Optimising global landscape evolution models with [10]Be

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10 **Abstract**

By simulating erosion and deposition, landscape evolution models (LEMs) offer powerful insights to Earth surface processes and dynamics. Stream-power based LEMs are often~~These models are typically~~ constructed from parameters describing drainage area (m), slope (n), substrate erodibility (K), hillslope diffusion (D), and a critical drainage area (A_c) that signifies the downslope transition from hillslope

15 diffusion to advective fluvial processes. In spite of the widespread success of such models, the parameter values are highly uncertain ~~have high degrees of uncertainty~~ mainly because the advection and diffusion equations amalgamate physical processes and material properties that span widely differing spatial and temporal scales. Here, we use a global catalogue of catchment-averaged cosmogenic [10]Be-derived denudation~~erosion (denudation)~~ rates with the aim to optimise a set of LEMs

20 ~~landscape evolution models~~ via a Monte Carlo based parameter search. We consider three model scenarios: advection-only, diffusion-only, and an advection-diffusion hybrid. In each case, we search for a parameter set that best approximates denudation~~erosion~~ rates at the global scale, and we directly compare denudation~~erosion~~ rates from the modelled scenarios with those derived from [10]Be data. We find that optimised ~~Optimised~~ ranges can be defined for many LEM parameters at the global scale. In

25 the absence of diffusion, $n \sim 1.3$, and with increasing diffusivity the optimal n increases linearly to a global maximum of $n \sim 2.3$. Meanwhile, ~~Meanwhile~~ we find that the diffusion-only model yields a

slightly better correlation than ~~somewhat outperforms~~ the advection-only model and is optimised when the concavity parameter ~~concavity~~ is raised to a power of 2. With these examples, we suggest that our approach provides baseline parameter estimates for large-scale studies spanning long timescales and diverse landscape properties. Moreover, our direct comparison of model-predicted versus observed denudation~~erosion~~ rates is preferable to methods that rely upon catchment-scale averaging or amalgamation of topographic metrics. We also seek to optimise K and D parameters in LEMs ~~landscape evolution models~~ with respect to precipitation and substrate lithology. Despite the potential bias due to factors such as lithology, these ~~These~~ optimised models allow us to effectively control for topography and target specifically the relationship between denudation ~~erosion rate~~ and precipitation. All models suggest a general increase in exponents with precipitation in line with previous studies. When isolating K under globally optimised models, we observe a positive correlation between K or D and precipitation > 1500 mm yr^{-1}, plus a local maximum at ~ 300 mm yr^{-1}, which is compatible with the long-standing hypothesis that semi-arid environments are among the fastest-eroding landscapes~~most erodible~~.

1 Introduction

To appreciate short-term changes in Earth surface processes, such as those induced by humans (Brown, 1981; Hooke, 2000), it is first necessary to understand long-term rates of denudation~~erosion~~ and deposition. Recognizing this, some recent studies (e.g., Simoes et al., 2010) derive erosion-transport rules from topography with an aim to predict macro-scale~~macroscale~~ patterns of denudation~~erosion~~ and sediment flux. At more restricted spatial scales, denudation~~scales, erosion~~ rates based on cosmogenic nuclides (e.g., ^{10}Be) show a modest exponential correlation with catchment-averaged slope, as does normalised steepness in stream profiles (Portenga and Bierman, 2011; Harel et al., 2016). Nevertheless, it is widely observed that steepness and stream power parameters are subject to considerable~~notable~~ variation wherever climate and/or lithology differ (Harel et al., 2016; Gailleton et al., 2021; Marder and Gallen, 2022), and that the parameters also covary. A ~~a~~ robust analysis must accommodate such interactions.

Earth's surface undergoes continuous modification through uplift and denudation~~erosion~~ over
55   timescales too long to observe directly, hence landscape evolution models (LEMs) are crucial~~vital~~
tools for building knowledge. Note that we use the term 'denudation' to denote the mass loss leading
to lowering of Earth's surface. LEMs are often employed over expansive scales of space and time in
order to study topographic response to changes in tectonics (e.g., Kooi and Beaumont, 1996; Garcia-
Castellanos et al., 2003), climate (e.g., Temme et al., 2009, Adams et al., 2020), and sea level (e.g.,
60   Pico et al., 2019; Ruetenik et al., 2019). And yet, large spatial and temporal scales require
generalisation of model parameters that accounts reliably for processes of hillslope diffusion and
advective fluvial erosion. Using LEMs to estimate denudation~~erosion~~ rates delivers the key advantage
of bridging scales and defining an empirically derived mechanism at the local (grid cell) scale ~~within~~
~~each catchment~~. This demands that denudation~~erosion~~ rates are integrated over scales matching the
65   topographic changes they describe. At the local to regional scale, recent studies have focused on
constraining LEM parameters via inversions that optimise rates of denudation~~erosion~~, deposition, and
topographic observations (e.g., Miller et al., 2013; Croissant and Braun~~et al.~~, 2014; Pedersen et al.,
2018; Barnhart et al., 2020). However, implementing many of these approaches at a global scale is
challenging in terms of both computational cost and because it often requires a compilation of a large
70   set of observables (such as knickpoints, depositional patterns, and denudation~~erosion~~ rates). In the
absence of computational power that can accurately simulate~~model~~ stratigraphy at the global scale,
and without constraints on global palaeo-topography, we settle for optimising LEM parameters with
respect to catchment-averaged denudation rates estimated with cosmogenic [10]Be. While LEM
parameters can be estimated via direct topographic analysis (e.g., Wobus et al., 2006, Clubb et al.,
75   2018, Mudd et al., 2018), this approach can lead to bias, as we discuss below~~erosion rates~~.

Here, we determine LEM parameter values that minimise the variance among [10]Be-derived
apparent denudation~~erosion~~ rates in the *OCTOPUS* v.2 global catalogue (Codilean et al., 2022), and
we analyse the capacity of LEMs to predict denudation~~erosion~~ rate given those optimised parameters.
Our LEM employs the common stream-power-plus-~~power plus~~diffusion formulation, which is
80   subject to important limitations, such as the neglect of fluvial deposition and mass wasting processes

(e.g., Whipple and Tucker, 1999). The trade-offs involved in this simplified approach, we believe, are justified by the record of success with simulating landscape processes at large scales and across a wide range of lithologies, drainage areas, and steepness (e.g., Gallen et al., 2013; Miller et al. 2013; Fox et al., 2014).

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1.1 Catchment-averaged denudation~~erosion~~ rates from cosmogenic [10]Be

Rates of catchment-scale denudation~~erosion~~ can be estimated by measuring the ~~from~~ abundances of cosmogenic radionuclides, such as [10]Be, ~~which is measured~~ in quartz-bearing river sand (Granger et al., 1996; von Blanckenburg, 2005). Such nuclides accumulate within minerals exposed to secondary

90 cosmic rays in the upper few metres of the bedrock subsurface and are lost via erosion and radioactive decay (Lal, 1991). The attenuation of cosmic rays with depth causes the nuclide production rate to decrease exponentially (at 2 m depth the [10]Be production rate is < 5 % that at the surface); hence, nuclide abundances measured in sediment are an inverse function of denudation~~erosion (or, strictly denudation)~~ rate.

95 The spatial variations observed in denudation~~erosion~~ rates across a range of climates and lithologies (Portenga and Bierman, 2011; Starke et al., 2020) suggest that the erosional~~erosion~~ processes driving the evolution of landscapes also vary. This has important implications for the interpretation of [10]Be-derived denudation~~erosion~~ rates and how we parameterise ~~parameterize erosion in~~ LEMs. Estimating catchment-averaged denudation rates~~erosion rate~~ from nuclide abundances in

100 river sand depends on a multitude of assumptions ~~at least two fundamental premises~~ (von Blanckenburg, 2005; Mudd, 2016). For example, we assume ~~: (1)~~ sediments were produced via long-term, steady bedrock erosion distributed uniformly across the catchment, and that~~; and (2)~~ sediments have experienced continuous exposure to cosmic rays at/near the surface. In detail, long-term steady erosion refers to at least one attenuation length (~ 0.6 m) of surface lowering ~~erosion~~ integrated over a

105 10^3–10^5 yr timescale. Abrupt ~~Hence, abrupt~~ bedrock erosion events, for instance, caused by bedrock landsliding or glacial quarrying may bias denudation~~erosion~~ rate estimates. Similarly, long intervals of ice cover or intermittent deep sediment burial contradict the requirement for continuous cosmic-ray

exposure. Other sources of potential discord relate to lithology, catchment size, and hypsometry, which are known to affect sediment transport dynamics and grain-size yields (Carretier et al., 2015; Riebe et al., 2015; Lukens et al., 2016; Zavala et al., 2020). The sources of deviation noted above are collectively responsible for the considerable variability observed in large compilations of [10]Be-derived denudationerosion rates (e.g., Portenga and Bierman, 2011; Harel et al., 2016). Catchment-wide denudationerosion rates are commonly determined and published for settings that do not comply strictly with the method's premises; these estimates are best referred to as *apparent* denudationerosion rates (Mudd, 2016). Nevertheless, [10]Be-derived data erosion rates currently offer the most widely distributed insight tointo long-term denudation, catchment-scale erosion rates on a global scale.

2 Methods

2.1 Stream power and hillslope diffusion

Stream power is represented by a non-linear advection equation derived from observations of river morphology and generalised relationships for bed shear stress (e.g., Howard et al., 1994; Whipple and Tucker, 1999; Lague, 2014). It affords a description of channel incision as a function of upstream drainage area (A) and local slope (S) for the portion of the catchment (above the critical drainage area, A_c) where fluvial advection dominates over hillslope diffusion and debris flow processes (e.g., Lague and Davy, 2003; Fontana et al., 2003; Whipple and Tucker, 1999). The stream power equation takes the form:

$$E_{predicted,advective} = K A^m S^n, \qquad (1)$$

where K is the advection coefficient or erosional efficiency that is (in our formulation) uniform across a catchment, and m and n determine the relative dependence of incision on drainage area and slope. The ratio m/n defines how elevation scales with drainage area at steady state relative to the concavity of the longitudinal channel profile and typically varies from 0.3 to 0.6 (Wobus et al., 2006; Whipple and Tucker, 1999); n determines the erosional nonlinearity, which is thought to relate to regional

discharge variability, as well processes governing incision thresholds, ~~flood variability~~ and typically

ranges from 1 to 4 (e.g., Lague, 2014). A global compilation of stream power parameters constrained

by topographic metrics (Harel et al., 2016) reports an optimised $n \sim 2.6$, albeit with considerable

scatter (also observed by Gailleton et al., 2021). ~~~~The value of n may also vary depending on the

location in the channel network—the steepest and fastest-eroding locales such as knickpoints can have

values closer to unity (Lague, 2014). In general, higher n results in larger erosional flux from steep

terrain, while higher m results in larger erosional flux from big rivers. However, if m and n both

increase (keeping their ratio, and K constant), a larger fraction of erosional flux will be sourced from

steeper main-stem channels.

The amalgamated outcomes of hillslope transport processes, such as rainsplash, soil creep, and

bioturbation, are primarily diffusive. Hence, to simulate hillslope processes, we include a diffusion

equation:

$$E_{predicted,diffusive} = D\left(\frac{d^2 z}{dx^2} + \frac{d^2 z}{dy^2}\right)^p, \qquad (2)$$

where D is diffusivity, which is reported to range from $\sim 4.4 \times 10^{-4}$ to 3.6×10^{-2} for linear diffusion

(e.g., Fernandes and Dietrich, 1997). We use an exponentiated~~a non-linear~~ form of the diffusion

equation in which concavity is raised to an exponent, p, ~~~~in order to harmonise with Gabet et al.

