioners can easily follow them and may find its applicability presumably without any particular difficulty. As I am interested in the most possible general settings of the solutions, I presented several representative figures to display the results with some physically plausible values of the composite model parameters that are exclusively based on the physics of the material and the dynamics of the flow. With this, the audience see the general, and broad picture of the model and its spectrum of applicabilities. As this ms is the direct extension of the previous eSurf paper (Pudasaini & Krautblatter, 2022), by constructing much stronger, broad and complete exact analytical solutions, this justifies suitability of the present ms to appear in eSurf.

It would be nice, but not all fundamentally novel exact analytical solutions must be validated right away at the time of constructing the solutions. It is just the question of time and will, soon or later researcher may use it to various purposes. This has been proven with many of our previous mass flow model equations, which become leading contributions in the field (see, e.g., https://doi.org/10.1029/2011JF002186; https://doi.org/10.1029/2019JF005204).

I have in fact presented the first-ever simple and complete general exact, analytical solutions for the avalanche motions, and have explicitly mentioned/discussed with examples in several figures how the mountain engineers and practitioners can use these solutions in solving applied problems that was not possible by any existing analytical solutions as the previous landslide velocity solutions are either applicable only to time, or only to spatial variation of the motion down the slope with restricted physics, but not including the variation of both the time and space which is exactly what is needed in real applications.

C: Furthermore, all calculations account for the velocity field but at no point do the results include the landslide's volume, thickness or shape, which are obviously quantities of interest for practical applications: is it implicit that the thickness is constant and uniform? or what does the model predict for its variations with x and t?

R: I agree that the landslide volume, thickness and shape are the quantities of practical interest. However, avalanche volume and thickness (and its gradient) are not of concern here that can be dealt with in a separate ms. Similarly, the effect of the shape may be included by dimensionally extending the present model to higher complexity, but not covered here. Even without the variation of avalanche thickness (in time and space), present exact analytical solutions can be used to solve many technical problems as the new solutions are far better than the widely used Voellmy and Burger’s solutions. This has been exclusively discussed in the present ms.

C: In consequence, I think that the manuscript would be easier to follow and more suitable for publication in ESurfD provided that more effort is made to relate its conclusions to (even simple) physical/geological configurations.

R: I fully understand the reviewer’s concern. I tried to improve the revised ms making it easier to follow, also relating its conclusions to potential mass flow problems, the figures and discussions are primarily designed for this.

General comments:

C: Below are some more precise remarks and questions about the manuscript:

R: Many thanks for the remarks and questions. I have improved the ms following all relevant suggestions.

Section 2

C: Presentation of the model in section 2 is rather confusing. In the following 'Results' sections, eqs (1) and (2) are referred to as radically different (though they only differ by a sign convention), and it looks like eq (1) stands for 'accelerated' and eq (2) for 'decelerated'. However from section 2, one gets the impression that equation (1) covers all cases (l.119 'we have the following two situations' and l.129 'alpha^a<0') and eq (2) is a subcase.

It would be much clearer to start from the beginning with either two distinct equations (say (a) and (d)) that include only positive coefficients, or (perhaps even simpler) a unique equation with two cases (alpha>0 and alpha<0). Additionally, a sketch presenting the physical system modelled by these equations would be most useful.

R: I understand the unease. I also thought that way. However, since the results are fundamentally different for the accelerating and decelerating flows, in my opinion, it is better to distinguish (1) and (2) and construct the corresponding solutions for these equations and streamline everything including the discussion, please see Lines 128-132, 174-186, 232-236, 286-290. I paid attention on the simplicity and structure. This way, the ms is much clearer and well structured. Physically, the model equations are exclusively discussed both for the accelerating and decelerating flows. I think these are clear without sketch. There are different situations that may led to accelerating or decelerating flows, e.g., with changing slope, friction, solid fraction, etc. So, making all those many different sketches may lead to confusion. But, as the landslide velocity will be increasing (accelerating flows), or decreasing (decelerating flows) with the sign (+, or -) of the total system force, in my opinion, it is better not to add several figures.

C: -l.89: Is the solid fraction supposed to be constant (and independent on the local velocity or other varying parameters)? if it is indeed the case it should be precised. Similarly, h_g is included among the other 'external' parameters, but it has to be intrinsically linked to the landslide dynamics: can the author detail the assumptions made here?

