Response to Reviewers: ESurf submission "Stochastic description of intermittent transport and aggregate derivation of the bedload flux"

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We would like to thank both of the anonymous reviewers for their detailed comments on the manuscript. We have made comprehensive changes in response to every reviewer comment. Below we have explained our revisions. We hope the revised manuscript is satisfactory to the Editors.

Response to Reviewer #1

1. "From this perspective, the present formulation is intrinsically the same as that used previously, for example, by Fan et al. (2014), who considered the motions mechanically, while simulated the transport process by switching the motions of the particle on and off (Fan et al., 2016). The authors may need to discuss this point explicitly."

   Please see the revised lines 143-149, where we explain precisely the relationship of our work to *Fan et al.* [2014, 2016].

2. "Keep this in mind, the starting point of this work, Eq. (5), can only be considered as a "formal description", because the entrainment and deposition of the grain are not formulated mechanically. That is, the start and end of the motions of a particle are not determined by the forces acting on it; thus no new information on the travel and resting times can be obtained based on incorporating this dichotomous Markov noise."

   We agree completely and have added an explicit statement mirroring this perspective at L357-359.

3. "The authors are also suggested to discuss the effects of the velocity distributions on their deduced results."

   We have added explicit discussion on the (lack of) effects of the velocity distributions at lines 408-409

4. "Wu et al. (2020) provided an explanation for the existence of the two different distributions, by pointing out that the long trajectories contribute to the Gaussian velocities, and the mixture of both long and short trajectories results in the exponential distribution; the long and short trajectories are distinguished by the shift of the hop distance-time scaling. Resorting to this result I think is important for clarifying some key issues in this work..."

   We have discussed the analyses of *Wu et al.* [2020] and *Wu et al.* [2021] at lines 49-51 alongside the complementary perspective of *Pierce* [2021].

5. "For the "overdamped" approximation, explained by the authors as "moving particles attain their steady-state velocities relatively quickly after entrainment", which is only valid for the description of the long trajectories of particle motions. This is because only the long trajectories have a well defined mean velocity (e.g. the "steady-state velocity"); and the mean velocity for the short trajectories can on the average increase with their travel times (Wu et al., 2020). Since the short trajectories can cover over 80% of the total trajectories in experiments (Wu et al., 2021), applying this "overdamped" approximation may not be appropriate."

   We have added explicit statements that the overdamped approximation is only possible for Gaussian velocities at lines 187-189 and 325-326. Additionally, we have added additional citations and discussion to support the overdamped approximation and explain the conditions when the acceleration phase might be acceptably neglected at lines 190-194 and 336-337.

Additionally, we have unpublished data showing that the overdamped approximation is reasonable for 5mm glass beads in transport. We have attached a figure summarizing these data in the response to reviewer #2, which can be found below.

6. "There are recent studies using different methods to theoretically address the motion period of the bedload particle transport, for example, as discussed above (Wu et al., 2020; Wu et al., 2021), the results of which are compared with measured data. In other words, how the particle velocity changes with time was proposed and further determined based on experimental measurements (i.e. other means of specifying the external forces acting on the particle, F(u) in this work). The authors can compare the part of their formulation on the particle motions with different results."

   We added citations to *Campagnol et al.* [2015] in several locations– lines 190 and 305: this paper is the only one to our knowledge that examines the evolution of the velocity statistics through time from entrainment.

7. "Could the derivation be started directly from the probability distribution function based on the continuum master equation (5)?"

   We could certainly use the joint distribution functions as the starting point for the calculation of the flux, rather than Eq. 13, but we preferred to emphasize the particle-scale origins of the flux, wherein particle concentrations are represented as arrays of discrete points using indicator functions, and probability distributions result from ensemble averages over these indicators.

Thank you sincerely for your effort reading and commenting on the manuscript. We hope that we have incorporated your suggestions to your satisfaction.

Response to Reviewer # 2

Major comments

1. " The paper is well written, clear and concise. The approach is sound and standard mathematical tools are briefly introduced before or after they are used, which helps to understand the main ideas behind technical derivations. Main equations however miss a detailed physical explanation, term by term, to be understood by the readership."

   Please see lines 205-209 and 161-172, where we have added detailed descriptions of the individual terms in the equations.

2. "The title should be more precise, several stochastic description of bedload having been already proposed."

   Thanks - we changed the title to "Stochastic description of intermittent transport and aggregate derivation of the bedload flux". We hope this is sufficiently descriptive. Thanks for your advice on this.

3. "A general concern is that the stochastic approach, although theoretically sound, is weakly linked to actual statistics of sediment transport by bedload, and thus the relevance of such complicated form of the bedload flux (eq 21!) is questionable for realistic transport conditions. In particular, there is no discussion on the actual values of Péclet number and the importance of considering both velocity fluctuations and entrainment/deposition as processes acting on similar time scales. There are considerable simplifications when decoupling both, so the authors should better point why such