(2021), which posits that denudation~~erosion~~ rate scales approximately with hillslope concavity

squared. For linear diffusion to satisfy mass conservation~~, which is mass conservative~~, the deposited

and eroded sediment should ~~in~~ balance. However, in the exponentiated formulation~~non-linear model~~,

negative concavities raised to a power of p can produce non-real components~~numbers~~. Thus, we

calculate~~produce~~ an average catchment-wide denudation~~erosion~~ rate in which we ignore deposited

sediment (areas of negative concavity) and take an average ~~only~~based on eroded sediment only~~in this~~

~~model~~.

160   For our joint advection-diffusion model, we follow the common approach of combining stream power with linear diffusion (i.e., $p = 1$ in Eq. *2*). When Equations (*1*) and (*2*) are combined, D and K covary, and it has been noted that a higher D/K ratio results in a more linear scaling between erosion rate and catchment-averaged slope (Forte et al., 2016). Hence, we divide (Eq. *3*) by K, which allows the D/K ratio to be optimised with respect to predicted erosion rate:

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$$\frac{E_{predicted}}{K} = A^m S^n + \frac{D}{K}\left(\frac{d^2 z}{dx^2} + \frac{d^2 z}{dy^2}\right), \quad (3)$$

With our advection-diffusion model formulation (Eq. *3*), we set out to solve simultaneously for globally optimised values of D/K, n and A_c. D/K and n determine the relative importance of advective versus diffusive processes in driving erosion; lower D/K and higher n (which implies higher m given

170   uniform *m/n*~~concavity~~) will result in the dominance of advection, whereas higher D/K and lower n will promote diffusion~~result in more diffusion-dominance~~. While varying m and n, we maintain their ratio constant at 0.45, a widely applied average channel concavity (e.g., Wobus et al., 2006; Harel et al., 2016), and in line with the global average of 0.42 reported by Gailleton et al. (2021) (despite

175   considerable variability).

    Based on previous modelling ~~results~~ (e.g., Roering and Kirchner~~et al~~., 2007), one might expect advection-dominant landscapes to be rougher in outline relative to the smoothing effects of diffusion. However, Theodoratos et al. (2018) show that multiple sets of parameters can give rise to equifinality. In our modelling framework, the D/K ratio covaries with n to determine the ratio of hillslope

180   denudation and ~~erosion versus the~~ total (fluvial + hillslope) denudation~~erosion~~—denoted here as $E_{predicted,diffusive}/E_{total}$, where E_{total} is the sum of $E_{predicted,diffusive}$ and $E_{predicted,advective}$. The ratio $E_{predicted,diffusive}/E_{total}$ is therefore a function of both n and D/K. In principle, this metric is inversely proportional to an ~~(inverse)~~ effective Peclet number for net denudation~~erosion~~ (Perron et al., 2008). For values of $E_{predicted,diffusive}/E_{total}$ close to unity, diffusive processes will dominate, while values closer to zero

185   represent advection dominance (Fig. 1).

2.2~~1~~ [10]Be-derived apparent denudation~~erosion~~ rates

~~——~~We conduct modelling experiments that employ randomly selected sets of LEM parameter values, and then compare our model outputs with the ~~*OCTOPUS* v.2~~ global catalogue of [10]Be-derived catchment-averaged denudation rates, *OCTOPUS* v.2 ~~erosion rates~~ (Codilean et al., 2022). In addition to apparent denudation~~erosion~~ rates (N = 4631), *OCTOPUS* includes topographic data and catchment outlines~~morphometries~~. Using the catchment boundaries in *OCTOPUS*, we clipped rasters from the *Hydrosheds* Shuttle Radar Topography Mission (SRTM) dataset, a global digital elevation model (DEM) with 3-arc-second resolution (Lehner et al., 2008), plus the National Elevation Dataset (Gesch et al., 2002) for catchments in Alaska north of 60 degrees. Local pits (i.e., lakes) within the catchments were filled using the priority-flood method of Barnes et al. (2014). In building the network of local upstream drainage areas for each cell, runoff is assumed to flow down the steepest descent in accordance with the *D8* flow routing algorithm. Slopes for every cell are computed along this steepest-descent flow path.

To determine the effect of DEM resolution, we test our models on 1-arc-second Copernicus DEM data, ~~SRTM data (Farr et al., 2004),~~ although for reasons of computational capacity we restrict our analyses to catchments with < 1.3~~13~~ million grid cells (N = 3414), and for the ~~scenarios~~ diffusion-only and advection-only scenarios. We find that DEM resolution is not decisively important to our results (Supplementary figs. S1, ~~pivotal (Appendix A, Fig. S1/~~S2).

About 24 % of the *OCTOPUS* dataset is not exploitable for our purposes: 689~~1~~ catchments are too small to be processed by the LEM (< 3 DEM cells in any dimension), and we exclude 33 of the very largest catchments due to the extreme computational cost. Multiple denudation~~erosion~~ rate measurements included in *OCTOPUS* refer to samples from different locations within the same larger catchment. For such cases, a separate catchment is defined only where the drainage area differs by > 5 %; otherwise, we amalgamate the data and derive a single average denudation~~erosion~~ rate. In total, 3640~~3618~~ catchment-wide apparent denudation~~erosion~~ rates ($E_{apparent}$) are used in our modelling

experiments, ranging from 0.028 km^2 to 430129,000 km^2 (11 to 5338 million grid cells). We do not separate ^{10}Be measurements conducted on different grain-size fractions.

215   Processing DEMs begins with computing the drainage network. To minimize the effects of disequilibrium perturbations (e.g., Schwanghart and Scherler, 2017), we smooth river profiles using a ~1 km averaging window. Un-smoothed profiles are also used, for comparison, and yield similar results, where the largest discrepancies lie in the advection-diffusion model. After catchment slopes and drainage areas are computed, the diffusion and stream power equations can be solved. The LEM is run for exactly one time-step on DEMs representing each of the 36403618 catchments; the sediment

220 flux to the catchment outlet is then averaged over the total drainage area to yield a LEM-predicted denudationerosion rate ($E_{predicted}$). Because different values of input LEM parameters typically yield different denudationerosion rates (with the exception of highly diffusive models, as described below), such models are then optimised by comparison with our catalogue of $E_{apparent}$ data.

225 **2.32 Monte Carlo simulations**

We use a brute-force Monte Carlo approach to investigate the parameter space by running randomly selected sets of parameters and testing the fit of modelled versus observed (^{10}Be-derived) denudationerosion rates. We adopt the philosophy of equifinality (e.g., Beven and Binley, 1992) to evaluate the model parameters applied in our LEMs; implicit in these assumptions is that multiple sets

230 of parameters may give rise to a similar, or equifinal, result (e.g., Csilléry et al., 2010). Hence, we report both the range of optimal parameters in addition to the best-fit model parameters.

     In our framework, no more than three parameters are modified and compared at any one time (Table 1). This is possible thanks to several simplifying steps (detailed below) that require fewer modelling runs (e.g., Theodoratos et al., 2018). The performance of the model with a given set of

235 parameters is evaluated based on the mismatch between $E^*_{predicted}$ and $E_{apparent}$ with respect to the likelihood function (Beven and Binley, 2014). Modelled and observed rates are compared directly, no regression is involved. In so doing, one or more local maxima representing an optimised parameter set may be identified in the space defined by parameter values versus the likelihood function. A range is

then defined within 1 % of the peak (for example, if the best-fit model has a score of 0.500, $r^2 = 0.500$

240 we report the range of parameters from models with scores $r^2 > 0.495$).

For each randomly selected set of parameter values (Table 1), the LEM computes a single time step, and the erosion in each grid cell is integrated. $E^*_{predicted}$ is then scaled by employing a log-transformation on all modelled catchments:

245 $$\log\left(K^*\right) = \frac{1}{N} \sum \left(\log\left(\frac{E_{predicted}}{K}\right) - \log\left(E_{apparent}\right) \right), \qquad (4)$$

$$\log\left(E^*_{predicted}\right) = \log\left(\frac{E_{predicted}}{K}\right) + \log\left(K^*\right), \qquad (5)$$

where N is the number of observations and K^* is K prior to log-transformation (see Eq. 7 below). The

250 performance of $log(E^*_{predicted})$ against $log(E_{apparent})$ is then calculated for each parameter set. This setup offers the advantage of limiting the number of parameters varied; it is not our aim to determine the absolute values of coefficients because these covary; instead, we focus on the ratio D/K (see Section 2.1).

After optimal values of A_c, D/K and/or n are found, the associated value of K can be corrected

255 for log-transformation using an unbiased estimator (after Ferguson, 1986 1989):

$$K = K^* e^{\frac{s^2}{2}}, \qquad (6)$$

where s^2 is an estimate of the variance:

260 $$s^2 = \frac{1}{N-1} \sum \left(\log\left(E_{apparent}\right) - \log\left(E^*_{predicted}\right) \right)^2, \qquad (7)$$

An equivalent form of Eqs. *4–7* is used for the diffusion-only model, simply by replacing K with D. Although we believe this log transformation is justified, we also provide ~~show~~ coefficients without Eqs. 6–7 (Supplementary fig. S7c) ~~4–7 applied in Appendix C~~.

Given that our denudation rate data span several orders of magnitude, we compare $\log(E_{apparent})$ and $\log(E^*_{predicted})$ using the Nash-Sutcliffe coefficient of efficiency (NSE):

$$NSE = 1 - \frac{\sum \left(E_{apparent} - E^*_{predicted}\right)^2}{\sum \left(E_{apparent} - mean\left(E_{apparent}\right)\right)^2} \qquad (8)$$

Optimised values are defined as the maximum NSE value only where they are surrounded by local maxima in the likelihood, which is the case in all experiments below. We also tested a version of NSE that incorporates measurement uncertainty, based on Harmel and Smith (2007), but it had little effect on the optimised parameters (Supplementary fig. S3, S4). Additionally, we calculate the Mean Absolute Error (MAE) to gauge the sensitivity to the likelihood function (Supplementary fig. S5, S6).

~~Given that our erosion rate data span several orders of magnitude, we compare $\log(E_{apparent})$ and $\log(E^*_{predicted})$ using the coefficient of determination (r^2) as implemented in the *scikit-learn Python* package (scikit-learn.org). This allows for direct comparison with previous studies that also use the r^2 metric in log-space to explore the controls on catchment-scale erosion rates (e.g., Portenga and Bierman, 2011; Harel et al., 2016). Optimised values are defined as the maximum r^2 value only if they are surrounded by local maxima in the likelihood, which is the case in all experiments below. Additionally, to gauge the sensitivity of results to the likelihood function, we provide results using Mean Absolute Error (MAE) for comparison in Appendix B.~~

2.4 The influence of lithology and precipitation on denudation

We subdivide the *OCTOPUS* catchments according to (1) areally-dominant lithology based on the GLiM global geologic map (Hartmann and Moosdorf., 2012), which gives a vectorized description of lithology compiled from a number of regional high-resolution geologic maps at a target resolution of 1:1,000,000; and (2) spatially-averaged mean annual precipitation (MAP) using the CHELSA dataset (Karger et al., 2016).