R: This is an important aspect, I should have clearly mentioned in the initial submission. The assumptions made in the present ms are explained in the base paper (Pudasaini and Krautblatter, 2022). This has now been detailed in the revised ms as (Lines 107-111) “We note that the solid volume fraction alpha_s is an intrinsic variable. For this, either an extra evolution equation can be considered, or in simplified situation, we can assume that the local variation of the solid volume fraction may be negligible. Here we follow the second choice. Similarly, for simplicity, we consider a physically plausible representative value for the free-surface gradient, h_g designated in due place. With these specifications, as in Pudasaini and Krautblatter (2022), it is possible to directly derive general exact analytical solutions to (1).”

C: -l.95: please associate more explicitely each term in alpha to its physical meaning. What does the term 'liquefaction' cover here?

R: Thanks for this useful suggestion. Following its description from Pudasaini and Krautblatter (2022), I have now explained each term in alpha as (Lines 96-106): “The first, second and third terms in α_s are the gravitational acceleration; effective Coulomb friction (which includes lubrication $(1-\gamma)$, liquefaction (α_s) (because if there is no solid or a substantially low amount of solid, the mass is fully liquefied, e.g., lahar flows); and the term associated with buoyancy, the fluid-related hydraulic pressure gradient, and the free-surface gradient. Moreover, the term associated with K describes the extent of the local deformation that stems from the hydraulic pressure gradient of the free surface of the landslide. Note that the term with $(1-\gamma)$, or γ, originates from the buoyancy effect. By setting $\gamma=0$ and $\alpha_s=1$, we obtain a dry landslide, grain flow, or an avalanche motion. For this choice, the third term on the right-hand side of alpha^a vanishes. However, we keep $\gamma$and α_s to also include possible fluid effects in the landslide (mixture).”

C: -l.121: 'the initial velocity u_0': doesn't is depend on the position x? Where is the condition verified?

R: This is a legitimate concern. The statement has been improved as (Lines 147-148): “Where the initial velocity u_0 refers to the situation associated with the particular segment of the avalanche track in which the condition $u_0 < \sqrt{\frac{\alpha^a}{\beta}}$ is satisfied at the uppermost position of the segment.”

C: -l.131: what is the 'decelerating velocity', and why is it obviously always larger in the case II.1 than II.2?

R: I agree that the term the “decelerating velocity” was not enough explained, which has been improved now and reads (Lines 157-160): “By definition the decelerating velocity, the velocity of the landslide when it decelerates”. Moreover, I have improved the following sentence explaining which velocity is larger: “Because of the higher negative total system force in II.2 than in II.1, the decelerating velocity in II.2 is always below the decelerating velocity in II.1.”

Section 3

C: -l.166: the similarity between equations (5) and (6) would be more obvious if expressed in a more uniform way (e.g. not switch from 1/exp(A) to exp(-A) and keep the same first factor).

R: I fully agree and changed (6) as suggested (Line 195-196).

C: -l.186: I do not understand here what the travel time is (from where to where? what is a sector?)

R: It has already been explained, but now has been improved as (Lines 215-219): “The travel time for the accelerating landslide in any sector (section) of the flow path. So, this is the travel time the landslide takes for travelling from the left end to the right end of the considered sector.” A sector is a section in the flow part. So, I changed “sector” to “sector (section)”.

Section 5

C: -l.265: if I am not mistaken, equations (1) and (2) are not dimensionless. Coefficients alpha and beta should therefore be given units.

R: Following the suggestion, I have mentioned the dimensions of alpha and beta at Lines 94-95, just below equation (1) where they first appeared: “$\alpha^a \left[m\,s^{-2}\right]$ and $\beta \left[m^{-1}\right]$” also for the velocity u [ms^{-1}].

C: What justifies the ranges adopted here? (and should the range for beta read 0.001-0.0025 or rather 0.0025-0.01?)

R: Thanks for checking the range. The text has been improved at Lines 299-309.

C: And how realistic are these values? Perhaps the author can give an example of common values for each physical control parameter (slope, gamma, mu...) and the resulting value of alpha.

R: We have improved the ms as follows (Lines 299-302): “Following the literature (see, e.g., Mergili et al., 2020; Pudasaini and Krautblatter, 2022), the representative values of physical parameters are: g = 9.81, ζ = 50°, γ = 1100/2700, δ = 20° (μ = 0.36), α_s = 0.65, K = 1, h_g = -0.05. This results in a typical value of α about 7.0.”

C: Same comment for beta: what values for the viscous drag coefficient are commonly used, typically in the abundant literature about shallow-layer ('Saint-Venant') models for landslides?