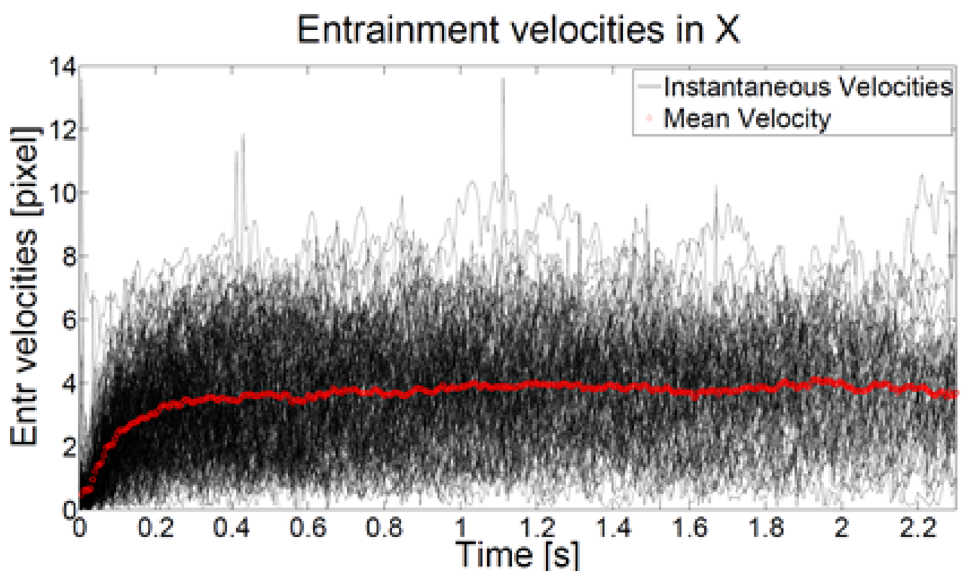

Figure 1: This figure demonstrates that the acceleration phase of particles following entrainment is typically short compared to the duration of trajectories.

coupled approach is necessary. By doing so, the authors should also consider comparing their results with existing experimental or numerical data."

We have provided estimates of the Péclet numbers seen in experiments in a new discussion paragraph at L390. We added discussion on the validity of neglecting the acceleration/deceleration phases of particle motions using the results of *Campagnol et al.* [2015] at L190-195.

We mentioned that we have some unpublished experimental data showing that the acceleration phase following entrainment is commonly more than ten times shorter than the period particles spend in motion. Please see the below figure. This figure shows the trajectories of 5mm glass beads in relatively weak transport which show Gaussian velocity distributions. The acceleration phase is visible in the trajectories, and is relatively short compared to the full trajectory between entrainment and either deposition or departure from the viewing window. We plan to work on this project in more detail in the future.

Minor comments

1. 12 : drop "really", and precise why /when fluctuations matter ?

   We emphasized that transport fluctuations are strongest in weak transport conditions that are characteristic of gravel-bed rivers in our edits at lines 14-19.

2. 16 : What is a "classic" description ? Deterministic ?

   We replaced "classic" with "deterministic" at L22.

3. 17-19 : I do not get the point here. The approach followed by the authors is also mainly kinematic in that no discussion is made on the forces (gravitational, drag, friction, collision,...) acting on particles.

   We have added additional discussion of what exactly incorporating forces into the motion state gives us at L135-149. This is an important next step toward a fully mechanistic description of particle transport at the grain scale.

4. The original "probabilistic" description ...

We added "stochastic" at L30.

5. 21 Later → replace by "more recently" (there were a lot a probabilistic studies between Einstein and Lisle)

   We rephrased the entire paragraph, which now appears near L35, to indicate the entire progression of the research, rather than skipping over the intermediate advancements as we did in the original manuscript."

6. 22 "by promoting his instantaneous steps to intervals of motion with constant velocity" I do not get the meaning of promoting hear.

   We replaced "promoting" with "replaces".

7. 75 - 85 No mention of Continuous Time Random Walks model is made. Authors should compare their approach with for instance [Schumer et al 2009]

   We added discussion of CTRW approaches at lines 32-36.

8. l115 : Better explain how this equation can be physically understood, notably the presence of k and ke with time derivatives.

   We added a description of how the mixed order time derivatives encode intermittency at lines 164-172.

9. l137 Is the overdamped approximation similar to adiabatic elimination of the fast variable ? A deeper discussion is needed here, notably the validity of such approximation with respect to typical bedload transport scales.

   We added additional discussion of the overdamped approximation with reference to *Campagnol et al.* [2015] and earlier Langevin models of bedload velocities at lines 190-194. See also the above figure.

10. l 143 : How does such expression compare with a spatio-temporal markov process, for instance eq 4.4 in Ancey & Heyman JFM 2014 ?

    We have left this for future investigation. The challenge, as we described before, is that $P(x,t)$ is a single-particle density and not the particle activity as considered in *Ancey and Heyman* [2014]. We are investigating the relationships between these formulations along the lines of *Ballio et al.* [2014].

11. l217 : Why would velocity fluctuation during motion decrease diffusion at small time scales ? I would have imagined the reverse.

    We fixed this typo at lines 277-279.

12. l230 Rewriting the Péclet in its usual form (diffusion time scale over advection time scale) would help understanding the transport process the authors are trying to characterize. In there definition of Peclet, the important length scale is the mean particle jump length. This should appear somewhere.

    We made a typo in our earlier reply to your comment. We consider *length-scales* of advection and diffusion. It is explained how the Péclet number emerges as a ratio of these lengthscales in the revised lines 290-295.

13. l264 : can you give an physical interpretation of why the flux is higher at the beginning ? Do we have a higher probability to sample particles in motion at short time scales ?

    We added a physical interpretation of why the flux is higher at lines 334-337. We added additional qualitative comparison to experimental data in a new discussion paragraph at L390.

14. Figure 4a : why is there a plateau between 1-100 s? Is the mean flux only dependent on Péclet and observation time ? If yes, can you make it appear clearly in eq 21. If not, what are the fixed parameter in this figure ? Can you compare with experimental/numerical (DEM) data ?

We added some speculation on why a plateau emerges at lines 335-337, and we added a statement of which variables are held constant at the end of the Fig4 caption.

15. Figure 4b If the distribution is Poissonian, you should be able to rescale it by its mean and have a single time-independent distribution. Could you plot this ?

    We added a sentence at L45 indicating that $\Lambda(T)$ is the only parameter of the flux.

Thank you again! We are most thankful for your supportive comments on the manuscript.

References

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