Precipitation differences between lithologic subgroups can be significant; for instance, averages range from 851 to 2077 mm yr^{-1} for unconsolidated sediment and metamorphic rocks, respectively. To address lithological variations in the presence of climatic differences between lithologic subgroups in the advection-only model, we attempt to isolate substrate effects with the form of the stream power equation given by Kooi and Beaumont (1996), which explicitly includes precipitation variations that are normally folded into K under the assumption that precipitation (P) scales linearly with discharge:

$$E_{predicted,advective} = K_{lith}(PA)^m S^n \qquad (9)$$

~~2.3 The influence of lithology and precipitation on erosion~~

~~We subdivided the *OCTOPUS* catchments according to (1) areally-dominant lithology based on the GLiM global geologic map (Hartmann et al., 2012), which gives a vectorized description of lithology compiled from a number of regional high-resolution geologic maps at a target resolution of 1:1,000,000; and (2) spatially-averaged mean annual precipitation (MAP) using the WorldClim bioclimatic dataset (Hijmans et al., 2005).~~

~~Precipitation differences between lithologic subgroups can be significant; for instance, averages of 650 to 1390 mm for unconsolidated sediment and metamorphic rocks, respectively. To address lithological variation in the presence of climatic differences between lithologic subgroups in the advection-only model, we attempt to isolate substrate effects with the form of the stream power equation given by Kooi and Beaumont (1996), which explicitly includes precipitation variations that are normally folded into K:~~

Although many factors influence K besides precipitation, we use K_{lith} in this case to denote the variable we are attempting to isolate. We do not attempt to correct for variable precipitation in calculating D; for instance, by devising an equivalent D_{lith} from Eq. 9, and it is only applied when calculating K.

$$E_{predicted,advective} = K_{lith}(pA)^m S^n; \qquad \text{~~(8)~~}$$

Although many factors influence K besides precipitation, we use K_{lith} in this case to denote the variable we are attempting to isolate. We do not attempt to correct for variable precipitation in calculating D; for instance, by devising an equivalent D_{lith} from Eq. 8.

3 Results

3.1 Advection-only model

We apply a stream power-based advection-only model (eq. 1) (excluding hillslope diffusion), with two free parameters: a slope exponent (n) and critical drainage area (A_c). Variations in m are fixed to n such that concavity is held constant at $m/n = 0.45$. We report the optimised values in terms of maximum value and an optimised range of values ($Q_{0.01}$) that are within 1 % of the maximum. (i.e., $Q_{0.01}$). The advection-only model (Fig. 2) is globally optimised at $n \sim 1.28\,31$ ($Q_{0.01} = 1.23$–$1.43\,17$–1.33; Fig. 2a) and at $A_c \sim 0.05\,06$ km^2 ($Q_{0.01} = 0.03$–0.07 km^2; Fig. 2b). We note that n changes by ~6 % ($n \sim 1.36$) these values do not change substantially when using the higher-resolution 1-arc-sec DEMs (Supplementary fig Appendix A, Fig. S1). The slight Optimal n increases to 1.47 ($Q_{0.01} = 1.23$–1.61), while optimal A_c decreases to 0.02 km^2 ($Q_{0.01} = 0.01$–0.04 km^2). The differences in n are likely the result of the sensitivity to higher catchment-averaged slopes, which naturally arises from higher resolution topographic data (Supplementary Table S1). in the 1-arc-sec DEMs (see supplementary Table S1). Additionally, both the 3-arc-sec and 1-arc-sec models show a consistent pattern of significant drop-off in r^2 values at 0.05–0.08 km^2, putting at least an upper limit on the optimal parameter value.

3.2 Diffusion-only model

The diffusion-only model (Fig. 3) is globally optimised with the hillslope diffusion exponent constant, $p \sim 2.0$ (NSE $r^2 = 0.51\,50$, Fig. 3b), and with. We find negligible dependence on DEM-resolution; $p \sim 2.0$ for the 1-arc-sec models (Supplementary fig. S2). We also find that the 1-arc-sec Copernicus DEM

has a lower overall performance than the 3-arc-sec SRTM DEM (see discussion in Supplementary fig.~~Appendix A, Figure~~ S2).

340

3.3 Advection-diffusion model

While the optimisation~~optimization~~ of our diffusion-only model ~~(Fig. 3)~~ with $p = 2$ (Fig. 3) is an intriguing result that invites further investigation (cf. Gabet et al., 2021), we retain linear diffusion ($p = 1$) in our advection-diffusion experiment because for $p \neq 1$ the model is (1) numerically unstable

345 when implemented in LEMs, and (2) it fails to accommodate hillslope deposition and hence ~~(i.e.,~~ does not conserve mass~~)~~.

The advection-diffusion model (Fig. 4) is globally optimised at $n \sim 2.3$~~02~~ ($Q_{0.01} = 1.$59–2.70~~51–2.57~~; Fig. 4c), and $D/K \sim 1.79 \times 10^{6}$~~$4.56 \times 10^{5}$~~ $m^{0.9n+1}$ ($Q_{0.01} = 2.39 \times 10^{4}$ to 3.44~~$9.01 \times 10^{3} - 1.94$~~ $\times 10^{7}$; Fig. 4d). For n and D/K, the $Q_{0.01}$ ranges are quite broad in part because both parameters are co-

350 dependent (Fig. 4a). Optimum $A_c \sim 0.03$~~04~~ km^2 ($Q_{0.01} = 0.$01–0.17~~02–0.37~~; Fig 4e) is similar to that from the advection-only models. The models are more diffusive when n is low and D/K is high, and $E_{predicted}$ is dependent mainly on catchment slope. This results in values clustering at NSE~~r^2~~ ~ 0.34~~35~~ (Fig. 4c, d, e) for models with high diffusion. Because D/K covaries with n (and m, Fig. 4a), we find that sediment transport derived from diffusional processes is maximised when $E_{predicted,diffusive}/E_{total}$ is $\sim$

355 0.43~~39~~ (Fig. 4f).

4 Discussion

In their benchmark study, Portenga and Bierman (2011) employ stepwise regression to relate their

360 compilation of [10]Be-derived denudation~~erosion~~ rates to a range of factors embracing topography, climate, lithology and seismicity. That study, along with the later inclusion of normalised steepness (e.g., Harel et al., 2016; Marder and Gallen, 2022), added substantially to our knowledge of how and why denudation~~erosion~~ rate varies. Our alternative approach here focuses upon the erosional processes at play in terms of advective and diffusive mass flux, rather than attempting~~attempt~~ to interpret the

machinations of landscape response to internal and external agents. A significant advantage is that explicit relations between m, n, topography, denudation~~topography, erosion~~, and LEM parameters are derived at the scale of the DEM grid cell within each catchment, and success is gauged from the absolute difference between modelled ($E^{*}_{predicted}$) and ^{10}Be-derived ($E_{apparent}$) denudation rates. In~~erosion rates — in~~ other words, we evaluate LEM parameters as they are commonly implemented in the models.

4.1 Optimised parameters for landscape evolution models

~~———~~We first consider some comparisons with previous work regarding advection-only approaches. Our optimised $A_c \sim 0.05$~~06~~ km^2 (Fig. 2b) for the 3-arc-sec resolution models falls near the minimum of the range applied in previous studies, such as Whipple and Tucker (1999), who suggest 0.059–0.14 km^2. Our optimised $n \sim 1.3$ (n ~ 1.4~~47~~ for the 1-arc-sec models) is much lower than the 2.6 reported by Harel et al. (2016), which is derived from regression of denudation~~erosion~~ rate and normalised steepness ($k_{s\,ref}$). Harel et al. (2016) then use the product of $k_{s\,ref}$ and a scaling drainage area to calculate M_χ. In principle, M_χ and n should be similar to our K and n values; however, n is derived from a regression of $E_{apparent}$ against M_χ and it is integrated based on each pixel within the catchment~~M_χ is integrated across the catchment, thus limiting the ability of these regressions to capture the inherent nonlinearity at the sub-catchment or sub-reach scale when $n \neq 1$~~. The large discrepancy between our globally optimised values of n and those of Harel et al. (2016) may stem from the inability of the latter method to accommodate inherent nonlinearities at the sub-catchment or sub-reach scale when $n \neq 1$, whereas our approach is designed to capture some of these nonlinear effects. This is particularly important in transient catchments, as spatial ~~likely arises from this integration across the catchment or reach scale, whereas here we compute erosion rates at each DEM grid cell, allowing us to better capture the nonlinear effects of stream power on erosion when $n \neq 1$. Spatial~~ heterogeneity in denudation~~erosion~~ rate is often controlled by steep areas in the catchment, such as knickpoints, and higher values of n amplify the proportion of denudation~~erosion~~ derived from steep areas relative to the rest of the catchment (Fig. 1).

While linear diffusion ($p = 1$) is commonly applied in landscape evolution studies (e.g., Forte et al., 2016~~Braun and Willett, 2013~~), our optimised $p \sim 2$ for the diffusion-only model is consistent with Gabet al. (2021) in which denudation~~erosion~~ rate correlates best with the square of hillslope

395 convexity. In response to Gabet et al. (2021), Struble and Roering (2021) point to a systematic underestimation of curvature in natural landscapes that may be an artefact of the numerical methods used for estimating curvature from DEMs. Gabet et al. (2021) employ high-resolution (~1 m) LIDAR data, but the broader point made by Struble and Roering (2021) poses a serious limitation for large-scale LEM analyses that are typically restricted to lower-resolution DEMs. In such cases, the need for

400 mass conservation and numerical stability are important considerations. And yet, a diffusion equation with exponent $p \neq 1$ is numerically unstable, physically unexplained, and does not accommodate deposition (the result of negative curvature). What does it say about the utility of running LEMs on natural landscapes if the optimised parameter value ($p \sim 2$) cannot be implemented? Struble and Roering (2021) suggest that $p \sim 2$ enhances the influence of steep, rapidly eroding areas on average

405 curvature, which are commonly underestimated by many methods. Below, we discuss how the influence of these steep areas may be approximated by the stream power equation coupled with linear diffusion.

Our advection-diffusion model allows us to explore aspects of how hillslope and river processes govern sediment flux in river catchments. Theoretically, for a catchment in a perfect state of

410 mass-flux equilibrium (or steady state), hillslopes and rivers are eroding at the same rate, so either one should be equally useful as a proxy for denudation~~erosion~~ rate. There would be no apparent advantage to combining advection and diffusion in the same model since they would both yield the same average denudation~~erosion~~ rate. Landscapes are, however, more often not at steady state (at least over timescales integrated by cosmogenic [10]Be) and the slight dominance of advective denudation~~erosion~~ in

415 our optimised model ($E_{predicted,diffusive}/E_{total} = 0.43$~~39~~, in Fig. 4f) suggests that transient signals disproportionately affect catchment-averaged denudation~~erosion~~ rates.

The positive relationship we observe between optimised n and relative diffusivity gives rise to a compelling possibility: as diffusivity increases, advective denudation~~erosion~~ becomes less important

as a proxy for the total average denudationerosion rate within the catchment, and more of a proxy for transience focused at the most rapidly eroding zones. However, in the absence of diffusion, river incision must account for all sediment that would be otherwise eroded diffusively from hillslopes. The increase in optimised n with diffusivity therefore represents the expanding role of steep transient zones in dictating the catchment-scale denudation rate. Our optimised result for the advection-diffusion model, $n \sim 2.3$ (Fig. 4b), is compatible with previous work suggesting that typically, $n > 1$ (e.g., Lague, 2014; Harel et al., 2016)erosion rate.

Our optimised result for the advection-diffusion model, $n \sim 2.02$ (Fig. 4b), is compatible with previous work suggesting that typically $n > 1$ (e.g., Lague, 2014; Harel et al., 2016). A value of $n \sim 2.02$ also has implications for the erosional response to climate change. When precipitation is included explicitly in the formulation (Eq. 8), erosion scales nearly linearly with changes in mean annual precipitation (assuming $m/n = 0.45$). We return to this point in Section 4.2 below.