R: We have improved the text with (Lines 302-309): “The value of β = 0.02 is often used in literature for mass flow simulations but without any physical justification, to validate simulations (Zwinger et al., 2003; Pudasaini and Hutter, 2007). With different modelling frame, considering some typical values of the flow depth on the order of 1 to 10 m, calibrated values of β cover the wide domain including (0.001, 0.03) (Christen et al., 2010; Frank et al., 2015; Dietrich & Krautblatter, 2019; Frimberger et al. 2021). Pudasaini (2019) provided an analytical solution and physical basis for the dynamically evolving complex drag in the mixture mass flow. This formulation shows that the values of β can vary widely, ranging from close to zero to the substantially higher values than 0.02. Similar values are also used by Pudasaini & Krautblatter (2022).”

-l.282-286: please introduce earlier (maybe within a sketch) what the 'lower portion of the track', 'transition zone', 'fan region' are regarding to the model.

R: These are often commonly used phrases in mass flows. However, following the suggestion, I have improved the descriptions. “lower portion of the track” has been defined as (Line 327): “where motion switches from accelerating to decelerating state”. I have improved the as (Line 329): “transition zone” as (Line 329-330) “transition zone (where the motion slows down substantially)”, and the “fan region” has been improved as “fan region (where the flow spreads and tends to stop and finally deposits)”.

C: The whole paragraph is written in such a way that it is very hard to make out the concrete situation that is modelled here. Maybe this can be reformulated starting from the example that is actually computed in figure 1, for which I do not understand the initial configuration (what is the length of the sliding mass? is the velocity u0 uniform?)

R: The initial configuration was mentioned later in that paragraph (Lines 342-343): “In Fig. 1, I have suitably chosen the time and spatial boundaries (or initial conditions) as x = 1500 m corresponding to t = 50 s for u0 = 50 ms−1.” Which has now been shifted at the place where the description for Fig. 1 first appears (Lines 332-334). As mentioned in the text [by referring equations (3) and (4) [Lines 302], and equations (7) and (8) [Lines 302] these are steady-state, equivalently mass point motions. So, the length does not appear, and by definition, the velocity is uniform. These are more or less (reduced) standard analytical solutions in avalanche motion.

-figure 1: if I am not mistaken, at this point of the analysis, u is a function of both space and time. If so, I do not understand what is plotted here: in figure (a) is it the velocity at a given position (and which), and in plot (b) at a given time?

In this figure as in the others, units are missing for u_0, alpha and beta.

R: The formal transformation of the solution from space to time has been explained at Section 3.2, Lines 201-203. So, these are two equivalent descriptions of landslide motions in time and space. The units of these quantities have now been mentioned at Line 94-95.

C: -l.307: I guess that 'ascending' and 'descending' refer here to the velocity, but 'ascending sector' sounds like it refers to an upward slope. 'Accelerating' and 'decelerating' might be more appropriate.

R: The text clearly explains as mentioned by the reviewer (Lines 353-354): “the ascending sector where the landslide accelerates, followed by the descending sector where it decelerates”. However, I have now improved the related phrases in the texts around. So, these terms are described in understandable way.

C: -l.312: please justify the transition from alpha=3.5 to alpha=-1.2: what would physically cause such a transition (kink in the slope for instance?)

R: I have improved the ms with the new text (Lines 359-362): “Such transition occurs when the previously accelerating motion turns into substantially decelerating motion. This can be caused, e.g., due to the decreasing slope or increasing friction (or both) when the landslide transits from the upper (say, left) segment to the lower (say, right) segment. In general, any parameter or set of parameters involved in the net driving force α^a can make the net driving force strongly negative.” More on this has been explained at Line 420-423.

C: - figure 2: same question as for figure 1 (and as for figs 3,4,5): to what position (a) and time (b) do the plots correspond? In the caption, the coordinates are given without units in two different coordinate spaces.

R: As made clear at Section 3.2 (Lines 201-203), these are two equivalent descriptions of the landslide motion in time and space. The units are given at Lines 94-95.

C: -l.332: what is a 'variable track'? Please give a physical example that would produce the results presented in the following figures (for instance, all other parameters being constant, what shape of the slope would lead to such successive values of alpha).

