Response to Reviewer # 1

Kevin Pierce, Marwan Hassan, Rui Ferreira

We would like to thank the first reviewer for their detailed comments on the manuscript. We have prepared revisions in response to all of the major points.

1. "From this perspective, the present formulation is intrincically the same as that usedis previously, for example, by Fan et al. (2014), who considered the motions mechanically, while simulated the transport process by switching the motions of the particle on and off (Fan et al., 2016). The authors may need to discuss this point explicitly."

   The underlying idea of particle movements as a Langevin equation while the rests are treated heuristically is certainly contained in *Fan et al.* [2016]. We will indicate this in the revised manuscript.

2. "Keep this in mind, the starting point of this work, Eq. (5), can only be considered as a "formal description", because the entrainment and deposition of the grain are not formulated mechanically. That is, the start and end of the motions of a particle are not determined by the forces acting on it; thus no new information on the travel and resting times can be obtained based on incorporating this dichotomous Markov noise."

   In the revised manuscript we will try to be more clear on the point that the motion-rest alternation is not treated mechanistically. A key contribution of the manuscript is to formalize the heuristic motion-rest alternation used in random-walk models like *Lajeunesse et al.* [2017] using the dichotomous Markov noise concept from the physics literature. We couple this formalism with descriptions of the movement phase based on Langevin models. This allows us to write probabilistic equations and find analytical solutions – additional benefits compared to *Fan et al.* [2016]. Perhaps in the future it will be possible to derive the dichotomous noise as an approximation of more detailed physics. We believe the manuscript establishes a starting point for this task.

3. "The authors are also suggested to discuss the effects of the velocity distributions on their deduced results."

   The Poissonian and monoscaling characteristics of the flux distribution both originate from the assumed independence of individual particle motions which was used to arrive at Eq. (16). They do not relate to the movement or resting time distributions or the particular forcing terms included in the Langevin model. We will attempt to better describe this in the revised manuscript.

4. "Wu et al. (2020) provided an explanation for the existence of the two different distributions, by pointing out that the long trajectories contribute to the Gaussian velocities, and the mixture of both long and short trajectories results in the exponential distribution; the long and short trajectories are distinguished by the shift of the hop distance-time scaling. Resorting to this result I think is important for clarifying some key issues in this work..."

   We agree it is worth describing that *Wu et al.* [2020, 2021] have attributed the shape of the velocity distribution to the balance of long and short hops in the velocity statistics. However this does not seem to be the last word on this problem. Another (probably related) view is that the shape of the velocity distributions relates to the amount of momentum dissipated by particle-bed collisions: this has been demonstrated to predict all distributions which have been reported so far from experiments, including the less common Gamma-like ones [*Pierce*, 2021]. Alternative explanations may still be possible, perhaps involving the interactions of particles with the vertical flow structure.

5. "For the "overdamped" approximation, explained by the authors as "moving particles attain their steady-state velocities relatively quickly after entrainment", which is only valid for the description of the long trajectories of particle motions. This is because only the long trajectories have a well defined mean velocity (e.g. the "steady-state velocity"); and the mean velocity for the short trajectories can on the average increase with their travel times (Wu et al., 2020). Since the short trajectories can cover over 80% of the total trajectories in experiments (Wu et al., 2021), applying this "overdamped" approximation may not be appropriate."

   We agree that the overdamped approximation should not be valid for exponential velocities. In fact one needs a forcing term that is linear in velocity to conduct an overdamped approximation whether it is approximately valid or not, see *Risken* [1984]. We plan to add an an explicit statement that the overdamped approximation is only possible for Gaussian velocities in the revised manuscript.

6. "There are recent studies using different methods to theretically address the motion period of the bedload particle transport, for example, as discussed above (Wu et al., 2020; Wu et al., 2021), the results of which are compared with measured data. In other words, how the particle velocity changes with time was proposed and further determined based on experimental measurements (i.e. other means of specifying the external forces acting on the particle, F(u) in this work). The authors can compare the part of their formulation on the particle motions with different results."

   Given that our formulation spans the local, intermediate, and global ranges of *Nikora et al.* [2001], while no one dataset also spans these ranges [e.g. *Pierce and Hassan*, 2020; *Pretzlav et al.*, 2021], a comparison to experimental data would be a challenging data compilation problem. All components of this work have however been tested independently in other studies: *Ancey and Heyman* [2014] and *Heyman et al.* [2016] tested the velocity model (2) for moving particles, while *Lajeunesse et al.* [2017] tested the motion-rest alternation component (1) at the global range. For this reason we have chosen to present a purely formal description that combines and generalizes earlier stat mech descriptions of bedload transport.

7. "Could the derivation be started directly from the probability distribution function based on the continuum master equation (5)?"

   We could certainly use the joint distribution functions as the starting point for the calculation of the flux, rather than Eq. 13, but we prefer to emphasize the particle-scale origins of the flux, wherein particle concentrations are represented as arrays of discrete points using indicator functions, and probability distributions result from ensemble averages over these indicators.

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