The best correlation between predicted and apparent denudationerosion rates occurs when $D/K \sim 1.79 \times 10^{6}$$^{4.56 \times 105}$ (Fig. 4d). This outcome broadly agrees with other studies that use K values in the range $\sim 10^{-8}$ to 10^{-5} m$^{(n-1)}$ yr^{-1} and D values in the expected range noted by Fernandes and Dietrich (1997) of 4.4×10^{-4} to 3.6×10^{-2} m^2 yr^{-1}. Whipple et al. (2017) reports an optimal D/K ratio of 5×10^2 from Himalayan catchments, although fixing $n = 1$ in their models is a limiting assumption because D/K covaries with n, as we show. The diffusion model employed here assumes that the long-term flux of hillslope material is similar to the amount transported in one time-step. In reality, the catchment may not be at steady state and the hillslope denudationerosion rate may change notably so as to change the rate of hillslope flux within individual catchments.

4.2 Erosion and precipitation

Correlating topographically-derived metrics with mean annual precipitation (MAP) on a global scale has been a long-standing goal (e.g., Ahnert, 1970). Harel et al. (2016) examine correlations between stream-power variables and climate as defined by the Köppen-Geiger scheme. They find that aridity yields the lowest n and that, in general, In general, they find that $k_{s\ ref}$ and temperature covary

inversely: warm deserts yield the highest $k_{s\,ref.}$ and polar regions the lowest, on average, albeit with ~~although they also emphasise the~~ large uncertainties. Because our approach compares model results to denudation~~erosion~~ rates, rather than using regression, we can more directly correlate K and n as they are~~as it is~~ implemented in LEMs under differing climate. This also means that our method is

450 potentially better suited for transient catchments.~~precipitation regimes.~~

We demonstrate a general increase in n and p with precipitation for both the advection model and the diffusion model (Fig. 5b, c), with the exception of the highest MAP bin ($\sim$ 2300–6500 mm yr^{-1}). Marder and Gallen (2022) also observe a levelling-off in n for the wettest environments along with a relatively lower correlation of K_{sn} vs erosion rate in these wetter environments compared to other environments. Our advection-

455 diffusion model shows a similar increase in n, aside from the lowest MAP bin (Fig. 5d). In the stream-power based models, the rising n with respect to precipitation suggests that wetter environments favour a nonlinear erosional response perhaps tied to heavy-tailed flood-frequency distributions. Extremely variable flow regime is characteristic of drylands (Zaman et al., 2012); and yet, in two of the three models (advection-only and diffusion-only), the exponent (n or p) is low. By contrast, only the advection-diffusion model yields a relatively

460 high n ($\sim$ 2.8) for drylands. This may indicate that the advection-diffusion model, which we suggest is more sensitive to transience, does a better job at simulating dryland catchments. However, we emphasize the relatively low performance of all models (NSE = 0.2–0.3) for the driest settings (Fig. 5b-d), which may reflect the challenge of capturing the erosional dynamics of drylands with such simple models.

It is difficult to differentiate changes in exponents from changes in model coefficients, due to

465 their inherent coupling in power-law functions (e.g., Syvitski et al., 2000). In response to this issue, we ~~We~~ calculate coefficients D (for diffusion-only) or K (advection~~for advection models~~) for each of the optimised models (i.e., with constant exponents) using Eq. (5) in catchments represented by twenty ~~that correspond to 40 different~~ MAP bins distributed regularly ~~such that each bin has an approximately equal number of data points~~ (N = 182±91 ~~± 1~~; Fig. 5e~~5~~). By looking at how the~~these~~ coefficients vary

470 within each bin, we effectively reduce the influence of topography and isolate the relationship between erosion rate and precipitation. We emphasize that, due to the wide range of data used, optimized models can have large uncertainties (dry regions having the highest uncertainty relative to the mean)

with residuals that are generally log-normally distributed (Supplementary fig. S8). We also trialled different drainage-area bin sizes, where a similar (albeit noisier) pattern remains with higher bin density (Supplementary fig. S7)~~For reference, we also show results which do not apply a log-transformation correction from Eq. *6*, in Appendix C~~.

Optimised coefficients show a local peak in denudation~~erosion~~ rates centred around 300~~330~~ mm yr^{-1} and then dipping overall from around 1100 to 1600 mm yr^{-1} before increasing again for extremely wet regions (Fig. 5e~~5b~~). These results agree well with the classic work of Langbein and Schumm (1958), which suggests that the fastest eroding environments are semi-arid (MAP ~ 250 mm yr^{-1}). This relationship is thought to be a product of the interplay between denudation~~erosion~~, vegetation, total precipitation, and storm frequency in semi-arid regions—an outcome reproduced by Istanbulluoglu and Bras (2006), who show a positive relationship between sediment transport and the effects of reduced sediment cover/increased runoff during drought.

While Langbein and Schumm (1958) had scant access to data from wetter settings, our results reveal an upward trend in coefficient values for MAP > 1500 mm yr^{-1} (Fig. 5e~~5~~). This is in line with Walling and Kleo's (1979) global study of sediment yield and climate, which also shows a further major peak at ~ 800 mm yr^{-1} and may be attributed to the most expansive agricultural production globally (e.g., Hyman et al., 2016). In contrast to Walling and Kleo (1979), which does not isolate the effects of variable land use, topography, and geology on sediment yields, our use of denudation rates based on ^{10}Be ~~long-term erosion rates~~ means that we can largely ignore the effects of land use. This may explain the subdued peak ~ 800 mm yr^{-1} in our data~~. However, when no log transformation is applied to the coefficients, the peak at 800 mm yr^{-1} rises while the peak at 300 mm yr^{-1} is suppressed~~.

Marder and Gallen (2022) find a nonlinear relationship between ^{10}Be-derived denudation rate and k_{sn} via regression; they also find that the exponent relating k_{sn} to erosion rate (here effectively equivalent to *n)* increases with precipitation. They took steps to exclude transient catchments, as transience may influence *n* due to the non-linear effects of stream power, as noted above. Likewise, we find *n* generally increases with precipitation, giving credence to the idea that our approach may account to some degree for transience. In particular, we envisage cases in which steep zones, such as

adjusting areas downstream of knickpoints, are responsible for a large proportion of the total catchment erosional flux, even when they may represent a small fraction of the drainage area (Willenbring et al., 2013). Where denudation is a nonlinear function of slope and drainage area, an average of these denudation rates is unlikely to be proportional to the integrated k_{sn}.

One acknowledged shortcoming of our approach (noted in Section 2.4) is that some lithologies may be over-represented in areas with higher or lower MAP. As we discuss in the next section, it is a challenge to discriminate biases due to lithology from those linked to precipitation in bins that span heterogeneous lithologies.

~~Marder and Gallen (2022) find a nonlinear relationship between [10]Be-derived erosion rate and k_{sn} via regression; they also find that n increases with increasing erosion rate. However, k_{sn} is calculated by integrating along different channel lengths (as with Harel et al., 2016), whereas we calculate erosion at every grid cell and so capture local variations in erosion rate more effectively. We envisage cases in which steep zones, such as knickpoints, are responsible for a large proportion of the total catchment erosional flux, even though they represent a small fraction of the drainage area (e.g., Willenbring et al., 2013). Where erosion is a nonlinear function of slope and drainage area, an average of these erosion rates is unlikely to be proportional to the integrated k_{sn}.~~

4.3 Erosion and lithology

We assessed the variability of~~The variability in~~ LEM parameters within each lithological bin ~~was assessed~~ following a similar approach to Section 4.2, with the additional step of employing K_{lith} (Eq. 9~~8~~) to isolate the effects of lithology on K in the advection-only model. The three-fold range in erodibility (Fig. 6e) is much lower than that reported elsewhere, in some cases by several orders of magnitude (Sklar and Dietrich, 2001; Garcia-Castellanos et al., 2018). This may be due to the greater focus on the differential erodibility within individual sites (Garcia-Castellanos et al., 2018). Despite our efforts to account for some of the covariation with MAP, our analysis inevitably smooths out some variability owing to the diversity of catchments incorporated within each lithological bin. ~~in the face of variable precipitation within lithologic subgroups.~~

We find that for the advection-only model (Fig. 6b), n is higher in metamorphic and plutonic-intermediate rocks, which also tend to be more resistant (e.g., Moosdorf et al., 2018). This is not surprising given that n is thought to be influenced by higher thresholds for rock detachment, which in turn can lead to more non-linear behaviour. More complex relationships involving hillslope diffusion may result in the general lack of a comprehensible pattern emerging in the advection-diffusion models (Figure 6c & 6d). The higher number of parameters used in the advection-diffusion model leads to broader distributions (Fig. 6d).

When looking at the variability in K, our overall ~~Our~~ results are somewhat ~~largely as~~ expected: all three models agree on the general tendency of sedimentary rocks being most erodible and plutonic/volcanic being least, although the relative magnitudes of these differences vary between models. Unconsolidated sedimentary rock is the first or second-most erodible of all according to the diffusion and advection-only models (Fig. 6e~~6b~~). Models disagree about the least erodible subcategories: basic plutonic, intermediate volcanic, or pyroclastic. The pyroclastic, plutonic intrusive, and unconsolidated sediment ~~and basic plutonic~~ rock-types show the most variance. For pyroclastic, the advection models suggest relatively low erodibility, whereas the diffusion-only models suggest moderate erodibility. Most models agree on the moderate erodibility of plutonic intrusive rocks, but due to the low sample size, the uncertainty is very high. Less expected is the relatively high erodibility of carbonate sedimentary rocks shown for all our models. For example, other studies argue that carbonates should be less erodible (Ott et al., 2021), and that volcanic rocks may be relatively more erodible than we have shown here (Moosdorf et al., 2018). Some of these variable findings possibly arise due to factors, such as fracture density and weathering condition, which are difficult to accommodate in large-scale analyses (Neely et al., 2019), and also contribute the large uncertainty (Supplementary fig. S9).

4.4 Erosion and drainage area

Are LEM parameters sensitive to catchment drainage area? Violin plots (Fig. 7) show exponents (n and p) values binned according to drainage area. The advection-diffusion model shows no dependence

on area, but for the advection-only and diffusion-only model, the exponent values generally increase with drainage area, n ranging from ~1.1 to ~1.5, and p ranging from ~ 1.8 to 2.2. Additionally, we note the extreme range in n (~ 1.3 to > 3) for small catchments in the advection-diffusion models, potentially indicating the influence of landslides in headwaters (e.g., Yanites et al., 2010). This dependence of n and p on drainage area may be difficult to discern from other effects, but one possibility may be the inherent link with precipitation. Larger catchments are more likely to cross large rainfall gradients, which as we have shown, gives rise to higher n or p.

4.5 Limitations and future considerations

An important limitation of our approach is that it fails to unravel the m/n ratio, which is known to vary widely (Gailleton et al., 2021). We ran several trials (using optimized $A_c = 0.05$ km^2 and 20 values of n linearly spaced from 0 to 4) and found that $m/n = 0.3$, 0.45 and 0.6 yielded negligible difference in fit (NSE = 0.48, 0.47, and 0.45, respectively) despite the similar values of optimized $n = 1.4, 1.2,$ and 1.2, respectively (Supplementary fig. S10). One option for improvement is to determine m/n from the river profile concavity using recent integral-based techniques (e.g., Harel et al., 2016, Gailleton et al., 2021) before running the suite of models. This may be especially useful for tuning the m/n ratio within specific regions.

Our approach cannot account for several factors that are known to bias [10]Be-derived denudation rates (e.g., von Blanckenburg, 2005; Dingle et al., 2018; Hippe et al., 2019; Struck et al., 2019). Such factors violate two key underpinning premises of the method: steady and uniform erosion across the catchment and continuous exposure of sediment at/near the surface (noted in Section 1.1). For example, sudden pulses of sediment from sources of deep-seated mass wasting can shift apparent denudation rates downward by diluting the nuclide abundances in the river sediment sample. Given that our LEM is deterministic, it is difficult to model the impact of landslides on nuclide inventories, particularly in small headwater catchments where their influence will be greater (e.g., Yanites et al., 2010). A second pertinent issue is that our modelling assumes the erosional flux is transported instantly to the catchment outlet without intermediate storage. And yet, river sediment is likely to

experience multiple episodes of erosion and deposition, especially in large lowland catchments where the volume of sediment storage expands greatly. Intermediate sediment storage can push [10]Be-derived denudation rates either upwards or downwards depending on the duration and depth of sediment burial (Wittmann and von Blanckenburg, 2016; Struck et al., 2018). The extent of bias in the *OCTOPUS* dataset (Codilean et al., 2022) is essentially unknown but it no doubt contributes to the data scatter we report here.