R: I have added the following text to elaborate it (Lines 382-387): “From the dynamic point of view, the variable track can be generated by changing values of one or more parameters involved in the net driving force α^a (or, α_d). For example, this can be due to the changing slope or basal friction. For simplicity, we may keep other parameters in α^a unchanged, but successively decrease the slope angle such that the values of α^a decreases accordingly. As α^a is the collective model parameter, without being explicit, it is more convenient to appropriately select the decreasing values of α^a such that each decreased value in α^a leads to the reduced acceleration of the landslide.” I can provide explicit values of all the model parameters in α^a including the slope angles, however, I think, it is not necessary to be too explicit. Because, then someone may also ask, what about keeping slope constant, but increasing the friction. Another question would be what would happen if we keep slope fixed but increase solid fractions, or decrease lubrications. There are so many possibilities to physically and mechanically changing values of α^a . So, I do not want to restrict different possibilities. It is up to the practitioners to physically appropriately determine the values of this collective parameter by properly supplying the values of the parameters involved in α^a. My aim is to present a broad and overall picture.

C: -l.344: repetitive explanations of all ascending/descending connections do not seem necessary, terms being self-explanatory.

R: I understand the concern. However, I think such explicit descriptions of connections help the practitioners to clearly distinguish different scenarios.

C: -l.357: I do not understand the sentences 'alpha values are relative to each other' and 'perceived as relatively negative to alpha^a'.

R: To avoid the confusion, I have removed the text (Lines 414-416): “However, note that the α values on the ascending and descending sectors are relative to each other. So, α_d on the descending sectors should be perceived as relatively negative to α^a in the ascending sector.” which does not affect the essence of the text around.

C: -l.363-370: the velocity is observed to change dramatically at the major kink, but this sounds intuitive if we impose a dramatic change in the value of alpha. Is this a realistic case?

R: Indeed, this verifies the physical representativeness of the new exact analytical solution.

C: The paragraph is concluded with the sentence 'this can be a scenario for a track': the section should start with the example of such a scenario, that is investigated here: what physical configuration (e.g. with alpha being controlled by the slope profile only) would lead to a brutal transition from alpha=6 to alpha=-0.15?

R: I have rewritten (Lines 430-432) “This can be a scenario for a track with multi-sectors of ever increasing slope, followed by a quick transition to a decreasing slope, again succeeded by multi-sectors of ever decreasing slopes, and finally mass deposition.” as (Lines 410-411): “Here, I discuss a scenario for a track with multi-sectors of ever increasing slope, followed by a quick transition to a decreasing slope, again succeeded by multi-sectors of ever decreasing slopes, and finally mass deposition. ” and put this in the beginning of the paragraph as suggested by the reviewer. This makes the presentation more structured.

I have carefully checked the parameter values for alpha. The text has been improved (Lines 420-423): “All these values (even in the outer range) are possible by changing the physical parameters appearing in alpha. For example, $\zeta = 50°$, $\gamma = 1100/2700$, $\delta = 20°$ ($\mu = 0.36$), α_s = 0.65, K = 1, h_g = -0.05 and g = 9.81 give alpha value of about 7, and g = 9.81, $\zeta = 1°$, $\gamma = 1100/2700$, $\delta = 33°$ (resulting in $\mu = 0.65$), α_s = 0.65, K = 1, h_g = -0.05 even give alpha about 1.8”. So, by just appropriately changing the friction and radically changing the slope we can get all the employed (positive and negative) values of alpha.

C: Overall, all situations studied here (figures 3,4,5) appear rather arbitrary and abstract. Though it is useful to demonstrate the capacities of the analytical model, it would be more convincing to apply them to concrete configurations: a first step would be to plot the slope profile that would lead to each calculated dynamics. Even better would be to compare the outcomes (e.g. runout distance) to other models in known, simple configurations (such as a constant slope followed by a horizontal plane). Numerical works of Mangeney et al. with Saint-Venant equations and Staron et al. with DEM simulations, experimental works on inclines or even simplified versions of field cases should be used as benchmarks to validate the results obtained here.

R: I appreciate for the relevant suggestions. It would be nice, but not all fundamentally novel exact analytical solutions must be validated right away at the time of constructing the solutions. It is the question of time and will, soon or later researcher may use it to various purposes. This has been proven with many of our previous analytical mass flow model equations, which become leading contributions in the field (see, e.g., https://doi.org/10.1029/2011JF002186; https://doi.org/10.1029/2019JF005204).