In addition to the complications with stream-power and diffusion-based models (noted in the Introduction), the inherent assumption that channel width increases monotonically with stream discharge (or its proxy, drainage area) is one that is violated widely. Channel width narrows at knickpoints (Whitbread et al., 2015; Yanites et al., 2018) and across diverse substrate erodibilities (Jansen et al., 2010; Croissant et al., 2017). The value of *n* extracted from transient landscapes will be influenced by this process, and a goal of future global analyses should be to incorporate more sophisticated rules for channel width evolution (e.g., Yanites et al., 2018).

The approach described here is sufficiently robust to incorporate many different types of models. We would like to see, for instance, an exploration of the way in which drainage is routed through catchments. Different flow routing schemes can give rise to notably different drainage networks and corresponding drainage areas (e.g., Endreny et al., 2003). Here we have used unidirectional 'D8' flow, but alternatives such as multiple flow routing can produce alternative results and perspectives (e.g., Pelletier, 2004). Future efforts may test a range of flow routing methods against denudation rate in order to test their efficacy.

We note that the three-fold range in erodibility (Fig. 6b) is much lower than that reported elsewhere, in some cases by several orders of magnitude (Sklar and Dietrich, 2001; Garcia-Castellanos et al., 2018). This may be due to the greater focus on the differential erodibility within individual sites (Garcia-Castellanos et al., 2018). Despite our efforts to account for some of the covarying with MAP, our analysis inevitably smooths out some variability owing to the diversity of catchments incorporated within each lithological bin. Moreover, erodibility is clearly a function of several additional factors

~~such as fracture spacing (e.g., Neely et al., 2019), and weathering conditions, which is challenging to address in a global analysis such as ours.~~

5. Conclusions

~~————~~We have examined the most widely used parameters applied to a set of three landscape evolution model set ups: (1) a stream-power based, advection-only model; (2) a diffusion-only model; and (3) an advection-diffusion hybrid model. We optimised the parameter values by comparing directly the catchment-averaged denudation~~erosion~~ rates predicted by our three models with a global catalogue of [10]Be-derived apparent denudation~~catchment-averaged erosion~~ rates (Codilean et al., 2022).

The diffusion-only model outperformed the advection-only model when applying $p \sim 2$. However, the physical implications and numerical limitations of this result make it impractical for implementation in LEMs~~landscape evolution models~~. Instead, we propose that linear diffusion coupled with fluvial denudation~~erosion~~ (advection-diffusion) captures a high proportion of sediment derived from rapidly eroding, steep areas in a similar sense to a diffusion model with exponent $p \sim 2$. In the advection-diffusion hybrid model, the best agreement between the predicted and apparent denudation~~erosion~~ rates is observed with $n \sim 2.3$~~0~~ (assuming a fixed concavity, $m/n = 0.45$), while the ratio of diffusivity/advection coefficient (D/K) is optimised at $\sim \underline{1.79 \times 10^{6}}{}^{4.56 \times 10^{5}}$.

The Monte Carlo method employed here is a simple and powerful means of identifying ideal parameter sets over large spatial scales and is especially useful for dealing with sparse datasets. We applied the same approach to elucidate differences in optimal LEM parameters when considering lithology and precipitation. By looking at the LEM coefficients, we were able to better account for the influence of topography when isolating the relationship between denudation~~erosion~~ rate and precipitation/ lithology. Of particular interest was a general upward trend in the coefficients (K and D) with respect to precipitation, and a local maximum~~maxima~~ centred at $\sim$ 300 mm yr[-1]. ~~300 mm/yr.~~ This local maximum~~maxima~~ may represent the higher erodibility of semi-arid environments ~~as~~ identified by Langbein and Schumm (1958)~~. Nevertheless, many other influences on erosion are yet to be~~

~~explored in a satisfactory and robust way. Future studies may use these parameter ranges as a baseline to inform large landscape evolution studies. Moreover, our methodology could be extended to incorporate more complexity into the canonical advection and diffusion-based equations applied here~~.

Nevertheless, many other influences on denudation are yet to be explored in a robust way. Future studies may use these parameter ranges as a baseline to inform large-scale landscape evolution studies. Moreover, our methodology could be extended to incorporate more complexity into the canonical advection and diffusion-based equations applied here.

Author contributions: GR conceived the study, performed data analysis, and devised the code. JDJ and PV assisted with framing the study, and LYM performed code analysis. All co-authors contributed to MS production.

Resource availability: The functions and notebooks for running this analysis are available from www.github.com/ruetg/lem_global_optimize.

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Tables

Table 1: Parameter values and ranges for the three model set ups.

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Model set	Parameter	Range	Sampling
Stream power only	n	0–4	Random, 1000 samples
	A_c	0.01–8 km^2	Random, log-uniform, 1000 samples
Diffusion only	p	0–4	Linear increment by 0.2
Stream power + diffusion	n	0–4	Random, 10,000 samples
	D/K	10^3–10^{10} $m^{0.9n+1}$	Random, log-uniform, 10,000 samples
	A_c	0.01–8 km^2	Random, log-uniform, 10,000 samples
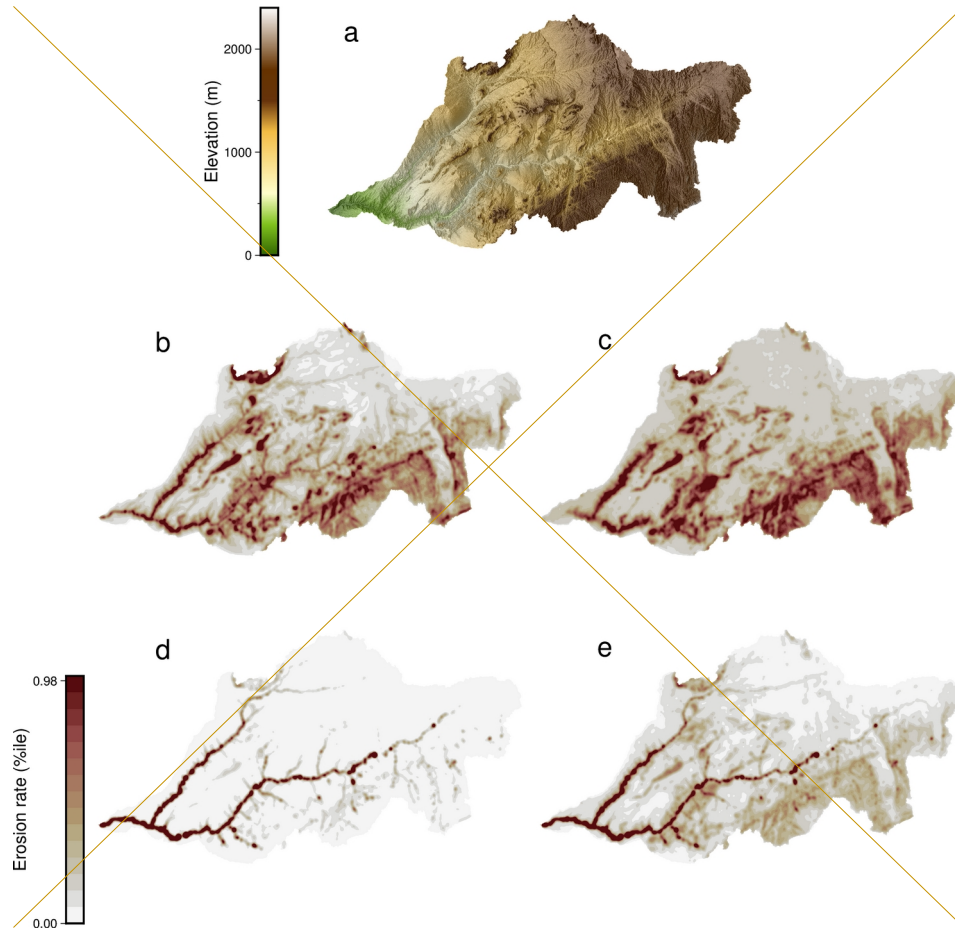
Figure 1: a) Catchment example (Swakop River, Namibia) clipped from a *Hydrosheds* DEM based on the shapefile provided in *OCTOPUS* v.2 (Codilean et al., 2022). Lower panels (b–e) show corresponding relative denudation rates (colour ramp spans 0–98 % of the range) for differing parameter values. No diffusion is included in (b) and (d), hence erosion is focused in the channels. In (c) and (e), a moderately high (10^7) diffusivity is used relative to advection, which causes erosion to be more focused on hillslopes.

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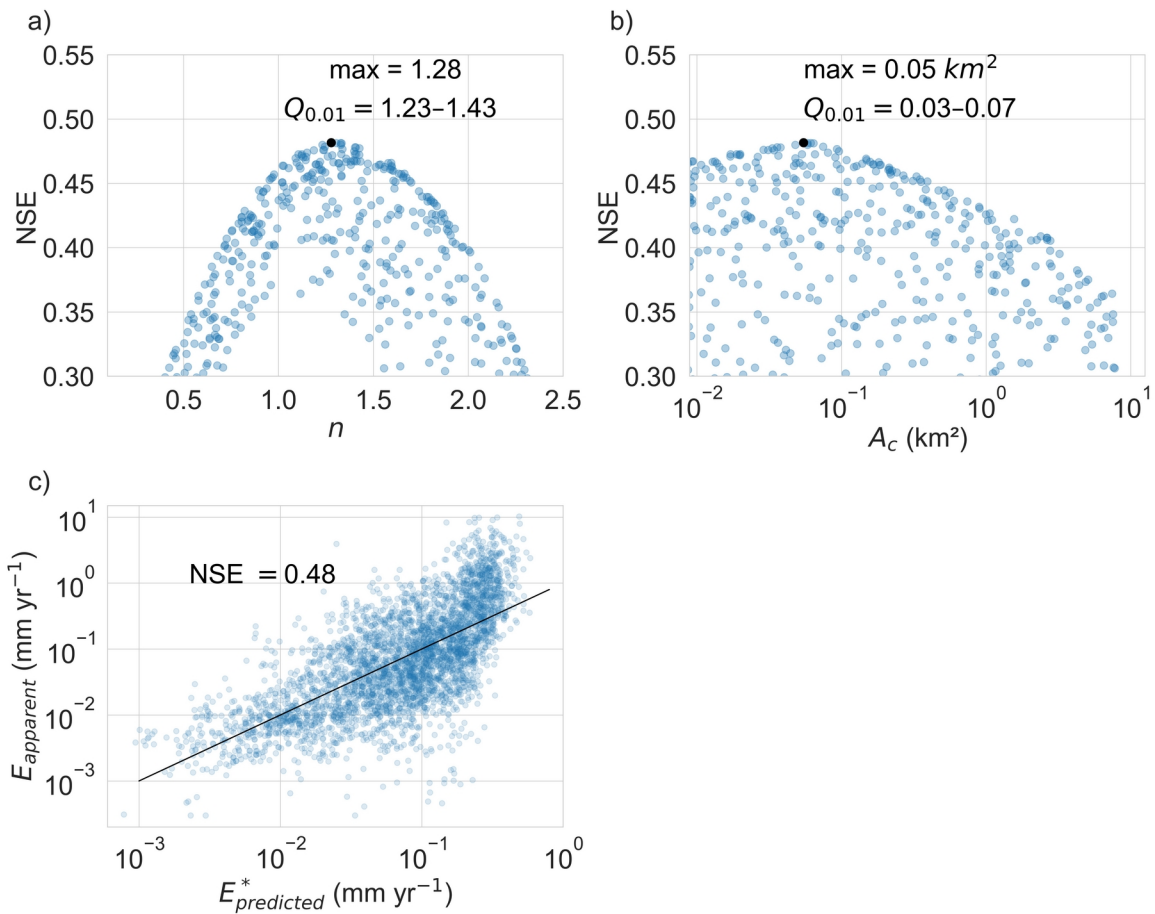

885 **Figure 2:** The advection-only model. a) Optimised $n = 1.28$ ($Q_{0.01} = 1.23–1.43$), b) Optimised $A_c = 0.05 \text{ km}^2$ ($Q_{0.01} = 0.03–0.07 \text{ km}^2$), c) Apparent vs predicted erosion rate, NSE = 0.48; no regression is performed, the black line indicates a 1:1 fit.

~~**Table 1:** Parameter values and ranges for the three model set ups.~~

~~Model set~~	~~Parameter~~	~~Range~~	~~Sampling~~
~~Stream power only~~	~~*n*~~	~~0–4~~	~~Random, 1000 samples~~

	A_c	0.01–8 km²	Random, log-uniform, 1000 samples
Diffusion only	p	0–4	Linear increment by 0.2
Stream power + diffusion	n	0–4	Random, 10,000 samples
	D/K	10^2–10^9 $m^{0.9n+1}$	Random, log-uniform, 10,000 samples
	A_c	0.01–8 km²	Random, log-uniform, 10,000 samples

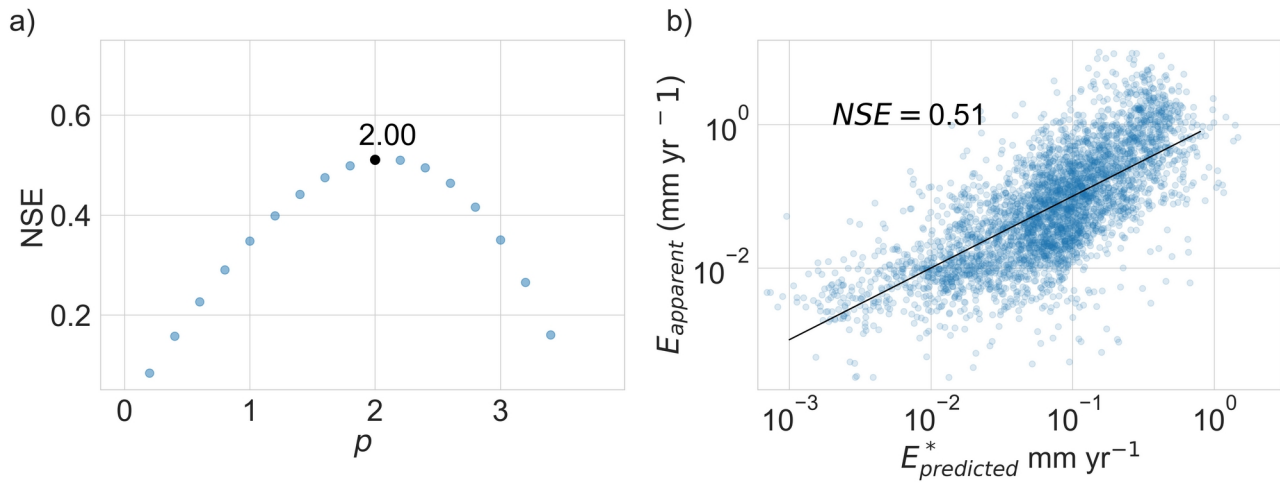



Figure 3: The diffusion-only model. a) The sole free parameter (p) is optimised at $p = 2.00$. b) Apparent vs predicted erosion rate, NSE = 0.51. The Black line represents a perfect 1:1 fit.

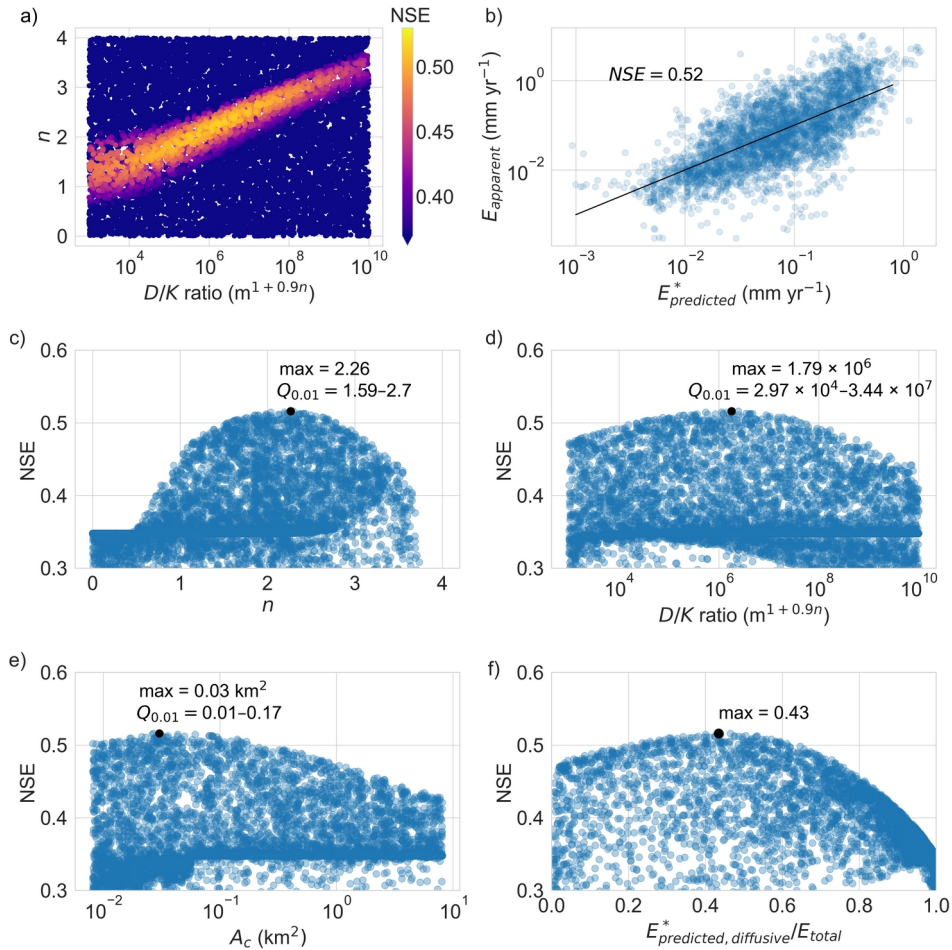



Figure 4: Model parameters representing variations in the relative dominance of advection vs diffusion. a) Covariance of D/K with n; when D/K is low (no diffusion), optimal n approaches ~1.3 (y intercept). b) The best correspondence between $E^*_{predicted}$ and $E_{apparent}$ is achieved with NSE = 0.52, where c) $n \sim 2.3$ ($Q_{0.01}$ = 1.59–2.7); d) $D/K \sim 1.79 \times 10^6$ ($Q_{0.01}$ = 2.97×10^4 to 3.44×10^7), and e) $A_c \sim$ 0.03 km² ($Q_{0.01}$ = 0.01–0.17 km²). Clustering at NSE ~ 0.34 in panels (c, d, e) represents parameter sets where diffusion dominates over advection. f) Sediment transport derived from diffusional processes is maximised when $E_{predicted,diffusive}/E_{total}$ is ~ 0.43.