As clearly mentioned in the ms, validation is not the focus here, that can be done later. I think the presented new results are sufficient to constitute a good new paper. I have in fact presented the first-ever simple and complete general exact, analytical solutions for the avalanche motions, and have explicitly mentioned/discussed with examples in several figures on how the mountain engineers and practitioners can use these solutions in solving applied problems that was not possible by any existing analytical solutions as the previous solutions are either applicable only to time, or only to spatial variation of motion down the slope, but not including the variation of both the time and space which is exactly what is needed in real applications. Practitioner’s benefit is the application aspect as the solutions can be utilized by any physical scientist.

Moreover, verifying and validating the numerical solutions is another important aspect of the exact analytical solution that has been explicitly mentioned in the ms. Based on the underlying physical parameters from the field and laboratory data, the values of collective model parameters alpha and beta are carefully estimated in several previous contributions whose values are used here with references, and are properly chosen. As made it clear in the ms, the values of alpha and beta can be made technically varying spatially by choosing their different values in different sectors of the landslide track by carefully and suitably dividing it into several sectors. Based on this, I have presented several figures exclusively demonstrating how to do this. Here, I aimed to present the broad picture on the dynamics of the newly constructed exact analytical solutions and their applications by displaying several figures for possible natural scenarios.

C: - figures 3 and 4: I am not convinced that the list of all kink coordinates brings much to the results (especially since their positions are imposed). Focus should be brought upon travel time or runout distance.

R: I agree with the reviewer. However, from the coordinates, the travel distances are evident. Similarly, as we have the information about velocity and distance from the figure, we can obtain the travel time. One can work on many different aspects of the analytical solutions as further contributions. Yet, I have improved the ms with the added text (Lines 468-569): “It is important to mention that from the coordinates, the travel distances are instantly obtained. Similarly, as we have the information about velocity and distance from the figure, we can directly construct the travel time.”

C: - figure 5: though keeping the same colors is useful, the two different solutions have to be distinguishable on the plot (e.g. dotted vs plain lines).

R: I also thought about this. However, to make a direct correspondence between the two sets of solutions, I prefer to use the same color and the same line style. I think, visually this is better. But, I am open for this.

C: Why is the second case totally unrealistic? Some landslides are known to travel more than 3.5 km and alpha could keep getting beyond that point.

R: To some extent, I agree with the reviewer. However, with this example, I just presented a scenario of possibly less realistic dynamics and travel distance of the landslide as it transits from accelerating to decelerating motion. With the upper solution, the mass does not stop even after several km long distance (as can be imagined with the negligible slope of the right most sector of the solution, but still has substantial velocity), indicating that this type of solution is less realistic to be applied for the frictional landslide motion. To tune down the previous text a bit, I have improved it as “seem to be less realistic” instead of “are less realistic” (Line 498).

C: -l.480: here again 'ascending' and 'descending' are equivocal and one might think that they refer to the shape of the front (i.e. h(x) and not u(x)), whose evolution it would be most interesting to plot here.

R: To avoid the confusion, I changed “ascending and descending fronts” to “ascending and descending velocity fronts”. The entire texts around have been improved accordingly.

C: -l.485: for the reader unfamiliar with the previous work, on what basis are these initial velocity profiles chosen? Once again concluding that the runout distance differ is most useful, but it is hard to relate the arbitrary 'initial' configurations to a practical situation (or, for that matter, to previous examples such as figure 5). Starting with the release of a given mass at zero velocity, how does the landslide end up in the 6a rather than 6b configuration?

R: I fully understand the concern. In the revised ms, I changed “The initial velocity distributions are chosen following Pudasaini and Krautblatter (2022).” to (Lines 548-550) “As a possible scenario (described in the figure captions) the initial velocity distributions are chosen following Pudasaini and Krautblatter (2022).” In fact, these figures are motivated by the potential practical applications. As discussed in the text, the main aim is to demonstrate that two landslides entering into the same decelerating zone, one with the accelerating front and another with decelerating front, can result in completely, even dramatically, different runouts and depositions. This is made possible with our new analytical solution. I think, the practitioners can follow this. The mass starting with zero velocity is analogous to the mass point motion. Results on this have been presented in the previous sections.

C: -section 5.2.3: the predictions of the model regarding the geometry of the deposit would indeed be of much interest, but the link between the results (velocity profile only) and the geomorphology (that is, the thickness profile of the deposit) remains only implicit here.

R: I agree. As further development of the present solutions, one may extend the methods presented here also to include the landslide depth, which, however, is out of scope here. To indicate this important aspect, I have included the text (Lines 610-611): “As further development of the present solutions, the methods presented here may be expanded to include the landslide depth and relate it to the landslide velocity.”