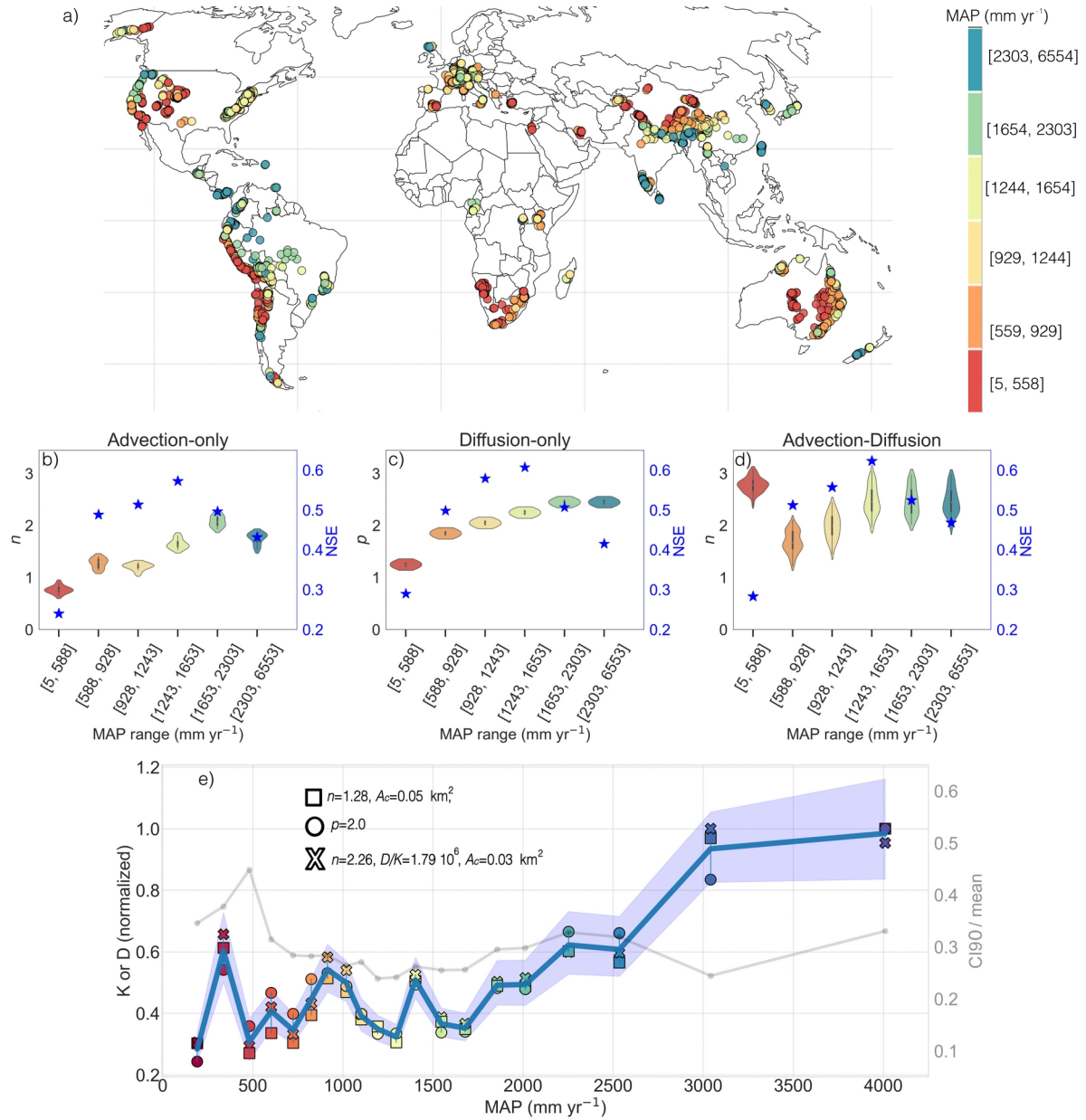
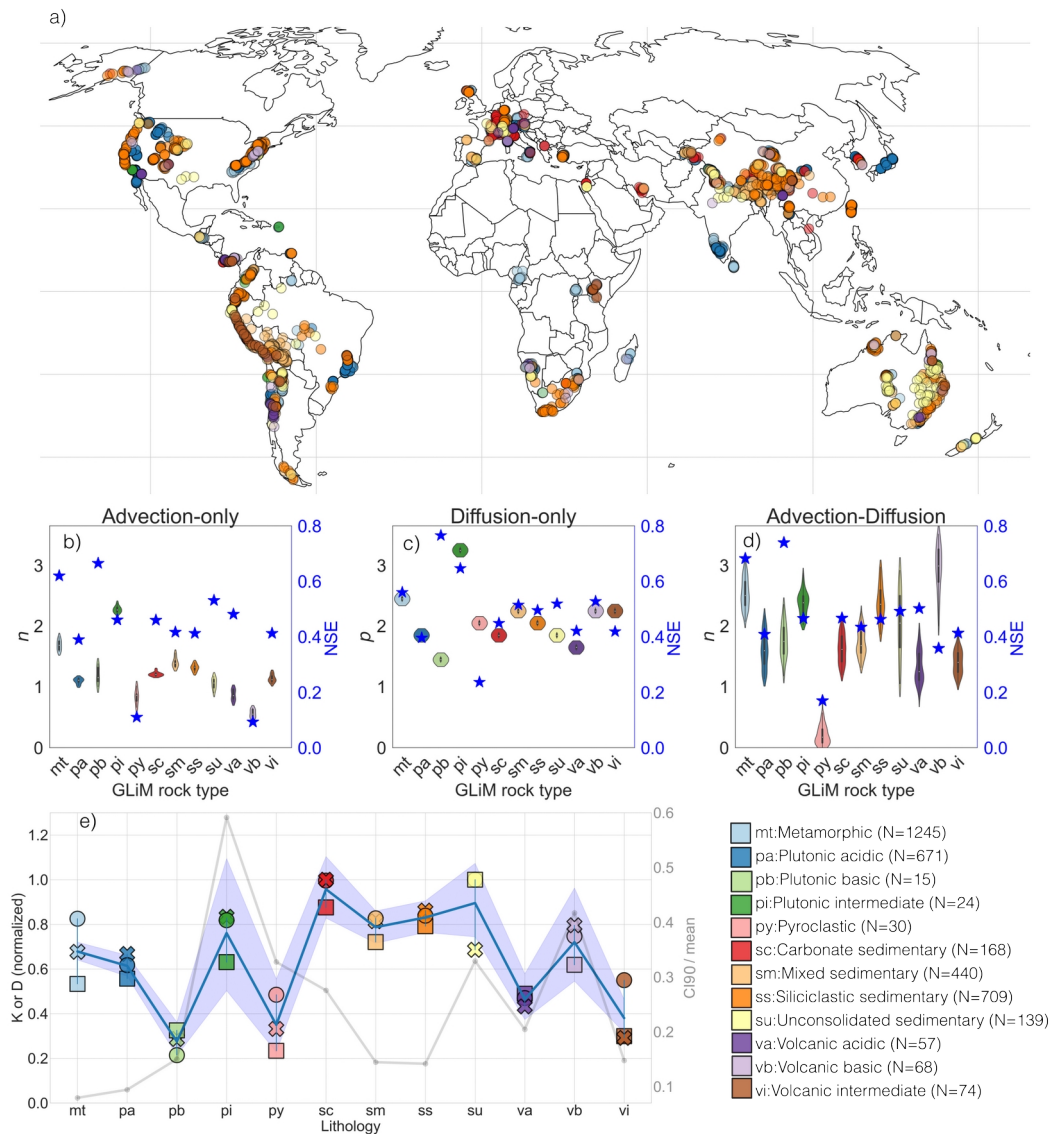
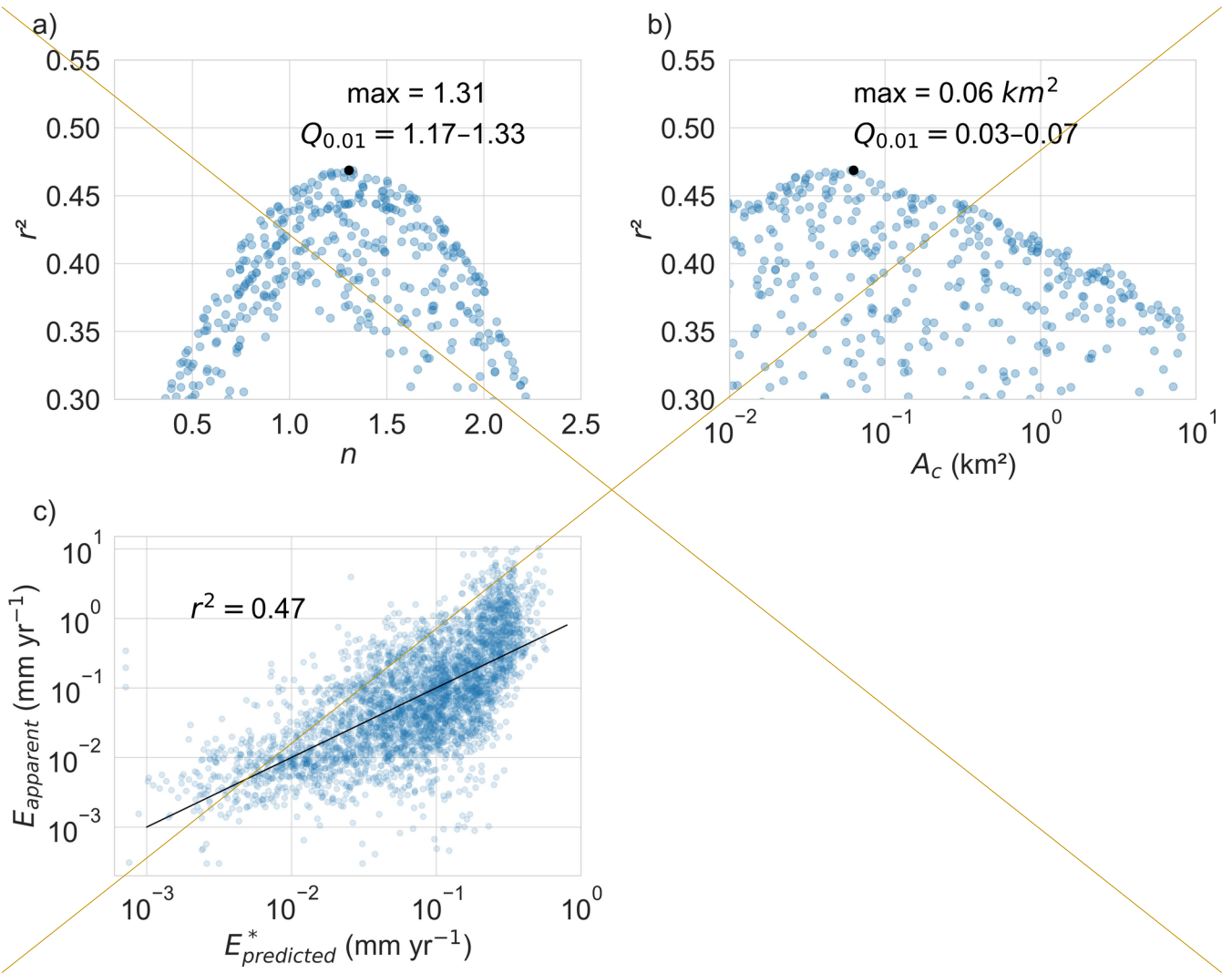



Figure 5: a) Global distribution of catchments in the *OCTOPUS* v.2 catalogue of ^{10}Be-derived apparent denudation rates (Codilean et al., 2022) coloured by mean annual precipitation (MAP). b-d) Violin plots (symmetric kernel density plots) of n or p values within the top 1% of model runs for the advection (b), diffusion (c), and advection-diffusion (d) models. Blue stars correspond to the NSE value for each bin. e) Coefficients for diffusion (circle), advection (square), and advection-diffusion (cross) calculated for each globally optimised model per MAP bin. Diffusion model plots represent only the global maximum within each bin, due to insufficient model runs to form a distribution. All panels use the same colour ramp, which corresponds to the MAP bin; blue shading represents the average bootstrapped 90% CI spanned by the 3 models, and heavy blue-line is the mean. The grey line represents the range spanned by the 90% confidence interval averaged among all 3 models relative to the mean (see Supplementary fig. S8).

 Figure 1: a) Catchment example (Swakop River, Namibia) clipped from a *Hydrosheds* DEM based on the shapefile provided in *OCTOPUS* v.2. Lower panels show corresponding relative erosion rates

(colour ramp spans 0–98 % of the range) for different parameter values. No diffusion is included in (b) and (d), hence erosion is focused in the channels. In (c) and (e), a moderately high (10^7) diffusivity is used relative to advection, which causes erosion to be more focused on hillslopes.

Figure 6: a) Global distribution of catchments in the *OCTOPUS* v.2 catalogue of [10]Be-derived apparent denudation rates (Codilean et al., 2022) coloured by dominant lithology. b-d) Violin plots of *n* or *p* values within the top 1% of model runs for the advection (b), diffusion (c), and advection-diffusion (d) models. Blue stars correspond to the NSE for each bin. e) Coefficients (normalised by their maximum values) for different best-fit models within 12 lithologic subsets (from Hartmann and Moosdorf, 2012). Coefficients for diffusion (circle), advection (square), and advection-diffusion (cross) calculated for each globally optimised model per lithologic bin; blue shading represents avg. bootstrapped 90% CI spanned by the 3 models, and heavy blue-line is the mean. All panels use the same colour ramp, which corresponds to the lithologic bin. The grey line represents the range spanned by the 90% confidence averaged among all 3 models relative to the mean (see Supplementary fig. S9).

 Figure 2: The advection-only model. a) optimised $n = 1.31$ ($Q_{0.01} = 1.17–1.33$), with a global maximum of 1.31, b) optimised $A_c = 0.06$ km² ($Q_{0.01} = 0.03–0.07$ km²), c) apparent vs predicted erosion rate yields $r^2 = 0.47$; no regression is performed, the black line indicates a perfect 1:1 fit.

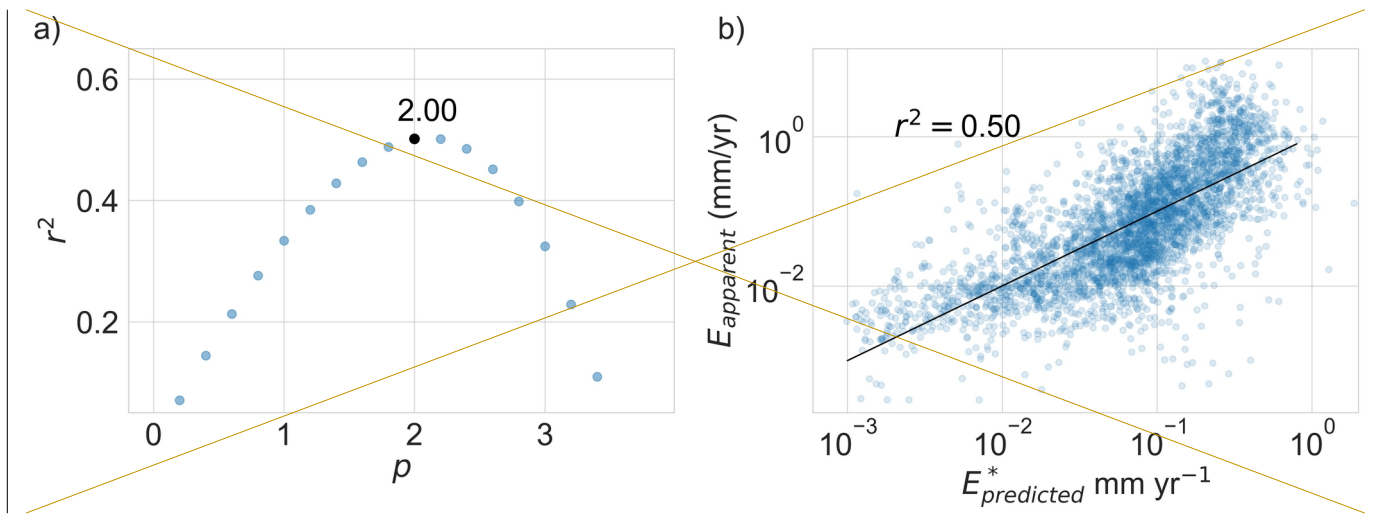



Figure 3: The diffusion-only model. a) the sole free parameter (p) is optimised at $p = 2.00$. b) Apparent vs predicted erosion rate yields $r^2 = 0.50$; no regression is performed, the black line indicates a perfect 1:1 fit.

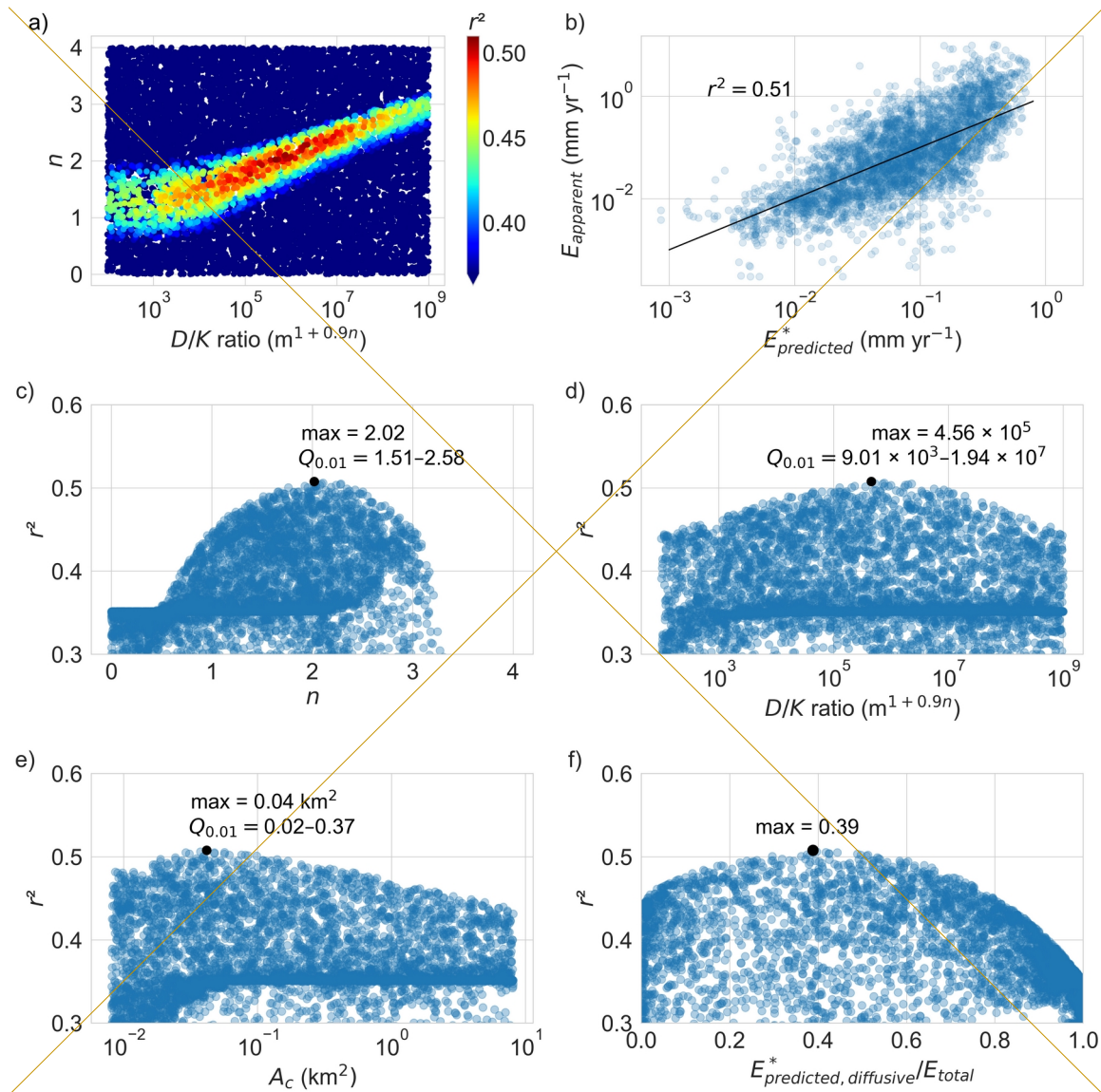



Figure 4: Model parameters representing variations in the relative dominance of advection vs diffusion. a) Covariance of D/K with n; when D/K is low (no diffusion), optimal n approaches ~1.3 (y-intercept). b) The best correspondence between $E^*_{predicted}$ and $E_{apparent}$ is achieved with $r^2 = 0.51$, where c) $n \sim 2.02$ ($Q_{0.01} = 1.51$–2.57); d) $D/K \sim 4.56 \times 10^5$ ($Q_{0.01} = 9.01 \times 10^3$–$1.94 \times 10^7$), albeit with a fairly

broad peak, and e) $A_e \sim 0.04$ km^2 ($Q_{0.01} = 0.02$–0.37 km^2). Clustering at $r^2 \sim 0.35$ in panels (c, d, e) represents parameter sets where diffusion dominates over advection. f) Sediment transport derived from diffusional processes is maximised when $E_{predicted,diffusive}/E_{total}$ is ~ 0.39.

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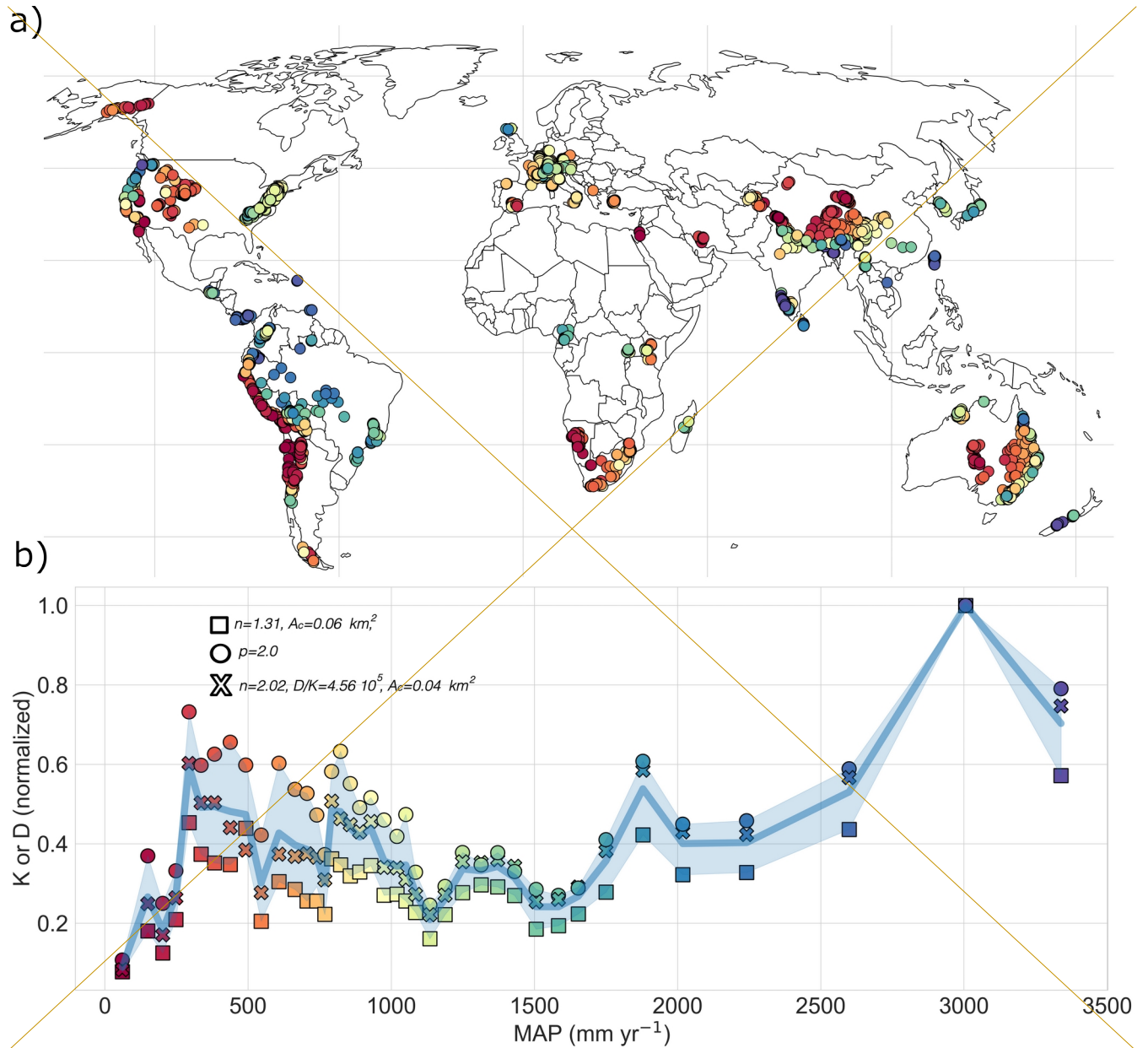
~~advection (square), and advection-diffusion (X) calculated for each globally optimised model per MAP bin. Both panels use the same colour ramp, which corresponds to the MAP bin; blue shading represents the range spanned by the 3 models, and heavy blue-line is the mean.~~
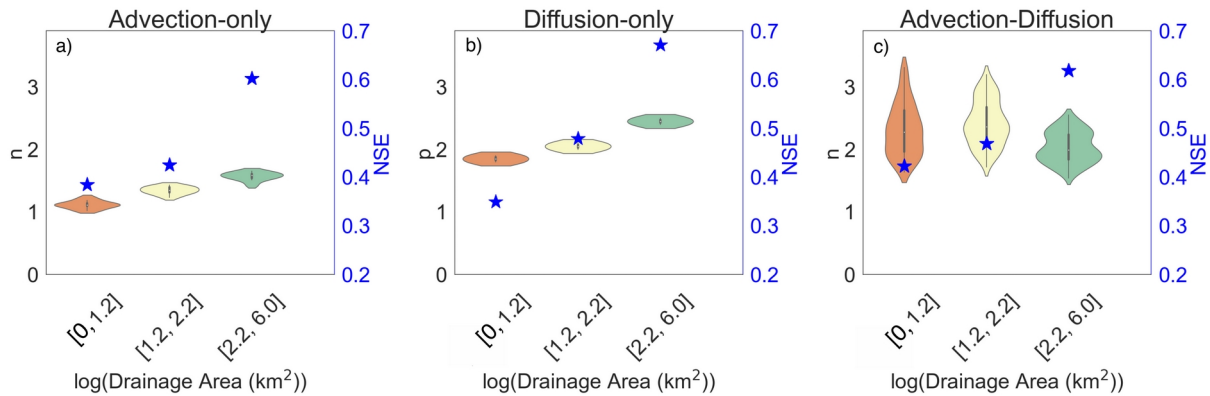
Figure 7: Distributions of *n* or *p* within each area-bin for (a) advection-only, (b) diffusion-only, and c) advection-diffusion models. Blue stars represent the corresponding NSE for each bin. The top 1 % of model runs are used.

models, and heavy blue-line is the mean. Both panels use the same colour ramp, which corresponds to the lithologic bin.